Casimir scaling from center vortices: Towards an understanding of the adjoint string tension

M. Faber*
Institut für Kernphysik, Technische Universität Wien, A-1040 Vienna, Austria

J. Greensite†
The Niels Bohr Institute, DK-2100 Copenhagen Ø, Denmark

Š. Olejník‡
Institute of Physics, Slovak Academy of Sciences, SK-842 28 Bratislava, Slovakia

(Received 10 October 1997; published 27 January 1998)

We argue that the approximate ''Casimir scaling'' of the string tensions of higher-representation Wilson loops is an effect due to the finite thickness of center vortex configurations. It is shown, in the context of a simple model of the Z_2 vortex core, how vortex condensation in Yang-Mills theory can account for both Casimir scaling in intermediate size loops and color-screening in larger loops. An implication of our model is that the deviations from exact Casimir scaling, which tend to grow with loop size, become much more pronounced as the dimensionality of the group representation increases. [S0556-2821(98)03206-8]

PACS number(s): 12.38.Aw, 11.15.Ha

I. INTRODUCTION

There is increasing numerical evidence [1,2,3,4] supporting the center vortex theory of quark confinement [5,6,7,8,9], which was put forward in the late 1970s. Briefly, a center vortex is a topological field configuration which is linelike (in D=3 dimensions) or surfacelike (in D=4 dimensions) having some finite thickness. Creation of a center vortex can be regarded, outside the linelike or surfacelike ''core,'' as a discontinuous gauge transformation of the background, with a discontinuity associated with the gauge group center. Creation of a center vortex linked to a Wilson loop, in the fundamental representation of SU(N), has the effect of multiplying the Wilson loop by an element of the gauge group center, i.e.

$$W(C)\to e^{i2\pi n/N}W(C), \quad n=1,2,\dots,N-1. \tag{1}$$

The vortex theory, in essence, states that the area law for Wilson loops is due to quantum fluctuations in the number of center vortices linking the loop.

Paradoxically, this emphasis on the center of the gauge group can be viewed both as a vital strength of the theory, and also as a fatal weakness. Both aspects are apparent when we consider the force between static quarks in an SU(N) gauge theory, whose color charge lies in the adjoint representation. The QCD vacuum will not tolerate a linear potential between adjoint quarks over an infinite range; this is simply because adjoint color charges can be screened by gluons. Asymptotically, the force between adjoint quarks must drop to zero, and this is exactly what happens in the center vortex theory. The adjoint representation transforms trivially under the group center; adjoint Wilson loops are unaffected by center vortices, unless the core of the vortex happens to overlap the perimeter of the loop. As a result, large loops have only perimeter falloff, and the force between adjoint quarks vanishes asymptotically. The argument extends to any color representation which transforms trivially under the Z_N center of the gauge group. The fact that center vortices make such a clear distinction between those color charges which *should* be confined, and those charges which should not, is one of the most attractive features of the theory.

The fatal weakness aspect was first pointed out in Ref. [10]. Consider the large-N limit, which has the factorization property $\langle AB\rangle=\langle A\rangle\langle B\rangle$, where A and B are any two gauge-invariant operators. Then

$$\langle W_A(C)\rangle=\langle W_F(C)W_F^{\dagger}(C)\rangle=\langle W_F(C)\rangle^2 \tag{2}$$

where $W_{A,F}(C)$ denotes, respectively, Wilson loops in the adjoint and fundamental representations. An immediate consequence is that confinement of fundamental representation quarks implies confinement in the adjoint representation, with string tension $\sigma_A=2\sigma_F$. This is possible because color screening by gluons is a $1/N^2$ suppressed process, so at large N the vacuum *can* support an adjoint string. But adjoint loops are insensitive to center vortices, as noted above. The apparent conclusion is that center vortices cannot be the confinement mechanism at large N.

Even more troubling is the fact that the existence of an adjoint string tension is not peculiar to large-N. Many numerical experiments in SU(2) and SU(3) lattice gauge theory have shown that flux tubes form, and a linear potential is established, between quarks in the adjoint (and higher) representations [11]. The string tension is representation-dependent, and appears to be roughly proportional to the eigenvalue of the quadratic Casimir operator of the representation. Thus, for an SU(2) gauge theory

$$\sigma_j\approx\frac{4}{3}j(j+1)\sigma_{1/2} \tag{3}$$

*Electronic address: faber@kph.tuwien.ac.at
†Electronic address: greensite@nbivms.nbi.dk
‡Electronic address: fyziolej@savba.sk

where σ_j is the string tension in representation j. The region where this relation is valid, from the onset of confinement to the onset of color screening, we call the ''Casimir-scaling regime'' [12,13]. Of course, the color charge of higher-representation quarks is eventually screened by gluons, and the force between quarks then depends only on the transformation properties of the representation with respect to the gauge group center; i.e. on the ''n-ality'' of the representation. Asymptotically, for an SU(2) gauge group,

$$\sigma_j = \begin{cases} \sigma_{1/2} & j = \text{half-integer} \\ 0 & j = \text{integer.} \end{cases} \tag{4}$$

Color screening, although it must occur for adjoint quarks at sufficiently large separation, is very difficult to observe in numerical simulations.[1] Existing Monte Carlo studies of the QCD string have mainly probed the Casimir scaling regime.

