Casimir energy of the Skyrmion due to kaon vacuum fluctuations

Joon-Il Kim and Byung-Yoon Park
Department of Physics, Chungnam National University, Taejon 305-764, Korea

(Received 22 September 1997; published 10 February 1998)

We study the Casimir energy associated with the kaon fluctuation about the static soliton configuration in the Skyrme model. Up to uncertainties due to the unknown counterterms, the Casimir energy of the kaon vacuum fluctuation turns out to be very small compared with that of the pion. In the chiral limit and in the SU(3) symmetric limit it is about -200 MeV, which is much smaller than a naive estimation $-\frac{2}{3}\times 900$ MeV based on the number of zero modes. It is shown that the Wess-Zumino term plays an essential role in this reduction of the Casimir energy. [S0556-2821(98)05505-2]

PACS number(s): 12.39.Dc, 12.39.Fe, 12.40.Yx

I. INTRODUCTION

In the early 1960s, Skyrme suggested describing baryons by a static soliton solution of an effective Lagrangian for the mesons [1]. The size and mass of the resulting soliton configuration are comparable to the typical size and mass of a nucleon. The conserved topological current is taken as the baryon number current, which was later proved by Witten [2] through the U(1) anomaly. Skyrme's idea was phenomenologically checked by Adkins, Nappi, and Witten [3]. It was shown that, when the soliton solution is quantized by introducing the collective coordinates associated with the zero modes with respect to the rotation in isospin space, the model can describe the nucleons at an accuracy as good as 30%. Since this revival, much work on various aspects has made the model one of the more successful phenomenological models for baryons [4].

On the other hand, the Skyrme model suffers from a problem: the physical constants of the meson Lagrangian do not seem compatible with the static properties of baryons. One could reproduce the experimentally measured baryon data by releasing the physical constants governing the meson dynamics to free parameters. For example, when the empirical value 93 MeV is used for the pion decay constant, the resulting soliton mass is about 1.5 GeV. In order to fit the masses of nucleons and deltas, the pion decay constant needs to be reduced down to 64.5 MeV in the two-flavor model [3]. Although this defect may be ascribed to the large-N_c aspect of the model or to the simplicity of the model Lagrangian, it certainly maculates a great virtue of the model.

A plausible solution to this problem is that some portion of the soliton mass of order N_c is canceled by the Casimir energy of order N_c^0, the energy of the vacuum fluctuation in the presence of the soliton [5]. Various methods have been attempted to evaluate the Casimir energy or equivalently the one-loop quantum correction [6–8]. Most of them result in a small Casimir energy of the order of 200 MeV. Recently, Moussallam and co-workers [9,10] showed that a negative and rather large Casimir energy of order 1 GeV can be obtained from the pion vacuum fluctuation. In their work, the Casimir energy is controlled by the low-energy behavior of the pion-soliton scattering phase shifts. The dominant contribution is due to the presence of six zero-modes with respect to the translation and rotation of the soliton. The main ultraviolet divergences are absorbed into the counterterms developed in the chiral perturbation theory [11], while their contribution to the finite piece of the Casimir energy is small. The one-loop corrections evaluated by Holzwarth and Walliser [12] strongly support the large and negative Casimir energy.

If the Casimir energy associated with the pion fluctuation is large, one can imagine a considerable amount of Casimir energy coming from the kaon vacuum. In Ref. [10], assuming SU(3) symmetry, the latter was roughly estimated as $\frac{2}{3}$ of the former by using the fact that there are four zero modes for the rotation of the soliton in the strangeness direction. Then the resulting total Casimir energy would be so large in magnitude that it could cancel out all the soliton mass of the leading order in $1/N_c$. Needless to say, this is another undesirable situation. In this paper, we clarify this point by evaluating explicitly the Casimir energy due to the kaon vacuum fluctuation by applying the same method developed in Ref. [10].

This paper is organized as follows. In Sec. II, we briefly describe the model Lagrangian and introduce the kaon fluctuation about the static soliton solution. Then the partial-wave phase shifts of the kaon-soliton scattering are obtained. The effects of zero modes and bound states on the phase shift are carefully analyzed, which reveals a special role of the Wess-Zumino term. In Sec. III, the Casimir energy is calculated by using the scale-dependent finite formula expressed in terms of the phase shift. Our numerical results are presented and discussed. A short conclusion is given in Sec. IV.

