Constraining the CKM angle γ and penguin contributions through combined $B \to \pi K$ branching ratios

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Motivated by recent CLEO measurements of $B \to \pi K$ modes, we investigate their implications for the CKM angle γ and a consistent description of these decays within the standard model. Interestingly it turns out that already the measurement of the combined branching ratios $B^{\pm} \to \pi^{\pm} K$ and $B_d \to \pi^{\mp} K^{\pm}$ allows us to derive stringent constraints on γ which are complementary to the presently allowed range for that angle. This range, arising from the usual fits of the unitarity triangle, is typically symmetric around $\gamma = 90°$, while our method can in principle *exclude* a range of this kind. Consistency within the standard model implies furthermore bounds on the ratio $r \equiv |T'|/|\tilde{P}|$ of the current-current and penguin operator contributions to $B_d \to \pi^{\mp} K^{\pm}$, and upper limits for the CP-violating asymmetry arising in that decay. Commonly accepted means to estimate r yield values at the edge of compatibility with the present CLEO measurements. [S0556-2821(98)01205-3]

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I. INTRODUCTION

Recently the CLEO Collaboration has presented a first measurement of some exclusive $B \to \pi K$ modes [1]. These modes are of particular interest since during the past years several strategies [2] have been proposed to use such decays for the extraction of angles of the unitarity triangle [3] of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [4], in particular for the angle γ which is an experimental challenge at B-factories. To this end flavor symmetries of strong interactions are used. Unfortunately electroweak penguin diagrams play in certain cases an important role and even spoil some of these methods [5,6]. Because of this feature rather complicated strategies [2,6,7] are needed that are in most cases—requiring, e.g., the geometrical construction of quadrangles among $B \to \pi K$ decay amplitudes—very difficult from an experimental point of view.

A much simpler approach to determine γ was proposed in [8]. It uses the branching ratios for the decays $B^+ \to \pi^+ K^0$, $B_d^0 \to \pi^- K^+$, and their charge conjugates. If the magnitude of the current-current amplitude T' contributing to $B_d^0 \to \pi^- K^+$ is known (we will discuss this point in more detail later), two amplitude triangles can be constructed with the help of these branching ratios that allow in particular the extraction of γ.

Since experimental data for $B^+ \to \pi^+ K^0$ and $B_d^0 \to \pi^- K^+$ are now starting to become available, we think it is an important and interesting issue to analyze the implications of these measurements for γ and the description of these decays within the standard model in general. So far the CLEO Collaboration has presented only results for the combined branching ratios

$$\mathrm{BR}(B^{\pm} \to \pi^{\pm} K) \equiv \frac{1}{2}[\,\mathrm{BR}(B^+ \to \pi^+ K^0) + \mathrm{BR}(B^- \to \pi^- \overline{K^0})], \tag{1}$$

$$\mathrm{BR}(B_d \to \pi^{\mp} K^{\pm}) \equiv \frac{1}{2}[\,\mathrm{BR}(B_d^0 \to \pi^- K^+) + \mathrm{BR}(\overline{B}_d^0 \to \pi^+ K^-)], \tag{2}$$

with rather large uncertainties:

$$\mathrm{BR}(B^{\pm} \to \pi^{\pm} K) = (2.3^{+1.1+0.2}_{-1.0-0.2} \pm 0.2) \times 10^{-5}, \tag{3}$$

$$\mathrm{BR}(B_d \to \pi^{\mp} K^{\pm}) = (1.5^{+0.5+0.1}_{-0.4-0.1} \pm 0.1) \times 10^{-5}. \tag{4}$$

At first sight one would think that measurements of such combined branching ratios are not useful with respect to constraining γ. However, as we will work out in this paper, this is not the case. First, nontrivial bounds on γ of the structure

$$0° \leq \gamma \leq \gamma_0 \ \vee \ 180° - \gamma_0 \leq \gamma \ \leq 180°, \tag{5}$$

where γ_0 is related to the ratio of the combined branching ratios (1) and (2), can be obtained. Second, the ratio r of the current-current amplitude $|T'|$ to the penguin amplitude $|\tilde{P}|$ contributing to $B_d \to \pi^{\mp} K^{\pm}$ can be constrained. Moreover, it is possible to derive a simple formula for the maximally allowed value of the magnitude of the direct CP-violating asymmetry,

$$\mathcal{A}_{CP}^{\mathrm{dir}}(B_d^0 \to \pi^- K^+) \equiv \frac{\mathrm{BR}(B_d^0 \to \pi^- K^+) - \mathrm{BR}(\overline{B}_d^0 \to \pi^+ K^-)}{\mathrm{BR}(B_d^0 \to \pi^- K^+) + \mathrm{BR}(\overline{B}_d^0 \to \pi^+ K^-)}, \tag{6}$$

which can be accommodated within the standard model. In the future, when the CLEO measurements will become more accurate, these constraints on γ, r, and $|\mathcal{A}_{CP}^{\mathrm{dir}}(B_d^0 \to \pi^- K^+)|$ should become more and more restrictive. If the amplitude ratio r should lie far off its standard model expectation and CLEO should measure a CP-violating asymmetry in $B_d \to \pi^{\mp} K^{\pm}$ that is considerably larger than the corresponding bounds on that observable obtained along the lines proposed in our paper, one would have indications for physics beyond the standard model.

