Quantum gravity near the apparent horizon and two-dimensional dilaton gravity

Ichiro Oda*

Edogawa University, 474 Komaki, Nagareyama City, Chiba 270-01, Japan

(Received 25 April 1997; published 13 January 1998)

We study Hawking radiation in a two-dimensional dilaton black hole by means of quantum gravity holding near the apparent horizon. First of all, we construct the canonical formalism of the dilaton gravity in two dimensions. Then the Vaidya metric corresponding to the dilaton black hole is established where it is shown that the dilaton field takes the form of a linear dilaton. Based on the canonical formalism and the Vaidya metric, we proceed to analyze the quantum properties of a dynamical black hole. It is found that the mass loss rate of Hawking radiation is independent of the black hole mass and at the same time the apparent horizon recedes to the singularity as shown in other studies of two-dimensional gravity. It is interesting that one can construct quantum gravity even near the origin in the spherical coordinate and draw the same conclusion with respect to Hawking radiation as the above-mentioned picture. Unfortunately, the present formalism seems to be ignorant of the contributions from the functional measures over the gravitational field, the dilaton, and the ghosts. [S0556-2821(98)02604-6]

PACS number(s): 04.70.Dy

I. INTRODUCTION

More than 20 years ago, Hawking [1] showed that black holes are not completely black and emit thermal radiation with a definite temperature through quantum mechanical pair creation of particles near the horizon in a gravitational field where one member of the pair drops in a black hole while the other escapes to infinity. This result was derived in the context of the ''semiclassical'' approach, where the effects of gravitation are still represented by a classical spacetime (M,g_{ab}), while matter fields are treated as quantum fields propagating in this classical spacetime. Subsequent investigations have focused on understanding the serious problems raised by Hawking radiation, concerning the fate of quantum information [2], the statistical mechanical picture of black hole thermodynamics [3], etc.

In recent papers, a quantum formalism has been proposed for the study of black hole quantum mechanics [4,5]. The critical idea behind this formalism is that some essential features of quantum black holes might be intimately related to the quantum mechanical behavior of the black hole horizon; thus, it might be sufficient to establish that quantum gravity holds particularly near the horizon to understand an overall picture of quantum black holes. Indeed, afterward, this formalism was fruitfully applied to several problems associated with quantum black holes in three [6] and four [7] spacetime dimensions.

In the course of the applications, we have wondered to what extent quantum gravity near the horizon would describe the quantum aspects of a black hole. To address this question, it is tempting to try to apply the formalism to a well-understood model of quantum black holes, that is, dilaton gravity in $1+1$ dimensions [Callan-Giddings-Harvey-Strominger (CGHS) model] [8]. Dilaton gravity in two dimensions enjoys the nice features of black hole formation and/or evaporation shared with a spherically symmetric black hole in $3+1$ dimensions. Therefore, this toy model has raised hopes that a satisfactory description of black hole evolution might be accounted for in a very simplified setting.

This article is organized as follows. In Sec. II, we construct the canonical formalism of two-dimensional dilaton gravity. In Sec. III, we derive the Vaidya metric corresponding to the dilaton black hole. The canonical formalism and the Vaidya metric are used to construct a quantum theory holding in the vicinity of the apparent horizon of the dilaton black hole in Sec. IV. In Sec. V, we analyze Hawking radiation from a purely quantum mechanical viewpoint. Here it is shown that the mass loss rate is independent of the black hole mass. The last section is devoted to a discussion where a comparison of the present formalism with fully quantized dilaton gravity over the whole spacetime region is commented on.

II. ADM CANONICAL FORMALISM OF TWO-DIMENSIONAL DILATON GRAVITY

We begin our investigations by constructing the Arnowitt-Deser-Misner (ADM) first-order canonical formalism of two-dimensional dilaton gravity.