II. MODEL LAGRANGIAN AND KAON FLUCTUATION

In describing kaon fluctuations about the static soliton solution, we follow the method of Callan and Klebanov [13]. We start with the Skyrme model Lagrangian

$$\mathcal{L}_{Sk} = -\frac{f_\pi^2}{4}\mathrm{Tr}(L_\mu L^\mu) + \frac{1}{32e^2}\mathrm{Tr}[L_\mu, L_\nu]^2 + \frac{f_\pi^2}{4}\mathrm{Tr}\mathcal{M}(U^\dagger + U - 2), \tag{1}$$

where $L_\mu = U^\dagger \partial_\mu U$ with $U(\boldsymbol{r},t) \in \mathrm{SU}(3)$. The quark mass matrix $\mathcal{M}$ can be written in terms of the meson masses as

$$\mathcal{M}=\mathrm{diag}(m_\pi^2,m_\pi^2,2m_K^2).$$

For simplicity, we take into account only the SU(3) symmetry-breaking term representing the meson mass difference. As for the pion decay constant f_π and the Skyrme parameter e, we fix them to the empirical values $f_\pi=93$ MeV and $e=4.75$ (which yield the correct axial coupling constant $g_A=1.33$) as in Ref. [14]. As for the meson masses, we consider three cases: (i) the chiral limit ($m_\pi=m_K=0$), (ii) the SU(3) symmetric limit ($m_\pi=m_K=138$ MeV), and (iii) the real world ($m_\pi=138$ MeV, $m_K=495$ MeV). The first two cases are to compare our results with the prediction of Ref. [10].

In the three-flavor Skyrme model, the Skyrme Lagrangian (1) should be supplemented by the Wess-Zumino term [2], which can be written only in an action term,

$$S_{\mathrm{WZ}}=-\frac{iN_c}{240\pi^2}\int d^5x\ \varepsilon^{\mu\nu\lambda\rho\sigma}\mathrm{Tr}(L_\mu L_\nu L_\lambda L_\rho L_\sigma), \tag{2}$$

where the integral is over a five-dimensional disk with space-time as its boundary, and $U(\boldsymbol{r},t)$ is continuously extended to the disk. Hereafter, the number of color N_c will be taken as 3.

The Lagrangian supports a static soliton solution under the ‘‘hedgehog’’ ansatz

$$U_0(\boldsymbol{r})=\begin{pmatrix} e^{i\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}F(r)} & 0\\ 0 & 1\end{pmatrix}, \tag{3}$$

where we have written the 3×3 matrix in a partitioned form

$$\begin{pmatrix} 2\times2 & 2\times1\\ 1\times2 & 1\times1\end{pmatrix},$$

and $\boldsymbol{\tau}$ denotes the 2×2 Pauli matrices. The profile function $F(r)$ minimizes the energy in the baryon number equal to one sector.

In order to generate the kaon fluctuation about the static soliton solution, we substitute the ansatz [13]

$$U=\sqrt{U_0}U_K\sqrt{U_0}, \tag{4}$$

where $\sqrt{U_0}=\exp[i\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}F(r)/2]$ and $U_K=\exp(i\lambda_qK_q)$ with q running from 4 to 7. Here λ denotes the generators of SU(3) normalized to $\mathrm{Tr}(\lambda_a\lambda_b)=2\delta_{ab}$. Explicitly, U_K can be written as

$$U_K=\exp\left[i\frac{\sqrt{2}}{f_\pi}\begin{pmatrix} 0 & K\\ K^\dagger & 0\end{pmatrix}\right], \tag{5}$$

where K is the standard complex isodoublet:

$$K=\frac{1}{\sqrt{2}}\begin{pmatrix} K_4-iK_5\\ K_6-iK_7\end{pmatrix}=\begin{pmatrix} K^+\\ K^0\end{pmatrix}.$$

Expanding in powers of K and keeping only terms up to second order in K, one can obtain the Lagrangian density

$$\mathcal{L}=\mathcal{L}_{Sk}(U_0)+(D_\mu K)^\dagger D^\mu K-m_K^2K^\dagger K+\cdots$$
$$+(iN_c/4f_\pi^2)B^\mu[K^\dagger D_\mu K-(D_\mu K)^\dagger K], \tag{6}$$

where the ellipsis represents a lengthy expression (see Ref. [13] for the explicit form) depending on

$$A_\mu=\tfrac{1}{2}(\sqrt{U_0}^\dagger\partial_\mu\sqrt{U_0}-\sqrt{U_0}\partial_\mu\sqrt{U_0}^\dagger),$$
$$V_\mu=\tfrac{1}{2}(\sqrt{U_0}^\dagger\partial_\mu\sqrt{U_0}+\sqrt{U_0}\partial_\mu\sqrt{U_0}^\dagger), \tag{6a}$$

and the covariant derivative is defined as

$$D_\mu K=\partial_\mu K+V_\mu K. \tag{6b}$$

The last term comes from the Wess-Zumino term (2), and B_μ is the baryon number current of the SU(2) soliton configuration.

