Hadronic couplings via QCD sum rules using three-point functions: Vacuum susceptibilities

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We develop a three-point formalism to treat vacuum susceptibilities used for the coupling of currents to hadrons within the method of QCD sum rules. By introducing nonlocal condensates, with the space-time structure taken from fits to experimental parton distrbutions, we show that one can treat hadronic coupling at zero or low momentum transfer as well as medium and asymptotic momentum transfers and obtain a general expression for the vacuum susceptibilities of the two-point formalism. The pion susceptibility, for which there has been a major uncertainty, is evaluated successfully with no new parameters. [S0556-2821(98)00107-6]

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I. INTRODUCTION

Hadronic couplings are essential ingredients in the study of hadronic decays and interactions, and the properties and interactions of hadrons in nuclear matter. In effective field theories these couplings are defined by three-point functions. Since the hadrons are complex systems and the strong interactions, given by QCD (quantum chromodynamics), require a nonperturbative treatment, the theoretical treatment of these three-point functions is quite challenging. In the present paper we discuss the application of the QCD sum rule method using a three-point approach for the coupling of currents to hadrons, and give a new interpretation of the vacuum susceptibilities used in the two-point approach. We apply this treatment to the parity-violating pion-nucleon coupling, for which theoretical estimates of the pion-induced susceptibility have met with difficulties, and discuss the isospin-violating pion-nucleon coupling.

In the method of QCD sum rules [1] complex hadronic systems are represented by local complex field operators so that standard two-point functions can be used for hadronic masses. The methods introduced by Shifman *et al.* allow a short-distance expansion and nonperturbative effects to be treated via operator product expansions (OPEs) using vacuum condensates whose values are determined by fits to experiment, as well as lattice gauge calculations. A review of the early work is given in Ref. [2].

Using these local field operators, one can also define three-point functions for hadronic coupling, similar to effective hadronic field theories. For medium and asymptotic momentum transfers the OPEs can be applied for form factors [3,4] and moments of wave functions (see Ref. [5] for review of the early work); however, at low momentum transfer the OPEs cannot be consistently applied, as was pointed out in the early work on photon couplings at low momentum for the nucleon magnetic moments [6,7]. In Ref. [6] the probem was solved by using a two-point correlator in an external electromagnetic field, with vacuum susceptibilities introduced as parameters for nonperturbative propagation in the external field. In Ref. [7] a three point formalism was used with the long-distance effects treated by bilocal corrections; and by assuming ρ-meson dominance results similar to Ref. [6] were obtained. Subsequently the magnetic susceptibility was calculated using the two-point formalism with extended vector meson dominance model treatments [8,9] with results similar to the phenomenological treatment of Ref. [6]. These methods were applied to the study of parton distribution functions [10,11] and radiative baryon decay [12], with explicit treatments of the bilocal operators. A detailed review of the relationship between the three-point and two-point external field treatments is given [13] for an extension to nonzero momentum transfer.

The external field method has also been used for the calculation of the axial coupling constant (g_A) [14–16], the parity-violating pion-nucleon coupling constant (g_W) [17] and the nucleon's tensor charge [18]. This two-point method, however, has two main problems: it cannot be used to extend the coupling to medium and high momentum transfer and there are additional parameters to be determined: the vacuum susceptibilities. This latter problem is seen to be crucial in the recent calculation of g_W, where a cancellation between perturbative and nonperturbative contributions is the dominant effect. Moreover, the phenomenological value obtained for the pion susceptibility from the study of $g_{\pi N}$, the strong pion-nucleon coupling constant, differs by as much as an order of magnitude from a theoretical estimate, as we discuss in Sec. II A. A three-point method used [19] for an estimate of $g_{\pi N}$ did not use the pion susceptibility. Also, for the calculation of the nucleon's tensor charge [18] it has been pointed out [20] that the treatment of the vacuum tensor susceptibility is subtle and different treatments can lead to very different results for the tensor charge.