The action that we start with has the well-known form [8]

$$S=\frac{1}{2G}\int d^2x\sqrt{-g}e^{-2\phi}[R+4(\nabla\phi)^2+4\lambda^2]-\frac{1}{2}\int d^2x\sqrt{-g}(\nabla f)^2, \tag{1}$$

with the dilaton field ϕ, the cosmological constant λ, and the single massless conformal matter field f. Differing from the original CGHS convention [8] where $G=\pi$, we will take $G=\frac{1}{2}$ in this paper. Related to this choice, we have modified the coefficient in front of the matter action from the CGHS value $-1/4\pi$ to $-\frac{1}{2}$, which is natural from the viewpoint of a spherically symmetric reduction of four-dimensional gravity [9].

*Email address: ioda@edogawa-u.ac.jp

Let us adopt the ADM splitting of $(1+1)$-dimensional spacetime given by

$$g_{ab}=\begin{pmatrix} -\alpha^2+\dfrac{\beta^2}{\gamma} & \beta \\ \beta & \gamma \end{pmatrix}. \tag{2}$$

Then the normal unit vector n^a orthogonal to the hypersurfaces $x^0=\text{const}$ reads

$$n^a=\left(\frac{1}{\alpha},-\frac{\beta}{\alpha\gamma}\right), \tag{3}$$

and the projection operator h^{ab} over the $x^0=\text{const}$ hypersurfaces becomes

$$h^{ab}=g^{ab}+n^a n^b=\begin{pmatrix} 0 & 0 \\ 0 & \dfrac{1}{\gamma} \end{pmatrix}. \tag{4}$$

In terms of the ADM parametrization (2), after some calculations the action (1) can be written as

$$S=\int d^2xL=\int d^2x\left[4\alpha\sqrt{\gamma}e^{-2\phi}\left\{\lambda^2-(n^a\partial_a\phi)^2+\frac{1}{\gamma}(\phi')^2+Kn^a\partial_a\phi-\frac{\alpha'}{\alpha\gamma}\phi'\right\}+\frac{1}{2}\alpha\sqrt{\gamma}\left\{(n^a\partial_a f)^2-\frac{1}{\gamma}(f')^2\right\}\right], \tag{5}$$

where the trace of the extrinsic curvature $K=g^{ab}K_{ab}$ is

$$K=\frac{1}{\sqrt{-g}}\partial_a(\sqrt{-g}n^a)=\frac{\dot{\gamma}}{2\alpha\gamma}-\frac{\beta'}{\alpha\gamma}+\frac{\beta}{2\alpha\gamma^2}\gamma', \tag{6}$$

and $\partial/\partial x^0=\partial_0$ and $\partial/\partial x^1=\partial_1$ are also denoted by an overdot and a prime, respectively. In deriving Eq. (5), we have used the formula [9,10]

$$R=2n^a\partial_a K+2K^2-\frac{2}{\alpha\sqrt{\gamma}}\left(\frac{\alpha'}{\sqrt{\gamma}}\right)'. \tag{7}$$

The action (5) indicates that α and β are nondynamical Lagrange multiplier fields due to the absence of the term including x^0 differentiation, and so we regard the massless matter field f, the dilaton field ϕ, and the "graviton" γ as dynamical fields. Then the canonical conjugate momenta can be read off from the action (5):

$$p_f=\sqrt{\gamma}n^a\partial_a f,$$

$$p_\phi=4\sqrt{\gamma}e^{-2\phi}(-2n^a\partial_a\phi+K),$$

$$p_\gamma=\frac{2}{\sqrt{\gamma}}e^{-2\phi}n^a\partial_a\phi. \tag{8}$$

Now it is straightforward to derive the Hamiltonian whose result is given by

$$H=\int dx^1(p_f\dot{f}+p_\phi\dot{\phi}+p_\gamma\dot{\gamma}-L)=\int dx^1(\alpha H_0+\beta H_1), \tag{9}$$