In Sec. II we set the stage for our discussion by giving the formulas for the decay amplitudes of $B^+\to\pi^+K^0$ and $B_d^0\to\pi^-K^+$ within the standard model. Quantitative estimates for the branching ratios of these decays are presented in Sec. III. There we also emphasize the importance of penguin diagrams with internal charm quarks to get results of the same order of magnitude as the recent CLEO measurements. The formula for γ_0 constraining γ through Eq. (5) is then derived in Sec. IV, where we also give analytical expressions for bounds on r and $|\mathcal{A}_{CP}^{\rm dir}(B_d^0\to\pi^-K^+)|$ following from a measurement of the combined branching ratios (1) and (2). In Sec. V we analyze the corresponding constraints arising from the present CLEO results and conclude our paper with a brief outlook in Sec. VI.

II. DESCRIPTION OF $B^+\to\pi^+K^0$ AND $B_d^0\to\pi^-K^+$ WITHIN THE STANDARD MODEL

Using a similar notation as in [6,9], the amplitudes for the decays under consideration can be written as [8]

$$A(B^+\to\pi^+K^0)=\mathcal{P}'+c_d\mathcal{P}'^{\rm C}_{\rm EW},\tag{7}$$

$$A(B_d^0\to\pi^-K^+)=-[(P'+c_uP'^{\rm C}_{\rm EW})+T']\equiv-[\tilde{P}+T'],\tag{8}$$

where $\mathcal{P}'$, P' denote QCD penguin amplitudes, $\mathcal{P}'^{\rm C}_{\rm EW}$, $P'^{\rm C}_{\rm EW}$ correspond to color-suppressed electroweak penguin contributions, and T' is the color-allowed $\bar{b}\to\bar{u}u\bar{s}$ current-current amplitude.[1] The primes remind us that we are dealing with $\bar{b}\to\bar{s}$ modes, the minus sign in Eq. (8) is due to our definition of meson states [9], and $c_u=+2/3$ and $c_d=-1/3$ are the up- and down-type quark charges, respectively. The amplitude $\tilde{P}$ in Eq. (8) is a shorthand notation for the penguin contributions to $B_d^0\to\pi^-K^+$, i.e.

$$\tilde{P}\equiv P'+c_uP'^{\rm C}_{\rm EW}.\tag{9}$$

Whereas it is straightforward to show that the current-current amplitude T' can be written in the standard model as

$$T'=e^{i\gamma}e^{i\delta_{T'}}|T'|,\tag{10}$$

where $\delta_{T'}$ is a CP-conserving strong phase and γ the usual angle of the unitarity triangle, the penguin amplitude $\tilde{P}$ is more involved. Here one has to deal with three different contributions corresponding to penguin diagrams with internal up-, charm-, and top-quark exchanges. Taking into account all three of these contributions and *not* assuming dominance of internal top quarks as is frequently done in the literature (we will comment on this point in Sec. III), it can be shown that the $\bar{b}\to\bar{s}$ penguin amplitude $\tilde{P}$ takes the following form [2,10]:

$$\tilde{P}=e^{i\pi}e^{i\delta_{\tilde{P}}}|\tilde{P}|.\tag{11}$$

Here $\delta_{\tilde{P}}$ is again a CP-conserving strong phase arising from final state interactions, while only a trivial CP-violating weak phase appears in Eq. (11) as $e^{i\pi}=-1$. The amplitude structure of Eq. (7) is analogous.

Consequently the amplitude for the decay of the neutral B_d^0 meson is given by

$$A(B_d^0\to\pi^-K^+)=e^{i\delta_{\tilde{P}}}|\tilde{P}|[1-e^{i\gamma}e^{i\delta}r],\tag{12}$$

where

$$r\equiv\frac{|T'|}{|\tilde{P}|}\tag{13}$$

and δ is defined as the difference of the strong phases of T' and $\tilde{P}$ through

$$\delta\equiv\delta_{T'}-\delta_{\tilde{P}}.\tag{14}$$

Taking into account phase space, the branching ratio for $B_d^0\to\pi^-K^+$ is given by

$$\mathrm{BR}(B_d^0\to\pi^-K^+)=\frac{\tau_{B_d^0}}{16\pi M_{B_d^0}}\Phi(M_{\pi^-}/M_{B_d^0},M_{K^+}/M_{B_d^0})|A(B_d^0\to\pi^-K^+)|^2,\tag{15}$$

where $\tau_{B_d^0}$ is the B_d^0 lifetime and

$$\Phi(x,y)=\sqrt{[1-(x+y)^2][1-(x-y)^2]}\tag{16}$$

the usual two-body phase space function. Using Eq. (12) gives

$$|A(B_d^0\to\pi^-K^+)|^2=|\tilde{P}|^2[1-2r\cos(\delta+\gamma)+r^2],\tag{17}$$

while we have, for the CP-conjugate process,

$$|A(\overline{B^0_d}\to\pi^+K^-)|^2=|\tilde{P}|^2[1-2r\cos(\delta-\gamma)+r^2],\tag{18}$$

corresponding to the replacement $\gamma\to-\gamma$. The present data (4) reported recently by CLEO is an average over B_d^0 and $\overline{B^0_d}$ decays that is given by

$$\mathrm{BR}(B_d\to\pi^\mp K^\pm)=\frac{\tau_{B_d}}{16\pi M_{B_d}}\Phi(M_\pi/M_{B_d},M_{K_u}/M_{B_d})\times\langle|A(B_d\to\pi^\mp K^\pm)|^2\rangle,\tag{19}$$

with

[1] Let us note that we have neglected a highly suppressed annihilation contribution in Eq. (7) which is expected to play an even less important role than the color-suppressed electroweak penguin diagrams [6].

$$\langle|A(B_d\to\pi^{\mp}K^{\pm})|^2\rangle\equiv\frac{1}{2}[|A(B_d^0\to\pi^-K^+)|^2+|A(\overline{B^0_d}\to\pi^+K^-)|^2]. \quad (20)$$