Nonlocal condensates have been shown to be useful for representing the bilocal vacuum matrix elements needed for the pion wave function [21] and pion form factor [22] over for low to medium momentum transfer. In this method one does not carry out an OPE for the power corrections but introduces new phenomenological parameters needed to characterize the space-time structure of the nonlocal condensates. The method is simple, but powerful. Although new phenomenological parameters are introduced, they are interesting in themselves. For example, in a study of parton distribution functions [23] the space-time scale of a nonlocal condensate was determined by a fit to experiment data.

In the present work we start with the standard three-point vertex functions for hadronic couplings and use nonlocal condensates to represent the bilocal operators. By comparison of terms appearing in the two-point external field expression with those in our hybrid expansion of the three-point function, we obtain a relationship between the nonperturbative elements in the two methods (Sec. II B). From this relationship, it is then possible to obtain the main result of this paper, namely an expression for the induced susceptibilities of the two-point method in terms of well-defined four-quark vacuum matrix elements, and make a simple estimate of their values, using the estimate of the space-time structure of the nonlocal quark condensate extracted from experimental data on quark distributions. Since the form assumed for the nonlocal condensates in Ref. [23] does not have satisfactory analytic properties, we choose a new form and refit the parameter needed for the present work.

In this study we make use of a factorization of four-quark operators which cannot be extended to the treatment of hadronic couplings in nuclear media [24]. Recently, we have shown [25] that the present knowledge of the in-medium Δ (1232) can constrain the unknown four-quark in-medium condensates. In a future publication [26] we demonstrate that the study of hadronic in-medium couplings using a QCD sum rule method with three-point functions enables us to extend our program.

In Sec. III we discuss how this method can be used for the study of the pion-nucleon coupling, the parity-violating pion coupling to nucleons and how the gauge-invariant method for calculating QED corrections in the QCD sum rule method [27] can be used for determining the QED isospin violations of coupling constants. Conclusions and discussion are given in Sec. IV.

II. COUPLING OF CURRENTS TO BARYONS: THREE-POINT VS TWO-POINT FORMULATION

In this section we give a discussion of the three-point vs two-point approach for hadronic couplings and show that by introducing the space-time structure of the condensates one can successfully use the sum rule method to derive new expressions for the induced susceptibilities of the two-point method. We also discuss the particular problem of the pion susceptibility, which is the main application of the present paper.

Although hadrons are complicated composite systems, both in effective hadronic field theories and in the sum rule methods hadrons are represented by local field operators. The coupling of a current $J^\Gamma(y)=\overline{q}(y)\Gamma q(y)$ to hadrons α,β is studied in such field theories by the three-point function:

$$V^{\Gamma}_{\beta\alpha}(p,q)=\int d^4x\int d^4y e^{ix\cdot p}e^{-iy\cdot q} \times\langle 0|T[\eta_\beta(x)J^\Gamma(y)\overline{\eta}_\alpha(0)]|0\rangle \tag{1}$$

where the quantity $\eta_\alpha(x)$ is a field operator representing the hadron α. In treatments in which QCD and electroweak interactions are explicit, as in the QCD sum rule method, the η operators must be composite with quark and gluon field constituents, so that the problem of coupling of currents to hadrons is intrinsically much more complex than the three-point functions of Eq. (1) for effective field theories. In this section we review how the couplings are represented by three-point functions and also by two-point functions in the sum rule method; and we show how the vacuum susceptibilities that appear in the two-point method can be evaluated in terms of four-quark condensates in the three-point approach.

A. QCD sum rule two-point method for coupling at low momentum

In this subsection we briefly review the two-point effective field approach [6] to hadronic couplings and the definitions of vacuum susceptibilities. In the present work we discuss only the coupling to nucleons and use as the composite field operator to represent the nucleon

$$\eta(x)=\epsilon^{abc}[u^a(x)^T C\gamma^\mu u^b(x)]\gamma^5\gamma^\mu d^c(x),$$
$$\langle 0|\eta(x)|\text{proton}\rangle=\lambda_p v(x), \tag{2}$$

