

Intermediate scales, μ parameter, and fermion masses from string models

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(Received 16 June 1997; published 23 January 1998)

We address intermediate scales within a class of string models. The intermediate scales occur due to the SM singlets S_i acquiring non-zero VEVs due to radiative breaking; the mass squared m_i^2 of S_i is driven negative at μ_{rad} due to $\mathcal{O}(1)$ Yukawa couplings of S_i to exotic particles (calculable in a class of string models). The actual VEV of S_i depends on the relative magnitude of the non-renormalizable terms of the type \hat{S}_i^{K+3}/M^K in the superpotential. We mainly consider the case in which the S_i are charged under an additional non-anomalous $U(1)'$ gauge symmetry and the VEVs occur along F - and D -flat directions. We explore various scenarios in detail, depending on the type of Yukawa couplings to the exotic particles and on the initial boundary values of the soft SUSY breaking parameters. We then address the implications of these scenarios for the μ parameter and the fermionic masses of the standard model. [S0556-2821(98)03905-8]

PACS number(s): 11.25.Mj, 12.15.Hh, 12.60.Jv

I. INTRODUCTION

One prediction of the weakly coupled heterotic string is the tree level gauge coupling unification at $M_{string} \sim g_U \times 5 \times 10^{17}$ GeV [1], where g_U is the gauge coupling at the string scale. M_{string} is the only mass scale that appears in the effective Lagrangian of such string vacua, and thus is one mass scale naturally provided by string theory.

However, one of the major obstacles to connecting string theory to the low energy world is the absence of a fully satisfactory scenario for supersymmetry (SUSY) breaking, either at the level of world-sheet dynamics or at the level of the effective theory. The SUSY breaking induces soft mass parameters which provide another scale in the theory that can hopefully provide a link between M_{string} and M_Z , the scale of electroweak symmetry breaking. For example, in models with radiative breaking one of the Higgs masses squared runs from an initial positive value m_0^2 at M_{string} to a negative value, of $\mathcal{O}(-m_0^2)$, at low energies, so that the electroweak scale is set by the soft supersymmetry breaking scale m_0 (and not by the intermediate scale at which the mass squared goes through zero).

In spite of this difficulty, string theory does provide certain generic and, for a certain class of string vacua, definite predictions. With the assumption of soft supersymmetry breaking masses as free parameters, the features of the string models, such as the explicitly calculable structure of the superpotential, provide specific predictions for the low energy physics.

For example, one can restrict the analysis to a set of string vacua which have $N=1$ supersymmetry, the standard model (SM) gauge group as a part of the gauge structure, and a particle content that includes three SM families and at least two SM Higgs doublets, i.e., the string vacua which have at least the ingredients of the minimal supersymmetric standard model (MSSM) and thus the potential to be realistic.¹ Such

vacua often predict an additional nonanomalous $U(1)'$ gauge symmetry in the observable sector. It has been argued [9] that for this class of string vacua with an additional $U(1)'$ broken by a single standard model singlet S , the mass scale of the $U(1)'$ breaking should be in the electroweak range (and not larger than a TeV). That is, if the $U(1)'$ is not broken at a large scale through string dynamics, the $U(1)'$ breaking may be radiative if there are Yukawa couplings of $\mathcal{O}(1)$ of S to exotic particles. The scale of the symmetry breaking is then set by the soft supersymmetry breaking scale m_0 , in analogy to the radiative breaking of the electroweak symmetry described above.

Recently, a model was considered [10] in which the two SM Higgs doublets couple to the SM singlet, and the gauge symmetry breaking scenarios and mass spectrum were analyzed in detail. A major conclusion of this analysis was that a large class of string models not only predicts the existence of additional gauge bosons and exotic matter particles, but can often ensure that their masses are in the electroweak range. Depending on the values of the assumed soft supersymmetry breaking mass parameters at M_{string} , each specific model leads to calculable predictions, which can satisfy the phenomenological bounds. In addition, the model considered in [10,11] forbids an elementary μ term for appropriate $U(1)'$ charges, but an effective μ is generated by the electroweak scale vacuum expectation value (VEV) of the singlet, thus providing a natural solution to the μ problem.

However, the qualitative picture changes if there are couplings in the renormalizable superpotential of exotic particles to two or more mirrorlike singlets S_i charged under the $U(1)'$. In this case, the potential may have D - and F -flat directions, along which it consists only of the quadratic mass terms due to the soft supersymmetry breaking mass squared parameters m_i^2 . If there is a mechanism to drive the linear combination m^2 that is relevant along the flat directions negative at $\mu_{rad} \gg M_Z$, the $U(1)'$ breaking is at an intermediate scale. On the other hand, if some individual m_i^2 are negative but m^2 remains positive, then the D -flat direction is not relevant and the breaking occurs near the electroweak scale, similar to the case of only one singlet.

A large number of string models have ingredients that can lead to such scenarios:

¹A number of such models (not necessarily consistent with gauge unification) were constructed as orbifold models [2,3] with Wilson lines, as well as models based on the free (world-sheet) fermionic constructions [4–7]. For a review and references see [8].

(i) SM singlets S_i which do not have renormalizable self-interactions of the superpotential (F flatness).

(ii) If such singlets S_i are charged under additional nonanomalous $U(1)'$ factors, more than one S_i with opposite relative signs for the additional $U(1)'$ charges may ensure D -flat directions. This is the case that we focus on in this paper. However, similar considerations hold for a single scalar S which carries no gauge quantum numbers and therefore has no D terms.

(iii) Most importantly, in a large class of models such S_i can couple to additional exotic particles via Yukawa couplings of $\mathcal{O}(1)$. Such Yukawa couplings can then lead to radiative breaking, by driving some or all of the soft m_i^2 parameters negative at $\mu_{rad} \gg M_Z$.

In the case of pure radiative breaking, the minimum of the potential occurs near the scale μ_{rad} , and so the nonzero VEV of S_i 's is at an intermediate scale. In principle, non-renormalizable terms in the superpotential compete with the radiative breaking. These terms are generically present in most string models. If such terms dominate at scales below μ_{rad} , they will determine the VEV of S_i . In this case, the order of magnitude of the VEV depends on the order of the non-renormalizable terms, but is also at an intermediate scale.

The purpose of this paper is to investigate the nature of intermediate scales in a class of string models. Intermediate scales are of importance, as they are often utilized in phenomenological models (e.g., for neutrino masses), and may also have important cosmological implications (e.g., in the inflationary scenarios [12]). In this paper, we also investigate the implications of intermediate scales for the standard model sector of the theory, specifically for the μ parameter, ordinary fermion masses, and Majorana and Dirac neutrino masses.

In Sec. II, we give a general discussion of radiative breaking along a flat direction and study two different mechanisms (radiative corrections and non-renormalizable terms) that stabilize the potential and fix an intermediate scale VEV. We also examine the implications for the low energy particle spectrum of such type of scenarios.

In Sec. III, we explore the range of μ_{rad} that can arise assuming that the flat direction has large Yukawa couplings to exotic fields (as is typically expected in string models) [13]. We consider three different models, with varied quantum numbers for the exotic fields, and in each case we examine the effect on μ_{rad} of different choices of boundary conditions for the soft masses. The relevant renormalization group equations, with exact analytic solutions and useful simplified approximations, are given in the Appendix.

In Sec. IV, we discuss the size and structure of non-renormalizable contributions [30] to the superpotential expected in string models [32,29,31]. These terms are relevant to fix the intermediate scale and can also play an important role in connection with the physics of the effective low-energy theory. In particular, in Sec. V we study how these contributions may offer a natural solution to the μ problem and generate a hierarchy of standard model ordinary fermion masses in rough agreement with observation. We also indicate that interesting neutrino masses can arise from such terms. Both the ordinary seesaw mechanism for Majorana

masses or naturally small (non-seesaw) Dirac or Majorana masses can be generated.

Finally, in Sec. VI we draw some conclusions.

II. INTERMEDIATE SCALE VEV

A well-known mechanism to generate intermediate scale VEVs in supersymmetric theories utilizes the flat directions generically present in these models [14]. The discussion in this section applies to a general class of supersymmetric models with flat directions; string models [15] discussed subsequently in general possess these features.

For example, consider a model with two chiral multiplets \hat{S}_1 and \hat{S}_2 that are singlets under the standard model gauge group, but carry charges Q_1 and Q_2 under an extra $U(1)'$.² If these charges have opposite signs ($Q_1 Q_2 < 0$), the scalar field direction S with

$$\langle S_1 \rangle = \cos \alpha_Q \langle S \rangle, \quad \langle S_2 \rangle = \sin \alpha_Q \langle S \rangle, \quad (1)$$

with

$$\tan^2 \alpha_Q \equiv \frac{|Q_1|}{|Q_2|}, \quad (2)$$

is D flat. If \hat{S}_1 and \hat{S}_2 do not couple among themselves in the renormalizable superpotential, the direction (1) is also F flat and the only contribution to the scalar potential along S is given by the soft mass terms $m_1^2 |S_1|^2 + m_2^2 |S_2|^2$. If we concentrate on the (real) component $s = \sqrt{2} \operatorname{Re} S$ along the flat direction,

$$s = s_1 \cos \alpha_Q + s_2 \sin \alpha_Q, \quad (3)$$

the potential is simply

$$V(s) = \frac{1}{2} m^2 s^2, \quad (4)$$

where

$$m^2 = m_1^2 \cos^2 \alpha_Q + m_2^2 \sin^2 \alpha_Q \\ = \left(\frac{m_1^2}{|Q_1|} + \frac{m_2^2}{|Q_2|} \right) \frac{|Q_1 Q_2|}{|Q_1| + |Q_2|}, \quad (5)$$

which is evaluated at the scale $\mu = s$. We assume that m^2 is positive at the string scale³ ($m^2 = m_0^2$ if we assume universality.) However, m^2 can be driven to negative values at the electroweak scale if \hat{S}_1 and/or \hat{S}_2 have a large Yukawa cou-

²We assume that the supersymmetry breaking is due to hidden sector fields that are not charged under the additional $U(1)'$: i.e., the $U(1)'$ belongs to the observable sector. Thus, the mixing of the $U(1)_Y$ and $U(1)'$ gauge kinetic energy terms, which can arise due to the one-loop (field theoretical) corrections or genus-one corrections in string theory [16], can be neglected in the analysis of the soft supersymmetry breaking mass parameters.