Combining Eqs. (17) and (18) yields

$$\langle|A(B_d\to\pi^{\mp}K^{\pm})|^2\rangle=|\tilde{P}|^2[1-2r\cos\delta\cos\gamma+r^2], \quad (21)$$

whereas the direct CP-violating asymmetry (6) can be expressed as

$$\mathcal{A}_{CP}^{\rm dir}(B_d^0\to\pi^-K^+)=2\frac{|\tilde{P}|^2}{\langle|A(B_d\to\pi^{\mp}K^{\pm})|^2\rangle}r\sin\delta\sin\gamma. \quad (22)$$

Taking into account that no nontrivial CP-violating weak phase is present in Eq. (7) implies

$$A(B^+\to\pi^+K^0)=A(B^-\to\pi^-\overline{K^0}), \quad (23)$$

so that we get

$$\mathrm{BR}(B^{\pm}\to\pi^{\pm}K)=\frac{\tau_{B_u}}{16\pi M_{B_u}}\Phi(M_\pi/M_{B_u},M_{K_d}/M_{B_u})\times|A(B^+\to\pi^+K^0)|^2. \quad (24)$$

The color-suppressed electroweak penguin contributions $\mathcal{P}'^{\rm C}_{\rm EW}$ and $P'^{\rm C}_{\rm EW}$ in Eq. (7) and (8) are expected to play a very minor role with respect to the QCD penguin amplitudes $\mathcal{P}'$ and P' as we will see explicitly in Sec. III [2]. Neglecting these contributions and using the $SU(2)$ isospin symmetry of strong interactions allows us to relate the penguin amplitude $\tilde{P}$ relevant for $B_d\to\pi^{\mp}K^{\pm}$ to the $B^{\pm}\to\pi^{\pm}K$ decay amplitude through

$$\tilde{P}=P'=\mathcal{P}'=A(B^+\to\pi^+K^0). \quad (25)$$

The magnitude of the right-hand side of this equation can be obtained from the measured $B^{\pm}\to\pi^{\pm}K$ branching ratio with the help of Eq. (24). Consequently we get the following relation:

$$\mathcal{A}_{CP}^{\rm dir}(B_d^0\to\pi^-K^+)=2\frac{\mathrm{BR}(B^{\pm}\to\pi^{\pm}K)}{\mathrm{BR}(B_d\to\pi^{\mp}K^{\pm})}r\sin\delta\sin\gamma, \quad (26)$$

where the very small phase space difference between $B^{\pm}\to\pi^{\pm}K$ and $B_d\to\pi^{\mp}K^{\pm}$ has been neglected and the relevant B lifetime and mass ratios have been set to unity.

III. SEMIQUANTITATIVE ESTIMATES

Let us have a brief look at the theoretical framework to describe the $B\to\pi K$ decays relevant for our analysis. They are described by low energy effective Hamiltonians taking the following form:

$$\mathcal{H}_{\rm eff}(\Delta B=-1)=\frac{G_{\rm F}}{\sqrt{2}}\left[V_{us}^*V_{ub}\sum_{k=1}^{2}Q_k^uC_k(\mu)+V_{cs}^*V_{cb}\sum_{k=1}^{2}Q_k^cC_k(\mu)-V_{ts}^*V_{tb}\sum_{k=3}^{10}Q_kC_k(\mu)\right], \quad (27)$$

where Q_k are local four-quark operators and $C_k(\mu)$ denote Wilson coefficient functions calculated at a renormalization scale $\mu=\mathcal{O}(m_b)$. The technical details of the evaluation of such Hamiltonians beyond the leading logarithmic approximation has been reviewed recently in [11], where the exact definitions of the current-current operators $Q_{1,2}^u$, $Q_{1,2}^c$, the QCD penguin operators $Q_3,\ldots,Q_6$, the electroweak penguin operators $Q_7,\ldots,Q_{10}$, and numerical values of their Wilson coefficients can be found. Note that the operators $Q_{1,2}^c$ contribute to $B\to\pi K$ modes only through penguinlike matrix elements (see, e.g., [12,13]) that are included by definition in the penguin amplitudes. A similar comment applies to effects of inelastic final state interactions that originate, e.g., from the rescattering process $B_d^0\to\{D_s^+D^-\}\to\pi^-K^+$. In our notation these contributions are related to penguin like matrix elements of the current-current operators and are also included in P'.

The color-allowed amplitude $\overline{T'}=e^{-2i\gamma}T'$ contributing to $\overline{B^0_d}\to\pi^+K^-$ is related to hadronic matrix elements of the current-current operators Q_1^u and Q_2^u given by [8]

$$\langle K^-\pi^+|Q_1^u|\overline{B^0_d}\rangle=\langle K^-\pi^+|(\overline{s}_\alpha u_\beta)_{\rm V\text{-}A}(\overline{u}_\beta b_\alpha)_{\rm V\text{-}A}|\overline{B^0_d}\rangle, \quad (28)$$

$$\langle K^-\pi^+|Q_2^u|\overline{B^0_d}\rangle=\langle K^-\pi^+|(\overline{s}u)_{\rm V\text{-}A}(\overline{u}b)_{\rm V\text{-}A}|\overline{B^0_d}\rangle, \quad (29)$$

where α and β denote $SU(3)_{\rm C}$ color indices and V-A refers to the Lorentz structure $\gamma_\mu(1-\gamma_5)$. As in the case of $Q_{1,2}^c$, penguin like matrix elements of the current-current operators $Q_{1,2}^u$ with up quarks runing as a virtual particles in the loops [12,13] contribute by definition to the penguin amplitudes $\overline{P'}=P'$, $\overline{P'^{\rm C}_{\rm EW}}=P'^{\rm C}_{\rm EW}$ and not to $\overline{T'}$. Introducing nonperturbative B parameters, Eqs. (28) and (29) can be written as