Now the problem is reduced to studying the motion of kaons moving in the static potentials provided by the SU(2) soliton. It will have energy eigensolutions

$$K(\boldsymbol{r},t)=K(\boldsymbol{r})\exp(-i\omega t), \tag{7}$$

in terms of which the kaon field operator can be expanded. Furthermore, the invariance of the soliton configuration $U_0(\boldsymbol{r})$ under simultaneous rotations in the spatial and isospin space ($\boldsymbol{\Lambda}=\boldsymbol{I}+\boldsymbol{L}$ with $\boldsymbol{I}=\frac{1}{2}\boldsymbol{\tau}$ being the kaon isospin operator) enables us to perform a partial-wave analysis. The kaon eigenmodes can be written as the product of a radial function $k(r)$ and the spinor spherical harmonics $\mathcal{Y}_{\Lambda\ell\Lambda_3}(\hat{\boldsymbol{r}})$. Finally, we are led to the Lagrangian for the radial function

$$L=-4\pi\int_0^\infty dr\ r^2\{h(r)k^{\dagger\prime}k'+[m_K^2-\omega^2f(r)+2\omega\lambda(r)$$
$$+V_{\mathrm{eff}}(r;\Lambda,\ell)]k^\dagger k\}, \tag{8}$$

where $f(r)=1+\frac{1}{2}s(r)+\frac{1}{4}d(r)$, $h(r)=1+\frac{1}{2}s(r)$, $d(r)=(F')^2$ (the prime denotes the derivative with respect to its argument), $s(r)=(\sin F/r)^2$, $c(r)=(\sin\frac{1}{2}F)^2$,

$$V_{\mathrm{eff}}(r)=-\tfrac{1}{4}(d+2s)-\tfrac{1}{2}s(s+2d)+\frac{4+s+d}{4r^2}[\ell(\ell+1)$$
$$+2c^2+4c\boldsymbol{I}\cdot\boldsymbol{L}]+\frac{3}{2r^2}\Big\{s[c^2+\boldsymbol{I}\cdot\boldsymbol{L}(2c-1)]$$
$$+\frac{d}{dr}[(c+\boldsymbol{I}\cdot\boldsymbol{L})F'\sin F]\Big\}-m_\pi^2c,$$

and $\lambda(r)=-(N_ce^2/8\pi^2)s(r)F'$. From here on, we measure the times and distances in units of $1/ef_\pi$ and the energy in units of ef_π.

From Lagrangian (8), the equation of motion for the classical eigenmodes is readily derived as

$$-\frac{1}{r^2}\frac{d}{dr}\left(h(r)r^2\frac{d}{dr}k\right)+[m_K^2-\omega^2f(r)-2S\omega\lambda(r)$$
$$+V_{\mathrm{eff}}(r;\Lambda,\ell)]k=0, \tag{9}$$

where S in the Wess-Zumino term linear in ω denotes the strangeness number of the solution. It is included to incorporate the fact that the positive-energy eigenmodes of the $S=+1$ sector are related to those of the negative-energy solutions of the $S=-1$ sector. (From now on, the energy appearing in the equation of motion is non-negative.) Thus the Wess-Zumino term acts as an attractive potential in the $S=-1$ channel, and as a repulsive one in the $S=+1$ channel.

In case of $\omega<m_K$, the equation may provide bound state solutions. With $m_\pi=138$ MeV and $m_K=495$ MeV, there are two bound states. The lowest one is found in the $(\Lambda,\ell,S)=(\frac{1}{2},1,-1)$ channel at the eigenenergy $\omega_1=143$ MeV, which can describe the lowest positive parity hyperons [13]. The other is in the $(\frac{1}{2},0,-1)$ channel at the eigenenergy $\omega_2=425$ MeV, which can be interpreted as $\Lambda(1405)$ negative parity resonance below the $\overline{K}N$ threshold.