In short, there is a linear potential between adjoint quarks in some finite range of distances, with approximate Casimir scaling of the string tensions, even at $N=2$. Casimir scaling should become exact, in a region extending from the confinement scale to infinity, in the $N\to\infty$ limit. Yet according to the center vortex theory, it would appear that string formation between adjoint quarks is impossible, at any N, at any distance scale. This has always seemed to us a good reason for discarding the vortex theory.

But suppose—and in our view the numerical evidence [1,2,3,4] is becoming persuasive—that center vortices really *are* the confining configurations, at least for quarks in the fundamental representation. Then either there is some other mechanism for inducing a linear potential between adjoint quarks, or else there must be a loophole in the ''fatal weakness'' arguments. The first alternative does not seem very economic, and in any case we have no insight, at present, in that direction. We will concentrate instead on the second possibility because there is, in fact, one possible loophole.

II. THICK VORTICES AND THE LOOP PERIMETER

The statement that adjoint loops are unaffected by center vortices contains one slight caveat: They are unaffected *unless* the vortex core somewhere overlaps the perimeter of the loop. At first sight this caveat seems irrelevant; for large loops the effect can only contribute to the perimeter falloff. But suppose that the vortex thickness is actually quite large: on the order of, and perhaps exceeding, the typical diameters of low-lying hadrons. What would be the effect of vortices on Wilson loops whose area is smaller than, or comparable to, the vortex cross section? We will study this question in the context of a simple model of vortex/perimeter overlaps, mainly in SU(2) lattice gauge theory.

In D=3 dimensions, or on a constant-time hypersurface in D=4 dimensions, a vortex is a closed tube of magnetic flux. For simplicity we consider planar and, in D=4 dimensions, spacelike Wilson loops. If the vortex is linked to a Wilson loop, with winding number =1, then the vortex pierces the minimal area of the loop an odd number of times. If the vortex is not linked to the loop, it either does not pierce the minimal area at all, or pierces it an even number of times. If, for the moment, we ignore the finite radii of the vortex tubes, then the effect of vortices on a Wilson loop is simple: For every instance where the minimal surface is pierced by a center vortex, insert a center element $-I$ somewhere along the loop, i.e.

$$W(C) = \mathrm{Tr}[UU\ldots U] \to \mathrm{Tr}[UU\ldots(-I)\ldots U]. \tag{5}$$

In principle we should place the $-I$ at the point of discontinuity of the gauge transformation which creates the vortex. However, since $-I$ commutes with everything, the placement is arbitrary; and this is related to the fact that the Dirac sheet of a center vortex can be moved about by gauge transformations.

Denote by f the probability that any given plaquette on the lattice is ''pierced'' by a vortex; i.e. a line running through the center of the vortex tube intersects the plaquette. The area law for Wilson loops is then trivially derived from the assumption that these probabilities, for plaquettes in a plane, are uncorrelated.[2] In that case, one has

$$\begin{aligned}\langle W(C)\rangle &= \prod_{x\in A}\{(1-f)+f(-1)\}\langle W_0(C)\rangle \\ &= \exp[-\sigma(C)A]\langle W_0(C)\rangle \end{aligned} \tag{6}$$

where the string tension is

$$\begin{aligned}\sigma &= -\frac{1}{A}\sum_{x\in A}\ln(1-2f) \\ &= -\ln(1-2f)\end{aligned} \tag{7}$$

and where $\langle W_0(C)\rangle$ is the expectation value of the loop with the constraint that no vortices pierce the minimal area. The quantity $\langle W_0(C)\rangle$ can be (and has been) computed from lattice Monte Carlo, cf. Refs. [1, 2]. In those computations, it is found that $W_0(C)$ does not have an area law falloff.

By the same argument, the string tension for loops in any j=half-integer representation is the same as for $j=1/2$, while the string tension for j=integer vanishes. Of course the argument is too simple in a number of respects, e.g. there is likely to be some short-range correlation between the f probabilities of nearby plaquettes. This point, however, is not crucial to the discussion. What is more important is that we have ignored the finite radii of the vortices. Equation (5) is only true if the core of the vortex, where it crosses the plane of the loop, is entirely contained in the minimal area of the loop. If the core overlaps the perimeter of the loop, then Eq. (5) cannot be quite right.