³In some string models it is in principle possible to obtain $m^2 < 0$ for some scalar field, depending on its modular weight [17].

pling to other fields in the superpotential.⁴ In this case, the potential develops a minimum along the flat direction and S acquires a VEV. From the minimization condition

$$\frac{dV}{ds} = \left(m^2 + \frac{1}{2} \beta_{m^2} \right) \Big|_{\mu=s} \quad s=0 \quad (6)$$

(where $\beta_{m^2} = \mu dm^2/d\mu$), one sees that the VEV $\langle s \rangle$ is determined by

$$m^2(\mu = \langle s \rangle) = -\frac{1}{2} \beta_{m^2}, \quad (7)$$

which is satisfied very close to the scale μ_{rad} at which m^2 crosses zero. This scale is fixed by the renormalization group evolution of parameters from M_{string} down to the electroweak scale and will lie at some intermediate scale. The precise value depends on the couplings of $\hat{S}_{1,2}$ and the particle content of the model, as we discuss in the next section.

The stabilization of the minimum along the flat direction can also be due to non-renormalizable terms in the superpotential, which lift the flat direction for sufficiently large values of s . If these terms are important below the scale μ_{rad} , they will determine $\langle s \rangle$. The relevant non-renormalizable terms⁵ are of the form⁶

$$W_{NR} = \left(\frac{\alpha_K}{M_{Pl}} \right)^K \hat{S}^{3+K}, \quad (8)$$

where $K=1,2,\dots$ and M_{Pl} is the Planck scale. The coefficients α_K will be discussed in Sec. IV. Depending on the $U(1)'$ charges, not all values of K are allowed. For example, if $Q_1 = -Q_2$, $U(1)'$ invariance dictates $W_{NR} \sim (\hat{S}_1 \hat{S}_2)^n \sim \hat{S}^{2n}$ and only odd values of K should be considered. If $Q_1 = \frac{4}{5}$, $Q_2 = -1/5$, $W_{NR} \sim (\hat{S}_1 \hat{S}_2^4)^n \sim \hat{S}^{5n}$, and so on.

Including the F term from Eq. (8), the potential along s is

$$V(s) = \frac{1}{2} m^2 s^2 + \frac{1}{2(K+2)} \left(\frac{s^{2+K}}{\mathcal{M}^K} \right)^2, \quad (9)$$

where $\mathcal{M} = C_K M_{Pl} / \alpha_K$, and the coefficient $C_K = [2^{K+1} / ((K+2)(K+3)^2)]^{1/(2K)}$ takes the values (0.29, 0.53, 0.67, 0.76, 0.82) for $K=(1,2,3,4,5)$. The VEV of s is then⁷

⁴Another case which often occurs is that in which, e.g., m_1^2 goes negative but m^2 remains positive. In that case the D flatness is not important: S_1 acquires an electroweak scale VEV while $\langle S_2 \rangle = 0$, so that the $U(1)'$ is broken at or near the electroweak scale, similar to the case discussed in [9,10].

⁵The notation for superfields and their bosonic and fermionic components follows that of [10].

⁶One can also have terms of the form $\alpha_K^K \hat{S}^{2+K} \Phi / M_{Pl}^K$, where Φ is a standard model singlet that does not acquire a VEV. These have similar implications as the terms in Eq. (8).

⁷For simplicity, we do not include in Eq. (9) soft terms of the type $(AW_{NR} + \text{H.c.})$ with $A \sim m_{soft}$. Such terms do not affect the order of magnitude estimates that follow.

$$\langle s \rangle = [\sqrt{(-m^2)} \mathcal{M}^K]^{1/(K+1)} = \mu_K \sim (m_{soft} \mathcal{M}^K)^{1/(K+1)}, \quad (10)$$

where $m_{soft} = \mathcal{O}(|m|) = \mathcal{O}(M_Z)$ is a typical soft supersymmetry breaking scale. In this equation, $-m^2$ is evaluated at the scale $\mu_K = \langle s \rangle$ and has to satisfy the necessary condition $m^2(\mu_K) < 0$. If non-renormalizable terms are negligible below μ_{rad} , no solution to Eq. (10) exists and $\langle s \rangle$ is fixed solely by the running m^2 .

The mass M_S of the physical field s in the vacuum $\langle s \rangle$ can be obtained easily in both types of breaking scenarios. In both cases, M_S is of the soft breaking scale or smaller and not of the intermediate scale $\langle s \rangle$. For pure radiative breaking,

$$M_S^2 \equiv \frac{d^2 V}{ds^2} \Big|_{s=\langle s \rangle} = \left(\beta_{m^2} + \frac{1}{2} \mu \frac{d}{d\mu} \beta_{m^2} \right) \Big|_{\mu=\langle s \rangle} \simeq \beta_{m^2} \sim \frac{m_{soft}^2}{16\pi^2}. \quad (11)$$

In the last expression we give an order of magnitude estimate: The RG beta function for m^2 is the sum of several terms of order m_{soft}^2 (multiplied by some coupling constants), and part of the $16\pi^2$ suppression can be compensated when all the terms are included.

In the case of stabilization by non-renormalizable terms,

$$M_S^2 = 2(K+1)(-m^2) \sim m_{soft}^2. \quad (12)$$

In the preceding discussion, we have ignored the presence of scalar fields other than s_1 and s_2 in the potential. In addition, there are extra degrees of freedom from the two singlets. The real field transverse to the flat direction, Eq. (3), is forced to take a very small VEV of order $m_{soft}^2 / \langle s \rangle$. The physical excitations along that transverse direction have (up to soft mass corrections) an intermediate scale mass

$$M_I^2 = g_1'^2 (Q_1^2 \langle s_1 \rangle^2 + Q_2^2 \langle s_2 \rangle^2). \quad (13)$$

The two pseudoscalar degrees of freedom, $\text{Im } S_1$, $\text{Im } S_2$, are massless: The potential is invariant under independent rotations of the phases of S_1 and S_2 so that the spontaneous breaking of this $U(1) \times U(1)$ symmetry gives two Goldstone bosons. One of the $U(1)$'s is identified with the gauged $U(1)'$ and the corresponding Goldstone boson is absorbed by the Z' , which has precisely the same intermediate mass given by Eq. (13). The other massless pseudoscalar remains in the physical spectrum and can acquire a mass if there are terms in the potential that break the other $U(1)$ symmetry explicitly (e.g., in the presence of AW_{NR} terms.). The fermionic part of the $Z'-S_1-S_2$ sector consists of three neutralinos $(\tilde{B}', \tilde{S}_1, \tilde{S}_2)$. The combination

$$\tilde{S} = \cos \alpha_Q \tilde{S}_1 + \sin \alpha_Q \tilde{S}_2 \quad (14)$$

is light, with mass of order m_{soft} if the minimum is fixed by non-renormalizable terms. If the minimum is instead determined by the running of m^2 , \tilde{S} is massless at the tree level but acquires a mass at one loop of order $m_{soft}/(4\pi)$. The two other neutralinos have masses $M_I \pm \frac{1}{2} M_1'$, where M_1' is the $U(1)'$ soft gaugino mass.

This pattern of masses can be easily understood; in the absence of supersymmetry breaking, a nonzero VEV along the flat direction breaks the $U(1)'$ gauge symmetry but leaves supersymmetry unbroken. Thus, the resulting spectrum is arranged in supersymmetric multiplets: One massive vector multiplet (consisting of the Z' gauge vector boson, one real scalar and one Dirac fermion) has mass M_I , and one chiral multiplet (consisting of the complex scalar S and its Weyl fermion partner \bar{S}) remains massless. The presence of soft supersymmetry breaking terms modifies the picture slightly, lifting the mass degeneracy of the components in a given multiplet by amounts proportional to the soft breaking.

The rest of the fields that may be present in the model can be classified into two types; those that couple directly in the renormalizable superpotential to $\hat{S}_{1,2}$ will acquire intermediate scale masses, and those which do not can be kept light. In particular, all the usual MSSM fields should belong to the latter class. The particle spectrum at the electroweak scale thus contains the usual MSSM fields and one extra chiral multiplet (S, \bar{S}) remnant of the $U(1)'$ breaking along the flat direction.⁸ The interactions among the light multiplet \hat{S} and MSSM fields are suppressed by powers of the intermediate scale. At the renormalizable level, the only interaction between the MSSM fields and the intermediate scale fields arises from the $U(1)'$ D terms in the scalar potential. The resulting effect after integrating out the fields which have heavy intermediate scale masses [18] is a shift of the soft masses of MSSM fields charged under the extra $U(1)'$:

$$\delta m_i^2 = -Q_i \frac{m_1^2 - m_2^2}{Q_1 - Q_2}. \quad (15)$$

The $U(1)'$ D -term contribution to the scalar quartic coupling of light fields charged under the $U(1)'$ drops out after decoupling these intermediate scale particles.

Non-renormalizable interactions between MSSM fields and the $S_{1,2}$ fields, which can play an important role (e.g., for the generation of the μ parameter and fermion masses), are discussed separately in Sec. IV.

Before closing this section, we remark in passing that a similar intermediate scale breaking can occur in the $H_{1,2}$ sector of the theory, where, in the absence of a fundamental μ parameter, the direction $H_1^0 = H_2^0$ is also flat. The condition $m_{H_1}^2 + m_{H_2}^2 > 0$ on the Higgs soft masses would prevent the formation of such a dangerous intermediate scale minimum. This is however not a necessary condition; the breaking could well occur first along the S flat direction, generating an effective μ parameter that can lift the $H_1^0 = H_2^0$ flat direction. The determination of which breaking occurs first would require an analysis of the effective potential in the early Universe.

⁸In the case of a single S and no additional $U(1)'$ there is also one extra chiral multiplet at the electroweak scale.

III. RADIATIVE BREAKING

Both mechanisms for fixing an intermediate VEV (purely radiative or by non-renormalizable terms) depend on the scale μ_{rad} at which some combination of squared soft masses is driven to negative values in the infrared. In this section we present several examples in which the breaking of the extra $U(1)'$ can take place naturally at an intermediate scale and examine the range of the scale μ_{rad} .

