Vanishing corrections on an intermediate scale and implications for unification of forces

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In a two-step breaking of a class of grand unified theories including $SO(10)$, we prove a theorem showing that the scale (M_I), where the Pati-Salam gauge symmetry with parity breaks down to the standard gauge group, has vanishing corrections due to all sources emerging from higher scales ($\mu > M_I$) such as the one-loop and all higher-loop effects, the grand unified theory threshold, gravitational smearing, and string threshold effects. The implications of such a scale for the unification of gauge couplings with small Majorana neutrino masses are discussed. In string inspired $SO(10)$, we show that $M_I \simeq 5\times 10^{12}$ GeV, needed for neutrino masses, with the GUT scale $M_U \simeq M_{str}$, can be realized provided certain particle states in the predicted spectrum are light. [S0556-2821(98)01701-9]

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I. INTRODUCTION

Grand unified theories (GUTs) based upon supersymmetric (SUSY) $SU(5)$, $SO(10)$, non-SUSY $SO(10)$ with intermediate symmetries, and those inspired by superstrings have been the subject of considerable interest over recent years. In order to solve the strong CP problem through the Peccei-Quinn mechanism and achieve the small neutrino masses [1] necessary to understand the solar neutrino flux [2] and/or the dark matter of the universe, an intermediate scale seems to be essential [3]. Such a scale might correspond to the spontaneous breaking of gauged $B\text{-}L$ contained in intermediate gauge symmetries such as $SU(2)_L\times SU(2)_R\times U(1)_{B\text{-}L}\times SU(3)_C(\equiv G_{2213})$ and $SU(2)_L\times SU(2)_R\times SU(4)_C$ ($\equiv G_{224}$) with [3–6] or without [7] parity, or even others such as $SU(2)_L\times U(1)_{I_{3R}}\times SU(4)_C$ and $SU(2)_L\times U(1)_{I_{3R}}\times U(1)_{B\text{-}L}\times SU(3)_C$. But it is well known that the predictions of a grand unified theory are more [8] or less [6,9] uncertain predominantly due to threshold [10] and gravitational smearing effects [11,12] originating from higher dimensional operators. The uncertainty in the intermediate scale prediction naturally leads to theoretical uncertainties in the neutrino mass predictions through the seesaw mechanism. Therefore, an intermediate scale, stable against theoretical uncertainties, would be most welcome from the point of more accurate predictions on neutrino masses.

Another problem in SUSY GUTs having a supergrand desert is the requirement of $\alpha_s(M_Z)\geqslant 0.12$ to achieve unification at $M_U\simeq 2\times 10^{16}$ GeV. Even though the problem is alleviated by unknown GUT threshold and gravitational corrections [13], realization of a natural grand unification scale $M_U\simeq M_{str}\simeq 5.6\times g_{str}\times 10^{17}$ GeV requires the presence of some lighter string states which could be the extra gauge bosons or Higgs scalars of a unifying symmetry, exotic vectorlike quarks and leptons with nonconventional hypercharge assignments [14–16], or a $SU(3)_C$ octet and weak $SU(2)$ triplet in the adjoint representation of the standard gauge group [17]. But, in the absence of an intermediate symmetry, the neutrino mass predictions may fall short of the solar flux requirements by two to three orders. Assuming boundary conditions at the string scale to be different from a GUT boundary condition, attempts have been made to bring down the values of intermediate scales relevant for larger neutrino masses [18].

The presence of a G_{224P} intermediate gauge symmetry, having only two couplings for $\mu > M_I$, would always guarantee gauge unification, and a demonstration of $M_I \simeq 10^{12}$–10^{14} GeV with $M_U\simeq M_{str}$ in SUSY inspired $SO(10)$ would solve at least two of the major problems: the string scale unification with $\alpha_s(M_Z)\simeq 0.11$ and neutrino masses needed for solar neutrino flux.

It has been shown recently that in all GUTs where G_{224P} breaks spontaneously at the highest intermediate scale, the $\sin^2\theta_W(M_Z)$ prediction is unaffected by GUT threshold and multiloop (two-loop and higher) radiative corrections emerging from higher mass scales [6]. As a single intermediate symmetry is more desirable from a minimality consideration, we confine to the single G_{224P} symmetry in two-step breakings of all possible GUTs including $SO(10)$ and prove a theorem showing that all higher-scale corrections to the intermediate scale (M_I) prediction vanish. In SUSY $SO(10)$ inspired by superstrings [19], we find that $M_I \simeq 10^{12}$–10^{14} GeV is possible with $M_U\simeq M_{str}$, provided certain states in the predicted spectrum are light.

II. THEOREM ON VANISHING CORRECTIONS ON THE INTERMEDIATE SCALE

We now state the following theorem and provide its proof.

Theorem. In all two-step breakings of grand unified theories, the mass scale (M_I) *corresponding to the spontaneous breaking of the intermediate gauge symmetry* $SU(2)_L\times SU(2)_R\times SU(4)_C\times P(g_{2L}=g_{2R})$ *has vanishing contributions due to every correction term emerging from higher scales* ($\mu > M_I$).