By ''core'' we are referring to the region of the center vortex which cannot be represented as a discontinuous gauge transformation, and where the local field strength associated with the vortex is nonzero. This region is a 4-dimensional volume, generated, e.g., by the propagation in time of a closed tube of finite radius. It may also be thought of as a thickened surface. Consider the creation of a center vortex

[1] At least, this is difficult at zero temperature. Color screening *has* been observed in certain finite-temperature studies, cf. Müller *et al.* in Ref. [11].

[2] Some related ideas are found in Ref. [14].

which is linked to a planar loop C parametrized by $x^\mu(\tau), \tau\in[0,1]$; the linking implies that the minimal area of the loop is pierced an odd number of times. For simplicity, let us suppose that the minimal area is pierced only once. Then there is some area K, in the plane of the loop, which is a 2-dimensional cross section of the vortex core. If K lies entirely within the minimal area of the loop, then (by definition) the effect of the vortex on the gauge fields $A_\mu(x)$ along loop C is simply that of a discontinuous gauge transformation

$$A_\mu(x(\tau))\rightarrow g^{-1}(x(\tau))A_\mu(x(\tau))g(x(\tau)) + ig^{-1}(x(\tau))\partial_\mu g(x(\tau)) \tag{8}$$

with the inhomogeneous term dropped at the point of discontinuity $x^\mu(\tau)$, $\tau=0,1$, and where, for SU(N),

$$g(x(0))=e^{i2\pi n/N}g(x(1)) \quad n=1,2,\dots,N-1. \tag{9}$$

The result is that the value of the fundamental representation Wilson loop $W(C)$ changes as shown in Eq. (1), despite the fact that the field strength of the vortex vanishes outside the core (note that if the area K were somehow shrunk to a point, then the vortex field strength would be singular at that point). On the other hand, if some segment of loop C intersects region K of the vortex core, then Eq. (8) is not valid on that segment, and the effect of the vortex on a Wilson loop is more complicated; the effect is *not* simply given by insertion of a center element at the point of gauge discontinuity, as shown in Eq. (5).

What is needed is a full-fledged theory of center vortices, perhaps something along the lines of the old Copenhagen vacuum [7], which would explain how Eq. (5) should be modified when the vortex core (or, more precisely, its cross section K) is not entirely enclosed within the minimal area of the loop. In lieu of that, we will just consider a simplified picture in which the center element $-I$ in Eq. (5) of the SU(2) gauge group is replaced by a group element G, which interpolates smoothly from $-I$, if the core is contained entirely with the loop, to $+I$, if the core is entirely exterior. Our assumption, for Wilson loops in any group representation j, is the following:

Assumption 1. The effect of creating a center vortex piercing the minimal area of a Wilson loop may be represented by the insertion of a unitary matrix G at some point along the loop

$$W(C)=\mathrm{Tr}[UU\dots U]\rightarrow \mathrm{Tr}[UU\dots G\dots U] \tag{10}$$

where

$$G(x,S)=\exp[i\alpha_C(x)\vec{n}\cdot\vec{L}] = S\exp[i\alpha_C(x)L_3]S^\dagger. \tag{11}$$

The L_i are group generators in representation j, $\vec{n}$ is a unit 3-vector, and S is an SU(2) group element in the j representation.

The parameter $\alpha_C(x)\in[0,2\pi]$ depends on what fraction of the vortex core is enclosed by the loop; thus it depends on both the shape of the loop C and the position $\vec{x}$ of the center of the vortex core, relative to the perimeter, in the plane of the loop. It does not depend on the group representation. If the core is entirely enclosed by the loop, then $\alpha_C(x)=2\pi$, conversely, if the core is entirely outside the minimal area of the loop, then $\alpha_C(x)=0$. In an Abelian theory, this first assumption would be completely correct, where Tr$[G]$ would be the value of the loop for a vortex created on a classical vacuum background. In a non-Abelian theory the assumption might be quantitatively correct for expectation values (i.e. averaging over group orientations S and over small quantum fluctuations U_μ around the vortex background); this would be quite sufficient for our purposes.

Generalizing a little further, if we create some number m of vortices in the loop, centered at positions $x_1,x_2,\dots,x_m$, then

$$\begin{aligned}W(C)\rightarrow &W[C;\{x_i,S_i\}]\\ &=\mathrm{Tr}[U\dots UG(x_a,S_a)U\dots UG(x_b,S_b)\\ &\quad\times U\dots UG(x_p,S_p)\dots U]\end{aligned} \tag{12}$$

where $a,b,\dots,p$ is some permutation of $12\dots,m$. We now make the second assumption of our model:

Assumption 2. The probabilities f that plaquettes in the minimal area are pierced by vortices are uncorrelated. The random group orientations associated with S_i are also uncorrelated, and should be averaged.