$$\langle K^-\pi^+|Q_1^u(\mu)|\overline{B^0_d}\rangle=\frac{1}{3}B_1(\mu)\mathcal{F}, \quad (30)$$

$$\langle K^-\pi^+|Q_2^u(\mu)|\overline{B^0_d}\rangle=B_2(\mu)\mathcal{F}, \quad (31)$$

where $\mathcal{F}$ corresponds to the ''factorized'' matrix element $\langle K^-|(\overline{su})_{\rm V\text{-}A}|0\rangle\langle\pi^+|(\overline{ub})_{\rm V\text{-}A}|\overline{B^0_d}\rangle$ and $B_k(\mu)\neq1$ parametrizes deviations from factorization. Consequently we get

$$\overline{T'}=-\frac{G_{\rm F}}{\sqrt{2}}V_{us}^*V_{ub}\left[\frac{1}{3}\frac{B_1(\mu)}{B_2(\mu)}C_1(\mu)+C_2(\mu)\right]B_2(\mu)\mathcal{F}, \quad (32)$$

which can be written by introducing the phenomenological color factor a_1 [14,15] as

$$\overline{T'} = -\frac{G_F}{\sqrt{2}} V_{us}^* V_{ub} a_1 \mathcal{F}. \tag{33}$$

The minus sign is due to our definition of meson states [see also the remark after Eq. (8)].

For the following discussion $|T'|$ plays an important role and can be written with the help of the ''factorization'' assumption as

$$|T'||_{\mathrm{fact}} = \frac{G_F}{\sqrt{2}} \lambda |V_{ub}| a_1 (M_{B_d}^2 - M_\pi^2) f_K F_{B\pi}(M_K^2; 0^+), \tag{34}$$

where $\lambda = 0.22$ is the Wolfenstein parameter [16] and $\mathcal{F}$ has been expressed in terms of quark-current form factors [14]. The presently allowed range for $|V_{ub}|$ is given by $(3.2 \pm 0.8) \times 10^{-3}$ [17]. Data from $\overline{B}_d^0 \to D^{(*)+}\pi^-$ and $\overline{B}_d^0 \to D^{(*)+}\rho^-$ decays imply $a_1 = 1.06 \pm 0.03 \pm 0.06$ [18]. From a theoretical point of view, a_1 is very stable for B decays and lies within the range $a_1 = 1.01 \pm 0.02$ [19]. Although the ''factorization'' hypothesis [20] is in general questionable, it may work with reasonable accuracy for the color-allowed current-current amplitude T' [21]. Using the form factor $F_{B\pi}(M_K^2; 0^+) = 0.3$ as obtained in the Bauer-Stech-Wirbel (BSW) model [14] yields

$$|T'||_{\mathrm{fact}} = a_1 \left[\frac{|V_{ub}|}{3.2 \times 10^{-3}} \right] \times 7.8 \times 10^{-9}\ \mathrm{GeV}. \tag{35}$$

In contrast to the case of T', the use of the factorization assumption is questionable for the penguin amplitude $\widetilde{P}$. Let us nevertheless use that approach to get some feeling for the expected orders of magnitudes. Following the formalism developed in [12,13,22] and using the Wolfenstein expansion [16] with $A = 0.810 \pm 0.058$ and $R_b \equiv |V_{ub}|/(\lambda |V_{cb}|) = 0.363 \pm 0.073$ [23], the $\overline{b} \to \overline{s}$ penguin amplitude (9) can be expressed as

$$\begin{aligned}\widetilde{P}|_{\mathrm{fact}} = -\frac{G_F}{\sqrt{2}} A \lambda^2 \Bigg[& \frac{1}{3}\overline{C_3} + \overline{C_4} + \frac{1}{3}\overline{C_9} + \overline{C_{10}} + \frac{2M_K^2}{m_s m_b} \\ & \times \left(\frac{1}{3}\overline{C_5} + \overline{C_6} + \frac{1}{3}\overline{C_7} + \overline{C_8} \right) + \frac{\alpha_s(m_b)}{9\pi} \\ & \times \left\{ \frac{10}{9} - G(m_c, k, m_b) \right\} \left(1 + \frac{2M_K^2}{m_s m_b} \right) \\ & \times \left\{ \overline{C_2} + \frac{\alpha_{\mathrm{QED}}}{\alpha_s(m_b)} \left(\overline{C_1} + \frac{1}{3}\overline{C_2} \right) \right\} \Bigg] \mathcal{F},\end{aligned} \tag{36}$$

where we have used in addition to the factorization approximation the equations of motion for the quark fields leading to the terms proportional to $M_K^2/(m_s m_b)$. The $\overline{C_k}$'s refer to $\mu = m_b$ and denote the next-to-leading order scheme-independent Wilson coefficient functions introduced by Buras *et al.* in [24]. The function $G(m_c, k, m_b)$ is related to one-loop penguin matrix elements of the current-current operators $Q_{1,2}^c$ with internal charm quarks and is given by

$$G(m_c, k, m_b) = -4 \int_0^1 \mathrm{d}x\, x(1-x) \ln\left[\frac{m_c^2 - k^2 x(1-x)}{m_b^2} \right], \tag{37}$$

where m_c is the charm quark mass and k denotes some average four-momentum of the virtual gluons and photons appearing in corresponding penguin diagrams [12,13]. Simple kinematical considerations at the quark level imply the following ''physical'' range for this parameter [25,26]:

$$\frac{1}{4} \lesssim \frac{k^2}{m_b^2} \lesssim \frac{1}{2}. \tag{38}$$

In the case of the $b \to s$ penguin processes considered here, the penguin like matrix elements of $Q_{1,2}^u$ are highly suppressed with respect to those of $Q_{1,2}^c$ by the CKM factor $|V_{us}^* V_{ub}|/|V_{cs}^* V_{cb}| = \lambda^2 R_b = \mathcal{O}(0.02)$ and have been neglected in Eq. (36). These terms may lead to CP asymmetries in $B^\pm \to \pi^\pm K$ that are at most of $\mathcal{O}(1\%)$ and consequently affect the relation (23) to a very small extent [12,22].

