Phase structure and nonperturbative states in a three-dimensional adjoint Higgs model

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The thermodynamics of a three-dimensional (3D) adjoint Higgs model is considered. We study the properties of the Polyakov loop correlators and the critical behavior at the deconfinement phase transition. Our main tool is a reduction to the 2D sine-Gordon model. The Polyakov loops appear to be connected with the soliton operators in it. The known exact results in the sine-Gordon theory allow us to study in detail the temperature dependence of the string tension, as well as to get some information about the nonperturbative dynamics in the confinement phase. We also consider symmetry restoration at high temperature which makes it possible to construct the phase diagram of the model completely. [S0556-2821(98)00802-9]

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I. INTRODUCTION

The adjoint Higgs model in three dimensions exhibits a number of features which probably are shared by four-dimensional gauge theories in the confining phase. This theory possesses a mass gap, although a part of the gauge symmetry remains unbroken. The charges of the unbroken subgroup are confined by a string of electric flux with the energy proportional to its length. The confinement, as well as the mass gap, arise nonperturbatively due to the Euclidean field configurations of the magnetic monopole type [1].

In three dimensions, the magnetic charge is a counterpart of the instanton number. The classical solutions which carry a unit of the magnetic charge are the well known 't Hooft–Polyakov monopoles [2]. The effects driven by these pseudoparticles can be studied at weak coupling by standard semiclassical techniques and the nonperturbative phenomena can be investigated in much detail, without any uncontrollable approximations.

At low energies, the relevant degrees of freedom in this model are gauge fields of the unbroken Abelian subgroup and monopoles. It is important to take into account the long-range interactions between pseudoparticles [1], so the vacuum of the theory is a Coulomb plasma of monopoles and antimonopoles, globally neutral and dilute at weak coupling. The monopole gas is conveniently described by a scalar field theory with a cosine interaction, the coupling being proportional to the mean monopole density [1]. The scalar field is dual to the photon in the sense of the usual electric-magnetic duality—the monopoles are the sources of this field.

When a probe charge is inserted in the vacuum, its electric field is screened by monopoles and form a tube with thickness of order of the correlation length in the Coulomb plasma. The surface spanned by the trajectory of the charge serves as a source of the dual scalar field. The action of the corresponding classical configuration is proportional to the area of this surface [1]. The surface appearing in the semiclassical calculations, therefore, can be interpreted as a world sheet of the string which confine the charges. More recently, some progress has been made in the dynamical consideration of such strings [3]. The arguments were given [3] that beyond the semiclassical approximation string world sheet fluctuates and the Wilson loop average $W(C)$ in the gauge theory can be represented as a sum over surfaces bounded by the contour C whose Boltzmann weight is determined by some string action.

The three-dimensional (3D) adjoint Higgs model possesses interesting thermodynamic properties. It undergoes a deconfinement phase transition and in the high temperature phase linear forces between static charges are replaced by the Coulomb logarithmic interaction. The universality arguments, as well as the renormalization group methods, were used to study this phase transition in a closely related model of lattice U(1) gauge theory [4]. The phase transition was shown to be of the Berezinskii-Kosterlitz-Thouless (BKT) type. The reasons are based on the dimensional reduction of the monopole plasma at finite temperature to a two-dimensional Coulomb gas which is known to undergo the BKT phase transition [5]. Because at weak coupling the effects of the monopoles are exponentially small, the confinement scale of the theory is very large and the dimensional reduction should work well even for rather low temperatures. Thus, the deconfinement phase transition can be accurately described within the two-dimensional theory.

In the present paper 3D adjoint Higgs model at finite temperature is studied in more detail. We shall be primarily interested in the behavior of the Polyakov loops which measure the free energy of static charged sources and play the role of an order parameter for the deconfinement phase transition [6]. We find the operators corresponding to them in the effective sine-Gordon model. In two dimensions, the sine-Gordon theory is completely integrable and many quantities in it can be calculated exactly. The dimensional reduction enables us to utilize some of these exact results.

The thermodynamics of 3D adjoint Higgs model is interesting by itself, but there exist some other motivations to study it. The point is that the dimensional reduction is expected to be a good approximation at comparably low temperatures. This fact allows us to use it in the study of the

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confinement phase. The theory considerably simplifies after passing from three to two dimensions and some principal questions become more tractable, in particular, the problem of string representation for Wilson loop averages. The temporal degrees of freedom of the confining string decouple under the dimensional reduction and a sum over surfaces reduces to a sum over paths which is more familiar in the field theory and can be investigated in more detail. Another important question which can be studied with the help of the dimensional reduction concerns a spectrum of light degrees of freedom in the confinement phase.

We also discuss the non-Abelian gauge symmetry restoration at high temperature [7]. This permits to examine the phase diagram of the model more completely. An extrapolation of the results obtained leads to interesting predictions about the phase structure in the nonperturbative strong coupling region.

II. 3D ADJOINT HIGGS MODEL

Before considering the thermodynamics we briefly describe the properties of the theory involved at zero temperature following Ref. [1]. The Euclidean action of the model has the form

$$S=\int d^3x\left[\frac{1}{4g^2}\,\mathcal{F}^a_{\mu\nu}\mathcal{F}^a_{\mu\nu}+\frac{1}{2}D_\mu\Phi^a D_\mu\Phi^a+\frac{1}{4}\lambda(\Phi^a\Phi^a-\eta^2)^2\right]. \tag{2.1}$$

The scalar field transforms in the adjoint representation. In this paper we consider SU(2) gauge group, but a generalization to $SU(N)$ with arbitrary N is also possible [8,9].

The non-Abelian symmetry of Eq. (2.1) is spontaneously broken to U(1), unless η^2 is not too small, when the transition to the symmetric phase can occur. This transition was recently studied numerically in much detail [10,11]. The perturbative spectrum of the model consists of the massless photon W and Higgs bosons with masses

$$m_W^2=g^2\eta^2, \quad m_H^2=2\lambda\eta^2, \tag{2.2}$$

respectively. In the perturbative regime,

$$m_H\sim m_W\gg g^2, \tag{2.3}$$

the massive fields decouple at low energies and we are left with free U(1) gauge theory. As it was shown in [1], this simple picture is spoiled by nonperturbative effects related to monopoles.

The monopole solutions have the form

$$\mathcal{A}^a_\mu=\varepsilon_{a\mu\nu}\frac{x_\nu}{r^2}[1-f(r)], \tag{2.4a}$$

$$\Phi^a=q\,\eta\frac{x_a}{r}[1-u(r)], \tag{2.4b}$$

where $r=|x|$ and $q=\pm 1$–is a magnetic charge. The functions f and u fall exponentially at the distances of order m_W^{-1} or m_H^{-1}. At the origin they behave so that the solution is nonsingular. The monopoles have a finite action and their contribution can be calculated by the conventional semiclassical techniques. In the dilute gas approximation the Boltzmann weight of a single pseudoparticle and their interactions are treated separately. The one-particle partition function ζ is then obtained by the loop expansion around the classical solution (2.4):

$$\zeta=\mathrm{const}\,\frac{m_W^{7/2}}{g}\,\mathrm{e}^{-(4\pi m_W/g^2)\epsilon(m_H/m_W)}. \tag{2.5}$$

The exponential is the classical action of the monopole. The dimensionless function $\epsilon(m_H/m_W)$ varies from $\epsilon(0)=1$ [12] to $\epsilon(\infty)=1.787\cdots$ [13]. The constant in the pre-exponential factor is determined by the loop corrections and is expected to be of order unity for the values of the parameters satisfying Eq. (2.3). However, it is known that the one-loop contribution diverges in the Bogomol'nyi-Prasad-Sommerfield (BPS) limit $m_H\to 0$ [14]. Hence, the BPS limit lies outside the region of applicability of the semiclassical approximation.