For the sake of concreteness, we consider three models in which one or both of the singlets couples to exotic superfields in the renormalizable superpotential:

Model (I): \hat{S}_1 couples to exotic $SU(3)$ triplets \hat{D}_1, \hat{D}_2 in the superpotential

$$W = h \hat{D}_1 \hat{D}_2 \hat{S}_1. \quad (16)$$

Model (II): \hat{S}_1 couples to exotic $SU(3)$ triplets \hat{D}_1, \hat{D}_2 and \hat{S}_2 to exotic $SU(2)$ doublets \hat{L}_1, \hat{L}_2 in the superpotential

$$W = h_D \hat{D}_1 \hat{D}_2 \hat{S}_1 + h_L \hat{L}_1 \hat{L}_2 \hat{S}_2. \quad (17)$$

Model (III): \hat{S}_1 couples to N_p identical pairs of MSSM singlets, charged under $U(1)'$, in the superpotential

$$W = h \sum_{i=1}^{N_p} \hat{S}_{ai} \hat{S}_{bi} \hat{S}_1. \quad (18)$$

We have analyzed the renormalization group equations (RGEs) of each model to determine the range of μ_{rad} as a function of the values of the parameters at the string scale. In principle, we could consider other models, such as a variation of model (II) in which the same singlet couples to the exotic triplets and doublets through $W = h_D \hat{D}_1 \hat{D}_2 \hat{S}_1 + h_L \hat{L}_1 \hat{L}_2 \hat{S}_1$, or a variation of model (III) in which the singlet couples to additional singlets that are not a set of N_p identical pairs through $W = \sum_{i,j} C_{i,j} \hat{S}_i \hat{S}_j \hat{S}_1$. For simplicity, we restrict our consideration to these three models, because they can be analyzed analytically.⁹

We assume gauge coupling unification at M_{string} , such that

$$g_3^0 = g_2^0 = g_1^0 = g_1'^0 = g_0, \quad (19)$$

which is approximately consistent with the observed gauge coupling unification.¹⁰ At the one-loop level, the singlets in model (III) do not affect the gauge coupling unification of the MSSM. Model (II) is also consistent with gauge coupling unification if the D_i, L_i are approximately degenerate in mass, because they have the appropriate quantum numbers to fit into multiplets of $SU(5)$. However, the presence of exotic

⁹Even simpler analytic examples, neglecting trilinear A terms, gaugino masses, and the running of the Yukawas, are given in the Appendix of [9].

¹⁰We assume a grand unified theory (GUT) normalization for the Abelian gauge couplings, such that $g_1 = \sqrt{k} g_Y$, where g_Y is the coupling usually called g' in the standard model and $k = \frac{5}{3}$. In general, the string models considered could have $k \neq \frac{5}{3}$.

TABLE I. Model (I). Singlet coupled to triplets: $W = h\hat{D}_1\hat{D}_2\hat{S}_1$.

Q_1, Q_2	A^0/m_0	$M_{1/2}/m_0$	μ_{rad} (GeV)	Q_1, Q_2	A^0/m_0	$M_{1/2}/m_0$	μ_{rad} (GeV)
-1,1	1.0	1.0	2.7×10^{10}	-1,1	0.3	0.3	2.0×10^3
-1,1	1.0	0.1	-	$-\frac{5}{4}, \frac{3}{4}$	1.0	1.0	3.3×10^8
-1,1	3.0	1.0	1.5×10^{15}	$-\frac{5}{4}, \frac{1}{3}$	1.0	1.0	2.1×10^4
-1,1	3.0	0.1	1.1×10^{15}	$-1, \frac{1}{3}$	1.0	1.0	2.3×10^5
-1,1	5.0	1.0	8.8×10^{16}	$-1, \frac{1}{4}$	1.0	1.0	2.1×10^3
-1,1	1.0	0.5	2.6×10^7	$-\frac{1}{2}, 1$	1.0	0.1	6.0×10^8
-1,1	0.5	0.5	2.4×10^6	$-\frac{1}{3}, 1$	1.0	0.1	1.5×10^{11}
-1,1	0.7	0.7	3.2×10^8	$-\frac{1}{3}, \frac{1}{2}$	1.0	0.1	2.3×10^5

triplets not part of an $SU(5)$ multiplet violates the gauge coupling unification in model (I). This problem can be resolved if there are other exotics which do not couple to S_1 but contribute to the running of the gauge couplings [e.g., additional $SU(2)$ doublets; i.e., model (I) is a limiting case of model (II) as h_L goes to zero]. The additional exotics will generally have electroweak scale masses so they will not precisely cancel the effects of the triplets except by accident. However, model (I) is still useful to illustrate the basic ideas.

For the sake of simplicity, we assume that the boundary conditions for the Yukawa couplings are given by

$$h^0 = g_0 \sqrt{2}, \quad (20)$$

as calculated in string models based on fermionic ($Z_2 \times Z_2$) orbifold constructions at a special point in moduli space.¹¹ Thus, the analysis presented below relies on large Yukawa couplings to exotic fields, which are a generic feature of a class of string models considered. However, the specific choice of exotic couplings in Eqs. (16), (17), and (18) is chosen for concreteness in order to illustrate different symmetry breaking scenarios.

In the analysis, we assume unification of gaugino masses at M_{string} ,

$$M_3^0 = M_2^0 = M_1^0 = M_1'^0 = M_{1/2}, \quad (21)$$

and universal¹² scalar soft mass-squared parameters

$$m_i^{02} = m_0^2. \quad (22)$$

¹¹An overall normalization factor of $g_0 \sqrt{2}$ at the string scale is required if the three-gauge-boson coupling is to be g_0 . In this class of string models, cubic couplings in a superpotential can contain additional factors of $(1/\sqrt{2})^n$, with $n \in \{0, 1, 2, 3\}$. The power n corresponds to the number of Ising fermion oscillator excitations paired with $\sigma_+ \sigma_-$ factors (i.e., sets of order-disorder operators) present in the product of vertex operators associated with the multiplets in the superpotential term.

¹²We do not consider nonuniversal soft mass-squared parameters, because it is possible to explore the range of μ_{rad} without this additional complication.

The first and third models have only one trilinear coupling, with initial value A^0 . We do consider the possibility of non-universal trilinear couplings A_D^0, A_L^0 in the analysis of the second model.

The RGEs of the models (16)–(18) are presented in a general form in the Appendix.¹³ We have solved the RGEs in each case for a range of boundary conditions to determine the range of μ_{rad} . Each of the models considered has the advantage that it is possible to obtain exact analytical solutions to the RGEs, which yield insight into the nature of the dependence of the parameters on their initial values. Exact solutions [19] are possible in these models because the RGEs for the Yukawa couplings are decoupled. In more complicated cases, e.g., if the same singlet couples to both triplets and doublets, no simple exact solutions exist. It is also useful to consider simpler semi-analytic solutions to the RGEs, in which the running of the gauge couplings and gaugino masses is neglected in the solutions of the RGEs of the other parameters. The exact and semi-analytic solutions are presented in the Appendix. The results of the renormalization group analysis are presented in Tables I–III for models (I)–(III), respectively. The evolution of the parameters of model (I) is shown in some representative graphs.

Model (I): In Table I, we present the results of the analysis of model (I). We first choose the $U(1)'$ charge assignment $Q_1 = -Q_2 = -1$ for the singlets \hat{S}_1 and \hat{S}_2 and investigate the nature of μ_{rad} as a function of the initial values of the dimensionless ratios A^0/m_0 and $M_{1/2}/m_0$. The scale dependence of the Yukawa coupling¹⁴ and the trilinear coupling are shown in Fig. 1. With this choice of $U(1)'$ charges and $A^0 = m_0$, the breaking scale is of the order 10^{10} GeV for values of $M_{1/2} = \mathcal{O}(m_0)$. However, radiative breaking (along the D -flat direction) is not achieved for small values of the initial gaugino masses, as is also shown in Fig. 2(a). The gaugino mass parameter $M_{1/2}$ governs the fixed point behavior of the soft mass-squared parameters (as was also found in

¹³The running of the $U(1)'$ gauge coupling depends on the charge assignments of all of the fields in the theory, and so is highly model dependent. For simplicity, we assume that the $U(1)'$ charge assignments are such that the evolution of g_1' is identical to that of g_1 .

¹⁴The evolution of the Yukawa coupling for large initial values demonstrates the fixed point behavior, as discussed in the Appendix.

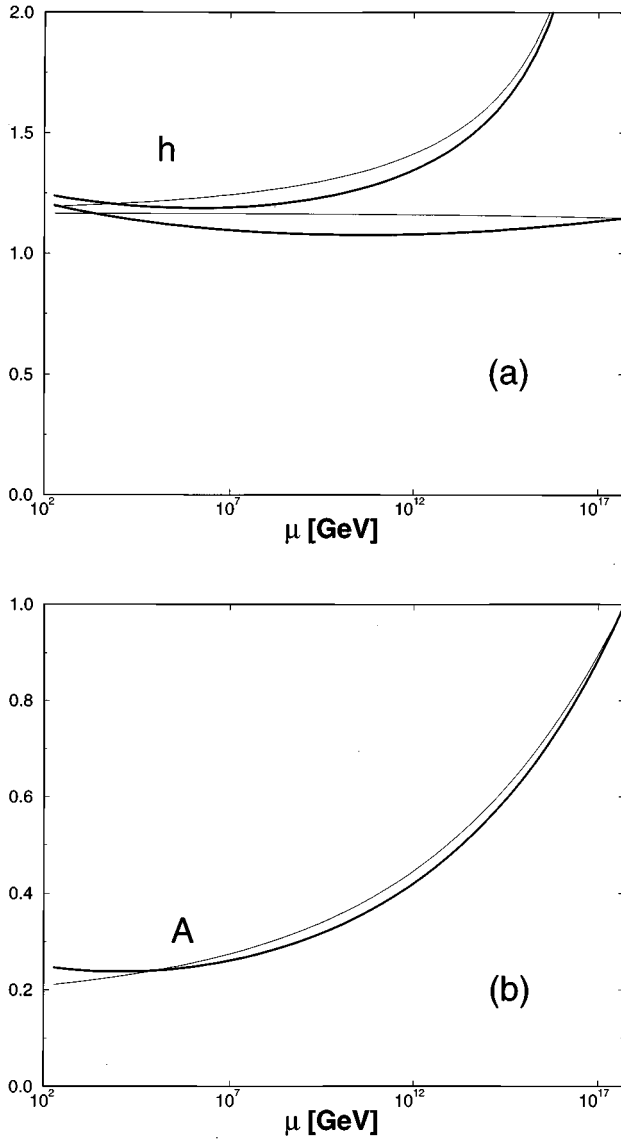


FIG. 1. (a) Scale dependence of the Yukawa coupling of model (I) for $h^0 = g_0\sqrt{2}$ and $h^0 = 10$. (b) Scale dependence of the trilinear coupling of model (I) in units of m_0 , with $M_{1/2} = 0.1m_0$. In each case $Q_1 = -Q_2 = -1$. Bold curves are for exact solutions, and light curves represent semi-analytic approximations.