In the limit of the unbroken SU(3) symmetry, the equation has zero-mode solutions with $\omega=0$ in the $(\frac{1}{2},1,\pm 1)$ channels. These zero modes are associated with the collective rotation of the soliton solution (3) in the strangeness direction. One can obtain them analytically by equating the infinitesimal collective rotation to the kaon fluctuation (5), for example, as

$$A^{\dagger}U_0A=\sqrt{U_0}U_K\sqrt{U_0}, \tag{10}$$

with $A=\exp[i\epsilon(\lambda_4-i\lambda_5)]$ and $K=k(r)\mathcal{Y}_{1/2,1,+1/2}(\hat{\boldsymbol{r}})$. Explicitly, the radial function of the zero-mode solution can be written as

$$k(r)=\frac{1}{\sqrt{\mathcal{N}}}\sin(\tfrac{1}{2}F) \tag{11}$$

up to a normalization constant $\mathcal{N}$.

For $\omega>m_K$, Eq. (9) describes the kaon-soliton scattering process in the soliton corotating frame. From the asymptotic behavior of the solutions, one can obtain the phase shifts $\delta_{\Lambda,\ell,S}$. Such work was done a long time ago for the purpose of investigating the hyperon resonances in the Skyrme model [15,16]. Here we repeat the calculation by paying special attention to $\delta_{\Lambda,\ell,S}(0)$, the phase shifts in the limit of the vanishing kaon momentum. They play the most important role in evaluating the Casimir energy later, and are strongly affected by the presence of the zero modes or bound states.

In Fig. 1, we present phase shifts of the kaon-soliton scattering process for a few Λ as a function of the kaon momentum p. The solid (dashed) curves are for the $S=-1$ $(+1)$ channels. Some channels show a clear resonance. As ℓ increases, they become broader and higher in energy. After the resonances, all the phase shifts show linearly rising behavior in the high momentum. This is due to the fact that the second derivative terms in space and time in the equation of motion (9) are not of the simple Laplace form ∂^2. The asymptotic behaviors can be grouped into four lines depending on whether $\Lambda=\ell+\frac{1}{2}$ or $\Lambda=\ell-\frac{1}{2}$, and on the strangeness number S. In general, as discussed in Ref. [16], the $\Lambda=\ell-\frac{1}{2}$ channel receives more attraction than the $\Lambda=\ell+\frac{1}{2}$ channel.

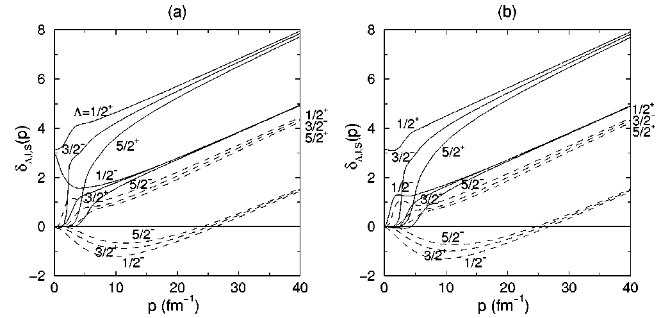

FIG. 1. Phase shifts $\delta_{\Lambda,\ell,S}(p)$ for a few low Λ. (a) $m_\pi=138$ MeV and $m_K=495$ MeV. (b) $m_\pi=m_K=138$ MeV [SU(3) symmetric limit]. Solid (dashed) curves are for $S=-1$ $(+1)$ channels.

In the case of $m_K=495$ MeV and $m_\pi=138$ MeV [Fig. 1(a)], $\delta_{1/2,1,-1}(0)$ and $\delta_{1/2,0,-1}(0)$ are set to π according to Levinson's theorem. Recall that there exists a single (doubly degenerate) bound state in each channel. In the SU(3) symmetric limit [Fig. 1(b)], only $\delta_{1/2,1,-1}(0)$ can be π, reflecting the presence of the zero mode in this channel.