The monopoles interact as Coulomb charges of the magnitude $\sqrt{4\pi}/g$. The vacuum of the theory is, therefore, a Coulomb gas of monopoles and antimonopoles. At weak coupling the monopole gas is dilute. The dependence of the monopole density on couplings was studied numerically in Ref. [10] and was found to be actually small in the perturbative regime. In the dilute gas approximation, the monopoles contribute to the correlation functions via their classical long-range fields. Obviously, only the Abelian fields of the solution (2.4) survive on the distances much larger than the monopole size. This can be checked by transforming the classical solution to the unitary gauge $\Phi^1=0=\Phi^2$. The remaining long-range component $A_\mu\equiv\mathcal{A}^3_\mu$ obeys the superposition principle for multimonopole configurations. Zero mode integration in the functional integral leads to the averaging over all configurations of this type.

Although the contribution of the pseudoparticles is exponentially small, it has more important consequences than powerlike perturbative corrections, and the monopoles should be retained in the low energy Abelian theory. Due to the Debye screening by monopoles the photon acquires the mass [1]

$$m_\gamma^2=\frac{32\pi^2\zeta}{g^2} \tag{2.6}$$

and the Wilson loop expectation values exhibit an area law behavior with the string tension [1,9]:

$$\sigma_0=\frac{g^2m_\gamma}{2\pi^2}. \tag{2.7}$$

III. PARTITION FUNCTION

The partition function of the system defined by the action (2.1) at the temperature T is conventionally represented by the functional integral with periodic boundary conditions in the imaginary time:

$$Z=\int [d\mathcal{A}][d\Phi]\mathrm{e}^{-S-S_{\mathrm{gf}}-S_{\mathrm{gh}}}, \tag{3.1}$$

where S_{gf} and S_{gh} are gauge fixing and ghost terms, respectively. All fields are periodic in x_0 with the period $\beta=1/T$. The integral over x_0 in Eq. (2.1) is also assumed to range from 0 to β.

At sufficiently low temperatures,

$$T \ll m_W, \tag{3.2}$$

only Abelian degrees of freedom are relevant. Apart from the free photons, we must also take into account the monopole contribution. The corresponding classical field configurations now do not coincide with Eq. (2.4), because they should respect the periodic boundary conditions. This can be easily achieved by considering the periodic chains of monopoles placed at the points with coordinates $x_\mu^{(n)}=x_\mu+\delta_{\mu\ 0}n\beta$. Since $\beta m_W \gg 1$, the distance between neighboring monopoles in the chain is much larger than their size and such classical configurations can be treated within the dilute gas approximation, so the one-particle partition function for the periodic monopole in this approximation is the same as in Eq. (2.5). To be more precise, one should also take into account the Coulomb repulsion of elementary monopole from its images in the chain, but it is convenient to consider this repulsion as a part of the interaction energy, when we take into account the multimonopole configurations. For the interaction energy in the gas of the monopoles, therefore, we have

$$S_{\mathrm{int}}=\frac{2\pi}{g^2}\sum_{a,b}\sum_{n}{}'\frac{q_a q_b}{|x_a-x_b^{(n)}|}, \tag{3.3}$$

where the first sum runs over all pseudoparticles, $q_a=\pm 1$ are their magnetic charges and the prime means that the term with $n=0$ is omitted for $a=b$.

The interaction energy is divergent in the infrared, unless the total magnetic charge Σq_a is equal to zero. So, strictly speaking, it is necessary to insert the delta function $\delta_{\Sigma q_a,0}$ in the summation over all monopole configurations. However, the neutrality condition in the Coulomb plasma is satisfied automatically [1], infrared divergencies cancel by themselves and there is no need to worry about them. These properties are essentially the consequences of the Debye screening.

The properly regularized sum

$$G(x)=\sum_{n=-\infty}^{+\infty}\frac{1}{\sqrt{\mathbf{x}^2+(x_0-n\beta)^2}} \tag{3.4}$$

defines the periodic Green function of the Laplace operator:

$$-\partial^2 G(x)=4\pi\delta(\mathbf{x})\sum_{n=-\infty}^{+\infty}\delta(x_0-n\beta). \tag{3.5}$$

Therefore, in the dilute gas approximation, the partition function has the following form:

$$Z=Z_{\mathrm{ph}}\sum_{N=0}^{\infty}\frac{\zeta^N}{N!}\sum_{q_a=\pm 1}\int\prod_{a=1}^{N}d^3x_a\ \exp\left(-\frac{2\pi}{g^2}\times\sum_{a,b}q_a q_b G(x_a-x_b)\right). \tag{3.6}$$

The Boltzmann factor corresponds to the interaction of monopoles and antimonopoles, and the fugacity is determined by the one-monopole partition function. We denote a free photon contribution by Z_{ph}:

$$Z_{\mathrm{ph}}=\int [DA]\exp\left(-\frac{1}{4g^2}\int_0^\beta dx_0\int d^2x F_{\mu\nu}F_{\mu\nu}\right). \tag{3.7}$$

Here $F_{\mu\nu}$ is the Abelian field strength $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$. We use the unitary gauge $\Phi^1=0=\Phi^2$, in which $A_\mu\equiv\mathcal{A}_\mu^3$. The correlation functions receive the contribution both from the free photon part and from the monopoles.

At low temperature the partition function (3.6) describes the globally neutral Coulomb plasma. But, as the temperature is raised, the monopoles form bound states with antimonopoles and the system passes to the molecular phase. The existence of the BKT phase transition from the plasma to the molecular phase can be demonstrated by the following simple argument due to Kosterlitz and Thouless [5]. The Green function (3.4) behaves at large spatial separation, $|\mathbf{x}|\gg\beta$, as

$$G(x)\simeq-\frac{2}{\beta}\ln(|\mathbf{x}|\mu), \tag{3.8}$$

where μ is an IR cutoff. Restricting ourselves to the two–particle partition function, we find that the mean squared separation between monopole and antimonopole diverges at low temperature as

$$\langle r^2\rangle\sim\int d^2x|\mathbf{x}|^2\mathrm{e}^{(4\pi/g^2)G(x_0,\mathbf{x})}\sim\int^\infty dr\, r^{3-8\pi/g^2\beta}.$$

But beyond the critical point,

$$T_c=\frac{g^2}{2\pi}, \tag{3.9}$$

the integral converges at large distances. Consequently, the mean separation becomes finite and monopoles and antimonopoles form the bound states.