where C is the charge conjugation operator, the u(x), d(x) are u,d-quark fields labelled by color, λ_p is a structure parameter and $v(x)$ is a Dirac spinor. For coupling of the current J^Γ to the proton, if one starts with $V^\Gamma(p,q)$ of Eq. (1), for low q there is no justification for an OPE in the y variable. This was dicussed at length in the early three-point function treatment of the nucleons magnetic dipole moment [7], but ignored in the treatment [19] of the pion coupling to nucleons and the N-Δ pionic coupling. To avoid this difficulty a two-point formulation of the QCD sum rule in an external electromagnetic field was introduced [6]. For an external current J^Γ the correlator

$$\Pi^\Gamma(p)=i\int d^4e^{ix\cdot p}\langle 0|T[\eta(x)\overline{\eta}(0)]|0\rangle_{J^\Gamma} \tag{3}$$

is used. As can be seen from Eq. (3) the microscopic evaluation of $\Pi^\Gamma(p)$ can be done using the operator product expansion, since the variable x is at short distance from the origin. This is done by an OPE of the quark propagator in the presence of the the J^Γ current

$$S^\Gamma_q(x)=\langle 0|T[q(x)\overline{q}(0)]|0\rangle_{J^\Gamma},$$
$$=S^{\Gamma,PT}_q(x)+S^{\Gamma,NP}_q(x), \tag{4}$$

where $S^{\Gamma,PT}_q(x)$ is the quark propagator coupled perturbatively to the current and $S^{\Gamma,NP}_q(x)$ is the nonperturbative quark propagator in the presence of the external current, J^Γ. (One should note that the external current should be taken to be $J^\Gamma\phi_\Gamma$, where ϕ_Γ is the value of the external field. In what follows, to simplify the notation, we will take $\phi_\Gamma=1$, which does not affect our results. For some manipulations, one will have to remember to put the external field back into the current, e.g., when taking the linear field limit.) The quantity $S^{\Gamma,NP}_q$(x) can be thought of as a nonlocal susceptibility; and it is essential to determine the space-time structure of this susceptibility to predict the coupling at higher momentum transfer, as we discuss below. For the two-point treatment at

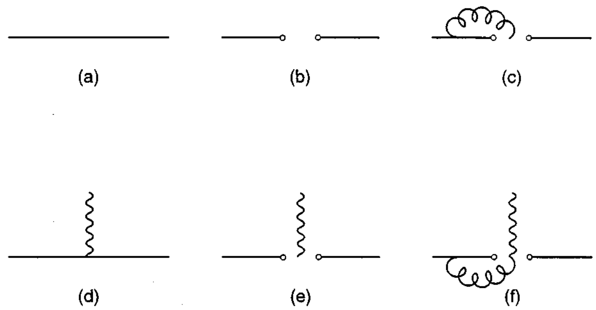

FIG. 1. Diagrammatic representation of terms appearing in the operator-product expansion of the two-point function in free space (a)–(c) and in an external field (d)–(f).

low momentum transfer the OPE for $S_q^{\Gamma,NP}(x)$ is justified as in the ordinary two-point function, giving

$$S_q^{\Gamma,NP}(x)=\frac{-\Gamma}{12}\langle 0|:\overline{q}\,\Gamma q:|0\rangle_{J^\Gamma}+\frac{x^2\Gamma}{3\times 2^6}\langle 0|:\overline{q}\,\sigma\cdot G\Gamma q:|0\rangle_{J^\Gamma}+\cdots . \quad (5)$$

Although the OPE can be justified and the sum rules can easily be derived in this external field two-point method, there is a major problem: new parameters appear whose determination must be carried out. For the new terms in the nonperturbative quark propagator in the external J^Γ current, given in Eq. (5) and illustrated in Figs. 1e and 1f, one can write

$$\langle 0|:\overline{q}\,\Gamma q:|0\rangle_{J^\Gamma}=-\chi^\Gamma\langle 0|:\overline{q}\,q:|0\rangle \quad (6)$$

and

$$\langle 0|:\overline{q}\,\sigma\cdot G\Gamma q:|0\rangle_{J^\Gamma}=-\chi_m^\Gamma\langle 0|:\overline{q}\,q:|0\rangle. \quad (7)$$

The lowest-dimensional diagrams for the microscopic evaluation of $\Pi^\Gamma(p)$ are shown in Fig. 2. Note that diagrams of Fig. 2b and 2c involve the susceptibilities χ^Γ and χ_m^Γ, respectively. These susceptibilities must be determined in order to predict the coupling constant from the sum rules.