In spite of the presence of the zero mode in the corresponding channel, $\delta_{1/2,1,+1}(p)$ vanishes in the limit of zero momentum. This is because the Wess-Zumino term acts repulsively for the $S=+1$ channel. In Fig. 2(a), the wave functions $k(x)$ of the $(\frac{1}{2},1,-1)$ channel (solid curve) and $(\frac{1}{2},1,+1)$ channel (dashed curve) are drawn at the kaon momentum $p=0.1$ (fm^{-1}) in the chiral limit. As reference curves, we draw the zero-mode ($p=0$ in the chiral limit) wave function (dotted curve) and the free wave solution $j_1(px)$ (dash-dotted curve) together. The attractive force between the soliton and the $S=-1$ kaon makes the wave function shift inward from the zero-mode solution, so that it has a node near the origin. Thus the wave function becomes out of phase with the free wave solution in the asymptotic region. Conversely, due to the repulsion coming from the Wess-Zumino term, the wave function of the $S=+1$ kaon is repelled outward, and becomes in phase with the free wave solution in the asymptotic region. For a higher kaon momentum where the strength of the Wess-Zumino term is increased, the trace of the zero mode becomes dimmer and will disappear [see Fig. 2(b)].

III. CALCULATING THE CASIMIR ENERGY

The Casimir energy associated with the kaon fluctuation is defined by the difference in the mode sum of the eigenenergies,

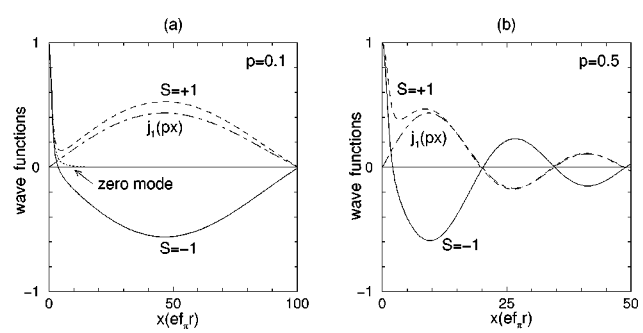

FIG. 2. Radial function $k(x)$ for the $\Lambda=\frac{1}{2}$, $\ell=1$, and $S=\pm 1$ channels at the kaon momenta (a) $p=0.1$ and (b) $p=0.5$ (in fm^{-1} units).

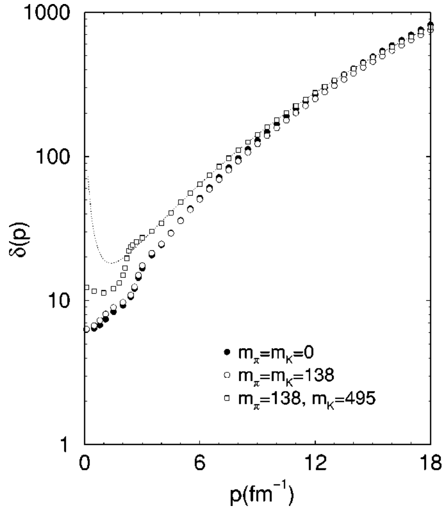

FIG. 3. Phase shift $\delta(p)$, the sum of all the partial wave phase shifts. The dotted line is a fit to the asymptotic formula (15).

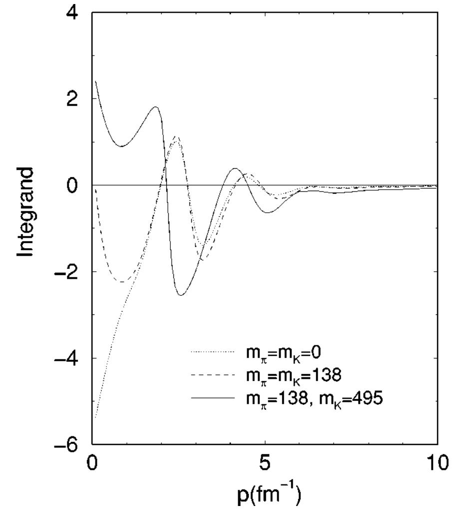

FIG. 4. Integrand of the integral in Eq. (16) as a function of the momentum.

$$E_C=\tfrac{1}{2}\sum_n \omega_n-\tfrac{1}{2}\sum_n \omega_n^0, \tag{12}$$

where $\omega_n(\omega_n^0)$ is the eigenenergy of the kaon field in the presence(absence) of the soliton. It can be expressed as [10,17,18]

$$E_C=\lim_{s\to 0}\frac{1}{2\pi}\int_{m_K}^{\infty} dE\; E^{1-2s}\delta'(E)+\frac{1}{2}\sum_{n=\text{b.s.}}\omega_n, \tag{13}$$

where $\delta(E)$ is the phase shift in the one space dimension related to the partial-wave phase shifts obtained in Sec. II as

$$\delta(E)=\sum_{\Lambda,\ell,S}(2\Lambda+1)\delta_{\Lambda,\ell,S}(E). \tag{14}$$

The sum in the last term of Eq. (13) runs over all the bound states (if any).