[10]), such that small gaugino masses do not drive m_1^2 sufficiently negative to overcome the fact that m_2^2 does not run significantly because it does not have any couplings in the superpotential. S_1 will acquire an electroweak scale VEV in this case, as was described in Sec. I. Increasing the value of A^0 increases μ_{rad} dramatically (up to 10^{17} GeV), for it drives m_1^2 negative at a higher scale; this behavior is also shown for the case of $A^0/m_0 = 3.0$, $M_{1/2}/m_0 = 0.1$ in Fig. 2(b). The breaking scale decreases significantly (in some cases, all the way to the TeV range) when both A^0/m_0 and $M_{1/2}/m_0$ are lowered simultaneously. This is to be expected, for this is equivalent to raising the initial value of the soft mass-squared parameters and keeping $A^0 = M_{1/2}$, in which case m_1^2 is driven negative at a lower scale.

For a given set of boundary conditions, it is also possible to raise or lower μ_{rad} by choosing different values of the

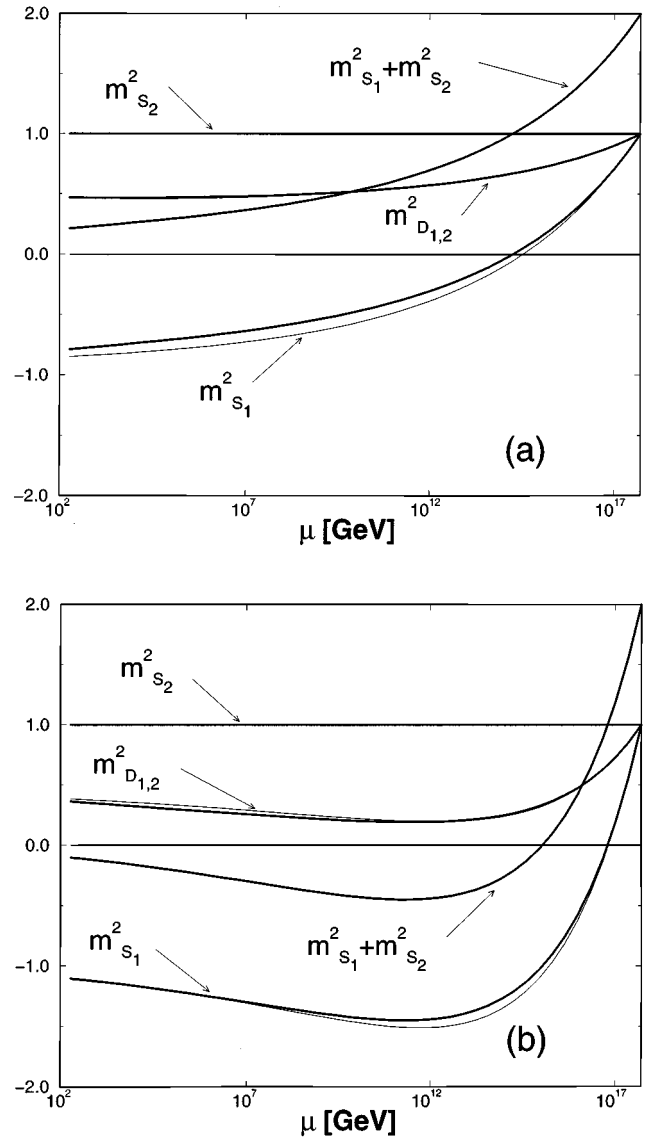


FIG. 2. (a) Scale dependence of the soft mass-squared parameters of model (I) in units of m_0^2 , with $A_0 = m_0$ and $M_{1/2} = 0.1m_0$. (b) Same, except $A_0 = 3m_0$ and $M_{1/2} = 0.1m_0$. In each case $Q_1 = -Q_2 = -1$. Bold curves are for exact solutions, and light curves represent semi-analytic approximations.

ratio of $|Q_1/Q_2|$, as can be seen from Eq. (5). In particular, $|Q_1/Q_2| > 1$ will increase the relative weight of m_2^2 and so decrease μ_{rad} , while $|Q_1/Q_2| < 1$ will increase the relative weight of m_1^2 and thus increase μ_{rad} . Several examples of this type are presented in Table I. The values of μ_{rad} for the examples with $M_{1/2}/m_0 = 1.0$ should be contrasted with the value of 10^{10} GeV obtained with $Q_1 = -Q_2 = -1$, and the values with $M_{1/2}/m_0 = 0.1$ should be compared with the result that radiative breaking does not occur in the case with equal and opposite $U(1)'$ charges.

Model (II): The results of the analysis of model (II) are presented in Table II. In this case, both m_1^2 and m_2^2 are driven negative due to the large Yukawa couplings in the superpotential. Thus, μ_{rad} is generally much higher than in the case of the model previously discussed, of order $10^{13} - 10^{17}$ GeV for $Q_1 = -Q_2 = -1$. The breaking scale increases with larger

TABLE II. Model (II). Singlets coupled to triplets and doublets: $W = h_D \hat{D}_1 \hat{D}_2 \hat{S}_1 + h_L \hat{L}_1 \hat{L}_2 \hat{S}_2$.

Q_1, Q_2	$A_D^0/m_0, A_L^0/m_0$	$M_{1/2}/m_0$	μ_{rad} (GeV)	Q_1, Q_2	$A_D^0/m_0, A_L^0/m_0$	$M_{1/2}/m_0$	μ_{rad} (GeV)
-1,1	1.0,1.0	1.0	1.0×10^{14}	-1,1	1.0,3.0	0.1	4.4×10^{15}
-1,1	1.0,1.0	0.1	1.1×10^{13}	-1,1	3.0,3.0	1.0	4.0×10^{16}
-1,1	3.0,1.0	1.0	1.1×10^{16}	-1,1	5.0,5.0	1.0	1.9×10^{17}
-1,1	3.0,1.0	0.1	1.1×10^{16}	-1,1	0.3,0.3	0.3	1.9×10^{12}
-1,1	1.0,3.0	1.0	4.4×10^{15}	-1,1	0.1,0.1	0.1	7.4×10^{11}

values of $M_{1/2}$ and A^0 , and the effects of the gaugino masses are negligible for sufficiently large values of A^0/m_0 . The breaking scale can be lowered to the range of 10^{11} GeV by decreasing the values of A^0/m_0 and $M_{1/2}/m_0$. In this model, changing the value of $|Q_1/Q_2|$ does not have a significant effect on μ_{rad} , as the soft mass-squared parameters of both singlets are driven to negative values.

Model (III): In Table III, we present the results of the analysis of model (III), in which S_1 couples to identical pairs of singlets charged under the $U(1)'$, and S_2 has no couplings in the renormalizable superpotential. In this case, the number of pairs of singlets is analogous to the group theoretical weight in the RGEs, such that m_1^2 is driven negative at some scale. While $N_p = 3$ gives the same weight as that of the first model with exotic triplets, the values of μ_{rad} shown in the first two entries of Table III demonstrate that this model does not mimic the first model. For example, the results show that to obtain radiative breaking for $A^0/m_0 = 1.0$, it is necessary to take large values of N_p (such as $N_p = 7$ for $\mu_{\text{rad}} \sim 10^4$ GeV). This is due to the fact that model (III) does not have the $SU(3)$ coupling, and so all of the parameters have a smaller gauge contribution. In particular, the Yukawa coupling is weaker in model (III), and so m_1^2 is not driven to negative values as quickly as in model (I). This model also differs from the previous models in that smaller values of $M_{1/2}$ yield larger values of μ_{rad} , often by many orders of magnitude. Increasing the value of A^0 raises the breaking scale dramatically even for small values of N_p , eventually dominating the effects of the gaugino masses.

Several examples are also presented in Table III in which the breaking scale is modified by choosing different values of $|Q_1/Q_2|$ for a given set of boundary conditions. As in the first model, the scale can be raised significantly (e.g., to

10^5 GeV from the case of no solution) by assigning charges such that $|Q_1/Q_2| < 1$. The value of μ_{rad} can also be lowered substantially (e.g., to 10^4 GeV from 10^{11} GeV) by choosing $|Q_1/Q_2| > 1$.

The results of this analysis demonstrate that within models in which only one of the singlets couples to exotic matter in the renormalizable superpotential [such as model (I) and model (III)], there is a broad range of values of the breaking scale μ_{rad} , from the TeV range up to around 10^{16} GeV. In many cases there is no D -flat solution, so that the $U(1)'$ breaking will be at the electroweak scale. While the non-renormalizable terms will be important if μ_{rad} is sufficiently high, in many cases the scale of radiative breaking will determine the VEV of S . If both singlets have trilinear couplings in the superpotential [such as in model (II)], the breaking is strongly radiative, such that non-renormalizable terms will dominate the symmetry breaking.

IV. INTERMEDIATE SCALE DUE TO NON-RENORMALIZABLE TERMS IN STRING MODELS

The scenarios discussed in a general particle physics context in the previous sections have interesting implications for string models. In particular, in a large class of string models the particle spectrum consists of SM singlets S_i whose (particular combination) ensures that they correspond to D -flat directions and F -flat directions at least for the renormalizable terms in the superpotential. On the other hand, it is often the case that these fields do have non-renormalizable terms in the superpotential, which along with the radiatively induced negative mass-squared terms yield intermediate scales with implications for the SM sector of the theory.

For specific examples we shall concentrate on the type of

TABLE III. Model (III). Singlet coupled to singlet pairs: $W = h \sum_{i=1}^{N_p} \hat{S}_{ai} \hat{S}_{bi} \hat{S}_1$.

Q_1, Q_2, N_p	A^0/m_0	$M_{1/2}/m_0$	μ_{rad} (GeV)	Q_1, Q_2, N_p	A^0/m_0	$M_{1/2}/m_0$	μ_{rad} (GeV)
-1,1,3	1.0	1.0	-	-1,1,8	1.5	0.1	3.9×10^{11}
-1,1,3	1.0	0.1	-	-1,1,10	1.5	1.0	8.7×10^{11}
-1,1,7	1.0	1.0	-	-1,1,3	3.0	1.0	7.9×10^{12}
-1,1,7	1.0	0.1	3.4×10^4	-1,1,3	3.0	0.1	3.8×10^{13}
-1,1,8	1.0	1.0	-	-1,1,4	3.0	1.0	4.8×10^{14}
-1,1,8	1.0	0.1	3.5×10^7	-1,1,4	3.0	0.1	4.2×10^{15}
-1,1,10	1.0	1.0	1.4×10^7	$-\frac{1}{3}, 1, 3$	1.0	0.1	6.4×10^4
-1,1,10	1.0	0.1	1.1×10^{10}	$-\frac{1}{2}, 1, 3$	1.5	0.1	1.0×10^5
-1,1,3	1.5	1.0	-	$-1, \frac{4}{5}, 8$	1.5	0.1	4.4×10^4
-1,1,8	1.5	1.0	3.2×10^8	$-1, \frac{2}{3}, 4$	3.0	0.1	3.6×10^{12}

non-renormalizable terms in a class of fermionic constructions. In such models, there are a number of SM singlets S_i which are in general charged under additional $U(1)'$ factors. The D flatness is ensured if the $U(1)'$ charges of at least two S_i 's have opposite signs. For the sake of concreteness we confine ourselves to the case of two S_i 's, with the D -flatness constraint satisfying Eq. (1). Since the S_i are massless states at M_{string} , they have no bilinear terms in the superpotential. We also require that in the superpotential the trilinear self-couplings of S_i and the trilinear terms of one S_i to the MSSM particle content be absent as well. This is often the case due to either (world-sheet) selection rules (as demonstrated below) and/or target space gauge coupling unification.