To prove the theorem we consider the two-loop breaking pattern in SUSY or non-SUSY GUTs,

$$\mathrm{GUT}\xrightarrow{M_U} G_{224P}\xrightarrow{M_I} G_{213}\xrightarrow{M_Z} U(1)_{em}\times SU(3)_C,$$

which may or may not originate from superstrings. Following the standard notation, we use the following renormalization group equations (RGEs) for the gauge couplings $\alpha_i(\mu)=g_i^2(\mu)/4\pi$:

$M_Z \leq \mu \leq M_I$

$$\frac{1}{\alpha_i(M_Z)}=\frac{1}{\alpha_i(M_I)}+\frac{a_i}{2\pi}\ln\frac{M_I}{M_Z}+\theta_i-\Delta_i, \quad i=Y,2L,3C \tag{2.1}$$

$M_I \leq \mu \leq M_U$

$$\frac{1}{\alpha_i(M_I)}=\frac{1}{\alpha_i(M_U)}+\frac{a_i'}{2\pi}\ln\frac{M_U}{M_I}+\theta_i'-\Delta_i', \quad i=2L,2R,4C \tag{2.2}$$

where Δ_i includes threshold effects at $\mu=M_Z(\Delta_i^Z)$ due to the top-quark and Yukawa couplings and superpartners in SUSY theories. It also includes threshold effects (Δ_i^I) due to heavy particles near the intermediate scale:

$$\Delta_i=\Delta_i^{(Z)}+\Delta_i^{(I)}, \quad i=Y,2L,3C \tag{2.3}$$

The second (third) term on the right-hand side of (2.1), (2.2) is the usual one-loop (multiloop) contribution.

The GUT threshold (Δ_i^U), gravitational corrections (Δ_i^{NRO}), or the string threshold effects (Δ_i^{str}) when the model is based upon string inspired $SO(10)$ [20] are contained in Δ_i':

$$\Delta_i'=\Delta_i^U+\Delta_i^{NRO}+\Delta_i^{str}, \quad i=2L,2R,4C. \tag{2.4}$$

In non-SUSY and SUSY GUTs, the Δ_i^{NRO} may emerge from higher dimensional operators scaled by the Planck mass [11] leading to a nonrenormalizable Lagrangian

$$\mathcal{L}_{NRO}=-\frac{\eta^{(1)}}{2M_{pl}}\,\mathrm{Tr}(F_{\mu\nu}\phi F^{\mu\nu})-\frac{\eta^{(2)}}{2M_{pl}^2}\,\mathrm{Tr}(F_{\mu\nu}\phi^2 F^{\mu\nu})+\cdots, \tag{2.5}$$

where M_{Pl} = Planck mass, and ϕ = Higgs field which is responsible for breaking the GUT symmetry to G_{224P}. For example, in $SO(10)$, $\phi=\overline{54}$. These operators lead to the modifications of the GUT-scale boundary conditions on gauge couplings,

$$\alpha_{2L}(M_U)+(1+\epsilon_{2L})=\alpha_{2R}(M_U)(1+\epsilon_{2R})=\alpha_{4C}(M_U)(1+\epsilon_{4C})=\alpha_G, \tag{2.6}$$

which imply

$$\Delta_i^{NRO}=-\frac{\epsilon_i}{\alpha_G}, \quad i=2L,2R,4C, \tag{2.7}$$

where α_G = GUT coupling and ϵ_i are known functions of the parameters $\eta^{(i)}$, the vacuum expectation value of ϕ, M_U, and M_{Pl}.

Using suitable combinations of gauge couplings and Eqs. (2.1), (2.2), we obtain the following analytic formulas:

$$\ln\frac{M_U}{M_Z}=\frac{(L_S B_I-L_\theta A_I)}{D}+\frac{(J_\theta B_I-K_\theta A_I)}{D}+\frac{(K_\Delta A_I-J_\Delta B_I)}{D}, \tag{2.8}$$

$$\ln\frac{M_I}{M_Z}=\frac{(L_\theta A_U-L_S B_U)}{D}+\frac{(K_\theta A_U-J_\theta B_U)}{D}+\frac{(J_\Delta B_U-K_\Delta A_U)}{D}, \tag{2.9}$$

$$D=A_U B_I-A_I B_U, \quad L_S=\frac{16\pi}{3\alpha(M_Z)}\left[\frac{\alpha(M_Z)}{\alpha_S(M_Z)}-\frac{3}{8}\right],$$

$$L_\theta=\frac{16\pi}{3\alpha(M_Z)}\left[\sin^2\theta_W(M_Z)-\frac{3}{8}\right], \tag{2.10}$$

$$A_U=2a_{4C}'-a_{2L}'-a_{2R}', \quad B_U=\tfrac{5}{3}a_{2L}'-a_{2R}'-\tfrac{2}{3}a_{4C}', \quad A_I=\tfrac{8}{3}a_{3C}-a_{2L}-\tfrac{5}{3}a_Y-A_U, \quad B_I=\tfrac{5}{3}(a_{2L}-a_Y)-B_U,$$