These two assumptions define our model. They are, no doubt, an oversimplification of the effects of vortex thickness, but we believe they at least provide a plausible picture of those effects.

According to the second assumption, we are justified in averaging independently every $G(x_a,S_a)$ over orientations in the group manifold specified by S_a. This is easily seen to give

$$\overline{G(\alpha)}=\int dSS\exp[i\alpha L_3]S^\dagger\equiv\mathcal{G}_j[\alpha]I_{2j+1}$$

$$\begin{aligned}\mathcal{G}_j[\alpha]&=\frac{1}{2j+1}\mathrm{Tr}\exp[i\alpha L_3]=\frac{1}{2j+1}\sum_{m=-j}^{j}\cos(\alpha m)\\ &=\frac{\sin\left[(2j+1)\frac{\alpha}{2}\right]}{(2j+1)\sin\left[\frac{\alpha}{2}\right]}\end{aligned} \tag{13}$$

where I_k is the $k\times k$ unit matrix. Then $W[C,\{x_i,S_i\}]$, averaged over all S_i, becomes

$$W[C;\{x_i\}]=\left\{\prod_i \mathcal{G}_j[\alpha_C(x_i)]\right\}\mathrm{Tr}[UU\dots U]. \tag{14}$$

Next, take the expectation value of Tr$[UU\dots U]$ for configurations U with the constraint that no vortices pierce the loop, denoted $\langle W_0(C)\rangle$. Equation (14) then goes to

$$\langle W[C;\{x_i\}]\rangle=\left\{\prod_i \mathcal{G}_j[\alpha_C(x_i)]\right\}\langle W_0(C)\rangle. \tag{15}$$

The last step is to sum over the number and position of vortices piercing the plane of the loop C, weighted by the appropriate probability factors, and we find

$$\begin{aligned}\langle W(C)\rangle &= \prod_x \{(1-f)+f\mathcal{G}_j[\alpha_C(x)]\}\langle W_0(C)\rangle \\ &= \exp\left[\sum_x \ln\{(1-f)+f\mathcal{G}_j[\alpha_C(x)]\}\right]\langle W_0(C)\rangle \\ &= \exp[-\sigma(C)A]\langle W_0(C)\rangle \end{aligned} \tag{16}$$

where

$$\sigma_C = -\frac{1}{A}\sum_x \ln\{(1-f)+f\mathcal{G}_j[\alpha_C(x)]\}. \tag{17}$$

The product and sum over positions x run over all plaquettes in the plane of the loop. The reason for not restricting x to lie strictly within the minimal area of the loop is, again, because the vortex core is finite. Denote the radius of the vortex core by R_c. If the center of the core lies outside the loop, but at a distance less than R_c from the perimeter, then it can still overlap the perimeter. One *can* restrict the sum to run over $x\in A'$, where A' includes all plaquettes inside the minimal area in the plane of the loop, as well as plaquettes in the plane outside the perimeter, but within a distance R_c of the loop.

Now σ_C is not exactly a string tension, because it depends, via $\alpha_C(x)$, on the shape (and the area) of the loop. If, however, there is some region where σ_C changes only slowly with area, then the potential will rise approximately linearly. In particular, consider the limit of very large loops. In that case, almost every vortex which affects the loop is entirely enclosed by the loop, and for these vortices $\alpha_C(x)\approx 2\pi$. Only those vortices near the perimeter have $\alpha_C(x)$ different from 2π, and as the loop becomes very large this is a negligible fraction of the total; in particular, $A'/A\approx 1$. This means that σ_C is an area-independent constant for large loops, and it can be seen from Eqs. (13) and (16) that

$$\sigma_C = \begin{cases} -\ln(1-2f) & j=\text{half-integer} \\ 0 & j=\text{integer} \end{cases} \tag{18}$$

which is the correct representation dependence of the asymptotic string tension.

Next, let us consider the case where $f\ll 1$, which is certainly true in the lattice theory at weak coupling, and also small or medium size loops, where $\alpha_C(x)$ is also typically small. For small α, we have, from Eq. (13),

$$\mathcal{G}_j[\alpha]\approx 1-\frac{\alpha^2}{6}j(j+1). \tag{19}$$

Then, making an expansion of the logarithm in Eq. (18) and applying Eq. (19),

$$\begin{aligned}\sigma_C &= f\frac{1}{A}\sum_{x\in A'}(1-\mathcal{G}_j[\alpha_C(x)]) \\ &= \frac{1}{A}\left\{\frac{f}{6}\sum_{x\in A'}\alpha_C^2(x)\right\}j(j+1)\end{aligned} \tag{20}$$

or just

$$\sigma_C = \frac{f}{6}\,\overline{\alpha_C^2}\,j(j+1) \tag{21}$$

where α_C^2 is an average value

$$\overline{\alpha_C^2} = \frac{1}{A}\sum_{x\in A'}\alpha_C^2(x). \tag{22}$$

We see that σ_C, for small f and small loops, is proportional to the eigenvalue of the quadratic Casimir operator.