Due to the monotonously rising high-energy behavior of the phase shifts, formula (13) ends up with ultraviolet divergences at $s=0$. Asymptotically, the phase $\delta(p)$ behaves as

$$\delta'(p)=3\overline{a}_0p^2+\overline{a}_1-\frac{\overline{a}_2}{p^2}+\cdots. \tag{15}$$

Terms of higher order in $1/p$ abbreviated by the ellipses do not cause any divergence problem. The troublesome divergences can be subtracted and replaced by the analytic continuation of them. This process leads us to a scale-dependent Casimir energy

$$\begin{aligned}E_C(\mu)=\frac{1}{2\pi}\Bigg\{&\int_0^\infty dp\Bigg[-\frac{p}{\sqrt{p^2+m_K^2}}(\delta(p)-\overline{a}_0p^3-\overline{a}_1p)\\&+\frac{\overline{a}_2}{\sqrt{p^2+\mu^2}}\Bigg]-\frac{3\overline{a}_0}{8}m_K^4\left(\frac{3}{4}+\frac{1}{2}\ln\frac{\mu^2}{m_K^2}\right)\\&-\frac{\overline{a}_1}{4}m_K^2\left(1+\ln\frac{\mu^2}{m_K^2}\right)-m_K\delta(0)\Bigg\}+\tfrac{1}{2}\sum_{n=\text{b.s.}}\omega_n,\end{aligned} \tag{16}$$

according to which the Casimir energy of the pion vacuum fluctuation was estimated in Refs. [9,10]. We are going to use the same formula to evaluate that of the kaon fluctuation. There can be further contributions from the counterterms, which are known only partly. We will assume that the Casimir energy coming from the counterterms is negligible as in Ref. [10]. As for the energy scale μ, we work with two different values: the ρ-meson mass $m_\rho=770$ MeV and the K^*-meson mass $m_{K^*}=892$ MeV.

The calculation is thus reduced to summing up the phase shifts over all the partial waves to obtain $\delta(p)$. The convergence of the sum requires high values of Λ. In our numerical work, we truncate the sum when $\delta_\Lambda(p)$ becomes less than 10^{-8}. This condition is usually fulfilled at $\Lambda\sim 20p$ (in fm^{-1} units). It would provide at least 10^{-5} accuracy to $\delta(p)$, tak-

TABLE I. The expansion coefficients of $\delta(p)$.

m_π	m_K	$\overline{a}_0$	$\overline{a}_1$	$\overline{a}_2$	$\delta(0)$
0	0	0.1286	3.898	2.31($\pm$0.05)	2π
138	138	0.1166	4.096	3.28($\pm$0.02)	2π
138	495	0.1166	6.162	12.98($\pm$0.03)	4π

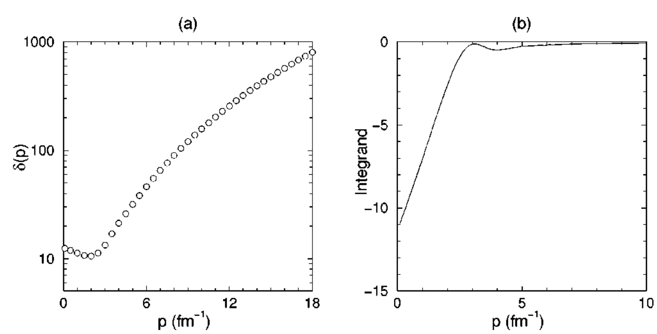

FIG. 5. (a) Phase shift $\delta(p)$ and (b) integrand obtained *without* the Wess-Zumino term.

TABLE II. Casimir energy of the kaon vacuum fluctuation.