$$J_\theta=2\pi[\theta_{2L}+\tfrac{5}{3}\theta_Y-\tfrac{8}{3}\theta_{3C}+\theta_{2L}'+\theta_{2R}'-2\theta_{4C}'], \quad K_\theta=2\pi[\tfrac{5}{3}(\theta_Y-\theta_{2L})+\theta_{2R}'+\tfrac{2}{3}\theta_{4C}'-\tfrac{5}{3}\theta_{2L}'],$$

$$J_\Delta=2\pi[\Delta_{2L}+\tfrac{5}{3}\Delta_Y-\tfrac{8}{3}\Delta_{3C}+\Delta_{2L}'+\Delta_{2R}'-2\Delta_{4C}'], \quad K_\Delta=2\pi[\tfrac{5}{3}(\Delta_Y-\Delta_{2L})+\Delta_{2L}'+\tfrac{2}{3}\Delta_{4C}'-\tfrac{5}{3}\Delta_{2L}']. \tag{2.11}$$

The first, second, and third terms on the right-hand side (RHS) of (2.8), (2.9) represent the one-loop, the multiloop, and the threshold effects, respectively. Each of these contains contributions originating from lower scales $\mu = M_Z - M_I$, and higher scales $\mu = M_I - M_U$. We now examine the contributions to $\ln(M_I/M_Z)$ term by term. In the presence of the G_{224P} gauge symmetry for $\mu \geqslant M_I$, $\alpha_{2L}(\mu) = \alpha_{2R}(\mu)$. Then Eq. (2.2) gives

$$a'_{2L} = a'_{2R}, \quad \theta'_{2L} = \theta'_{2R}, \quad \Delta'_{2L} = \Delta'_{2R} \tag{2.12}$$

where the G_{224P} symmetry implies

$$\Delta^U_{2L} = \Delta^U_{2R}, \qquad \Delta^{NRO}_{2L} = \Delta^{NRO}_{2R}, \quad \Delta^{str}_{2L} = \Delta^{str}_{2R}, \tag{2.13}$$

The restoration of left-right discrete symmetry in the presence of $SU(4)_C$ in G_{224P} plays a crucial role in giving rise to a vanishing contribution due to every type of higher scale corrections.

A. One-loop contributions

Using Eq. (2.12) we find that B_U and A_U are proportional to each other,

$$B_U = \tfrac{2}{3}(a'_{2L} - a'_{4C}) = -\tfrac{1}{3}A_U, \tag{2.14}$$

$$D = \tfrac{5}{3}(a_{2L} - a_Y)A_U - (\tfrac{8}{3}a_{3C} - a_{2L} - \tfrac{5}{3}a_Y)B_U,$$

$$= \frac{4A_U}{9}(3a_{2L} + 2a_{3C} - 5a_Y). \tag{2.15}$$

Then B_U or A_U cancel out from the denominator and the numerator of the one-loop term in Eq. (2.9) leading to

$$\left(\ln \frac{M_I}{M_Z}\right)_{one\ \ loop} = \frac{12\pi}{\alpha d}\left(\sin^2\theta_W - \frac{1}{2} + \frac{1}{3}\frac{\alpha}{\alpha_S}\right), \qquad d = 3a_{2L} + 2a_{3C} - 5a_Y. \tag{2.16}$$

The fact that a'_i $(i = 2L, 2R, 4C)$ are absent from Eq. (2.16) demonstrates that the scale M_I is independent of the one-loop contribution to the gauge couplings emerging from higher scales, $\mu = M_I - M_U$. But these coefficients do not cancel out from $\ln(M_U/M_Z)$, which assumes the form

$$\ln \frac{M_U}{M_Z} = \frac{12\pi}{\alpha d}\left(\sin^2\theta_W - \frac{1}{2} + \frac{\alpha}{3\alpha_S}\right) + X, \tag{2.17}$$

$$X = \frac{6\pi}{\alpha d}\left[a_{3C}\left(1 - \frac{8}{3}\sin^2\theta_W\right) + a_{2L}\left(\frac{5}{3}\frac{\alpha}{\alpha_S} - 1 + \sin^2\theta_W\right) + \frac{5}{3}a_Y\left(\sin^2\theta_W - \frac{\alpha}{\alpha_S}\right)\right] \Big/ (a'_{4C} - a'_{2L}). \tag{2.18}$$

The first term on the RHS of Eq. (2.17) is the one-loop contribution in Eq. (2.16).

We also note that for any standard weak doublet (H),

$$a^{(H)}_{3C} = 0, \quad 3a^{(H)}_{2L} = 5a^{(H)}_Y,$$

which keeps the one-loop term in Eq. (2.16) unchanged. Thus, the scale M_I is predominantly unaffected by the presence of any number of light doublets with masses $< M_I$, degenerate or nondegenerate.