The analysis of the previous section has shown that couplings to exotic particles with a Yukawa coupling of $\mathcal{O}(1)$ can ensure a radiative breaking for S_i 's. On the other hand, in general there are non-renormalizable self-couplings of \hat{S}_i 's in the superpotential. It is convenient to rewrite Eq. (8) as¹⁵

$$W_K = \hat{S}^3 \left(\frac{\hat{S}}{M} \right)^K, \tag{23}$$

where we have absorbed the coefficient α_K in the definition of mass scale M . [M is related to \mathcal{M} in Eq. (9) as $\mathcal{M} = C_K M$.] For simplicity we have not displayed the dependence of M on K or the detailed form of the operators.

If p_1 and p_2 are the unique relative primes defined by $p_2/p_1 = |Q_1|/|Q_2|$, then, as discussed in Sec. II, $U(1)'$ invariance permits values of $K > 0$ in Eq. (23) such that $3 + K = (p_1 + p_2)n$, where n is an integer. World-sheet selection rules further constrain K through restrictions on n [20–24]. For example, in the free fermionic construction n must be an even integer in the case of only two S_i , thus limiting K to only odd values, independent of the values of the p_i .

Fermionic world-sheet selection rules further require that both singlets S_i must originate from twisted world-sheet supersymmetric (i.e., Ramond) sectors of a model for *any* non-renormalizable terms of the form (23) to appear in the superpotential. In contrast, for a renormalizable trilinear self-coupling \hat{S}^3 term to appear, one of the two S_i must have its origin in the untwisted Neveu-Schwarz sector while the other comes from a Ramond sector [22,23]. Thus, renormalizable ($K=0$) and non-renormalizable ($K>0$) terms of the form (23) are mutually exclusive.

The coefficients of the non-renormalizable couplings can be calculated in a large class of string models. For the free fermionic construction, coefficient values can be cast in terms of the $(K+3)$ -point string amplitude, A_{K+3} , in the following form:¹⁶

$$\left(\frac{1}{M} \right)^K \equiv \left(\frac{\alpha_K}{M_{Pl}} \right)^K, \tag{24}$$

¹⁵In these terms we have already chosen the D -flatness constraint, and thus the non-renormalizable self-coupling is expressed in terms of the S field [defined in Eq. (1)] only.

¹⁶For the explicit calculation of the non-renormalizable terms in a class of fermionic models, see [23,25].

$$= (2\alpha')^{K/2} A_{K+3} \tag{25}$$

$$= (2\alpha')^{K/2} \left(\frac{g}{2\pi} \right)^K g \eta C_K I_K \tag{26}$$

$$= M_{Pl}^{-K} \left(\frac{4}{\sqrt{\pi}} \right)^K g \eta C_K I_K \tag{27}$$

where g is the gauge coupling at M_{string} , $\eta = \sqrt{2}$ is a normalization factor (defined so that the three-gauge-boson and two-fermion—one-gauge-boson couplings are simply g), $2\alpha' \equiv (64\pi)/(M_{Pl}^2 g^2)$ is the string tension [1], C_K is the coefficient of $\mathcal{O}(1)$ that encompasses different renormalization factors in the operator product expansion (OPE) of the string vertex operators (including the target space gauge group Clebsch-Gordon coefficients), and $\alpha_K \equiv (4/\sqrt{\pi})^K g \eta C_K I_K$. I_K is a world-sheet integral of the type

$$I_K = \int d^2z_3 \cdots d^2z_{K+2} \times f_K(z_1 = \infty, z_2 = 1, z_3, \dots, z_{K+2}, z_{K+3} = 0), \tag{28}$$

where z_i is the world-sheet coordinate of the vertex operator of the i^{th} string state. As a function of the world-sheet coordinates, f_K is a product of correlation functions formed respectively from the spacetime kinematics, Lorentz symmetry, ghost charge, local non-Abelian symmetries, local and global $U(1)$ symmetries, and (non-)chiral Ising model factors in each of the vertex operators for the $3+K$ fields. All correlators but the Lorentz and Ising ones are of exponential form. For non-Abelian symmetries and for $U(1)$ symmetries and ghost systems these exponential correlators have the respective generic forms

$$\left\langle \prod_i e^{i\vec{Q}_i \cdot \vec{J}} \right\rangle = \prod_{i < j} z_{ij}^{\vec{Q}_i \cdot \vec{Q}_j} \quad \text{and} \quad \left\langle \prod_i e^{iQ_i H} \right\rangle = \prod_{i < j} z_{ij}^{Q_i Q_j} \tag{29}$$

where $z_{ij} = z_i - z_j$. In this language, Q_i is imaginary for ghost systems.

While the Lorentz correlator is non-exponential, it is nevertheless trivial and contributes a simple factor of $z_{12}^{-1/2}$ to f_K . On the other hand, the various Ising correlators are generically non-trivial. This makes I_K difficult to compute. In fact, Ising correlators generally prevent a closed form expression for an integral I_K [23]. Nevertheless, Ising fermions may be necessary in fermionic models for obtaining realistic gauge groups and (quasi-)realistic phenomenology [23,7]. Thus, although the Ising correlation functions make I_K increasingly difficult to compute as K grows in value, Ising correlation functions generally enter string amplitudes.

From [23,25] we infer that $I_1 \sim 70$ and $I_2 \sim 400$. In [23,25], the non-renormalizable terms for which I_1 and I_2 were calculated involved only one and two MSSM singlets, respectively. However, we do not expect that the values of $I_1^{singlets}$ or $I_2^{singlets}$ associated with terms composed totally of S -type singlets will generically vary significantly from the values obtained when some non-singlets are involved.

For a $K=1$ term composed solely of non-Abelian singlets¹⁷ carrying $U(1)'$ charge [6], we have explicitly calculated a value for $I_1^{singlets}$. For comparative purposes we relate our $I_1^{singlets}$ to the associated four-point string amplitude $A_4^{singlets}$ via the normalization

$$A_4^{singlets} = \frac{g}{2\pi} \frac{1}{4} I_1^{singlets}. \quad (30)$$

This is the same normalization as in [25], where the value of I_1 was 77.7. The four singlet case produces

$$I_1^{singlets} = 2\sqrt{2} \int d^2z |z|^{-1} |1-z|^{-3/2}. \quad (31)$$

By shifting $z \rightarrow z+1$ and converting the world-sheet coordinate z to polar coordinates (r, θ) , the integral can be expressed as

$$I_1^{singlets} = 4\sqrt{2} \int_0^\infty dr \int_0^\pi d\theta \frac{1}{1+r^2-2r \cos \theta}. \quad (32)$$

Integrating over the angle θ results in

$$I_1^{singlets} = 4\sqrt{2} \int_0^\infty dr \frac{1}{r} \frac{2\sqrt{r}}{r+1} K\left(\frac{2\sqrt{r}}{r+1}\right) \quad (33)$$

$$= 8\sqrt{2} \int_0^\infty dl \frac{2}{l^2+1} K\left(\frac{2l}{l^2+1}\right), \quad (34)$$

where K is the complete elliptic integral of the first kind.

Numerical approximation of Eq. (32) (after splitting integration over r into two separate regions $0 \leq r \leq 1$ and $0 \leq r \leq \infty$) via Mathematica yields a value of $I_1^{singlets} = 63.7$. As a test of the numerical approximation, we can also expand K in powers of $2\sqrt{r}/(r+1)$ [or in powers of $2l/(l^2+1)$] and then integrate the first two (or more) terms in this series. This latter approach yields (for two terms) $I_1^{singlets} \approx (9/\sqrt{2})\pi^2 \approx 62.8 \pm 10\%$, in excellent agreement with our numerical approximation.¹⁸ Thus the non-singlet factor in the four-point string amplitude of [25] causes I_1 to be about 20% larger than $I_1^{singlets}$.

¹⁷The four states forming this $K=1$ superpotential term were denoted H_{30} , H_{32} , H_{37} , and H_{39} in Table 2 of [6]. The first two of these states originate in one sector of the model, while the latter two reside in a second sector. This is the general pattern also followed in [23,25].

¹⁸ $x \equiv 2\sqrt{r}/(r+1)$ is within the range of convergence $0 \leq x < 1$ of the series expansion,

$$K(x) = \frac{\pi}{2} \left\{ 1 + \left(\frac{1}{2}\right)x + \left(\frac{1 \times 3}{2 \times 4}\right)^2 x^2 + \dots \right\}, \quad (35)$$

for all values of r except for $r=1$. At $r=1$, x reaches the end point of convergence, $x=1$, for which $\lim_{x \rightarrow 1} K(x) \rightarrow \infty$. As consistency between our two estimates of $I_1^{singlets}$ indicates, inclusion of this end point in the range of integration of the series expansion still permits using the series expansion.

It is expected that the interference terms in I_K are generically such that $I_K < I_1^K$, and thus $M > M_1$. In particular, for $K=1$ we obtain $M_1 \sim 3 \times 10^{17}$ GeV using $I_1 \sim 70$ and, for $K=2$, $M_2 \sim 7 \times 10^{17}$ GeV using $I_2 \sim 400$.

V. NON-RENORMALIZABLE COUPLING TO THE MSSM PARTICLES

The flat direction S can have a set of non-renormalizable couplings to MSSM states that offer solutions to the μ problem [26] and yield mass hierarchies between generations [28]. The non-renormalizable μ -generating terms are of the form

$$W_\mu \sim \hat{H}_1 \hat{H}_2 \hat{S} \left(\frac{\hat{S}}{M}\right)^P. \quad (36)$$

In addition, the effective soft SUSY-breaking B term, $BH_1H_2 + \text{H.c.}$ in the Higgs potential, which is necessary for a correct electroweak symmetry breaking, can appear via mixed F terms from a superpotential¹⁹

$$W_B \sim \hat{H}_1 \hat{H}_2 \hat{S} \left(\frac{\hat{S}}{M}\right)^P + \hat{S}^3 \left(\frac{\hat{S}}{M}\right)^K, \quad (37)$$

or from supersymmetry breaking terms [27] in the potential of the type

$$V \sim AH_1H_2S \left(\frac{S}{M}\right)^P + \text{H.c.}, \quad (38)$$

where $A \sim m_{soft}$. In both cases, when the effective μ parameter is of the order of the electroweak scale, $B \sim m_{soft}^2$ automatically.