		$\mu = m_\rho = 770$ MeV				$\mu = m_{K*} = 892$ MeV			
m_π	m_K	$E_C^{\text{subt.}}$	$E_C^{\text{asym.}}$	$E_C^{\text{b.s.}}$	E_C	$E_C^{\text{subt.}}$	$E_C^{\text{asym.}}$	$E_C^{\text{b.s.}}$	E_C
0	0	-175	–	–	-175	-186	–	–	-186
138	138	-120	-69	–	-189	-135	-64	–	-199
138	495	$+20$	-481	$+568$	$+107$	-40	-395	$+568$	$+133$

ing into account the degeneracy factor $(2\Lambda+1)$ and the contribution from the neglected tail. In Fig. 3, we present the numerical result on $\delta(p)$ as a function of p in the range of $0<p\leqslant 18$ fm^{-1}. The data show a hump at the momentum $p\sim 2$ (fm^{-1}) where the $\Lambda^\pi = \frac{1}{2}^-$ channel has a resonance. It becomes more prominent for the case of the larger kaon mass. After the hump, $\delta(p)$ quickly reaches the asymptotic behavior (15). The dotted curve in Fig. 2 is obtained by fitting the data in the range of 8 fm$^{-1}\leqslant p \leqslant 18$ fm^{-1} to the asymptotic formula.

For a larger kaon momentum than $p_{\max}\sim 18$ (fm^{-1}), and for a large Λ, our numerical process becomes unstable. However, the data given in Fig. 3 are sufficient to determine the expansion coefficients $\overline{a}_i$ ($i=0$,1, and 2). In Table I, we list the coefficients obtained by fitting $\delta(p)$ in the range 8 fm$^{-1}\leqslant p \leqslant 18$ fm^{-1} to the asymptotic formula (15). The coefficients $\overline{a}_0$ and $\overline{a}_1$ show little dependence on the range of the kaon momentum used in the fitting process. On the other hand, $\overline{a}_2$ varies within the error range presented in the parentheses. Note that $\overline{a}_0$ for two cases of $m_\pi = 138$ MeV (but with different kaon masses) are equal up to the four digits presented in the table, which implies that it is a constant depending only on the soliton profile function $F(r)$.

Once the asymptotic expansion coefficients are determined, we can carry out the integral to obtain the Casimir energy $E_C(\mu)$. Shown in Fig. 4 is the integrand of the integral in Eq. (16). Here the energy scale μ is chosen as the K^*-meson mass. Compared with the one shown in Ref. [10] (Fig. 1), they show more complicated structures. One can see that the ultraviolet divergences are subtracted well, and that the dominant contribution of the continuum spectrum to the Casimir energy is determined by the low-energy behavior of the phase shifts.

In Table II, we summarize our numerical results for the Casimir energy due to the kaon vacuum fluctuation. We present separately the subtracted piece contribution of the continuous spectrum [$E_C^{\text{subt.}}$, the first line in Eq. (16)], the renormalized asymptotic piece of the continuous spectrum contribution [$E_C^{\text{asym.}}$, the second line in Eq. (16)], and the bound-state contribution ($E_C^{\text{b.s.}}$). The dependences on the energy scale μ and on the pion mass m_π are rather small. The Casimir energy associated with the kaon fluctuation in the strangeness direction is about -200 MeV in the SU(3) symmetric limit, and about $+100$ MeV with $m_K = 495$ MeV. They are very small compared with the Casimir energy of the pion fluctuation that amounts ~ -900 MeV. Especially, the value ~ -200 MeV in the SU(3) limit is much smaller than what was naively estimated as $\sim -\frac{2}{3}\times 900$ MeV in Ref. [10] by counting the number of zero modes.

One can easily note that this reduction is closely related to the fact that $\delta(0)$ is not 4π but 2π in the SU(3) limit, for which the Wess-Zumino term is most responsible. In order to check this point, we carry out the same calculation without the Wess-Zumino term. For a comparison, we present the numerical results in Fig. 5: (a) the phase shift $\delta(p)$ and (b) the integrand after subtracting the ultraviolet divergences. Now $\delta(0)$ becomes 4π, reflecting directly the existence of four zero modes. The integrand is very similar to that of Ref. [10] for the pion fluctuation, and has a simpler structure than that of Fig. 4.

Table III summarizes the numerical results obtained without the Wess-Zumino term. Note that (i) the value of the expansion coefficient $\overline{a}_0$ remains unchanged and (ii) that $\delta(0)=4\pi$ in all the cases considered. In the chiral limit, the Casimir energy is evaluated as -460 MeV with $\mu = 770$ MeV. This is more than twice the value evaluated with the Wess-Zumino term, and is close to the naive estimation of Ref. [10]. Again, the Casimir energy shows a weak dependence on the pion mass. In the case of a massive kaon, when the Wess-Zumino term is turned off, only a single bound state is found in each $(\frac{1}{2},1,\pm 1)$ channel with an eigenenergy 352 MeV. It is interesting to see that the final result on the Casimir energy varies little from the value obtained with the Wess-Zumino term in the last case. This emphasizes the importance of the value $\delta(0)$ in evaluating the Casimir energy.