B. Two-loop and higher-loop effects

Using the second term in the RHS of Eqs. (2.9), (2.14), and (2.15), the coefficients a'_i and terms containing θ'_i cancel out, leading to

$$\left(\ln \frac{M_I}{M_Z}\right)_{multiloop} = \frac{K_\theta A_U - J_\theta B_U}{D} = \frac{2\pi}{d}(5\theta_Y - 3\theta_{2L} - 2\theta_{3c}) \tag{2.19}$$

showing that all multiloop contributions to the gauge couplings originating from $\mu = M_I - M_U$ are absent in $\ln(M_I/M_Z)$. But these multiloop effects do not cancel out from the unification mass,

$$\left(\ln \frac{M_U}{M_Z}\right)_{multiloop} = \left(\ln \frac{M_I}{M_Z}\right)_{multiloop} + X_\theta, \tag{2.20}$$

where the first term on the RHS of Eq. (2.20) is the same as in Eq. (2.19),

$$X_\theta = \frac{9\pi}{4d(a'_{4C} - a'_{2L})}\left[\left\{\frac{5}{3}\left(\theta_{2L} + \frac{5}{3}\theta_Y - \frac{8}{3}\theta_{3C}\right) + \frac{10}{3}(\theta'_{2L} - \theta'_{4C})\right\}(a_{2L} - a_Y) - \left(\frac{8}{3}a_{3C} - a_{2L} - \frac{5}{3}a_Y\right)\left\{\frac{5}{3}(\theta_Y - \theta_{2L}) + \frac{2}{3}(\theta'_{4C} - \theta'_{2L})\right\}\right]. \tag{2.21}$$

C. Threshold effects

Including threshold effects at $\mu = M_Z$, M_I, and M_U, we separate J_Δ and K_Δ into three different parts,

$$J_\Delta = J_\Delta^U + J_\Delta^I + J_\Delta^Z,$$

$$K_\Delta = K_\Delta^U + K_\Delta^I + K_\Delta^Z,$$

where

$$J_\Delta^U = 2\pi(\Delta_{2L}^U + \Delta_{2R}^U - 2\Delta_{4C}^U),$$

$$J_\Delta^i = 2\pi(\Delta_{2L}^i + \tfrac{5}{3}\Delta_Y^i - \tfrac{8}{3}\Delta_{3C}^i), \quad i = I, Z,$$

$$K_\Delta^U = 2\pi(\Delta_{2R}^U + \tfrac{2}{3}\Delta_{4C}^U - \tfrac{5}{3}\Delta_{2L}^U),$$

$$K_\Delta^i = \frac{10\pi}{3}(\Delta_Y^i - \Delta_{2L}^i), \quad i = I, Z. \tag{2.22}$$

Using the parity restoration constraint gives

$$K_\Delta^U = \frac{4\pi}{3}(\Delta_{4C}^U - \Delta_{2L}^U) = -\frac{1}{3} J_\Delta^U$$

and

$$J_\Delta^U B_U - K_\Delta^U A_U = 0. \tag{2.23}$$

Using Eq. (2.23) in the third term in Eq. (2.9) gives

$$\left(\ln \frac{M_I}{M_Z}\right)_{threshold} = -\frac{9}{4d}\left(K_\Delta^I + \frac{J_\Delta^I}{3} + K_\Delta^Z + \frac{J_\Delta^Z}{3}\right). \tag{2.24}$$

Thus, it is clear that the would-be dominant source of uncertainty due to GUT-threshold effects has vanished from $\ln(M_I/M_Z)$ which contains contributions from only lower thresholds at $\mu = M_Z$ and $\mu = M_I$. But the GUT-threshold contributions do not cancel out from $\ln(M_U/M_Z)$ which has the form

$$\left(\ln \frac{M_U}{M_Z}\right)_{threshold} = \left(\ln \frac{M_I}{M_Z}\right)_{threshold} + X_\Delta, \tag{2.25}$$

where

$$X_\Delta = 2\pi \frac{(\Delta_{4C}^U - \Delta_{2L}^U)}{(a'_{4C} - a'_{2L})} + \frac{9}{4d}\left[(K_\Delta^I + K_\Delta^Z)\left(\frac{8}{3} a_{3C} - a_{2L} - \frac{5}{3} a_Y\right) - \frac{5}{3}(J_\Delta^I + J_\Delta^Z)(a_{2L} - a_Y)\right] \Big/ (a'_{4C} - a'_{2L}). \tag{2.26}$$

D. Gravitational smearing and string threshold effects

In the presence of left-right discrete symmetry in G_{224P}, $\Delta_{2L}^{NRO} = \Delta_{2R}^{NRO}$ and $\Delta_{2L}^{str} = \Delta_{2R}^{str}$. The analysis of II C holds true in these cases also leading to

$$J_\Delta^{NRO} B_U - K_\Delta^{NRO} A_U = 0,$$

$$J_\Delta^{str} B_U Y - K_\Delta^{str} A_U = 0,$$

$$\left(\ln \frac{M_I}{M_Z}\right)_p = 0, \quad p = NRO, \text{ string}$$

$$\left(\ln \frac{M_U}{M_Z}\right)_{NRO} = \frac{2\pi}{\alpha_G} \frac{(\epsilon_{2L} - \epsilon_{4C})}{(a'_{4C} - a'_{2L})},$$

$$\left(\ln \frac{M_U}{M_Z}\right)_{str} = 2\pi \frac{(\Delta_{4C}^{str} - \Delta_{2L}^{str})}{(a'_{4C} - a'_{2L})}. \tag{2.27}$$