Generational up, down, and electron mass terms appear, respectively, via

$$W_{u_i} \sim \hat{H}_2 \hat{Q}_i \hat{U}_i^c \left(\frac{\hat{S}}{M}\right)^{P'_{u_i}}, \quad W_{d_i} \sim \hat{H}_1 \hat{Q}_i \hat{D}_i^c \left(\frac{\hat{S}}{M}\right)^{P'_{d_i}},$$

$$W_{e_i} \sim \hat{H}_1 \hat{L}_i \hat{E}_i^c \left(\frac{\hat{S}}{M}\right)^{P'_{e_i}}, \quad (39)$$

with i denoting the generation number.²⁰

Majorana and Dirac neutrino terms may also be present via

¹⁹Although the values of the M in the two terms of Eq. (37) are expected to be of the same order of magnitude, they may vary somewhat. For simplicity, we ignore the distinction.

²⁰Alternatively, non-renormalizable chiral supermultiplet mass terms can be generated through anomalous $U(1)'$ breaking [29,30]. Typically, in that case the analogue of $\langle s \rangle / M \sim 1/10$, so that larger values of P' are required.

TABLE IV. Fermion mass ratios with the top quark mass normalized to 1. The values of u -, d -, and s -quark masses used in the ratios (with the t -quark mass normalized to 1 from an assumed mass of 170 GeV) are estimates of the modified minimal subtraction (MS) scheme current-quark masses at a scale $\mu \approx 1$ GeV. The c - and b -quark masses are pole masses. An additional mass constraint for stable light neutrinos is $\sum_i m_{\nu_i} \leq 6 \times 10^{-11}$ (i.e., 10 eV), based on the neutrino contributions to the mass density of the universe and the growth of structure [36].

m_u	:	m_c	:	m_t	=	3×10^{-5}	:	7×10^{-3}	:	1
m_d	:	m_s	:	m_b	=	6×10^{-5}	:	1×10^{-3}	:	3×10^{-2}
m_e	:	m_μ	:	m_τ	=	0.3×10^{-5}	:	0.6×10^{-3}	:	1×10^{-2}
m_{ν_e}	:	m_{ν_μ}	:	m_{ν_τ}	=	$< 6 \times 10^{-11}$:	$< 1 \times 10^{-6}$:	$< 1 \times 10^{-4}$

$$W_{L_i L_i}^{(\text{Maj})} \sim \frac{(\hat{H}_2 \hat{L}_i)^2}{M} \left(\frac{\hat{S}}{M} \right)^{P''_{L_i L_i}}, \quad W_{L_i \nu_i^c}^{(\text{Dir})} \sim \hat{H}_2 \hat{L}_i \hat{\nu}_i^c \left(\frac{\hat{S}}{M} \right)^{P'_{L_i \nu_i^c}},$$

$$W_{\nu_i^c \nu_i^c}^{(\text{Maj})} \sim \hat{\nu}_i^c \hat{\nu}_i^c \hat{S} \left(\frac{\hat{S}}{M} \right)^{\bar{P}_{\nu_i^c \nu_i^c}}. \quad (40)$$

($\hat{\nu} \in \hat{L}$ represents the neutrino doublet component and we have introduced neutrino singlets $\hat{\nu}^c$.)

When the VEV $\langle S \rangle$ is fixed solely by the running of m^2 , the size of the μ parameter will be determined by the scale μ_{rad} and the value of P in Eq. (36), $\mu_{eff} \sim \mu_{rad}^{P+1}/M^P$. For example, for $P=1$ a reasonable $\mu_{eff} \sim 1$ TeV would correspond to $\mu_{rad} \sim 10^{10}$ GeV. On the other hand, concrete order of magnitude estimates can be made when the VEV is fixed by non-renormalizable self-interactions of S . Generally, if $\mu_{rad} \leq 10^{12}$ GeV, running is the dominant factor, whereas, if $\mu_{rad} \geq 10^{12}$ GeV, the non-renormalizable operators (NROs) dominate instead. With NRO-dominated $\langle S \rangle \sim (m_{soft} M^K)^{1/(K+1)}$, the effective Higgs μ term takes the form

$$\mu_{eff} \sim m_{soft} \left(\frac{m_{soft}}{M} \right)^{(P-K)/(K+1)}. \quad (41)$$

The phenomenologically preferred choice among such terms is clearly $P=K$, yielding a K -independent $\mu_{eff} \sim m_{soft}$.

Both of these intermediate scale scenarios are to be contrasted to the case in which $\langle S \rangle$ is at the electroweak scale. Then, $\mu_{eff} \sim m_{soft}$ can be generated by a renormalizable ($P=0$) term [10].

Quark and lepton masses can have hierarchical patterns generated through

$$m_{u_i} \sim \langle H_2 \rangle \left(\frac{m_{soft}}{M} \right)^{P'_{u_i}/(K+1)}, \quad m_{d_i} \sim \langle H_1 \rangle \left(\frac{m_{soft}}{M} \right)^{P'_{d_i}/(K+1)},$$

$$m_{e_i} \sim \langle H_1 \rangle \left(\frac{m_{soft}}{M} \right)^{P'_{e_i}/(K+1)}. \quad (42)$$

In Eqs. (42) we ignore the running of the effective Yukawas below $\langle S \rangle$ (or below M_{Pl} for m_i) because such effects are small compared to the uncertainties in M .

Comparison of the physical fermion mass ratios [33] in Table IV with theoretical K and P dependent mass values in Table V suggests that the set

$$P'_1 \equiv P'_{u_1} = P'_{d_1} = P'_{e_1} = 2,$$

$$P'_2 \equiv P'_{u_2} = P'_{d_2} = P'_{e_2} = 1, \quad (43)$$

when used in tandem with $K=5$ or $K=6$, could produce a fairly realistic hierarchy for the first two generations in the

TABLE V. Non-renormalizable MSSM mass terms via $\langle S \rangle$. For $m_{soft} \sim 100$ GeV, $M \sim 3 \times 10^{17}$ GeV.

P or P'	$K=1$	$K=2$	$K=3$	$K=4$	$K=5$	$K=6$	$K=7$
$\left(\frac{m_{soft}}{M} \right)^{1/(K+1)}$	2×10^{-8}	7×10^{-6}	1×10^{-4}	8×10^{-4}	3×10^{-3}	6×10^{-3}	1×10^{-2}
$\langle S \rangle$ (GeV)	$K-1$	5×10^9	2×10^{12}	4×10^{13}	2×10^{14}	8×10^{14}	2×10^{15}
		5×10^7	1×10^5	7×10^3	1×10^3	400	200
$\frac{\mu_{eff}}{m_{soft}}$	K	1	1	1	1	1	1
	$K+1$	2×10^{-8}	7×10^{-6}	1×10^{-4}	8×10^{-4}	3×10^{-3}	6×10^{-3}
	0	1	1	1	1	1	1
	1	2×10^{-8}	7×10^{-6}	1×10^{-4}	8×10^{-4}	3×10^{-3}	6×10^{-3}
	2	3×10^{-16}	5×10^{-11}	2×10^{-8}	6×10^{-7}	7×10^{-6}	4×10^{-5}
$\frac{m_{QL}}{\langle H_i \rangle}$	3	6×10^{-24}	3×10^{-16}	2×10^{-12}	5×10^{-10}	2×10^{-8}	2×10^{-7}
	4	1×10^{-31}	2×10^{-21}	3×10^{-16}	4×10^{-13}	5×10^{-11}	1×10^{-9}
	5	2×10^{-39}	2×10^{-26}	5×10^{-20}	3×10^{-16}	1×10^{-13}	9×10^{-12}
							2×10^{-8}
							2×10^{-10}

tan $\beta \equiv \langle H_2 \rangle / \langle H_1 \rangle \sim 1$ limit.²¹ Alternatively, taking the tan $\beta \sim 50$ limit would suggest slightly higher values for K (while keeping the same set of P' values).

Presumably m_t is associated with a renormalizable coupling ($P'_{u_3} = 0$). The other third family masses do not fit quite as well: They are too small to be associated with renormalizable couplings, but somewhat larger than is expected for $P'_{d_3} = P'_{e_3} = 1$ for $K=5$ or $K=6$. However, given the roughness of the estimates and the simplicity of the model, the overall pattern of the masses is quite encouraging. It is also possible that m_b and m_τ are associated with some other mechanism, such as non-renormalizable operators involving the VEV of an entirely different singlet.

There is an obvious constraint on a string model that could produce a generational mass hierarchy along these lines, containing $P'_1 - 1 = P'_2 = P'_{u_3} + 1 = 1$ fermion mass terms, in tandem with a $P=K=5$ or 6 μ term. A combination of world-sheet selection rules and $U(1)'$ charges must prevent μ -generating terms with $P < 5$ from appearing, while allowing the low order P'_i fermion mass terms. If $U(1)'$ charges could be assigned by fiat to each state, then the $U(1)'$ symmetry should be able to accomplish this by itself. However, $U(1)'$ charge assignments are related to modular invariance and thus they cannot be freely chosen for many states. World-sheet selection rules must likely play a role in constraining P .

The neutrino mass terms in Eqs. (40) offer various possibilities²² for achieving small neutrino masses [36], some not involving a traditional seesaw mechanism [37]. Very light non-seesaw doublet neutrino Majorana masses are possible via $W_{L_i L_i}^{(\text{Maj})}$ of the form

$$m_{L_i L_i} \sim \frac{\langle H_2 \rangle^2}{M} \left(\frac{m_{\text{soft}}}{M} \right)^{P''_{L_i L_i} / (K+1)} \sim \langle H_2 \rangle \left(\frac{m_{\text{soft}}}{M} \right) \times \left(\frac{m_{\text{soft}}}{M} \right)^{P''_{L_i L_i} / (K+1)} \ll 1 \text{ eV.} \quad (44)$$

The upper bound on neutrino masses from this term (i.e., the case of $P''_{L_i L_i} = 0$) is around 10^{-4} eV (using $\langle H_2 \rangle \sim m_{\text{soft}} = 100$ GeV and $M = 3 \times 10^{17}$ GeV), which is too small to be relevant to dark matter or Mikheyev-Smirnov-Wolfenstein (MSW) conversions in the Sun [36].