As was stressed in [12], a consistent calculation using next-to-leading order (NLO) Wilson coefficients requires the inclusion of the penguinlike matrix elements of the current-current operators discussed above. The point is that the renormalization scheme dependences of these matrix elements cancel those of the C_k's leading to the scheme independent Wilson coefficients $\overline{C_k}$. On the other hand, using leading order (LO) Wilson coefficients C_k^{LO}, these matrix elements have to be dropped so that we have, in this case,

$$\begin{aligned}\widetilde{P}|_{\mathrm{fact}}^{\mathrm{LO}} = -\frac{G_F}{\sqrt{2}} A \lambda^2 \Bigg[& \frac{1}{3} C_3^{\mathrm{LO}} + C_4^{\mathrm{LO}} + \frac{1}{3} C_9^{\mathrm{LO}} + C_{10}^{\mathrm{LO}} \\ & + \frac{2M_K^2}{m_s m_b} \left(\frac{1}{3} C_5^{\mathrm{LO}} + C_6^{\mathrm{LO}} + \frac{1}{3} C_7^{\mathrm{LO}} + C_8^{\mathrm{LO}} \right) \Bigg] \mathcal{F}.\end{aligned} \tag{39}$$

Using numerical values for the Wilson coefficients, we find that the contribution of the electroweak penguin operators to $\widetilde{P}$ is below the $\mathcal{O}(1\%)$ level so that the approximation of neglecting the $P_{\mathrm{EW}}'^{\mathrm{C}}$ contributions [see the comment before Eq. (25)] seems to be on solid ground. Evaluating the branching ratios for the penguin mode $B^\pm \to \pi^\pm K$ corresponding to Eqs. (36) and (39), we find (as can be seen already in the tables given in [12]; see also [27]) that the penguins with internal charm quarks lead to a dramatic enhancement. This feature can be seen in Fig. 1, where we show the dependence of $\mathrm{BR}(B^\pm \to \pi^\pm K)|_{\mathrm{fact}}$ on k^2 for $A = 0.81$, $F_{B\pi}(M_K^2; 0^+) = 0.3$, and $\tau_{B_u} = 1.6$ ps. Using Eq. (33) with $a_1 = 1$, these branching ratios correspond to the amplitude ratios r shown in Fig. 2, where we have chosen $R_b = 0.36$ to evaluate these plots. Consequently in this rather simple model calculation the penguin matrix elements with internal charm quarks lead to an enhancement of $\mathrm{BR}(B^\pm \to \pi^\pm K)$ by a factor of ~ 3 and to a reduction of r by ~ 2. Interestingly the CLEO result (3) does already rule out the LO curve in Fig. 1. The NLO result, however, still has some dependence on k^2 which will disappear once a nonperturbative calculation of the matrix elements becomes available.

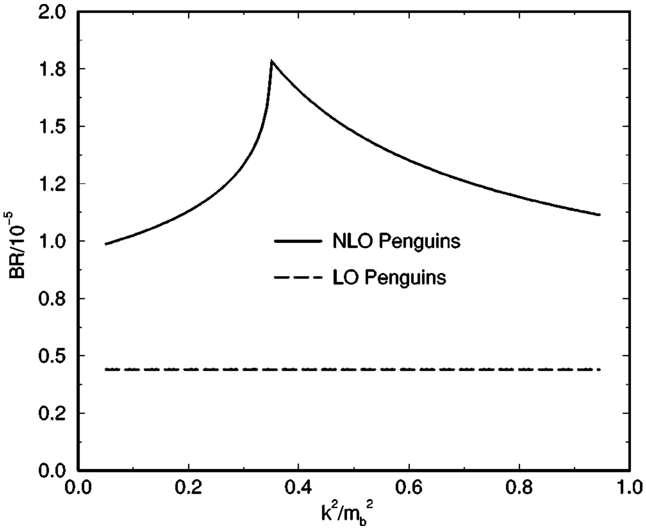

FIG. 1. The dependence of $\mathrm{BR}(B^\pm \to \pi^\pm K)|_{\mathrm{fact}}$ on k^2 for $\Lambda_{\overline{\mathrm{MS}}}^{(4)} = 0.3$ GeV. The difference between the NLO and LO curves is explained in the text.

Still the agreement with the present CLEO data is remarkable although it is a bit on the lower side.

In a similar spirit we can arrive at some estimate of the CP-conserving strong phase defined in Eq. (14). We obtain $\delta = 0°$ if we use Eq. (39) for $\tilde{P}$. Including the important penguin matrix elements with internal charm quarks through Eq. (36) gives values of δ within the range $-30° \lesssim \delta \lesssim 0°$. In spite of all the caveats connected with factorization we still consider it safe to extract the sign of $\cos\delta$ from this discussion. Hence we have very probably $\cos\delta > 0$.

Finally we want to stress that none of the crude estimates discussed above are needed for the analysis presented in Sec. V. The only purpose to include these results in our paper is to update previous theoretical work given the new input from CLEO.