Thus, the theorem is proved, demonstrating explicitly that $\ln(M_I/M_Z)$ does not have any modification due to corrections to the gauge coupling constants at higher scales for $\mu > M_I$. When the Higgs scalars, fermions, or gauge bosons of the full G_{224P} representations are taken into account, their contributions to $\ln(M_I/M_Z)$ vanish exactly. The origin behind all cancellations is the G_{224P} symmetry and the relation between the gauge couplings,

$$\frac{1}{\alpha_Y(\mu)} = \frac{3}{5}\frac{1}{\alpha_{2L}(\mu)} + \frac{2}{5}\frac{1}{\alpha_{4C}(\mu)}, \quad \mu \geq M_I.$$

Since no specific particle content has been used in proving the vanishing corrections, the theorem holds true with or without SUSY and also in superstring based models.

Another stability criterion of M_I with respect to contributions from lower scale corrections is that, up to one-loop level, it remains unchanged by the presence of any number of light weak doublets having masses from M_Z to M_I.

The other byproduct of this analysis is on the stability of M_U with respect to $16_H + \overline{16}_H$ pairs. In all correction terms for $\ln(M_U/M_Z)$, the higher scale one-loop coefficients appear in the combination $a'_{4C} - a'_{2L}$. We note that for any 16_H (or $\overline{16}_H$),

$$(a'_{4C})_{16_H} = (a'_{2L})_{16_H},$$

which keeps the value of $a'_{4C} - a'_{2L}$ unaltered. Thus, the value of M_U is almost unaffected by the presence of any number of pairs of $16_H \oplus \overline{16}_H$ between $\mu = M_I - M_{GUT}$. This has relevance for SUSY $SO(10)$ and string inspired models.

III. PREDICTIONS IN NON-SUSY $SO(10)$

The stability of M_I in non-SUSY $SO(10)$, under the variation of $\eta^{(1)}$ in Eq. (2.5) was demonstrated in Ref. [21] by accurate numerical estimation. According to the present theorem $\ln(M_I/M_Z)$ is not only independent of the five-dimensional operator and $\eta^{(1)}$, but also of other higher dimensional operators in Eq. (2.5) and parameters arising from the GUT scale. Similarly, the vanishing GUT threshold correction to M_I, obtained in the accurate numerical evaluation of Ref. [22], is a part of the present theorem. Imposing the parity restoration criteria for $\mu \geqslant M_I$ [23], the minimal non-SUSY $SO(10)$ with $\underline{54}$, $\underline{126}$, and $\underline{10}$ representations, $\sin^2\theta_W = 0.2316 \pm 0.0003$, $\alpha_s(M_Z) = 0.118 \pm 0.007$, and $\alpha^{-1}(M_Z) = 127.9 \pm 0.1$, predicts [21–23],

$$M_I = 10^{13.6 \pm 0.16^{+0.5}_{-0.4}}\ \text{GeV},$$

$$M_U = 10^{15.02 \pm 0.25 \pm 0.48 \pm 0.11(0.25)}\ \text{GeV},$$

where the first (second) uncertainties are due to those in the input parameters (threshold effects). In the case of M_I, the threshold uncertainties are due to those at M_Z and M_I thresholds only. The third uncertainty due to the five-dimensional operator in Eq. (2.5), which is absent in M_I, has been calculated for $\eta^{(1)} = \pm 5(\pm 10)$. In spite of the addition of a number of extra $\underline{126}$ and $\underline{10}$ dimensional Higgs fields to build a model for degenerate and seesaw contributions to the neutrino masses in $SO(10)$ introducing $SU(2)_H$ horizontal symmetry, the scale M_I, according to the present theorem, is identical to that in the minimal model with the same predictions on the nondegenerate neutrino masses [24]. The proton lifetime predictions in the minimal model including the NRO contribution is

$$\tau_{p \to e^+ \pi^0} = 1.44 \times 10^{32.1 \pm 0.7 \pm 1.0 \pm 1.9 \pm 0.45(1.0)}\ \text{yr},$$

which might be testified by the next generation of experiments.

IV. INTERMEDIATE SCALE IN SUSY $SO(10)$

In the conventional SUSY $SO(10)$ employing the Higgs supermultiplets $\underline{54}$, $16_H \oplus \overline{16}_H$, and $\underline{10}$, in the usual fashion, it is impossible to achieve M_I substantially lower than M_U. When $126_H \oplus \overline{126}_H$ are used instead of $16_H \oplus \overline{16}_H$, no intermediate gauge group containing $SU(4)_C$ has been found to be possible in Ref. [25]. But the possibilities of other intermediate gauge symmetries in string inspired SUSY $SO(10)$ including $G_{2213}(g_{2L} \neq g_{2R})$ have been demonstrated [25,26] by using extra light G_{2213} submultiplets not needed for spontaneous symmetry breaking, but predicted to be existing in the spectrum [19].