²¹In Table V we have used the computed value of $M_1 \sim 3 \times 10^{17}$ GeV as the value for all M . To test the validity of this approximation, we have also determined $m_{Q,L} / \langle H_i \rangle$ and $\mu_{\text{eff}} / m_{\text{soft}}$ for $K=2$ using $M_2 \sim 7 \times 10^{17}$ GeV and for $K=3$ using an extrapolated M_3 value of 11×10^{17} GeV. For $P < 5$ and $P' < K+5$, the better estimates of M_2 and M_3 reduce $m_{Q,L} / \langle H_i \rangle$ and $\mu_{\text{eff}} / m_{\text{soft}}$, respectively, only by factors of $\mathcal{O}(1)$ in comparison to the values of $m_{Q,L} / \langle H_i \rangle$ and $\mu_{\text{eff}} / m_{\text{soft}}$ given in the $K=2$ and $K=3$ columns of Table V. However, larger values of P and P' yield increasing significant reductions in $m_{Q,L} / \langle H_i \rangle$ and $\mu_{\text{eff}} / m_{\text{soft}}$, respectively, when the better estimates of M_2 and M_3 are used.

²²Other applications of non-renormalizable operators to neutrino mass include [34] and [35].

If $W_{L_i L_i}^{(\text{Maj})}$ is not present, a superpotential term like $W_{L_i \nu_i^c}^{(\text{Dir})}$ can naturally yield heavier physical Dirac neutrino masses of the form

$$m_{L_i \nu_i^c} \sim \langle H_2 \rangle \left(\frac{m_{\text{soft}}}{M} \right)^{P'_{L_i \nu_i^c} / (K+1)}. \quad (45)$$

For example, for $K=5$ the experimental neutrino upper mass limits given in Table IV allow $P'_{L_1 \nu_1^c} \geq 4$, $P'_{L_2 \nu_2^c} \geq 3$, and $P'_{L_3 \nu_3^c} \geq 2$. Masses corresponding to $P'_{L_i \nu_i^c} = 4$ or 5 ($m_{L_i \nu_i^c} = 0.9$ eV or 10^{-2} eV, respectively) are in the range interesting for solar and atmospheric neutrinos, oscillation experiments, and dark matter.

Neutrino singlets can acquire a Majorana mass through $W_{\nu_i^c \nu_i^c}^{(\text{Maj})}$,

$$m_{\nu_i^c \nu_i^c} \sim m_{\text{soft}} \left(\frac{m_{\text{soft}}}{M} \right)^{\bar{P}_{\nu_i^c \nu_i^c} - K / (K+1)}, \quad (46)$$

which can be very large or small, depending on the sign of $\bar{P}_{\nu_i^c \nu_i^c} - K$. Laboratory and cosmological constraints depend on the ν_i^c lifetimes (if it decays), cosmological production and annihilation rates, and mixings with each other and with doublet neutrinos. These in turn depend on other couplings, such as $W_{L_i \nu_i^c}^{(\text{Dir})}$ or renormalizable couplings not associated with the mass. Generally, however, the constraints are very weak due to the absence of normal weak interactions, especially for heavy ν_i^c ($\bar{P}_{\nu_i^c \nu_i^c} \leq K$).

If both $W_{L_i \nu_i^c}^{(\text{Dir})}$ and $W_{\nu_i^c \nu_i^c}^{(\text{Maj})}$ terms are present, the standard seesaw mechanism can produce light neutrinos via diagonalization of the mass matrix for Eqs. (45), (46). The light mass eigenstate is

$$m_{\text{seesaw}}^{\text{light}} \sim m_{L_i \nu_i^c}^2 / m_{\nu_i^c \nu_i^c} \sim m_{\text{soft}} \left(\frac{m_{\text{soft}}}{M} \right)^{(2P'_{L_i \nu_i^c} + K - \bar{P}_{\nu_i^c \nu_i^c}) / (K+1)}, \quad (47)$$

while the heavy mass eigenstate is to first order $m_{\nu_i^c \nu_i^c}$ as given by Eq. (46). Various combinations of K , $P'_{L_i \nu_i^c}$ and $\bar{P}_{\nu_i^c \nu_i^c}$ produce viable masses for three generations of light neutrinos. For example, with $K=5$ and $P'_{L_i \nu_i^c} = P'_i = \{2, 1\}$ for $i=1, 2$, respectively (the values of K and $P'_{i=1,2}$ discussed above for the quarks and electrons), and with either $P'_{L_3 \nu_3^c} = 1$ or $P'_{L_3 \nu_3^c} = P'_{u_3} = 0$ (involving a renormalizable Dirac neutrino term), the light eigenvalues of the three generations fall into the hierarchy of 3×10^{-5} eV, 1×10^{-2} eV, and either 1×10^{-2} eV or 5 eV for $\bar{P}_{\nu_i^c \nu_i^c} = P'_{L_i \nu_i^c} + 1$. This range is again of interest for laboratory and non-accelerator experiments.

VI. CONCLUSIONS

We have explored the nature of intermediate scale scenarios for effective supergravity models as derived within a class of string vacua. In particular, we explored a class of string models which, along with the SM gauge group and the MSSM particle content, contain massless SM singlet(s) S_i . In addition, we assumed that the effects of supersymmetry breaking are parametrized by soft mass parameters.

The necessary condition for the intermediate mass scenario is the existence of D -flat and F -flat directions in the renormalizable part of the S_i sector. In this case, the only renormalizable terms of the potential are due to the soft mass-squared parameters m_i^2 . If the running of the soft mass parameters is such that the effective mass squared, along the flat direction, becomes negative at $\mu_{rad} \gg M_Z$, the S_i 's acquire a non-zero VEV at an intermediate scale. (Another possibility is that individual mass squares, but not the effective combination for the D -flat direction, are negative. Then the VEV is of the order of the electroweak scale.)

Importantly, in a large number of string models, in particular for a class of fermionic constructions, there exist SM singlets S_i with flat directions at the renormalizable level, which couple to additional exotic particles via Yukawa couplings of $\mathcal{O}(1)$. Such Yukawa couplings in turn ensure the radiative breaking, by driving the soft m_i^2 parameters negative at $\mu_{rad} \gg M_Z$.

For simplicity we confined the concrete analysis to the case in which there is an additional $U(1)'$ symmetry, and two SM singlets $S_{1,2}$ have opposite signs of the $U(1)'$ charges, thus ensuring D flatness for $|Q_1||S_1|^2 = |Q_2||S_2|^2$ [similar results are expected for the case of a single standard model singlet and no additional $U(1)'$]. In the analysis of radiative breaking we considered three types of Yukawa couplings [of $\mathcal{O}(1)$] of S_i to the exotic particles and a range of the boundary conditions on soft mass parameters at M_{string} . For a large range of parameters we obtained μ_{rad} in the range $10^5 - 10^{16}$ GeV (or at $\mu_{rad} \sim M_Z$).

In addition, we discussed the competition between the effects of the pure radiative breaking ($\langle S \rangle \sim \mu_{rad}$) and the stabilization of vacuum due to the non-renormalizable terms in the superpotential of the type $\hat{S}^{K+3}/\mathcal{M}^K$ [$\langle S \rangle \sim (m_{soft}\mathcal{M}^K)^{1/(K+1)}$]. Non-renormalizable terms in the superpotential are generic (and calculable) in string models. For a class of fermionic constructions $M \sim M_{string}$. These terms are dominant for $(m_{soft}\mathcal{M}^K)^{1/(K+1)} < \mu_{rad}$.

In the case of the pure radiative breaking, the mass of the Higgs field (and its fermionic partner) associated with a non-zero VEV of S is light and of order $M_Z/(4\pi)$. On the other hand, the breaking due to the non-renormalizable terms implies a light Higgs field and the supersymmetric partner both with the mass of order M_Z .

The non-renormalizable couplings of S_i 's to the MSSM particles in the superpotential in turn provide a mechanism to obtain an effective μ parameter and the masses for quarks and leptons. In the case of the pure radiative breaking the precise values of the μ parameter and the lepton-quark masses crucially depend on μ_{rad} . When the non-renormalizable terms dominate, these parameters assume specific values in terms of K and the order P of the non-renormalizable term by which they are induced. In particular,

$\mu = \mathcal{O}(m_{soft})$ for $K=P$, thus providing a phenomenologically acceptable value for the μ parameter. [Another possibility is that in which the $U(1)'$ is broken at the electroweak scale and the effective μ is generated by a renormalizable term.] We are able to obtain interesting hierarchies for the quark and lepton masses for appropriate values of P . Also, small (non-seesaw) Dirac or Majorana neutrino masses can be obtained, or the traditional seesaw mechanism can be incorporated, depending on the nature of the non-renormalizable operators.

In conclusion, the string models provide an important framework in which the intermediate scales can naturally occur and provide interesting implications for the μ parameter and the fermion mass hierarchy of the MSSM sector.

ACKNOWLEDGMENTS

This work was supported in part by U.S. Department of Energy Grant No. DOE-EY-76-02-3071. We thank P. Steinhardt and I. Zlatev for useful discussions.

APPENDIX: RENORMALIZATION GROUP ANALYSIS

If the standard model singlet S_1 [with $U(1)'$ charge Q_1] couples in the superpotential

$$W = h \hat{S}_1 \hat{E}_1 \hat{E}_2 \quad (A1)$$

to a set of exotic fields $\hat{E}_{1,2}^\alpha$ (in general non-singlets under the standard model group; the index α is a multiplicity index not necessarily associated with a gauge symmetry), the one-loop RGEs for couplings and soft masses can be integrated analytically.

The RGE equations have the general form²³ [$t = (1/16\pi^2) \ln(\mu/M_{str})$]

$$\frac{dg_a}{dt} = b_a g_a^3, \quad \frac{dM_a}{dt} = 2b_a g_a^2 M_a, \quad (A2)$$

where the index a runs over the different gauge group factors, with gauge coupling g_a and gaugino mass M_a , and $b_a = \sum_R S(R_a) - 3C(G_a)$. The sum extends over chiral multiplets with $S(R_a)$ the Dynkin index of the corresponding representation and $C(G_a)$ the quadratic Casimir invariant of the adjoint representation. With the MSSM particle content, $b_3 = -3$, $b_2 = 1$, and $b_1 = \frac{33}{5}$. In the case of two fundamental $SU(5)$ multiplets added to the MSSM particle content, $b_3 = -2$, $b_2 = 2$, and $b_1 = \frac{38}{5}$. In writing these equations we are neglecting the possible kinetic mixing [38,16] between $U(1)_Y$ and $U(1)'$.