There is no Debye screening in the molecular phase of the monopole gas. On the other hand, the Debye screening of monopoles is responsible for the linear confining forces between electric charges. So, the BKT phase transition is associated with the deconfinement of electric charge in the adjoint Higgs model. The temperature dependence of the order parameter for the deconfinement phase transition—the Polyakov loop—is discussed in the next section.

IV. POLYAKOV LOOPS

The Polyakov loop is a phase factor associated with the contour which closes due to the periodic boundary conditions

$$L(\mathbf{x})=\exp\left(\frac{i}{2}\int_0^\beta dx_0 A_0(x_0,\mathbf{x})\right). \tag{4.1}$$

It describes a static charge inserted in the vacuum at the point $\mathbf{x}$. We consider charge 1/2 Polyakov loops. The reason is that they correspond to the matter field in the fundamental representation of SU(2)—after the symmetry breaking the latter splits into the two fields of charge $\pm 1/2$.

The correlation functions of Polyakov loops play a distinguished role in gauge theories at finite temperature, since they measure the free energy of the static charge sources [6]. The expectation value of the Polyakov loop is equal to zero in the confinement phase, because the energy of a single charged particle is infinite. In principle, the Polyakov loop should acquire a nonzero expectation value in the deconfinement phase, but it is not the case for the model under consideration due to the infrared divergencies related to the low dimensionality of the problem [4]. More appropriate parameter is the two-point correlator of the Polyakov loops, which is related to the interaction potential between particles of opposite charge [6]:

$$\langle L(\mathbf{x})L^\dagger(\mathbf{y})\rangle = \mathrm{e}^{-\beta V(\mathbf{x}-\mathbf{y})}. \tag{4.2}$$

In the confinement phase the potential grows linearly at large separation between charges, which is equivalent to the screening of the Polyakov loops:

$$\langle L(\mathbf{x})L^\dagger(\mathbf{y})\rangle \sim \mathrm{e}^{-\beta\sigma|\mathbf{x}-\mathbf{y}|}. \tag{4.3}$$

The screening length determines the string tension σ. The deconfinement transition is associated with the disappearance of the screening, and in the deconfinement phase the two-point correlator has a power-law behavior [4]

$$\langle L(\mathbf{x})L^\dagger(\mathbf{y})\rangle \sim |\mathbf{x}-\mathbf{y}|^{-\beta\alpha}, \tag{4.4}$$

indicating the Coulomb logarithmic interaction of charged particles.

In the dilute gas approximation, the monopoles contribute to the Polyakov loop via their classical fields. The long range field of the 't Hooft–Polyakov solution (2.4) in the unitary gauge coincides with that of a Dirac monopole

$$\widetilde{F}_{\mu\nu}=q\varepsilon_{\mu\nu\lambda}\frac{x_\lambda}{r^3}. \tag{4.5}$$

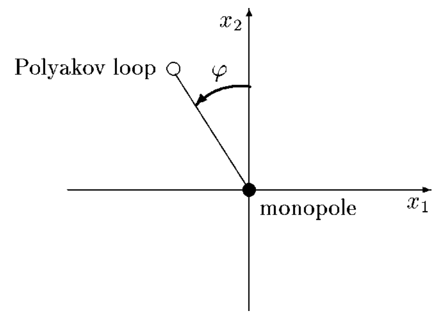

FIG. 1. The definition of $\varphi(\mathbf{x})$.

Here and below we mark by tilde the fields in the infinite Euclidean space which do not obey periodic boundary conditions. The gauge potentials for the Dirac monopole can be chosen as follows:

$$\widetilde{A}_0=-q\left(1-\frac{x_2}{r}\right)\frac{x_1}{x_0^2+x_1^2}, \tag{4.6a}$$

$$\widetilde{A}_1=q\left(1-\frac{x_2}{r}\right)\frac{x_0}{x_0^2+x_1^2}, \tag{4.6b}$$

$$\widetilde{A}_2=0. \tag{4.6c}$$

The Dirac string in this gauge is directed along the x_2 axis.

The relevant classical configurations at finite temperature are the periodic chains of monopoles. For them,

$$A_\mu(x_0,\mathbf{x})=\sum_{n=-\infty}^{+\infty}\widetilde{A}_\mu(x_0+n\beta,\mathbf{x}). \tag{4.7}$$

To calculate the contribution of the periodic monopole to the Polyakov loop, we need not perform the summation in (4.7) explicitly, since

$$\int_0^\beta dx_0 A_0(x_0,\mathbf{x})=\sum_{n=-\infty}^{+\infty}\int_0^\beta dx_0\widetilde{A}_0(x_0+n\beta,\mathbf{x})$$
$$=\int_{-\infty}^{+\infty}dx_0\widetilde{A}_0(x_0,\mathbf{x}).$$

Substituting the field of the monopole (4.6) for $\widetilde{A}_0$ we find:

$$\int_0^\beta dx_0 A_0(x_0,\mathbf{x})=\left(2\arctan\frac{x_2}{x_1}-\pi\right)q. \tag{4.8}$$

This result shows that the contribution of a single monopole to the Polyakov loop $L(\mathbf{x})$ is equal to $\mathrm{e}^{iq\varphi(\mathbf{x})}$, where $\varphi(\mathbf{x})$ is an angle of the direction from the monopole position to the point $\mathbf{x}$ with x_2 axis, as depicted in Fig. 1. The contributions of individual monopoles sum up and, for example, for the two-point correlator of Polyakov loops we get

$$
\langle L(\mathbf{x})L^\dagger(\mathbf{y})\rangle=\frac{Z_{\rm ph}}{Z}\sum_{N=0}^{\infty}\frac{\zeta^N}{N!}\sum_{q_a=\pm1}\int\prod_{a=1}^{N}d^3x_a
\times\exp\bigg[-\frac{2\pi}{g^2}\sum_{a,b}q_aq_bG(x_a-x_b)
+i\sum_a q_a[\varphi_a(\mathbf{x})-\varphi_a(\mathbf{y})]
-\frac{g^2}{16\pi}\int_0^\beta d\xi_0\int_0^\beta d\eta_0[G(\xi_0-\eta_0,\mathbf{0})
-G(\xi_0-\eta_0,\mathbf{x}-\mathbf{y})]\bigg]. \tag{4.9}
$$

The last term in the exponential represents a perturbative contribution of the free photons. The generalization of this formula to multipoint correlation functions is obvious.

The partition function (3.6) has the convenient functional representation [1]

$$
Z=Z_{\rm ph}\int[d\chi]\exp\bigg\{-\int_0^\beta dx_0\int d^2x\bigg[\frac{g^2}{32\pi^2}(\partial\chi)^2
-2\zeta\cos\chi\bigg]\bigg\}. \tag{4.10}
$$

The expansion of this path integral in ζ generates the grand canonical partition function for monopoles (3.6), which can be checked using Eq. (3.5) in the calculation of Gaussian integrals over χ.