In the present analysis, in addition to the usual $\underline{54}$ with all components at the GUT scale, the pair $16_H + \overline{16}_H$ with desired components at G_{224P} breaking scale, and the bidoublet $\phi(2,2,1) \subset \underline{10}$ near M_Z while (2,2,6) is at M_U, we examine the effects of other components in $\underline{45}$, or in $16_H + \overline{16}_H$ not absorbed by intermediate scale gauge bosons, being lighter and having masses between 1 TeV–M_I.

The adjoint representation $\underline{45}$ contains the left-handed triplet $\sigma_L(3,1,1)$, the right-handed triplet $\sigma_R(1,3,1)$, and also $\sigma^{(C)}(1,1,15)$ under G_{224P}. Under the standard gauge group, σ_R and $\sigma^{(C)}$ decompose as

$$\sigma_R(1,3,1) = \sigma_R^{(+)}(1,1,1) + \sigma_R^{(-)}(1,-1,1) + \sigma_R^{(0)}(1,0,1)$$

$$\sigma^{(C)}(1,1,15) = \sigma_3^{(C)}(1,\tfrac{2}{3},3) + \sigma_{\bar{3}}^{(C)}(1,-\tfrac{2}{3},\overline{3}) + \sigma_8^{(C)}(1,0,8) + \sigma_S^{(C)}(1,0,1).$$

The representation 16_H contains the G_{224P} submultiplets $\chi^{(L)}(2,1,4)$ and $\chi^{(R)}(1,2,\overline{4})$ and the latter decomposes under standard model gauge group as

$$\chi^{(R)}(1,2,\overline{4}) = \chi_1^{(R)}(1,-1,1) + \chi_S^{(R)}(1,0,1) + \chi_{\bar{3}}^{(R)}(1,-\tfrac{2}{3},\overline{3}) + \chi_{\bar{3}}^{(R)\prime}(1,-\tfrac{1}{3},\overline{3}).$$

To make the model simpler, we assume some of these lighter components from $\underline{45}$ or the pair $16_H \oplus \overline{16}_H$ to be either at $M_C \simeq 1$ TeV while others are at M_I. In that case all the equations for $\ln(M_I/M_Z)$ and $\ln(M_U/M_Z)$ derived in Sec. II hold with the replacements

$$\ln\frac{M_I}{M_Z} \to \ln\frac{M_I}{M_C}, \quad \ln\frac{M_U}{M_Z} \to \ln\frac{M_U}{M_C}, \quad \theta_i \to \theta_i^C,$$

$$\alpha_i \to a_i^c (i = Y, 2L, 3C) \quad \text{and} \quad d \to d_C$$

in Eqs. (2.15), (2.16). In addition, there are contributions to the mass scales due to evolutions from $M_Z - M_C$. We present them here only up to one loop. The two-loop, threshold, and gravitational corrections will be estimated elsewhere [27].

$$\left(\ln\frac{M_I}{M_C}\right)_{one\text{-}loop} = \frac{12\pi}{\alpha d_C}\left(\sin^2\theta_W - \frac{1}{2} + \frac{\alpha}{3\alpha_s}\right) - R\ \ln\frac{M_C}{M_Z},$$

$$\left(\ln\frac{M_U}{M_C}\right)_{one\text{-}loop} = \left(\ln\frac{M_I}{M_C}\right)_{one\text{-}loop} + X_C + Y, \tag{4.1}$$

where

$$Y = \frac{5}{8 d_C (a'_{4C} - a'_{2L})}\left[(a_{2L}^C - a_Y^C)(3a_{2L} + 5a'_Y - 8a_{3C}) - (a_{2L} - a_Y)(3a_{2L}^C + 5a_Y'^C - 8a_{3C}^C)\right] \times \ln\frac{M_C}{M_Z},$$

$$d_C = d(a_i \to a_i^c) = 3a_{2L}^c + 2a_{3C}^c - 5a_Y^c,$$

$$X_C = X(a_i \to a_i^c),$$

$$R = \frac{d}{d_c}.$$

We find that when the components under the standard gauge group given in Table I are at $M_C \simeq 1$ TeV, the intermediate mass scale $M_I = 5 \times 10^{12} - 2 \times 10^{14}$ GeV can be achieved with $M_U = M_{str} \simeq 6 \times 10^{17}$ GeV. It has been emphasized that the $SU(3)_C$ octet and $SU(2)_L$ weak triplet, being in the standard model adjoint representation and continuous moduli of strings, have a natural justification to keep them light [17]. In our case $\sigma^{\pm}$, σ_3, $\sigma_{\bar{3}}$, and $\sigma^{(c)}$ belong to the adjoint representations (1,3,1) and (1,1,15) of G_{224}, which in turn are contained in the adjoint representation $\underline{45} \subset SO(10)$. One

TABLE I. Predictions for mass scales in the string inspired $SO(10)$ model.