This result can be readily extended to any SU(N) group. In the general case there are $N-1$ types of center vortices, corresponding to the $N-1$ phase factors of Eq. (1). To the nth type, we associate probability f_n to pierce a plaquette, and a group factor

$$G[x,S] = S\exp[i\vec{\alpha}_C^n(x)\cdot\vec{H}]S^\dagger \tag{23}$$

where the $\{H_i,\ i=1,...,N-1\}$ are the generators spanning the Cartan subalgebra.[3] Following the same steps as before:

$$\begin{aligned}\langle W(C)\rangle &= \prod_x\left\{1-\sum_{n=1}^{N-1}f_n(1-\mathcal{G}_r[\vec{\alpha}_C^n(x)])\right\} \\ \mathcal{G}_r[\vec{\alpha}] &= \frac{1}{d_r}\,\text{Tr}\,\exp[i\vec{\alpha}\cdot\vec{H}]\end{aligned} \tag{24}$$

with d_r the dimension of representation r. Vortices of type n and type $N-n$ have phase factors in Eq. (1) which are complex conjugates of one another; they may be regarded as the same type of vortex but with magnetic flux pointing in opposite directions, so that

$$f_n = f_{N-n} \quad\text{and}\quad \mathcal{G}_r[\vec{\alpha}_C^n(x)] = \mathcal{G}_r^*[\vec{\alpha}_C^{N-n}(x)] \tag{25}$$

and therefore

$$\begin{aligned}\langle W(C)\rangle &= \prod_x\left\{1-\sum_{n=1}^{N-1}f_n(1-\text{Re}\,\mathcal{G}_r[\vec{\alpha}_C^n(x)])\right\} \\ \sigma_C &= -\frac{1}{A}\sum_x\ln\left\{1-\sum_{n=1}^{N-1}f_n(1-\text{Re}\,\mathcal{G}_r[\vec{\alpha}_C^n(x)])\right\}.\end{aligned} \tag{26}$$

Expanding the logarithm to leading order in f_n, expanding $\mathcal{G}_r[\vec{\alpha}]$ to leading order in $\vec{\alpha}$, and using the identity

$$\frac{1}{d_r}\,\text{Tr}(H_iH_j) = \frac{C_r^{(2)}}{N^2-1}\,\delta_{ij} \tag{27}$$

one finds

[3]It is possible that only vortices with the smallest magnitude of center flux have substantial probability; i.e. $f_1=f_{N-1}$ is finite, all other f_n are negligible. This is a dynamical issue which we cannot resolve here, and so we consider the general case that includes all possible f_n.

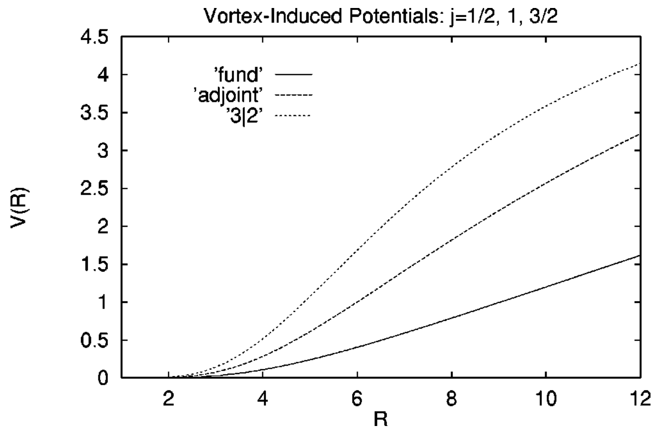

FIG. 1. Interquark potential $V(R)$ induced by center vortices, according to the model discussed in the text, for quark charges in the $j=\frac{1}{2},1,\frac{3}{2}$ representations.

$$\sigma_C=\frac{1}{A}\left\{\sum_x\sum_{n=1}^{N-1}\frac{f_n}{2(N^2-1)}\vec{\alpha}_C^n(x)\cdot\vec{\alpha}_C^n(x)\right\}C_r^{(2)} \quad (28)$$

where $C_r^{(2)}$ is the eigenvalue of the quadratic Casimir operator of the SU(N) group in representation r.