Correlation functions also admit the functional representation. Our present goal is to find the operators in the sine-Gordon theory (4.10) corresponding to the Polyakov loops. The correlation functions which reduce to the form similar to Eq. (4.9) have been considered in Ref. [15] for 2D lattice model, closely related to the sine-Gordon theory. Motivated by these results, we propose the following identification:

$$
L(\mathbf{x})=\exp\bigg(\frac{g^2}{8\pi}\int_0^\beta d\xi_0\int_{\mathbf{x}}d\xi_i\varepsilon_{ij}\partial_j\chi\bigg), \tag{4.11a}
$$

$$
L^\dagger(\mathbf{y})=\exp\bigg(\frac{g^2}{8\pi}\int_0^\beta d\xi_0\int^{\mathbf{y}}d\xi_i\varepsilon_{ij}\partial_j\chi\bigg). \tag{4.11b}
$$

Here the contour of integration over ξ_i goes to infinity in the (x_1,x_2) plane or can end on the Polyakov loop of the opposite charge. These formulas should be understood as operator equalities—any correlation functions of the Polyakov loops in the U(1) gauge theory with monopoles are equal to the correlation functions of the operators on the right-hand side, where the averaging over χ is defined by the partition function (4.10).

The identification (4.11) is established by developing the correlation functions of the operators (4.11) in ζ. As a result, one recovers correlation functions for the Coulomb gas of type (4.9). In the specific case of the two-point correlator,

$$
\bigg\langle\exp\bigg(\frac{g^2}{8\pi}\int_0^\beta d\xi_0\int_{\mathbf{x}}^{\mathbf{y}}d\xi_i\varepsilon_{ij}\partial_j\chi\bigg)\bigg\rangle
=\frac{Z_{\rm ph}}{Z}\sum_{N=0}^{\infty}\frac{\zeta^N}{N!}\sum_{q_a=\pm1}\int\prod_{a=1}^{N}d^3x_a\bigg\langle\exp\bigg(i\sum_a q_a\chi(x_a)
+\frac{g^2}{8\pi}\int_0^\beta d\xi_0\int_{\mathbf{x}}^{\mathbf{y}}d\xi_i\varepsilon_{ij}\partial_j\chi\bigg)\bigg\rangle_0, \tag{4.12}
$$

where $\langle\cdots\rangle_0$ denotes the Gaussian average over χ. This average after some transformations can be represented in the following form:

$$
\bigg\langle\exp\bigg(i\sum_a q_a\chi(x_a)+\frac{g^2}{8\pi}\int_0^\beta d\xi_0\int_{\mathbf{x}}^{\mathbf{y}}d\xi_i\varepsilon_{ij}\partial_j\chi\bigg)\bigg\rangle_0
=\exp\bigg(-\frac{2\pi}{g^2}\sum_{a,b}q_aq_bG(x_a-x_b)
+\frac{i}{2}\sum_a q_a\int_0^\beta d\xi_0\int_{\mathbf{x}}^{\mathbf{y}}d\xi_i\varepsilon_{ij}\partial_jG(\xi-x_a)
-\frac{g^2}{32\pi}\int_0^\beta d\xi_0\int_{\mathbf{x}}^{\mathbf{y}}d\xi_i\varepsilon_{ij}\int_0^\beta d\eta_0
\times\int_{\mathbf{x}}^{\mathbf{y}}d\eta_k\varepsilon_{kl}\partial_j\partial_lG(\xi-\eta)\bigg)
=\exp\bigg[-\frac{2\pi}{g^2}\sum_{a,b}q_aq_bG(x_a-x_b)
+i\sum_a q_a[\varphi_a(\mathbf{x})-\varphi_a(\mathbf{y})]-\frac{g^2}{16\pi}\int_0^\beta d\xi_0
\times\int_0^\beta d\eta_0[G(\xi_0-\eta_0,\mathbf{0})-G(\xi_0-\eta_0,\mathbf{x}-\mathbf{y})]\bigg]. \tag{4.13}
$$

This result is a simple generalization of the formula which has been used in Ref. [16] to construct a path-integral version of soliton operators in 2D sine-Gordon theory. In the derivation we have assumed that the regularization is used in which $\delta(0)=0$. Substitution of this expression in Eq. (4.12) reduces the latter to the form (4.9).

The average (4.12) in the sine-Gordon theory reproduces not only the monopole contribution to the correlator of the Polyakov loops, but the whole answer containing also the free photon part. It is interesting that the same property holds for the string representation proposed in Ref. [3] for the Wilson loop averages at zero temperature—this representation automatically encounters the perturbative photon contribution. The Gaussian nature of the functional averages in each order of the expansion in ζ ensures the validity of the identification (4.11) for arbitrary correlation functions. The operators (4.11) possess a number of peculiar properties which are discussed in the next section.

V. DIMENSIONAL REDUCTION

The partition function (4.10) defines the interacting field theory in three dimensions. Although the nonlinearity is caused by instantons and the coefficient before cosine ζ, is of order $\exp(-\mathrm{const}\; m_W/g^2)$, the interaction in the theory is not weak. The point is that ζ is the dimensional parameter and the strength of interaction depends on the scale. At very large distances, of order of the inverse photon mass $1/m_\gamma$ defined by Eq. (2.6), the effects of interaction are not small and they cannot be neglected. The consideration of a system at the finite temperature, actually, introduces a characteristic scale $\beta = 1/T$. Some simplifications do occur, when this scale is much smaller than the correlation length, i.e., when the temperature is not extremely low:

$$m_\gamma \ll T. \tag{5.1}$$

As before, the interaction cannot be completely neglected, rather some degrees of freedom become irrelevant. The fields with nontrivial dependence on the time coordinate x_0 decouple, since their Matsubara frequencies are large compared to the characteristic mass scale determined by m_γ. This argument justifies the dimensional reduction procedure, typical for the field theory at finite temperature. As a result of this procedure, we are left with the two-dimensional sine-Gordon model:

$$S_{\text{reduced}} = \int d^2x \left[\frac{g^2\beta}{32\pi^2}(\partial\chi)^2 - 2\zeta\beta \cos\chi \right]. \tag{5.2}$$

In the Coulomb gas picture, the dimensional reduction can be understood as a substitution of the large distance asymptotics (3.8) for the exact three-dimensional Green function in Eq. (3.6). The discussion at the end of Sec. III demonstrates that the time dependence is irrelevant for the description of the deconfinement phase transition [4]. Moreover, the condition (5.1) shows that the dimensional reduction is a good approximation in a wide range of temperatures below the phase transition. This circumstance allows us to use the reduced theory in the study of the confinement phase. The dimensional reduction actually leads to substantial simplification, since 2D sine-Gordon model is a classic example of completely integrable field theory and some of its properties are known exactly.

The conventional sine-Gordon action is obtained after the rescaling of the field $\phi(\mathbf{x}) \equiv \chi(\mathbf{x}) g\sqrt{\beta}/4\pi$:

$$S_{\text{SG}} = \int d^2x \left[\frac{1}{2}(\partial\phi)^2 - 2\mu \cos b\phi \right], \tag{5.3}$$

where

$$b^2 = \frac{16\pi^2 T}{g^2} = 8\pi \frac{T}{T_c}. \tag{5.4}$$

This parameter plays the role of a Plank constant in the sine-Gordon theory [17]. Consequently, the low-temperature limit of the theory is semiclassical.