SM submultiplets $M_Z - M_I$	SM submultiplets $M_C - M_I$	G_{224P} submultiplets $M_I - M_U$	a_i^c	a_i'	M_I (GeV)	M_U (GeV)
ϕ_u, ϕ_d	$\sigma_R^{\pm}, \sigma_3, \sigma_{\overline{3}}$ or $\sigma_R^{\pm}, \chi_3, \chi_{\overline{3}}$	$\sigma_L, \sigma_R, \sigma^c$, $\chi_L, \chi_R, \overline{\chi_L}$, $\overline{\chi_R}, \phi$	$\begin{pmatrix} \frac{47}{5} \\ 1 \\ -2 \end{pmatrix}$	$\begin{pmatrix} 7 \\ 2 \end{pmatrix}$	$10^{12.5}$	$10^{17.6}$
ϕ_u, ϕ_d	$\chi_1, \chi_3, \chi_{\overline{3}}$, χ_3'	$\chi_L, \underline{\chi_R}, \overline{\chi_L}$, χ_R, ϕ	$\begin{pmatrix} \frac{42}{5} \\ 1 \\ -\frac{3}{2} \end{pmatrix}$	$\begin{pmatrix} 5 \\ -2 \end{pmatrix}$	$10^{14.3}$	$10^{17.8}$

set of our solutions in Table I corresponds to the first three of them being as light as $M_C \simeq 1$ TeV while the fourth component, the $SU(3)_C$ octet component in $\sigma^c(1,1,15)$, is at M_I. We have also found a completely different type of solution where the $SU(2)_R$ triplet components and $\chi_3 \oplus \chi_{\overline{3}} \subset 16_H + \overline{16}_H$, but not absorbed by $SU(4)_C$ gauge bosons, are near 1 TeV. In that case all the components in $\sigma^c(1,1,15)$ are at M_I. The neutrinos acquire small Majorana masses by a seesaw mechanism using a $SO(10)$ singlets as explained in Ref. [26]. None of the lighter scalar degrees of freedom near 1 TeV are needed to acquire vacuum expectation values as the spontaneous symmetry breakings of gauge symmetries like $SO(10)$, G_{224P}, and G_{213} occur following the standard procedure through the vacuum expectation values of well known scalar components which are neutral under the residual gauge groups.

The left-handed neutrinos acquire small Majorana neutrino masses via a seesaw mechanism where the right-handed neutrino mass M_N, rather then M_I, occurs in the seesaw formula, in both SUSY [26] and non-SUSY theories. But since M_N is of the same order as M_I with $M_N \leqslant M_I$ in a large class of models, the right-handed Majorana mass is also made correspondingly uncertain whenever M_I is affected by larger uncertainties, especially due to the GUT-threshold effects with nondegenerate components of scalar representations [8] and gravitational effects due to higher dimensional operators [12,21]. This occurs in models where parity is broken at the GUT scale, but G_{224} or G_{2213} with $g_{2L} \neq g_{2R}$ [8], or even $SU(2)_L \times U(1)_R \times SU(4)_C (\equiv G_{214})$ [29], breaks at the intermediate scale. With G_{2213P} at the intermediate scale, these corrections do not vanish, although they are reduced. But in the $SO(10)$ and other GUTs, or string inspired models with G_{224P} (but not G_{2213P}) surviving down to the intermediate scale, all major sources of uncertainties emerging from higher-scale corrections are absent in M_I and, therefore, correspondingly in M_N, even though the latter is still undetermined within one order of magnitude below M_I. It is to be emphasized that in such models, the order-of-magnitude estimation of right-handed Majorana neutrino masses are much more accurate as compared to other models with intermediate scales. Consequently, the left-handed–Majorana-neutrino-mass prediction is more precise in these models. Further it is not true that imposition of the left-right symmetry at the intermediate scale always leads to vanishing higher-scale corrections. The vanishing correction occurs only in the presence of the left-right symmetric G_{224P} gauge symmetry for $\mu > M_I$. Mohapatara [30] has proved a theorem on vanishing corrections due to GUT-threshold effects originating from degenerate components of $SO(10)$ Higgs representations in the presence of other types of gauge symmetry. The present theorem emphasizes vanishing corrections due to all sources emerging from $\mu > M_I$ in the presence of G_{224P} only.

V. SUMMARY AND CONCLUSIONS

We have shown that all higher-scale corrections on the intermediate-scale prediction (M_I), corresponding to the G_{224P} gauge symmetry breaking, vanish exactly. Such corrections are due to one-loop, two-loop, and higher-loop effects, GUT-threshold, and gravitational smearing effects originating from higher-dimensional operators. In string inspired SUSY GUTs, the string-loop threshold effects have also vanishing contributions to M_I. In non-SUSY $SO(10)$ models, the intermediate scale has been predicted earlier and we emphasize that $M_I \simeq 10^{13.6}$ GeV is quite stable leading to more precise neutrino mass predictions. The predicted proton lifetime can be testified by future experiments. The G_{224P} symmetry having only two gauge couplings guarantees unification, but the problem in SUSY $SO(10)$ is the realization of $M_I \ll M_U$. We find solutions to this problem with $M_I \simeq 5 \times 10^{12} - 2 \times 10^{14}$ GeV and $M_U \simeq M_{str} \simeq 6 \times 10^{17}$ GeV provided certain states in the adjoint representation $\underline{45}$ and/or $16_H + \overline{16}_H$ have masses near 1 TeV. The light states in $16_H + \overline{16}_H$ may emerge naturally from the modes not absorbed by heavy $SU(2)_R \times SU(4)_C$ gauge bosons. String-scale unification might be possible in the case of another intermediate symmetry, such as G_{2213}, with parity broken at the GUT scale, when the submultiplet $\sigma^c(1,1,0,8)$ is at the intermediate scale [28]; but only in the present case of G_{224P} intermediate symmetry, the scale M_I has all higher-scale corrections vanishing and neutrino mass predictions in SUSY $SO(10)$ are expected to be more precise.