As an example of the difficulty let us consider the external pion field with the current $J^\pi = ig_\pi\overline{q}\,\tau_3\gamma_5 q$ $(\Gamma^\pi = ig_\pi\tau_3\gamma_5)$. We define the local pion susceptibility χ^π

$$\langle 0|:\overline{q}\,\Gamma^\pi q:|0\rangle_\pi=-\chi^\pi\langle 0|:\overline{q}\,q:|0\rangle, \quad (8)$$

and nonlocal pion susceptibility

$$\langle 0|:\overline{q}\,(x)\Gamma^\pi q(0):|0\rangle_\pi=-\chi^\pi \mathrm{H}(x)\langle 0|:\overline{q}\,q:|0\rangle. \quad (9)$$

The phenomenological function H(x) in Eq. (9) represents the entire OPE of Eq. (5). Note that H(0) = 1.

A value for the pion susceptibility has been recently extracted [17] in a study of strong and parity-violating π-N coupling constant, $g_{\pi NN}$. The following problem with the application of PCAC (partial conservation of axial vector current) to this problem was observed in Ref. [17]: The application of PCAC to the determination of the vacuum pion susceptibility with the two-point method and an external pion field gives [14]

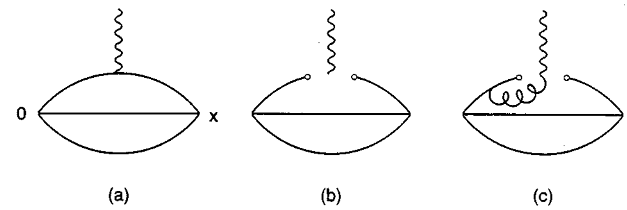

FIG. 2. Lowest dimension diagrams for evaluation of the two-point function in an external field as given in Eq. (3).

$$\chi^\pi\langle 0|:\overline{q}\,q:|0\rangle=\frac{f_\pi^2 m_\pi^2}{\sqrt{2}2m_q^2}, \quad (10)$$

while from PCAC it is known that

$$\langle 0|:\overline{q}\,(0)\Gamma^\pi q(0):|\pi(k)\rangle=\frac{f_\pi m_\pi^2}{\sqrt{2}m_q}. \quad (11)$$

From Eqs. (8), (10), (11) it is seen that there is more than an order of magnitude discrepancy between the two-point external-field method and standard PCAC, since $f_\pi/m_q \simeq 20$. In fact the application of Eq. (10) gives $\chi_\pi a \simeq 45\ \mathrm{GeV}^2$, while the result of the analysis of $g_{\pi NN}$

$$\chi^\pi a = 1.88\ \mathrm{GeV}^2, \quad (12)$$

with $a \equiv -(2\pi)^2\langle 0|:\overline{q}\,q:|0\rangle$. The error in the value of $\chi^\pi a$ is estimated to be about 20%. The value of $45\ \mathrm{GeV}^2$ is inconsistent with the sum rules for both the strong and parity-violating coupling constants, while the value $\chi^\pi a = 1.88\ \mathrm{GeV}^2$ is consistent with experiment for both the strong and weak coupling. We derive this susceptibility in the next section using our three-point method.