The parameter μ depends on the renormalization scheme, i.e., on a definition of the operator $\cos b\phi$. The conventional regularization procedure is based on the normal ordering. Since the dimension of the operator $:\cos b\phi:$ is $b^2/4\pi$, the coupling μ differs from its ''bare'' value $\zeta\beta$ by the factor $\Lambda^{-b^2/4\pi}$, where Λ is an UV cutoff. It is worth mentioning that the dimensional arguments clarify the origin of the BKT phase transition—at the critical point the dimension of the operator $:\cos b\phi:$ is 2, and the perturbation in Eq. (5.3) becomes marginal.

The temperature defines the natural UV cutoff of 2D theory. Actually, the dimensional reduction is valid only at distances much larger than the inverse temperature, on smaller scales the dynamics is governed by the full 3D theory. In particular, the exact propagator $G(x)$ cannot be replaced by its large distance asymptotics for $|\mathbf{x}| \sim \beta$. However, due to the renormalizability all the effects of the dynamics at short distances can be absorbed into a multiplicative renormalization of μ. Thus, the UV cutoff is proportional to the temperature with some numerical coefficient depending on the renormalization scheme. For the normal ordering prescription this coefficient is calculated in the Appendix:

$$\Lambda = \frac{e^\gamma}{2} T, \tag{5.5}$$

where $\gamma = 0.5772\cdots$ is the Euler constant. Thus, the renormalized value of μ is given by

$$\mu = \frac{\zeta}{T}\left(\frac{e^\gamma}{2}T\right)^{-b^2/4\pi} = \frac{g^2 m_\gamma^2}{32\pi^2 T}\left(\frac{e^\gamma}{2}T\right)^{-4\pi T/g^2}. \tag{5.6}$$

A. Polyakov loops and solitons

The peculiar property of the sine-Gordon theory is the presence of solitons in the spectrum of physical excitations. Classically, solitons are finite energy solutions of the equations of motion. They are very massive in the semiclassical region, but all more light particles can be viewed as soliton-antisoliton bound states [17]. The existence of solitons is closely related to the hidden U(1) symmetry generated by the topological current

$$j_i = \frac{b}{2\pi}\varepsilon_{ij}\partial_j\phi, \tag{5.7}$$

which is identically conserved. Solitons carry a unit charge corresponding to this current.

According to the results of the previous section, we associate with the Polyakov loops the following operators:

$$L(\mathbf{x}) = \exp\left(\frac{g^2}{8\pi T}\int_{\mathbf{x}} d\xi_i \varepsilon_{ij}\partial_j\chi\right) = \exp\left(\frac{2\pi}{b}\int_{\mathbf{x}} d\xi_i\varepsilon_{ij}\partial_j\phi\right), \tag{5.8a}$$

$$L^\dagger(\mathbf{y}) = \exp\left(\frac{g^2}{8\pi T}\int^{\mathbf{y}} d\xi_i \varepsilon_{ij}\partial_j\chi\right) = \exp\left(\frac{2\pi}{b}\int_{\mathbf{y}} d\xi_i\varepsilon_{ij}\partial_j\phi\right). \tag{5.8b}$$

It turns out that these operators have topological charges 1 and -1, respectively. There are many possibilities to demonstrate this fact. For example, one can use operator product expansion (OPE) of the topological currents

$$j_i(\mathbf{x})j_k(0)=-\frac{b^2}{8\pi^3}\left(\delta_{ik}-2\frac{x_ix_k}{|\mathbf{x}|^2}\right)\frac{1}{|\mathbf{x}|^2}+\cdots \qquad (5.9)$$

to show that

$$j_i(\mathbf{x})L(0)=-\frac{1}{2\pi}\frac{x_i}{|\mathbf{x}|^2}L(0)+\cdots, \qquad (5.10a)$$

$$j_i(\mathbf{x})L^\dagger(0)=\frac{1}{2\pi}\frac{x_i}{|\mathbf{x}|^2}L^\dagger(0)+\cdots. \qquad (5.10b)$$

Defining the topological charge operator

$$Q=\oint_C dx_i\varepsilon_{ik}j_k(\mathbf{x}), \qquad (5.11)$$

where the contour C encircles counterclockwise the origin, we readily find that L ($L^\dagger$) behaves as charge 1 (-1) operator under U(1) transformations generated by Q:

$$QL(0)=L(0), \qquad (5.12a)$$

$$QL^\dagger(0)=-L^\dagger(0). \qquad (5.12b)$$

On the other hand, the Polyakov loop, by its definition, creates an electric charge. This means that the following identification holds: topological charge=2×electric charge, and the U(1) symmetry of the sine-Gordon model corresponds to the invariance of the original three-dimensional theory under global gauge transformations.

The fact that Polyakov loops create solitons has important consequences. In particular, the large distance behavior of the two-point correlator of the Polyakov loops is governed by the lightest state with the topological charge one, i.e., by the soliton:

$$\langle L(\mathbf{x})L^\dagger(\mathbf{y})\rangle\sim \mathrm{e}^{-M|\mathbf{x}-\mathbf{y}|}. \qquad (5.13)$$

Here M is a soliton mass. Comparing this expression with Eq. (4.3) we find that the temperature dependence of the string tension is determined by the mass of the soliton

$$\sigma(T)=TM. \qquad (5.14)$$

In the zero temperature limit the semiclassical approximation is valid. The classical mass of the soliton is equal to $8\sqrt{2\mu}/b$ [17], and we obtain

$$\sigma(0)=\frac{8T\sqrt{2\mu}}{b}=\sqrt{\frac{8g^2\zeta}{\pi^2}}=\sigma_0,$$

where σ_0 is given by Eq. (2.7). Thus, the result of Ref. [9] is recovered.

In fact, the soliton mass in the sine-Gordon theory is known exactly [18]

$$M=\frac{2\Gamma(p/2)}{\sqrt{\pi}\Gamma[(p+1)/2]}\left(\frac{\pi\Gamma[1/(p+1)]}{\Gamma[p/(p+1)]}\mu\right)^{(p+1)/2}, \qquad (5.15)$$

where

$$p=\frac{b^2}{8\pi-b^2}=\frac{T}{T_c-T}. \qquad (5.16)$$

After some algebra we obtain for the string tension

$$\sigma(T)=\sigma_0\left(\frac{m_\gamma}{\mathrm{e}^\gamma T}\right)^{T/(T_c-T)}\left(\frac{\Gamma[(T_c-T)/T_c]}{\Gamma[(T_c+T)/T_c]}\right)^{T_c/(2T_c-2T)}\times\frac{\Gamma^2[(2T_c-T)/(2T_c-2T)]}{\Gamma[(2T_c-T)/(T_c-T)]}. \qquad (5.17)$$

It is worth mentioning that the dependence of the soliton mass on μ and, consequently, of the string tension on m_γ, follows from a dimensional consideration. The remaining factor, which is a function of b^2 or, equivalently, of the adimensional ratio T/T_c, cannot be found by elementary methods.