IV. CONSTRAINTS ON γ, r, AND $|\mathcal{A}_{CP}^{\mathrm{dir}}(B_D^0 \to \pi^- K^+)|$

In this section we will derive some simple relations allowing to constrain the CKM angle γ by measuring the combined branching ratios $\mathrm{BR}(B^\pm \to \pi^\pm K)$ and $\mathrm{BR}(B_d \to \pi^\mp K^\pm)$ specified in Eqs. (1) and (2), respectively. Such measurements allow us moreover to restrict the range of r and to give upper bounds for the direct CP asymmetry $|\mathcal{A}_{CP}^{\mathrm{dir}}(B_d^0 \to \pi^- K^+)|$. To this end the quantity

$$R \equiv \frac{\langle |A(B_d \to \pi^\mp K^\pm)|^2 \rangle}{|\tilde{P}|^2} \tag{40}$$

turns out to be very useful. Neglecting the small phase space difference between $B_d \to \pi^\mp K^\pm$ and $B^\pm \to \pi^\pm K$ and using Eqs. (19), (24), and (25) yields

$$R = \frac{\mathrm{BR}(B_d \to \pi^\mp K^\pm)}{\mathrm{BR}(B^\pm \to \pi^\pm K)}, \tag{41}$$

where the ratio of the relevant B-meson lifetimes and masses has been set to unity as in Eq. (26). Consequently R can be fixed through the measured branching ratios (3) and (4). Using Eq. (21) gives

$$C \equiv \cos\delta \cos\gamma = \frac{1-R}{2r} + \frac{1}{2} r. \tag{42}$$

In the following considerations we will keep δ as a free parameter leading to the relation $|\cos\gamma| \geq |C|$ which implies

$$\gamma_0 = \arccos(|C|) \tag{43}$$

for the range (5). Since C is given by the product of two cosines, it has to lie within the range $-1 \leq C \leq +1$. As R is fixed through Eq. (41), this range has the following implication for r:

$$|1 - \sqrt{R}| \leq r \leq 1 + \sqrt{R}. \tag{44}$$

The magnitude of the direct CP-violating asymmetry (22) in $B_d^0 \to \pi^- K^+$ can be expressed with the help of R and C as

$$|\mathcal{A}_{CP}^{\mathrm{dir}}(B_d^0 \to \pi^- K^+)| = 2 \frac{r}{R} \sqrt{\sin^2\gamma - C^2 \tan^2\gamma}. \tag{45}$$

Keeping r and R fixed, this CP asymmetry takes its maximal value

$$|\mathcal{A}_{CP}^{\mathrm{dir}}(B_d^0 \to \pi^- K^+)|_{\max} = 2 \frac{r}{R} (1 - |C|) \tag{46}$$

for

$$\gamma_{\max} = \arccos(\sqrt{|C|}) \vee \gamma_{\max} = 180° - \arccos(\sqrt{|C|}), \tag{47}$$

where C is expressed in terms of r and R in Eq. (42).

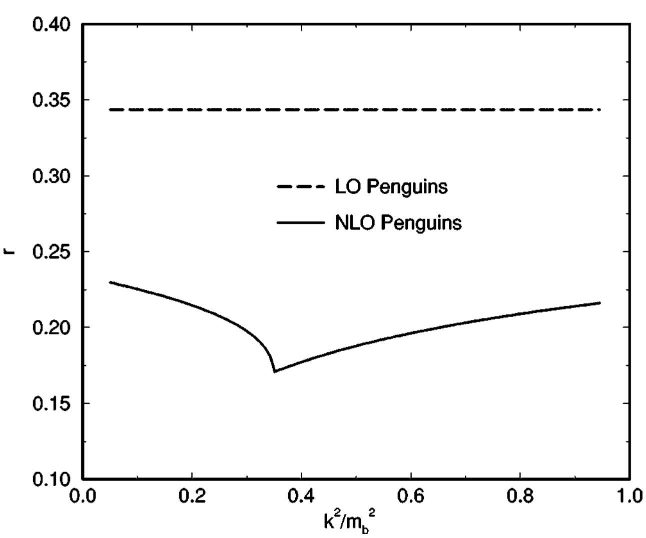

FIG. 2. The dependence of $r|_{\mathrm{fact}}$ on k^2 for $\Lambda_{\overline{\mathrm{MS}}}^{(4)} = 0.3$ GeV. The difference between the NLO and LO curves is explained in the text.

V. IMPLICATIONS OF THE CLEO MEASUREMENTS

In this section we shall discuss the implications of the recent CLEO measurements using the relations derived in the last section. In particular it will become clear that an experimental improvement will make these constraints much more stringent as they appear using present data.

The recent CLEO measurements given in Eqs. (3) and (4)

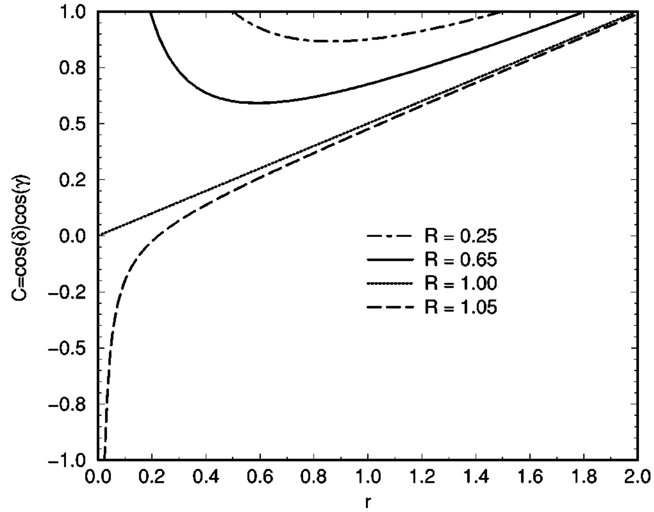

FIG. 3. The dependence of C on the amplitude ratio r for various values of R.

allow us to determine the value of R. Putting the numbers and adding the errors in quadrature gives

$$R = 0.65 \pm 0.40\ . \tag{48}$$

In Fig. 3 we show the dependence of the quantity C defined by Eq. (42) on the amplitude ratio $r = |T'|/|\tilde{P}|$ for various values of R within that experimentally fixed range. This figure illustrates nicely the constraints on r given in Eq. (44) arising from the fact that C has to lie within the range $-1 \leqslant C \leqslant +1$.