B. QCD sum rule three-point method for coupling at low momentum

Let us now return to the three-point function formulation, Eq. (1), which we write as

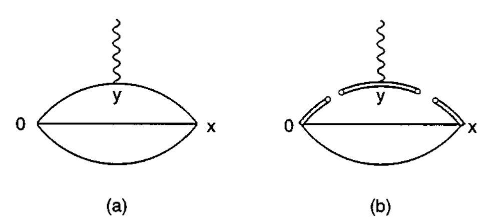

FIG. 3. Two- and four-quark diagrams corresponding to Eqs. (15) and (16), respectively, for evaluating the the coupling constant with the three-point function.

$$V^{\Gamma}(p,q)=\int d^4x\int d^4y e^{ix\cdot p}e^{-iy\cdot q}V^{\Gamma}(x,y)$$

$$V^{\Gamma}(x,y)=\langle 0|T[\eta(x)J^{\Gamma}(y)\overline{\eta(0)}]|0\rangle. \tag{13}$$

We write $V^{\Gamma}(x,y)$ as

$$V^{\Gamma}(x,y)=V^{\Gamma 2q}(x,y)+V^{\Gamma 4q}(x,y)+V^{\Gamma 6q}(x,y)+V^{\Gamma 8q}(x,y), \tag{14}$$

where the four terms contain two-quark matrix elements only, four-quark, six-quark and eight-quark matrix elements, respectively. Using the current given by Eq. (2), for which we take $\Gamma=g_{\pi}\gamma_5$ for the pion current, we find for the two-quark terms

$$V^{\Gamma 2q}(x,y)=-i2\epsilon^{abc}\epsilon^{b'a'c'}\gamma^5\gamma_{\mu}S_d^{ce}(x-y)\Gamma S_d^{ec'}(y)\gamma_{\nu}\gamma^5\mathrm{Tr}[S_u^{aa'}(x)\gamma^{\mu}C(S_u^{bb'}(x))^TC\gamma^{\nu}], \tag{15}$$

which corresponds to Fig. 3a. The four-quark terms are

$$V^{\Gamma 4q}(x,y)=-i2\epsilon^{abc}\epsilon^{b'a'c'}\langle 0|\gamma^5\gamma_{\mu}d^c(x)\overline{d^e}(y)\Gamma d^e(y)d^{c'}(0)\gamma_{\nu}\gamma^5|0\rangle\mathrm{Tr}[S_u^{aa'}(x)\gamma^{\mu}C(S_u^{bb'}(x))^TC\gamma^{\nu}], \tag{16}$$

where we only show the four-quark condensate term shown in Fig. 3b, since it is the only term used in the present paper. We do not consider the six- or eight-quark condensates in the present work.

Note that Fig. 3b for the three-point formulation corresponds to Figs. 2b and 2c plus the other terms in the OPE for $S_d^{\Gamma,NP}(x)$ of the two-point method. More generally $S_q^{\Gamma,NP}(x)$ for the two-point method is given in the three-point method by

$$S_q^{cc'\Gamma,NP}(x)=-i\int d^4y\langle 0|:q^c(x)\overline{q^e}(y)\Gamma q^e(y)\overline{q^{c}}'(0):|0\rangle \tag{17}$$

in a linear external field approximation, where the $q_{\mu}=0$ limit has been taken. Note that in principle the space-time structure, as well as the magnitude of the nonlocal susceptibility, can be determined from the expression Eq. (17), and the q^2 dependence can be obtained by carrying out the Fourier transform in the *x*-variable. If we assume vacuum saturation for intermediate states [1] only the scalar condensates contribute, and we obtain

$$S_q^{cc'\Gamma,NP}(x)\simeq\Gamma(-i)\int d^4y\langle 0|:\overline{q^e}(y)q^c(x):|0\rangle \times\langle 0|:\overline{q^{c}}'(0)q^e(y):|0\rangle. \tag{18}$$