B. Deconfinement phase transition

The temperature dependence of the string tension is shown in Fig. 2. At low temperatures, $m_\gamma\ll T\ll T_c$, the string tension falls rapidly:

$$\sigma\simeq\sigma_0\left(\frac{m_\gamma}{\mathrm{e}^\gamma T}\right)^{T/T_c}\sim\sigma_0\,\exp\left(-\frac{4\pi^2\epsilon(m_H/m_W)m_WT}{g^4}\right). \qquad (5.18)$$

The increase of $\sigma(T)$ at $T\sim m_\gamma$, certainly, is an artifact of the dimensional reduction which is inapplicable at such small temperatures. It is interesting that the low-temperature behavior of the string tension is completely determined by the dimensional arguments.

Let us turn to the critical behavior. The string tension has an essential singularity at the point of the phase transition:

$$\sigma=\sigma_0\sqrt{\frac{\pi}{2}}\,\mathrm{e}^{3/2-\gamma}\left[\frac{m_\gamma^2}{4\mathrm{e}^{2\gamma}T_c(T_c-T)}\right]^{T/(2T_c-2T)}\times\left[1+O\left(\frac{T_c-T}{T_c}\right)\right]. \qquad (5.19)$$

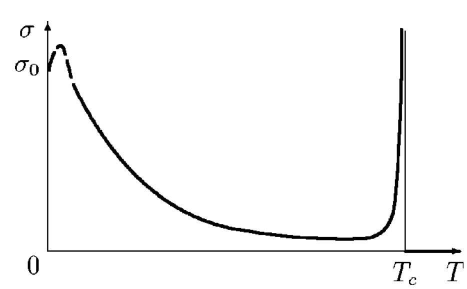

FIG. 2. The string tension σ(T).

The string tension decreases up to the temperature very close to the transition point

$$T_m \simeq T_c - \frac{m_\gamma^2}{4e^{2\gamma-1}T_c},$$

where it reaches a vanishingly small value,

$$\sigma_{\min} = \sigma_0 \sqrt{\frac{\pi}{2}} \exp\left(2-\gamma-\frac{2e^{2\gamma-1}T_c^2}{m_\gamma^2}\right)\left[1+O\left(\frac{m_\gamma^2}{T_c^2}\right)\right].$$

Then the string tension grows rapidly and at the critical point turns to infinity. This result is essentially nonperturbative. It cannot be obtained from the dimensional consideration. Were the dimensional analysis sufficient, the critical behavior of the string tension would be determined by the second factor in Eq. (5.17) and the string tension would vanish at $T = T_c$. This is not the case. Actually, the third factor has a stronger singularity and becomes more important at the temperatures very close to the point of the phase transition.

C. Confinement phase

Certainly, the reduced 2D theory is much simpler than the original 3D model. On the other hand, the dimensional reduction is a good approximation up to very low temperatures, and one might think that the results obtained for the reduced model can be even extrapolated to zero temperature. The fact that the string tension obtained from the mass of the soliton in two dimensions agrees with the result of the semiclassical analyses of Wilson loops directly in the three-dimensional theory [1,9] supports this assumption. There are two points we are going to discuss.

(i) Perturbatively, the only light particle in the 3D adjoint Higgs model is the photon with the mass m_γ. Other degrees of freedom have substantially larger masses m_W and m_H. It was conjectured that the dynamics in the intermediate region may be nontrivial and there are excitations with the masses $m_\gamma \ll m \ll m_W$ [3]. The known exact spectrum of the reduced theory provides a way to check this conjecture, since nonperturbative degrees of freedom should manifest themselves somehow at finite but small temperatures, $T_c \gg T \gg m_\gamma$.

(ii) Another interesting problem is the string representation for the Polyakov loop correlators.

We begin with the consideration of the spectrum. The semiclassical spectrum of 2D sine-Gordon theory [17] turns out to be exact [19] and consists of solitons, antisolitons, and their bound states (breathers). The lightest bound state can be identified with the perturbative excitation of the sine-Gordon field [17] which, in turn, corresponds to a dual photon of the original 3D theory.

Solitons and antisolitons disappear from the spectrum at $T \to 0$, since their mass determines the string tension by Eq. (5.14) and the string tension has the finite zero temperature limit. The exact masses of the breathers are given by the following expression [17,19]:

$$m_n = 2M \sin\frac{\pi n p}{2} \left(n = 1,2,\ldots;\ n < \frac{1}{p}\right), \qquad (5.20)$$

where M is the soliton mass and p is defined in Eq. (5.16). The number of bound states depends on the coupling constant b, i.e., on the temperature. If the breathers survive the zero temperature limit, their number becomes infinite and they form a linear ''Regge trajectory'':

$$m_n \to n m_\gamma. \qquad (5.21)$$

A possible interpretation of these particles is that they are threshold bound states of n photons. The existence of excitations with masses which can be arbitrarily large compared to m_γ, but which are much smaller than m_W, gives the strong evidence of a nontrivial dynamics in the intermediate region.

Probably the most important feature of U(1) gauge theory with monopoles is its connection to string theory. On the level of classical equations of motion for the Wilson loop, a surface spanned by the loop parametrize naturally corresponding classical solution [1]. The arguments were given that, beyond the semiclassical approximation, this surface fluctuates and Wilson loop average can be represented as a sum over random surfaces with some weight depending on the surface [3]. This representation will be practically useful, if the effective string action has a simple form in some reasonable approximation.

The same reasoning should be valid for the two-point correlator of Polyakov loops $\langle L(\mathbf{x})L^\dagger(\mathbf{y})\rangle$. In this case, the string world sheet is spanned by the contour $\mathcal{L}_\mathbf{x}\Gamma_{\mathbf{xy}}\mathcal{L}_\mathbf{y}^{-1}\Gamma_{\mathbf{xy}}^{-1}$, where $\mathcal{L}_\mathbf{z} = \{(x_0,\mathbf{z})|0 \leqslant x_0 \leqslant \beta\}$ and $\Gamma_{\mathbf{xy}}$ is some curve connecting $\mathbf{x}$ with $\mathbf{y}$ in the (x_1,x_2) plane. We expect that at sufficiently high temperatures, $\sqrt{\sigma_0} \ll T \ll T_c$, the temporal fluctuations of the string are irrelevant, in other words, the typical surfaces have the form $[0,\beta]\times\Gamma_{\mathbf{xy}}$. So, the sum over random surfaces is replaced by a sum over paths:

$$\langle L(\mathbf{x})L^\dagger(\mathbf{y})\rangle = \int_{\mathbf{X}(0)=\mathbf{x}}^{\mathbf{X}(1)=\mathbf{y}} [d\mathbf{X}] e^{-S[\mathbf{X}(\tau)]}. \qquad (5.22)$$

An important remark concerning this representation is in order. According to Ref. [3], the string world sheet for a given field configuration $\chi(x)$ is defined by the equation $\cos\chi(X) = -1$. This is consistent with semiclassical analyses [3]. On the other hand, in the reduced 2D sine-Gordon model the same equation, $\cos b\phi(\mathbf{X}) = -1$, is a conventional definition of a soliton path [20] for a field configuration $\phi(\mathbf{x})$. Thus, the paths in the functional integral (5.22) are interpreted as soliton trajectories. It would be interesting to find such representation for the two-point correlator of the Polyakov loop operators (5.8). Being the counterpart of the string representation for the Wilson loop averages, it may help to clarify the dynamics of the confining string.