The results (21) and (28) might be termed "Casimir proportionality," since the nonperturbative part of the interquark potential, which is due to vortices, is proportional to the quadratic Casimir of SU(N) for small loops. But this does not yet imply Casimir scaling of string tensions. The parameters $\vec{\alpha}_C^n(x)$ depend on loop size, and there is no particular reason to suppose that σ_C is constant in the adjoint representation or, equivalently, that the adjoint potential is linear in some range. Even if the adjoint potential *were* approximately linear in some interval, it is not obvious that the string tension for the fundamental representation, in the same range of distances, would have reached its asymptotic value. To study this issue, we will return to the SU(2) example.

III. LINEAR POTENTIALS AND $\alpha_C(x)$

It may be possible to measure $\alpha_C(x)$ in computer simulations, by the methods introduced in Refs. [1, 2]. In the meantime, it is worthwhile to ask whether there exists some reasonable ansatz for $\alpha_C(x)$ which would lead to both Casimir proportionality *and* linear potentials in some region.

To set things up, let us consider a long rectangular $R\times T$ loop in the $x-t$ plane, with $T\gg R$, in group representation j. The time-extension T is huge but fixed, so we can characterize loops C just by the width R. Let x denote the x coordinate of the center of a vortex, where it pierces the $x-t$ plane. From the previous discussion, the interquark potential induced by vortices is easily seen to be

$$V_j(R)=-\sum_{n=-\infty}^{\infty}\ln\{(1-f)+f\mathcal{G}_j[\alpha_R(x_n)]\}, \quad (29)$$

where $x_n=n+\frac{1}{2}$ (the choice of x_n comes from the fact that the vortex centers lie in the dual lattice, piercing the middle of plaquettes). The problem is to find some reasonable ansatz for $\alpha_R(x)$.

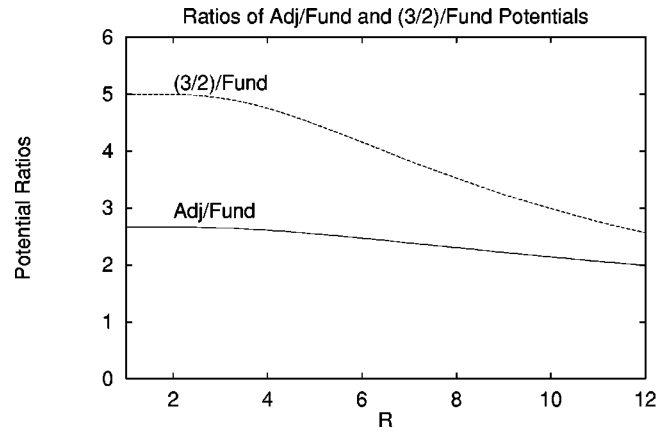

FIG. 2. Ratios of $V_{3/2}(R)$ (upper curve) and $V_1(R)$ (lower curve) to the potential $V_{1/2}(R)$ of fundamental representation quark charges.

Suppose the timelike sides of the loop are at $x=0$ and $x=R$. Then, to guide the search for an ansatz, there are a few simple conditions that $\alpha_R(x)$ must satisfy:

(1) Vortices which pierce the plane far outside the loop do not affect the loop. This means that for fixed R, as $x\to\pm\infty$, we must have $\alpha_R(x)\to 0$.
(2) If the vortex core is entirely contained within the loop, then $\alpha_R(x)=2\pi$. This translates as follows: Let x be inside the loop, and d be the distance from x to the nearest of the timelike sides. Then it must also be the case that $\alpha_R(x)\to 2\pi$ as $d\to\infty$.
(3) As $R\to 0$, the percentage of any vortex core which is contained inside the loop must also go to zero. Thus $\alpha_R(x)\to 0$ as $R\to 0$.

There are an infinite number of functional forms which would meet these conditions, but a simple 2-parameter (a,b) ansatz is the following: First define

$$y(x)=\begin{cases} x-R & \text{for } |R-x|\leq|x| \\ -x & \text{for } |R-x|>|x| \end{cases} \quad (30)$$

whose magnitude is the distance of the vortex center to the nearest timelike side of the loop, taken negative if the vortex center is inside the loop, and positive outside. Then choose

$$\alpha_R(x)=\pi\left[1-\tanh\left(ay(x)+\frac{b}{R}\right)\right] \quad (31)$$

which fulfills all three requirements.

Figure 1 shows the potentials for the $j=\frac{1}{2},1,\frac{3}{2}$ representations, for the choice of parameters $f=0.1$, $a=0.05$, $b=4$, in the range $R\in[1,12]$.[4] Note that the fundamental and adjoint potentials are roughly linear in a range from 5 or 6 to 12 lattice spacings. Figure 2 plots the ratios $V_1(R)/V_{1/2}(R)$, and $V_{3/2}(R)/V_{1/2}(R)$. As expected, these ratios start out at the

[4]Strictly speaking, R takes on only integer values in the lattice formulation, but we have plotted $V_j(R)$ over the continuous interval.