In Eq. (18) the nonlocal susceptibility is approximately given by nonlocal condensates:

$$\langle 0|:\overline{q(0)}q(y):|0\rangle\equiv \mathrm{g}(y^2)\langle 0|:\overline{q(0)}q(0):|0\rangle, \tag{19}$$

which gives

$$S_q^{\Gamma,NP}(x)\simeq\Gamma G(x)(\langle 0|:\overline{q(0)}q(0):|\rangle/12)^2,$$

$$G(x)=(-i)\int d^4y\,\mathrm{g}(y^2)\mathrm{g}((x-y)^2). \tag{20}$$

The function $\mathrm{g}(y^2)$ must be chosen to give satisfactory analytic properties as well as consistency with experimental constraints. Recently, this unknown phenomenological function $\mathrm{g}(y^2)$ has been fit to the experimental sea-quark distribution [23] using a three-point formulation of deep inelastic scattering in the scaling region. For the space-time structure for $\mathrm{g}(y^2)$ we use

$$\mathrm{g}(y^2)=\frac{1}{(1+\kappa^2y^2/8)^2} =\int_0^{\infty}d\alpha\,\mathrm{f}(\alpha)e^{-y^2\alpha/4},$$

$$\mathrm{f}(\alpha)=\frac{4}{\kappa^4}\alpha e^{-2\alpha/\kappa^2}. \tag{21}$$

This dipole form is physically reasonable and avoids the undesirable delta function in the Borel mass which is given by a Gaussian form. The Jung-Kisslinger monopole form is not satisfactory for the four-quark nonlocal condensate, but from the range of best fits found in Ref. [23] we estimate that $\kappa^2\simeq(0.15-0.2)$ GeV^2, corresponding to the quark condensate nonlocality of about 0.2 fm, obtained by equating the first moment of $\mathrm{f}(\alpha)$ for the dipole form with that of the monopole form used in Ref. [23]. This range of values for κ^2 is obtained by fits to the low-x sea-quark distributions comparable to those in Ref. [23]; and the narrowness of the range is due to the sensitivity to this parameter.

Using the form of Eqs. (19), (21) in Eq. (20) we obtain

$$G(x)\simeq-\frac{2^7\pi^2}{\kappa^4A(A+4)}\left[1-\frac{2+A}{\sqrt{A^2+4A}}\ln\left(\frac{\sqrt{A^2+4A}+A}{\sqrt{A^2+4A}-A}\right)\right], \tag{22}$$

with $A=\kappa^2x^2/2^3$.

Let us apply this to the determination of χ^{π}. From Eqs. (9), (20), (22) we find (taking the $x=0$ limit) that

$$\chi^{\pi}a\simeq\frac{G(0)a^2}{3\times 2^4\pi^2}\simeq\frac{2a^2}{9\kappa^4} \simeq(1.7-3.0)\ \mathrm{GeV}^2, \tag{23}$$

in agreement with the value $\chi^{\pi}\simeq 1.88$ GeV^2, found in Ref. [16] and discussed in the previous section. If we use the value $\chi^\pi a \simeq 1.88\ \mathrm{GeV}^2$, we find that $\kappa^2 \simeq 0.19\ \mathrm{GeV}^2$. Note that although there is about a 20% error in the phenomenological value of χ^π, the results are very sensitive to κ.

Finally we would like to point out that from Eqs. (20), (22) the space-time structure of the nonlocal vacuum susceptibilities is given. This enables one to derive the current-hadron vertices for low momentum transfer. The method can be immediately extended to medium momentum transfer for applications to form factors, hadronic interactions and so forth, by carrying out the Fourier transform in the y-variable instead of taking the $q=0$ limit.

III. QCD SUM RULE THREE-POINT METHOD FOR PARITY AND ISOSPIN VIOLATIONS OF PION-NUCLEON VERTICES

The QCD sum rule determination of the weak parity-violating and isospin violating pion-nucleon couplings is done by calculating Z_0 and photon loop corrections to the diagrams used for the strong coupling, some of which are shown in Fig. 3. By using a three-point formulation as described in the previous section one can carry out this program without introducing unknown new vacuum susceptibilities to the extent that the factorization of four quark vacuum matrix elements is justified. We briefly describe this procedure.