For the Yukawa coupling in Eq. (A1),

$$\frac{dh}{dt} = (T+2)h^3 - h \sum_a r_a g_a^2, \quad (A3)$$

²³The local (or global) symmetry associated with the multiplicity in α for the $E_{1,2}$ fields permits us to write the Yukawa coupling h and the soft masses of these fields with no α indices.

TABLE VI. Coefficients in RGEs for coupling of \hat{S}_1 to triplets, doublets, and N_p pairs of identical MSSM singlets, via the superpotential $W = h\hat{S}_1\hat{E}_1\hat{E}_2$.

$E_i \sim (SU(3), SU(2), U(1)_Y, U(1)')$	T	$C_3(E_i)$	$C_2(E_i)$	$C_1(E_i)$	$C_{1'}(E_i)$
$D \sim (3, 0, Y_D, Q_D)$	3	$\frac{4}{3}$	0	$3Y_D^2/5$	Q_D^2/k_1
$L \sim (0, 2, Y_L, Q_L)$	2	0	$\frac{3}{4}$	$3Y_L^2/5$	Q_L^2/k_1
$S_i \sim (0, 0, 0, Q_{S_i})$	N_p	0	0	0	$Q_{S_i}^2/k_1$

where $T = \sum_\alpha \delta_\alpha^\alpha$, $r_a = 2[C_a(S_1) + C_a(E_1) + C_a(E_2)]$. In Table VI, we list the values of T and $C_a(E_i)$ for specific examples of E_i .

For the associated soft trilinear coupling,

$$\frac{dA}{dt} = 2(T+2)Ah^2 - 2\sum_a r_a g_a^2 M_a. \quad (\text{A4})$$

Finally, for the soft masses of the scalar components of S_1 and $D_{1,2}$,

$$\begin{aligned} \frac{dm_1^2}{dt} &= 2Th^2\sigma^2 - 8\sum_a g_a^2 C_a(S_1)M_a^2 \\ &+ 2\sum'_a k_a^{-1} g_a^2 Q_a(S_1) \text{Tr}[Q_a m^2], \end{aligned} \quad (\text{A5})$$

$$\begin{aligned} \frac{dm_{E_{1,2}}^2}{dt} &= 2h^2\sigma^2 - 8\sum_a g_a^2 C_a(E_{1,2})M_a^2 \\ &+ 2\sum'_a k_a^{-1} g_a^2 Q_a(E_{1,2}) \text{Tr}[Q_a m^2]. \end{aligned} \quad (\text{A6})$$

We used $\sigma^2 = m_1^2 + m_{E_1}^2 + m_{E_2}^2 + A^2$, the primed summation extends only to Abelian gauge group factors, and the k_a are normalization factors for the Abelian groups (e.g., $k_1 = 5/3$ in a GUT normalizaton).

The solutions for this set of equations²⁴ are

$$g_a^2(t) = \frac{g_0^2}{1 - 2b_a g_0^2 t}, \quad (\text{A7})$$

$$M_a(t) = M_{1/2} \frac{g_a^2(t)}{g_0^2}, \quad (\text{A8})$$

$$h^2(t) = \frac{E(t)h_0^2}{1 + (T+2)h_0^2 F(t)}, \quad (\text{A9})$$

$$A(t) = A_0 \epsilon_f(t) + M_{1/2} [H_2(t) - (T+2)h_0^2 H_3(t) \epsilon_f(t)], \quad (\text{A10})$$

$$\begin{aligned} m_1^2(t) &= [1 - 3TR_f(t)]m_0^2 - TR_f(t)\epsilon_f(t)A_0^2 \\ &- 2TR_f(t)\epsilon_f(t) \frac{H_3(t)}{F(t)} A_0 M_{1/2} \\ &+ M_{1/2}^2 \left\{ I_1(t) - TR_f(t) \frac{J(t)}{F(t)} \right. \\ &\left. + T(T+2) \left[\frac{H_3(t)}{F(t)} \right]^2 R_f^2(t) \right\}, \end{aligned} \quad (\text{A11})$$

$$\begin{aligned} m_{E_{1,2}}^2(t) &= [1 - 3R_f(t)]m_0^2 - R_f(t)\epsilon_f(t)A_0^2 \\ &- 2R_f(t)\epsilon_f(t) \frac{H_3(t)}{F(t)} A_0 M_{1/2} \\ &+ M_{1/2}^2 \left\{ I_{E_{1,2}}(t) - R_f(t) \frac{J(t)}{F(t)} \right. \\ &\left. + (T+2) \left[\frac{H_3(t)}{F(t)} \right]^2 R_f^2(t) \right\}, \end{aligned} \quad (\text{A12})$$

where

$$E(t) = \prod_a [1 - 2b_a g_0^2 t]^{r_a/b_a}, \quad (\text{A13})$$

$$F(t) = 2 \int_t^0 E(t') dt', \quad (\text{A14})$$

$$\epsilon_f(t) = \frac{1}{1 + (T+2)h_0^2 F(t)}, \quad (\text{A15})$$

$$R_f(t) = h_0^2 F(t) \epsilon_f(t), \quad (\text{A16})$$

$$H_2(t) = -2 \sum_a r_a g_a^2(t) t, \quad (\text{A17})$$

$$H_3(t) = -2tE(t) - F(t), \quad (\text{A18})$$

$$I_k(t) = 2 \sum_a C_a(k) \frac{1}{b_a} \left[1 - \frac{1}{(1 - 2b_a g_0^2 t)^2} \right] \quad (\text{A19})$$

$$\begin{aligned} J(t) &= 2 \int_t^0 E(t') [H_2^2(t') + I_1(t') + I_{E_1}(t') \\ &+ I_{E_2}(t')] dt'. \end{aligned} \quad (\text{A20})$$

²⁴Assuming universality of the soft masses at the string scale, $\text{Tr}[Q_a m^2] = 0$ at all scales for non-anomalous Abelian groups.

It is instructive to write some of these functions in terms of the Yukawa coupling at its (pseudo)fixed point

$$h_f^2(t) \equiv \frac{E(t)}{(T+2)F(t)}. \quad (\text{A21})$$

This is the running Yukawa coupling for large h_0 . It determines the infrared fixed point at the electroweak scale, and when $h_0 \sim 1$ it is expected that $h(t)$ approaches $h_f(t)$ for sufficiently large $-t$ [see Fig. 1(a)]. When $h(t) \approx h_f(t)$ is a good approximation, many terms simplify in the previous analytic solutions. In particular,

$$\epsilon_f(t) \equiv 1 - \frac{h^2(t)}{h_f^2(t)} \rightarrow 0, \quad R_f(t) \equiv \frac{1}{(T+2)} \frac{h^2(t)}{h_f^2(t)} \rightarrow \frac{1}{(T+2)}. \quad (\text{A22})$$

In addition to the exact analytical solutions presented above, it is useful to consider approximate analytical solutions to the RGEs. In this semi-analytic approach, the running gauge couplings and gaugino masses are replaced by their average values:

$$\bar{g}_a = \frac{1}{2}(g_a(M_Z) + g_0), \quad (\text{A23})$$

$$\bar{M}_a = \frac{1}{2}(M_a(M_Z) + M_{1/2}). \quad (\text{A24})$$

With these approximations, the RGEs for the Yukawa coupling, soft trilinear coupling, and the soft masses can be solved easily. The Yukawa coupling has the approximate solution

$$h^2(t) = \frac{\bar{g}^2}{1 - X(t)}, \quad (\text{A25})$$

in which

$$\bar{g}^2 = \frac{1}{T+2} \sum_a r_a \bar{g}_a^2, \quad (\text{A26})$$

$$X(t) = X_0 e^{2(T+2)\bar{g}^2 t}, \quad (\text{A27})$$

$$X_0 = 1 - \left(1 - \frac{\bar{g}^2}{h_0^2}\right). \quad (\text{A28})$$

The approximate solution for the trilinear coupling is given by

$$A(t) = \frac{A_0 X(1-X)}{X_0(1-X)} + \tilde{m}_\lambda \frac{X}{1-X} \left[\frac{1}{X} - \frac{1}{X_0} + \ln \frac{X}{X_0} \right], \quad (\text{A29})$$

where

$$\tilde{m}_\lambda = \frac{1}{(T+2)\bar{g}^2} \sum_a r_a \bar{g}_a^2 \bar{M}_a, \quad (\text{A30})$$

and the other quantities are defined above.

If the $U(1)'$ factors are neglected, the soft scalar mass-squared parameters have the following approximate solutions:

$$m_1^2 = \left(1 - \frac{T}{T+2}\right) m_0^2 + \frac{T}{T+2} \Sigma(t) + 2T \bar{g}^2 \bar{m}^2 \ln \frac{X}{X_0}, \quad (\text{A31})$$

$$m_{E,1,2}^2 = \left(1 - \frac{1}{T+2}\right) m_0^2 + \frac{1}{T+2} \Sigma(t) + \left(2 \bar{g}^2 \bar{m}^2 - 8 \sum_a C_a(E_{1,2}) \bar{M}_a^2 \bar{g}_a^2\right) \ln \frac{X}{X_0}, \quad (\text{A32})$$

in which

$$\bar{m}^2 = \frac{4}{(T+2)\bar{g}^2} \sum_a C_a(E_{1,2}) \bar{M}_a^2 \bar{g}_a^2, \quad (\text{A33})$$

and

$$\begin{aligned} \Sigma(t) = & (\bar{m}^2 - \tilde{m}_\lambda^2) \frac{1 - \frac{X}{X_0}}{1 - X} + \frac{X(1-X_0)}{X_0(1-X)} \Sigma_0 - \frac{X(1-X_0)}{X_0} (A_0 \\ & - \tilde{m}_\lambda^2) \frac{1 - \frac{X}{X_0}}{(1-X)^2} + \frac{X(1-X_0)}{X_0(1-X)^2} 2(A_0 - \tilde{m}_\lambda) \tilde{m}_\lambda \ln \frac{X}{X_0} \\ & + \frac{X}{1-X} \ln \frac{X}{X_0} \left(\bar{m}^2 + \tilde{m}_\lambda^2 \frac{1}{1-X} \ln \frac{X}{X_0} \right). \end{aligned} \quad (\text{A34})$$

The semi-analytic solutions are valid in the limit of small initial gaugino masses, such that the contribution of the gauginos to the evolution of the trilinear coupling and the soft mass-squared parameters is small.

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