At present, we do not know how to construct the representation (5.22) with a reasonably simple action $S[\mathbf{X}(\tau)]$. We know only one example of the simple sum-over-path representation for correlation functions—the case when the operators are described by a local field theory. The sine-Gordon solitons are actually described by a local theory, namely, by the massive Thirring model [21,22]

$$S=\int d^2x\left(\overline{\psi}\gamma_\mu\partial_\mu\psi+M_T\overline{\psi}\psi+\frac{g_T}{2}\overline{\psi}\gamma_\mu\psi\overline{\psi}\gamma_\mu\psi\right), \tag{5.23}$$

where $g_T=4\pi^2/b^2-\pi$. However, the solitons obey Fermi statistics and soliton operators [22] (more precisely, their Euclidean-space counterparts [16]) differ from the Polyakov loops (5.8) by an additional factor:

$$\psi(\mathbf{x})=\begin{pmatrix} :\exp\left(\frac{2\pi}{b}\int^{\mathbf{x}}d\xi_i\varepsilon_{ij}\partial_j\phi+\frac{i}{2}b\phi(\mathbf{x})\right): \\ -i:\exp\left(\frac{2\pi}{b}\int^{\mathbf{x}}d\xi_i\varepsilon_{ij}\partial_j\phi-\frac{i}{2}b\phi(\mathbf{x})\right): \end{pmatrix}, \tag{5.24a}$$

$$\overline{\psi}(\mathbf{x})=\begin{pmatrix} i:\exp\left(\frac{2\pi}{b}\int_{\mathbf{x}}d\xi_i\varepsilon_{ij}\partial_j\phi+\frac{i}{2}b\phi(\mathbf{x})\right): \\ :\exp\left(\frac{2\pi}{b}\int_{\mathbf{x}}d\xi_i\varepsilon_{ij}\partial_j\phi-\frac{i}{2}b\phi(\mathbf{x})\right): \end{pmatrix}. \tag{5.24b}$$

These operators have an interesting interpretation in the gauge theory. The operator $\exp(\pm i/1b\phi)=\exp(\pm 1/2\chi)$ creates a monopole with magnetic charge $\pm 1/2$. So, from this point of view, the soliton can be regarded as a ''dyon''—the superposition of magnetic and electric charges, both equal to 1/2 in magnitude.

The free fermion propagator has the well known sum-over-path representation with an additional integration over Grassmannian world-line variables [23]. The world-line action in this representation is supersymmetric, although the supersymmetry is broken by boundary conditions. This allows us to speculate that a modification of the Wilson loop by particular monopole operators in 3D theory has a string representation with supersymmetric world-sheet action. The loop corrections due to the four-fermion term in Eq. (5.23) can be regarded as contact string interactions. Unfortunately, in the zero temperature limit both the mass M_T and the coupling constant g_T becomes infinite.

VI. SYMMETRY RESTORATION

In the previous sections we have dealt with the low-energy Abelian theory completely disregarding the massive fields. The only remnant of the spontaneously broken non-Abelian gauge symmetry relevant for this consideration was the presence of magnetic monopoles with finite action. This approximation, valid at sufficiently low temperatures, becomes more and more worse as the temperature is raised, since the thermal fluctuations decrease the mass scale of the complete non-Abelian theory. Ultimately, W bosons become massless and the non-Abelian symmetry is restored. In the perturbative region (2.3), the critical temperature should be very high, because the zero temperature value of the Higgs boson vacuum average is large and thermal fluctuations which decrease it to zero should be sufficiently strong. In this section we study the symmetry restoring phase transition in the framework of thermal perturbation theory.

We add to the action (2.1) the gauge fixing term

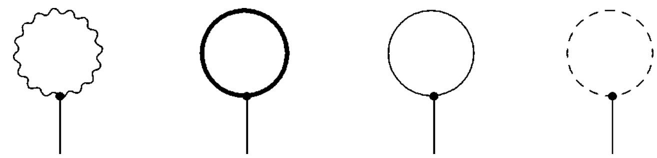

FIG. 3. Tadpole diagrams corresponding to the contribution of W bosons, complex scalar field $\Phi^{\pm}=(\Phi^1\pm i\Phi^2)/\sqrt{2}$, Higgs boson, and ghosts to the vacuum expectation value of the Higgs field.

$$S_{\mathrm{gf}}=\frac{1}{2\alpha g^2}\int d^3x(\partial_\mu\mathcal{A}^a_\mu+\alpha g^2\varepsilon^{abc}v^b\Phi^c)^2, \tag{6.1}$$

where α is a gauge parameter. This is a conventional choice which is convenient because it cancels a mixing between W bosons and the complex scalar field $\Phi^{\pm}=(\Phi^1\pm i\Phi^2)/\sqrt{2}$ coming from the covariant derivative of Φ squared. We always assume that the vacuum value of the scalar field is directed along the third axis in the color space and denote it by v: $v^a=\delta^{a3}\eta$. The ghost term in the chosen gauge is

$$S_{\mathrm{gh}}=\int d^3x[\partial_\mu\overline{c^a}D_\mu c^a+\alpha g^2\eta(\Phi^3\overline{c^a}c^a-\Phi^a\overline{c^a}c^3)]. \tag{6.2}$$

The reason for the symmetry restoration phase transition is the presence of the thermal corrections to the vacuum average $\langle\Phi^3\rangle$ which is equal to η at zero temperature. In order to find them, we expand Φ^3 as $\Phi^3(x)=\eta+\sigma(x)$ and calculate $\langle\sigma\rangle$. To the lowest order of perturbation theory, $\langle\sigma\rangle$ is given by four tadpole diagrams depicted in Fig. 3. It is convenient to denote by $F(m^2)$ the contribution of a scalar loop:

$$F(m^2)=T\sum_n\int\frac{d^2p}{(2\pi)^2}\frac{1}{\mathbf{p}^2+\omega_n^2+m^2}-(T=0)(\omega_n=2\pi nT). \tag{6.3}$$

The zero temperature part is subtracted because temperature independent contribution to $\langle\Phi^3\rangle$ merely renormalize η. We can, actually, denote by η the renormalized value of the Higgs condensate. Then the sum of the diagrams depicted in Fig. 3 is found to be