IV. CONCLUSION

In this paper, we studied the Casimir energy of the Skyrmion associated with the kaon fluctuation. The Casimir energy was calculated by investigating the low-energy behavior of the soliton-kaon scattering phase shifts after the ultraviolet

TABLE III. Casimir energy evaluated without the Wess-Zumino term. The energy scale is taken as $\mu = 770$ MeV.

m_π	m_K	$\overline{a}_0$	$\overline{a}_1$	$\overline{a}_2$	$\delta(0)$	$E_C^{\text{subt.}}$	$E_C^{\text{asym.}}$	$E_C^{\text{b.s.}}$	E_C
0	0	0.1286	2.902	5.52	4π	-485	–	–	-485
138	138	0.1166	3.039	6.72	4π	-314	-225	–	-539
138	495	0.1166	5.106	12.1	4π	-26	-580	$+704$	$+98$

divergences were carefully subtracted. The Casimir energy from the kaon vacuum fluctuation turned out to be very small compared with that of the pion fluctuation. The main reason for this reduction in the Casimir energy is not the larger mass of kaons than pions but the presence of the Wess-Zumino term in the Lagrangian governing the kaon dynamics.

We could not incorporate the contributions from the unknown counterterms, and hope to report on this issue in future publications. However, we believe that the Casimir energy is saturated by contributions from the low-energy continuum spectrum and the bound states as in the case of the pion fluctuation.

ACKNOWLEDGMENTS

We are grateful to Professor Mannque Rho for valuable comments. This work was supported by the Korea Science and Engineering Foundation through the SRC program.

[1] T. H. R. Skyrme, Proc. R. Soc. London, Ser. A **260**, 127 (1961); Nucl. Phys. **31**, 550 (1962).
[2] E. Witten, Nucl. Phys. **B223**, 422 (1983); **B223**, 433 (1983).
[3] G. S. Adkins, C. Nappi, and E. Witten, Nucl. Phys. **B228**, 552 (1983).
[4] For a review, see I. Zahed and G. E. Brown, Phys. Rep. **142**, 1 (1986); U.-G. Meissner, *ibid.* **161**, 213 (1988); B. Schewesinger, H. Weigel, G. Holzwarth, and A. Hayashi, *ibid.* **173**, 173 (1989); *Proceedings of the International Workshop on Baryons as Skyrme Solitons, Siegen, 1992*, edited by G. Holzwarth (World Scientific, Singapore, 1992); D.-P. Min, Y. Oh, B.-Y. Park, and M. Rho, Int. J. Mod. Phys. E **4**, 47 (1995).
[5] M. Nowak, M. Rho, and I. Zahed, *Chiral Nuclear Dynamics* (World Scientific, Singapore, 1996).
[6] L. C. Biedenharn, Y. Dothan, and M. Tarlini, Phys. Rev. D **31**, 649 (1985); H. Schnitzer, Nucl. Phys. **B261**, 546 (1986).
[7] I. Zahed, A. Wirzwa, and U.-G. Meissner, Phys. Rev. D **33**, 830 (1986).
[8] A. Dobado and J. Terron, Phys. Lett. B **247**, 581 (1990).
[9] B. Moussallam and D. Kalafatis, Phys. Lett. B **272**, 196 (1991).
[10] B. Moussallam, Ann. Phys. (N.Y.) **225**, 264 (1993).
[11] J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) **158**, 142 (1984).
[12] G. Holzwarth and H. Walliser, Nucl. Phys. **A587**, 721 (1995).
[13] C. G. Callan and I. Klebanov, Nucl. Phys. **B262**, 365 (1985); C. G. Callan, K. Hornbostel, and I. Klebanov, Phys. Lett. B **202**, 269 (1988).
[14] A. D. Jackson and M. Rho, Phys. Rev. Lett. **51**, 751 (1983).
[15] M. Karliner and M. P. Mattis, Phys. Rev. Lett. **56**, 428 (1986); Phys. Rev. D **34**, 1991 (1986).
[16] N. N. Scoccola, Phys. Lett. B **236**, 245 (1990).
[17] J. Schwinger, Phys. Rev. **94**, 1362 (1951).
[18] R. F. Dashen, B. Hasslacher, and A. Neveu, Phys. Rev. D **10**, 4114 (1974); **10**, 4130 (1974).