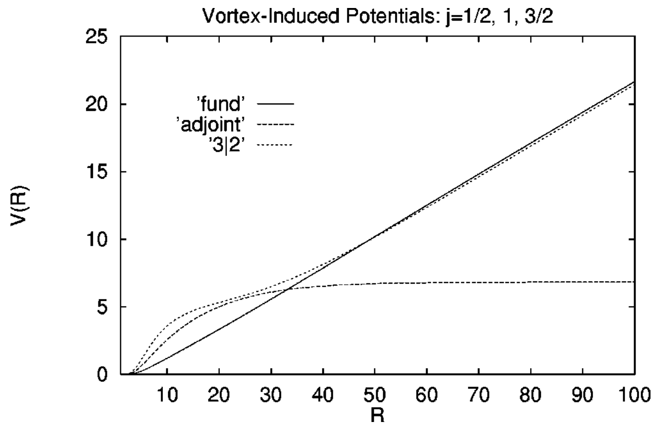

FIG. 3. Same as Fig. 1, for the range $R \in [1,100]$.

ratios of the corresponding Casimirs, 8/3 and 5 respectively. It can be seen that in this interval the adjoint/fundamental ratio drops only slowly, from 8/3 to about 2, while in the same interval the $(j=3/2)$/fundamental ratio drops more precipitously, from 5 to about 2.5. Figure 3 is again a plot of all three potentials, but this time over the range [1,100]. There are two things to notice in this last figure. First, we see color screening set in as expected, around $R=25-30$ lattice spacings. The adjoint potential goes to a constant, while the $j=3/2$ potential closely parallels the fundamental potential. Secondly—and this was not expected—it can be seen that the the slope of the fundamental potential, in the Casimir scaling region between 6 to 12 lattice spacings, is very close to its asymptotic value.

We have therefore found, in this rather simple model of the vortex core, the kind of Casimir scaling which is seen in Monte Carlo simulations. A natural question is to what extent this scaling depends on a very special choice of parameters. The answer is that Casimir scaling, although it is not found for any choice of parameters, is generic in a large region of the parameter space. For example, the Casimir scaling region of Fig. 3 can be scaled up by any factor F simply by setting $a \to a/F, b \to bF$. It is also quite easy to think up other functional forms for $\alpha_R(x)$ which satisy the above three conditions [e.g. $\alpha_R(x)=\beta(x)-\beta(x-R)$, with $\beta(x)\to \pm\pi$ in the limits $x\to\pm\infty$]. The possibilities are, of course, infinite. Some functions work better than others, but the existence of an approximate Casimir scaling region of some finite extent seems to be fairly common. The crucial ingredient is the thickness of the vortices, which would be on the order of $1/a$ for the choice of $\alpha_R(x)$ in Eq. (31). What we find is that the thickness of the vortices must be quite large; larger, in fact, than the Casimir region itself, in order to see approximate Casimir scaling of the adjoint string tension.

IV. CONCLUSIONS

We have presented a scenario whereby the Casimir scaling of higher-representation string tensions is explained in terms of the finite thickness of center vortices. We do not claim to have *proven* that vortex thickness is the origin of Casimir scaling, but this explanation now appears to be very plausible, particularly if center vortices turn out to be the true QCD confinement mechanism.

Numerical tests of our scenario are in order. If center vortices give rise to an adjoint string tension, and if an adjoint loop is evaluated only in those configurations where no vortex links the loop, then the string tension should vanish. This was found to be the case for the fundamental string tension, in Refs. [1, 2], and should be testable for the adjoint string, at least in principle, by the methods explained in those articles. It may also be possible to calculate $\alpha_R(x)$ from Monte Carlo simulations of fundamental loops, use that information in Eq. (29) to derive the adjoint potential, and compare the derived adjoint potential with the corresponding Monte Carlo data.

For every set of parameters in which we see Casimir scaling in our simple model, the deviations from exact Casimir scaling are much greater for the $j=3/2$ potential as compared to those of the $j=1$ potential. It would therefore be very interesting to compare Fig. 2 with the actual Monte Carlo data. Calculation of the $j=3/2$ potential in the scaling region of SU(2) gauge theory, by lattice Monte Carlo techniques, is a computationally intensive problem, but we believe it is feasible.