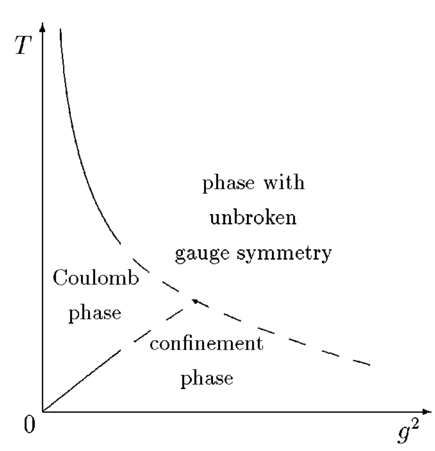

FIG. 4. The phase diagram of 3D adjoint Higgs model.

$$\langle\sigma\rangle=-\frac{1}{m_H^2}\{2g^2\eta[2F(m_W^2)+\alpha F(\alpha m_W^2)]+2\lambda\eta F(\alpha m_W^2)$$
$$+3\lambda\eta F(m_H^2)-2\alpha g^2\eta F(\alpha m_W^2)\}. \quad (6.4)$$

The loop integral (6.3) is easily calculable by Poisson resummation [24]:

$$F(m^2)=\int\frac{d^2p}{(2\pi)^2}\frac{1}{\sqrt{\mathbf{p}^2+m^2}}\frac{1}{e^{\beta\sqrt{\mathbf{p}^2+m^2}}-1}$$
$$=\frac{1}{2\pi\beta}\int_{m\beta}^{\infty}\frac{d\xi}{e^{\xi}-1}. \quad (6.5)$$

First of all, this formula shows that at the temperatures much smaller than the mass scale the corrections to the condensate are exponentially small. This result justifies the disregarding of the temperature corrections to the masses and the vacuum expectation value of the Higgs field in the previous sections. The corrections become comparable to η at a very high temperature—$T\gg m$. In this regime the function $F(m^2)$ can be expanded as

$$F(m^2)=\frac{T}{4\pi}\ln\frac{T^2}{m^2}+O(m). \quad (6.6)$$

Thus, we find with logarithmic accuracy,

$$\langle\Phi^3\rangle=\eta+\langle\sigma\rangle\simeq\eta-\frac{T\eta}{4\pi m_H^2}(5\lambda+4g^2)\ln\frac{T^2}{m_W^2}. \quad (6.7)$$

The dependence on α drops out from the final answer, as it should.

At the point of the phase transition $\langle\Phi^3\rangle$ turns to zero. The critical temperature

$$T_*=\frac{4\pi m_W^2}{g^2\{[5+8(m_W^2/m_H^2)]\ln(m_W/g^2)+O[\ln\ln(m_W/g^2)]\}}, \quad (6.8)$$

is very high in the perturbative region–$T_*\gg m_W$, as expected. The symmetry restoration phase transition, most probably, is the second order one, since the condensate of the Higgs field is continuous at the critical point in the one-loop approximation.

VII. DISCUSSION

The phase structure of 3D adjoint Higgs model can be studied in detail at weak coupling. The system undergoes the deconfinement transition of the BKT type and the second order phase transition associated with the restoration of the non-Abelian gauge symmetry. The temperatures of these transitions differ in the order of magnitude: $T_*\gg T_c$. But T_* rapidly decreases with the increase of the coupling g^2 at fixed m_W and m_H, while T_c grows. So, at $g^2\sim m_W$ the lines of deconfinement and symmetry restoration phase transitions meet at the triple point (Fig. 4). At sufficiently large g^2 the non-Abelian symmetry is restored directly in the confinement phase. The strong coupling behavior cannot be described by perturbative methods and we have nothing to say about the phase transition between confinement and symmetric phases, but numerical results of Refs. [10,11] show that at zero temperature this is a first order transition terminating at some point, so that symmetric and confinement phases are connected by analytical continuation. Probably, the same is true for the thermal phase transition as well.

Our consideration of the critical behavior at the deconfinement phase transition is based on the operator representation for the Polyakov loops in the effective sine-Gordon model describing the Coulomb gas of the monopoles. After the dimensional reduction Polyakov loop operators turn out to be closely connected with the solitons in 2D sine-Gordon theory. Using the known exact value of the soliton mass [18] we found the string tension as a function of temperature, Eq. (5.17). The string tension exhibits rather unusual behavior near the phase transition. It grows rapidly in the vicinity of the transition point and at the critical temperature diverges as $\exp(-\ln\Delta/2\Delta)$, where $\Delta=(T_c-T)/T_c$.

The interpretation of the known facts about 2D sine-Gordon model enables us to draw some conclusions about the properties of the confinement phase of 3D adjoint Higgs model. There are strong arguments in favor of nontrivial dynamics at the intermediate scales between m_γ and m_W. Perhaps, the most interesting result is a possible appearance of strings with a world-sheet supersymmetry, but this is only a conjecture which requires more serious confirmations.

ACKNOWLEDGMENTS

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APPENDIX: UV SCALE OF THE REDUCED THEORY

The normal ordered operator $:\cos b\phi:$ is defined by the two-point function in the free theory:

$$\langle :e^{ib\phi(\mathbf{x})}::e^{-ib\phi(\mathbf{y})}:\rangle_0\equiv|\mathbf{x}-\mathbf{y}|^{-b^2/2\pi}. \quad (A1)$$

On the other hand, Eq. (4.10) determines the Gaussian average in three dimensions to be

$$\langle e^{i\chi(x)}e^{-i\chi(y)}\rangle_0=e^{(4\pi/g^2)[G(0)-G(x-y)]}, \quad (A2)$$

where $G(x)$ is defined by Eq. (3.4). The Coulomb gas representation of the monopole partition function (3.6) implies the particular regularization of the divergent quantity $G(0)$. It is defined by Eq. (3.3):

$$G(0)-G(x-y)=\sum_{n\neq 0}\frac{1}{\beta|n|}-\sum_{n}\frac{1}{\sqrt{(\mathbf{x}-\mathbf{y})^2+(x_0-y_0-n\beta)^2}}=\frac{2}{\beta}\ln\frac{e^{\gamma}|\mathbf{x}-\mathbf{y}|}{2\beta}+O(e^{-2\pi|\mathbf{x}-\mathbf{y}|/\beta}). \quad \text{(A3)}$$

Here γ is the Euler constant. The dimensional reduction is equivalent to the omitting of exponential terms. Hence, we obtain

$$\langle e^{i\chi(\mathbf{x})}e^{-i\chi(\mathbf{y})}\rangle_0=\left(\frac{e^{\gamma}}{2}T\right)^{-8\pi T/g^2}|\mathbf{x}-\mathbf{y}|^{-8\pi T/g^2}\equiv\Lambda^{-b^2/2\pi}|\mathbf{x}-\mathbf{y}|^{-b^2/2\pi}. \quad \text{(A4)}$$

Comparing this equation with the correlation function of the normal ordered operators (A1) we find that the definition of the operator $\cos b\phi$ in the monopole partition function is connected with the normal ordering prescription by the multiplicative renormalization

$$\cos b\phi=\cos\chi=\Lambda^{-b^2/4\pi}{:}\cos b\phi{:}, \quad \text{(A5)}$$

and the UV scale is given by

$$\Lambda=\frac{e^{\gamma}}{2}T. \quad \text{(A6)}$$

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