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[1] M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, Proceedings of the Workshop, Stony Brook, New York, 1979, edited by P. van Niewenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979); T. Yanagida, in *Proceedings of the Workshop on Unified Theory and Baryon Number in the Universe, Tsukuba, Japan, 1979*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1997).

[2] S. P. Mikheyev and A. Yu. Smirnov, Yad. Fiz. **42**, 1441 (1975); L. Wolfenstein, Phys. Rev. D **17**, 2369 (1979).

[3] R. N. Mohapatra and G. SenJanovic, Phys. Rev. Lett. **44**, 912 (1980); Phys. Rev. D **23**, 165 (1981).

[4] R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11**, 566 (1975).

[5] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974).

[6] M. K. Parida and P. K. Patra, Phys. Rev. Lett. **68**, 754 (1992); **66**, 858 (1991).

[7] D. Chang, R. N. Mohapatra, and M. K. Parida, Phys. Rev. Lett. **52**, 1072 (1984); D. Chang, R. N. Mohapatra, J. Gipson, R. E. Marshak, and M. K. Parida, Phys. Rev. D **31**, 1718 (1985); P. Langacker and M. Luo, *ibid.* **44**, 817 (1991); N. G. Deshpande, E. Keith, and P. Pal, *ibid.* **46**, 2261 (1992).

[8] V. V. Dixit and Mark Sher, Phys. Rev. D **40**, 3675 (1989); M. K. Parida and C. C. Hazra, *ibid.* **40**, 3074 (1989).

[9] R. N. Mohapatra and M. K. Parida, Phys. Rev. D **47**, 264 (1993).

[10] L. Hall, Nucl. Phys. **B178**, 75 (1981); P. Langacker and N. Polonsky, Phys. Rev. D **47**, 4028 (1993).

[11] Q. Shafi and C. Wetterich, Phys. Rev. Lett. **52**, 875 (1981); C. T. Hill, Phys. Lett. **135B**, 47 (1984).

[12] M. K. Parida and P. K. Patra, Phys. Rev. D **39**, 2000 (1989); Phys. Lett. B **234**, 45 (1990).

[13] T. Dasgupta, P. Mamales, and P. Nath, Phys. Rev. D **52**, 5366 (1995); D. Ring, S. Urano, and R. Arnowitt, *ibid.* **52**, 6623 (1995).

[14] R. Barbieri, G. Dvali, and A. Strumia, Phys. Lett. B **333**, 79 (1994); Nucl. Phys. **B435**, 102 (1995).

[15] A. Font, L. Ibanez, and F. Quevedo, Nucl. Phys. **B345**, 389 (1990); D. Lewellen, Nucl. Phys. **B337**, 61 (1990).

[16] K. Dienes and A. Faraggi, Phys. Rev. Lett. **75**, 2646 (1995); Nucl. Phys. **B457**, 409 (1995).

[17] C. Bachas, C. Fabre, and T. Yanagida, hep-ph/9507219.

[18] K. Benakli and G. Senjanovic, Phys. Rev. D **54**, 5734 (1996).

[19] S. Chauduri, S. W. Chung, G. Hockney, and J. Lykken, Nucl. Phys. **B456**, 89 (1995).

[20] V. S. Kaplunovsky, Nucl. Phys. **B307**, 145 (1988).

[21] P. K. Patra and M. K. Parida, Phys. Rev. D **44**, 2179 (1991).

[22] Dae-Gyu Lee, R. N. Mohapatra, M. K. Parida, and M. Rani, Phys. Rev. D **51**, 229 (1995).

[23] M. K. Parida, Pramana, J. Phys. **41**, 271 (1993); *ibid.* **45**, 209 (1995).

[24] D. Caldwell and R. N. Mohapatra, Phys. Rev. D **50**, 3477 (1994).

[25] M. Bando, J. Sato, and T. Takahasi, Phys. Rev. D **52**, 3076 (1995).

[26] Dae-Gyu Lee and R. N. Mohapatra, Phys. Rev. D **52**, 4125 (1995).

[27] M. K. Parida (unpublished).

[28] R. N. Mohapatra (private communication).

[29] M. K. Parida and P. K. Patra, Phys. Rev. D **39**, 2000 (1989); M. K. Parida and M. Rani, *ibid.* **49**, 3704 (1994).

[30] R. N. Mohapatra, Phys. Lett. B **285**, 235 (1992).