Finally, there is the question raised years ago in Ref. [10]: how can center vortices explain confinement at large N, where the Casimir scaling region extends to infinity? We have seen, in the scenario outlined here, that the size of the Casimir scaling region depends on the thickness of center vortices. Already in SU(2) gauge theory, the existence of a Casimir regime implies that the diameter of the vortices substantially exceeds the separation length at which the heavy quark potential begins to grow linearly (i.e. the ''confinement scale''). We therefore expect that as N increases, the diameter of vortices relative to the confinement scale will also slowly increase, probably as log(N). At $N=\infty$ the vortex core would be infinite in extent, and the discontinuous gauge transformation exterior to the core would be pushed off to infinity. It is not clear that the term ''center vortex'' remains a useful description of the relevant configurations in this limit.

Note added. We have recently learned that a related proposal, namely, that center vortices might lead to a (breakable) string potential between massive gluons, was advanced by Cornwall [15]. We thank Mike Cornwall for bringing this reference to our attention.

ACKNOWLEDGMENTS

We would like to thank J. Ambjørn and L. Del Debbio for discussions. This work was supported in part by Fonds zur Förderung der Wissenschaftlichen Forschung P11387-PHY (M.F.), the U.S. Department of Energy under Grant No. DE-FG03-92ER40711 and Carlsbergfondet (J.G.), the ''Action Austria-Slovak Republic: Cooperation in Science and Education'' (Project No. 18s41) and the Slovak Grant Agency for Science, Grant No. 2/4111/97 (Š.O.).

[1] L. Del Debbio, M. Faber, J. Greensite, and Š. Olejník, Phys. Rev. D **55**, 2298 (1997).

[2] L. Del Debbio, M. Faber, J. Greensite, and Š. Olejník, ''New Developments in Quantum Field Theory,'' Proceedings of the Zakopane meeting, hep-lat/9708023; in Lattice 97, Proceedings of the International Conference, Edinburgh, Scotland, hep-lat/9709032; *31st International Symposium on the Theory of Elementary Particles*, Proceedings of the Buckow meeting (in preparation).

[3] T. Kovács and E. Tomboulis, in Lattice 97 [2], hep-lat/9709042.

[4] K. Langfeld, H. Reinhardt, and O. Tennert, hep-lat/9710068.

[5] G. 't Hooft, Nucl. Phys. **B138**, 1 (1978).

[6] G. Mack, in *Recent Developments in Gauge Theories*, edited by G. 't Hooft *et al.* (Plenum, New York, 1980).

[7] H. B. Nielsen and P. Olesen, Nucl. Phys. **B160**, 380 (1979); J. Ambjørn and P. Olesen, *ibid.* **B170**, 60 (1980); **B170**, 265 (1980).

[8] J. M. Cornwall, Nucl. Phys. **B157**, 392 (1979).

[9] R. P. Feynman, Nucl. Phys. **B188**, 479 (1981).

[10] J. Greensite and M. Halpern, Phys. Rev. D **27**, 2545 (1983).

[11] J. Ambjørn, P. Olesen, and C. Peterson, Nucl. Phys. **B240** [FS12], 198 (1984); **B240** [FS12], 533 (1984); C. Michael, in *Lattice '91*, Proceedings of the International Symposium, Tsukuba, Japan, 1991, edited by M. Fukugita *et al.* [Nucl. Phys. B (Proc. Suppl.) **26**, 417 (1992)]; Nucl. Phys. **B259**, 58 (1985); N. A. Cambell, I. H. Jorysz, and C. Michael, Phys. Lett. **167B**, 91 (1986); M. Faber and H. Markum, in *Field Theory on the Lattice*, Proceedings of the International Workshop, Seillac, France, 1987, edited by A. Billoire *et al.* [Nucl. Phys. B (Proc. Suppl.) **4**, 204 (1988)]; M. Müller, W. Beirl, M. Faber, and H. Markum, in *Lattice '91*, p. 423; G. Poulis and H. Trottier, Phys. Lett. B **400**, 358 (1997).

[12] L. Del Debbio, M. Faber, J. Greensite, and Š. Olejník, in *Lattice '96*, Proceedings of the International Symposium, St. Louis, Missouri, edited by C. Bernard *et al.* [Nucl. Phys. B (Proc. Suppl.) **53**, 141 (1997)], hep-lat/9607053.

[13] L. Del Debbio, M. Faber, J. Greensite, and Š. Olejník, Phys. Rev. D **53**, 5891 (1996).

[14] M. Chernodub, M. Polikarpov, and M. Zubkov, in *Lattice '93*, Proceedings of the International Symposium, Dallas, Texas, edited by T. Draper *et al.* [Nucl. Phys. B (Proc. Suppl.) **34**, 256 (1994)].

[15] J. M. Cornwall, in *Proceedings of the Workshop on Non-Perturbative Quantum Chromodynamics*, edited by K. A. Milton and M. A. Samuel (Birkhäuser, Boston-Basel-Stuttgart, 1983).