As is obvious from the discussions in Sec. II and III, r can in principle be determined in a direct way. Using Eq. (25), the denominator is fixed by the penguin decay $B^\pm \to \pi^\pm K$. The numerator is more difficult to obtain. One possibility is to use Eq. (35), leading to

$$r = 0.16 \times a_1 \times \left[\frac{|V_{ub}|}{3.2\times 10^{-3}}\right] \sqrt{\left[\frac{2.3\times 10^{-5}}{\mathrm{BR}(B^\pm \to \pi^\pm K)}\right]\left[\frac{\tau_{B_u}}{1.6\ \ \mathrm{ps}}\right]}. \tag{49}$$

However, this expression relies on factorization and uncertainties become hard to estimate. Another way would be to relate $|T'|$ to some current-current-dominated process. Assuming flavor $SU(3)$, a possible mode is $B^\pm \to \pi^\pm \pi^0$ which receives only color-allowed and color-suppressed current-current and negligibly small electroweak penguin contributions. Including factorizable $SU(3)$ breaking we obtain [8]

$$|T'| = \lambda \frac{f_K}{f_\pi} \sqrt{2} |A(B^\pm \to \pi^\pm \pi^0)|, \tag{50}$$

where we have neglected the color-suppressed current-current contributions. An interesting experimental consistency check would be the comparison of Eqs. (35) with (50). Unfortunately the $B^\pm \to \pi^\pm \pi^0$ mode has not yet been measured. However, recently the CLEO Collaboration has reported the following upper limit for the corresponding branching ratio [1]:

$$\mathrm{BR}(B^\pm \to \pi^\pm \pi^0) < 2.0 \times 10^{-5}, \tag{51}$$

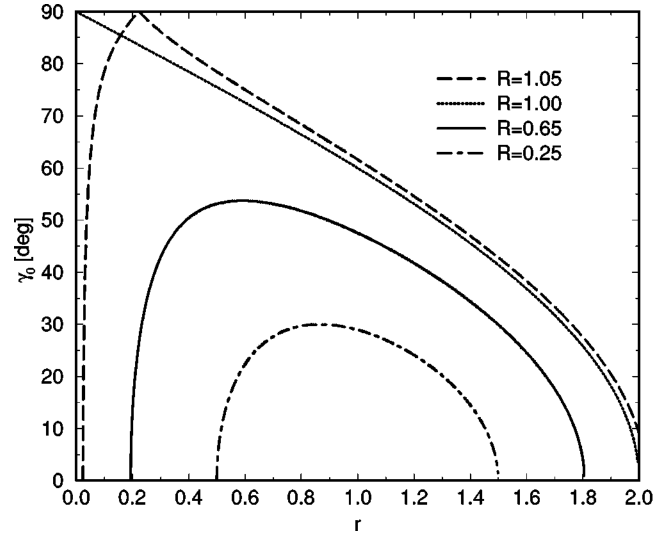

FIG. 4. The dependence of γ_0 constraining the CKM angle γ through Eq. (5) on the amplitude ratio r for various values of R.

which can be translated easily into an upper bound on r. Using the minimal and central values 1.1×10^{-5} and 2.3×10^{-5} of the branching ratio in Eqs. (3), (50) leads to

$$r \lesssim 0.51 \quad \text{and} \quad r \lesssim 0.35\ , \tag{52}$$

respectively. However, given all the assumptions leading to these bounds, we shall not exclude the possibility of having a larger value of r within the limits (44) in what follows and consider all quantities as functions of r.

In Fig. 4 we plot the constraint on γ as a function of r. As usual we shall assume that γ ranges between $0°$ and $180°$ which is determined from CP violation in the kaon system [17]. Because of the two possibilities for the sign of $\cos\delta$, we have to discuss two cases. For positive $\cos\delta$ the sign of $\cos\gamma$ is the same as the one of C. Thus for $R<1$ we can constrain the angle γ between $0°$ and $\gamma_0 < 90°$. For the small window $R>1$, which is still allowed due to the large experimental uncertainty, C becomes negative for $r < \sqrt{R-1}$, implying that γ lies within the range $90° < 180° - \gamma_0 \leqslant \gamma \leqslant 180°$ in that case. If $\cos\delta$ is negative, the situation reverses.

It is obvious that the constraint on γ becomes more restrictive the smaller the value of R is. As can be seen from Fig. 4, $R=1$ is an important special case. The point is that for $R<1$ one can always constrain γ independent of r, while $R>1$ requires some knowledge about r. For $R<1$ the maximal value of γ_0 is given by

$$\gamma_0^{\max} = \arccos(\sqrt{1-R}). \tag{53}$$

In particular, if R is significantly smaller than 1, we may place stringent restrictions on γ. For instance, taking the central value of the CLEO measurement we have $\gamma_0^{\max} = 54°$; for a value at the lower end of Eq. (48) we have even $\gamma_0^{\max} = 30°$. If one is to take the lower limit in Eq. (52) corresponding to $R = 0.65$ serious, one finds $\gamma_0 < 48°$.