A. Parity-violating pion-nucleon coupling

At the present time experiments have not detected parity-violations predicted from the one-pion exchange weak interaction. The parity-violating pion-nucleon coupling constant, $f_{\pi NN}$ might be much smaller than expected from quark models with the standard electroweak theory. In the sum rule approach the parity-violating pion-nucleon coupling is determined by starting with $V^\pi(p,q)$ defined by Eq. (13) with the current $J^\pi(y)$ used for $J^\Gamma(y)$ and all Z_0 loops included up to the desired order. Taking the limit of massive gauge bosons, so that the weak interaction becomes a four-fermion interaction with an effective Hamiltonian

$$H_w = \frac{G_F}{2\sqrt{2}} N^\mu N_\mu$$

$$N^\mu = \overline{q}\gamma^\mu \tau_3 \left(1 - \frac{4}{3}(1+\tau_3)\sin^2\ \theta_W - \gamma_5\right) q. \qquad (24)$$

In this low-energy limit of the standard model it was shown in Ref. [17] that the only nonvanishing weak contributions are in the two spectator quarks, which are not interacting with the pion field. The lowest dimensional diagrams (again without gluon condensates) are shown in Fig. 3. In the limit of $q^\mu = 0$ we find for the three-point function

$$V^\pi(p,q=0) = \frac{G_F\ \sin^2\ \theta_W g_{\pi q}}{3^2 2^8 \pi^6}\left(\frac{17}{3} - \gamma\right)\left[p^6\ \ln(-p^2) + \frac{4G(p^2)a^2}{2^4 3\pi^2} p^4\ \ln(-p^2)\right]. \qquad (25)$$

This expression can be readily derived from the results of Ref. [17] and the results of Sec. II of the present paper. Since this expression includes the entire operator product expression there is no need to determine the higher-dimensional susceptibilities, such as the mixed susceptibility of Eq. (7), which was a significant uncertainty in the calculation of Ref. [16]. The main result, that the parity-violating π-N coupling constant, $f_{\pi NN}$, is much smaller than expected from quark models, is still valid, but the experimental value of the strong constant, $g_{\pi NN}$, is not used. In other words one can predict both the strong and weak pion-nucleon coupling.

B. Isospin-violating pion-nucleon coupling

A new analysis of low-energy pion-nucleon scattering data [28] that has shown a large isospin violations in the elastic π-N amplitudes which are consistent [29] with isospin violations in π-N coupling constants. The QCD sum rule calculation with the three-point method is done as in the calculation of the parity-violating coupling just discussed with the replacement of H_W by the electromagnetic interaction and also including the effects of the current quark mass differences and the isospin splitting of the u and d condensates. With the development of a gauge-invariant theory for electromagnetic corrections in the sum rule method [27] it is now possible to carry out this calculation. The calculation is quite complicated, however for the electromagnetic corrections, which involve the three-loop diagrams resulting from photon exchange insertions in the diagrams of Fig. 2. These calculations are being carried out for the octet mass splittings [30], however, and will be extended to the calculation of the π-N isospin violations.

IV. CONCLUSIONS

The three-point function method is usually avoided in QCD sum rule treatments of meson-hadron coupling at low momentum transfer Q due to the fact that the OPE is valid only at high Q. There have been extensive previous studies of the problem of treating long distance bilocal operators for electromagnetic coupling. In the present work we have shown that the three-point function method can be extended to such low-Q processes by introducing nonlocal condensates, whose parametrization has been shown in Ref. [23] to be phenomenologically related to deep inelastic scattering processes. The extension of the three-point method in this fashion provides a convenient method for extending the evaluation of hadron coupling constants to high dimension without encountering a divergent operator product expansion.

We applied the three-point method to solve the outstanding problem of calculating the vacuum susceptibility for pion-nucleon coupling, encountered in previous applications of the two-point function to this problem. We find a vacuum susceptibility of $\chi^\pi a = 1.7-3.0\ \mathrm{GeV}^2$, close to the value found in Ref. [17].

We conclude that the three-point method with nonlocal condensates to represent long-distance effects is a viable approach for calculating low momentum-transfer processes in the QCD sum rule approach, and we have suggested applications to parity and isospin violating couplings. Another application of the three-point method of great interest is to hadron couplings in nuclei, which will be considered in a future paper [26].

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