In Fig. 5 we show the dependence of $|\mathcal{A}_{CP}^{\mathrm{dir}}(B_d^0 \to \pi^- K^+)|$ on r for various values of R within the experimental range (48). In contrast to the case of γ it is impossible to constrain that CP asymmetry without any knowledge of r.

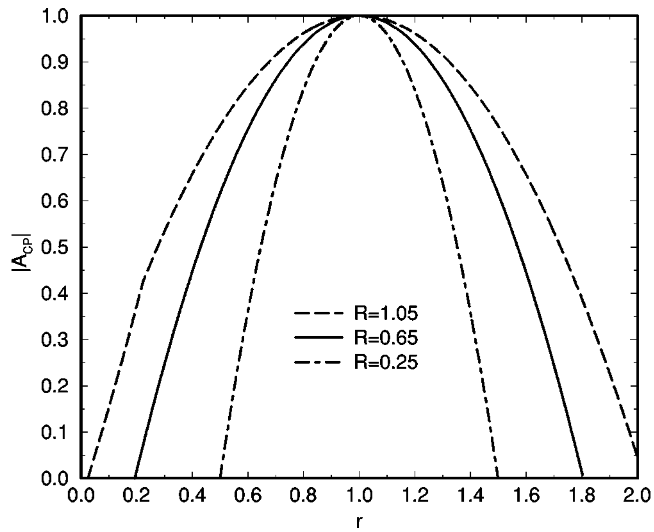

FIG. 5. The dependence of the maximal value (46) of $|\mathcal{A}_{CP}^{\rm dir}(B_d^0\to\pi^-K^+)|$ on the amplitude ratio r for various values of R.

Coming back to the previous example, we have $|\mathcal{A}_{CP}^{\rm dir}(B_d^0\to\pi^-K^+)|\leq 0.35$ for $R=0.65$ and r bounded by the lower value of Eq. (52).

VI. CONCLUSIONS AND OUTLOOK

In the present paper we have shown that a measurement of the combined branching ratios (1) and (2) allows one to obtain useful constraints on γ and direct CP violation in $B_d\to\pi^\mp K^\pm$ even with rather large experimental uncertainties. Needless to say, an improvement on the experimental side will sharpen the bounds on γ substantially; in particular it would be useful to further constrain R to the region $R<1$. Obviously another important step would be a separate measurement of the $B^+\to\pi^+K^0$, $B^-\to\pi^-\overline{K^0}$ and $B_d^0\to\pi^-K^+$, $\overline{B_d^0}\to\pi^+K^-$ branching ratios which may lead to a determination of γ as proposed in [8].

Looking at the bounds on γ we have derived in the present paper, they are complementary to what is obtained from a global fit of the unitarity triangle using experimental data on $|V_{cb}|$, $|V_{ub}|/|V_{cb}|$, B_d^0–$\overline{B_d^0}$ mixing, and CP violation in the neutral K-meson system [17,23]. Typically that range for γ using present data is

$$40°\lesssim\gamma\lesssim 140°. \quad (54)$$

Note that the allowed range is symmetric around $\gamma=90°$, while in our approach we *exclude* a range symmetric with respect to 90° for $R<1$. For instance, taking the central value $R=0.65$, we have $0\leq\gamma\leq\gamma_0^{\rm max}=54°$ or $126°\leq\gamma\leq 180°$ depending on the sign of $\cos\delta$. In order to be compatible, this means that γ has to be either between 40° and 54° or between 126° and 140°. Based on the discussion of Sec. III we conclude that the former range is the preferred one since most probably $\cos\delta>0$. In the case of the central value of Eq. (54), i.e., $\gamma=90°$, we have $C=0$ and get therefore the relation $r=\sqrt{R-1}$ between r and R independent of the value of δ. Note that in this case necessarily $R>1$. Using our bound (52) on r implies thus $1<R\lesssim 1.25$ for $\gamma=90°$. If r should be of $\mathcal{O}(0.2)$ as expected, we would practically fix R to be $1<R\lesssim 1.04$. However, this corresponds to the upper end of the present CLEO range. If γ is close to 90°, future measurements either have a value of R close to unity or it will become increasingly difficult to accommodate the situation within the standard model.

Although some of our bounds are independent of the ratio r, this quantity is still one of the main ingredients of the presented approach. The range implied by pure consistency given in Eq. (44) is quite generous. Using other input to access $|T'|$ and $|\tilde{P}|$, such as factorization for the color-allowed current-current amplitude or data on $B^\pm\to\pi^\pm\pi^0$, consistently indicates small values of r. It is interesting to note that these smallish values are already at the edge of compatibility with the CLEO measurements.

Another important experimental task is to search for direct CP violation in $B_d\to\pi^\mp K^\pm$ which would immediately rule out ''superweak'' models of CP violation [28]. Ruling out these scenarios with the help of that CP asymmetry is, however, not the only possibility; if one should measure CP-violating effects in $B_d\to\pi^\mp K^\pm$ that are inconsistent with the upper limits on $|\mathcal{A}_{CP}^{\rm dir}(B_d^0\to\pi^-K^+)|$ obtained along the lines proposed in our paper, one would also have indications for physics beyond the standard model.

In conclusion, we have demonstrated that the combined $B\to\pi K$ branching ratios reported recently by the CLEO Collaboration may lead to stringent constraints on γ that are complementary to the presently allowed region of that angle obtained with the help of the usual indirect methods to determine the unitarity triangle. These measurements provide in addition a powerful tool to check the consistency of the standard model description of these decays and to search for ''new physics.'' In this respect direct CP violation in $B_d\to\pi^\mp K^\pm$ is also expected to play an important role. Once more data come in confirming values of $R<1$, the $B\to\pi K$ modes discussed in our paper may put the standard model to a decisive test and could open a window to ''new physics.''

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