where the constraints are explicitly of the form

$$H_0=\frac{1}{2\sqrt{\gamma}}p_f^2-4\sqrt{\gamma}e^{-2\phi}\lambda^2-\frac{4}{\sqrt{\gamma}}e^{-2\phi}(\phi')^2-\left(\frac{4}{\sqrt{\gamma}}e^{-2\phi}\phi'\right)'+\frac{1}{2\sqrt{\gamma}}(f')^2+\frac{\sqrt{\gamma}}{2}e^{+2\phi}p_\phi p_\gamma+\gamma\sqrt{\gamma}e^{+2\phi}p_\gamma^2, \tag{10}$$

$$H_1=\frac{1}{\gamma}p_f f'+\frac{1}{\gamma}p_\phi\phi'-2p_\gamma'-\frac{1}{\gamma}p_\gamma\gamma'. \tag{11}$$

Note that H_0 and H_1 are generators corresponding to the time translation and the spatial displacement, respectively. Also let us notice that α and β are certainly the Lagrange multiplier fields as mentioned before. At this stage, it is easy to derive the ADM surface term via the dual Legendre transformation by following Regge and Teitelboim [11], though we now omit the details since it is not so important for later discussions.

III. VAIDYA METRIC

In this section, we will construct the Vaidya metric to the two-dimensional dilaton black hole. This Vaidya metric will be used in later sections when we wish to discuss Hawking radiation arising from a dynamical black hole.

As a simple illustration, let us recall how to build the Vaidya metric to the Schwarzschild black hole in four dimensions. Neglecting the irrelevant angular parts (θ,φ), Schwarzschild geometry has the famous form

$$ds^2=-\left(1-\frac{2M}{r}\right)dt^2+\frac{1}{1-\dfrac{2M}{r}}dr^2. \tag{12}$$

Introducing the advanced time coordinate $v=t+r^*$ with the tortoise coordinate $dr^*=dr/(-g_{00})$, in the (v,r) coordinates the Schwarzschild metric can be transformed to

$$ds^2=-\left(1-\frac{2M}{r}\right)dv^2+2dv\,dr. \tag{13}$$

A generalization of a constant mass M to the mass function $M(v)$ gives rise to the Vaidya metric corresponding to the Schwarzschild black hole. The reason why we prefer the Vaidya metric to the Schwarzschild one is that the former satisfies the classical field equations as it is when there is a flow of matter with a form of the energy-momentum tensor $T(v)$, while in the latter the mass function must have a complicated dependence on the coordinates to satisfy them, which makes the following analysis ugly. In other words, the Vaidya form of a black hole provides us a convenient playground to discuss the properties of a dynamical black hole.

In order to have a close relationship with the work of CGHS [8], let us start with their black hole solution in light cone coordinates [12],

$$ds^2 = -e^{2\rho} dx^+ dx^- = -\frac{1}{\frac{M}{\lambda} - \lambda^2 x^+ x^-} dx^+ dx^-. \quad (14)$$

Transforming the coordinates from (x^+, x^-) to the (v, r) which are related to each other,

$$x^+ = \frac{\sqrt{M}}{\lambda\sqrt{\lambda}} e^{\lambda v},$$
$$x^- = -\frac{1}{\sqrt{\lambda M}} \left(e^{2\lambda r} - \frac{M}{\lambda} \right) e^{-\lambda v}, \quad (15)$$

the two-dimensional line element (14) reduces to

$$ds^2 = -\left(1 - \frac{M}{\lambda} e^{-2\lambda r}\right) dv^2 + 2dv\, dr. \quad (16)$$

Then the Vaidya metric corresponding to the dilaton black hole can be obtained by promoting a constant mass to the mass function $M(v)$ depending on only the v coordinate. Here it is worthwhile to notice that in the newly introduced coordinates (v, r), the dilaton field, which was given by $\phi = \rho$ in light cone coordinates (x^+, x^-), takes a remarkably simple form, that is, a linear dilaton form

$$\phi = -\lambda r. \quad (17)$$

This would lead to a great advantage in analyzing the constraints as well as the field equations below.

Indeed, it is verified that the solutions (16) and (17) are an extremum of the action (1) near the apparent horizon,

$$r_{\rm AH} = -\frac{1}{2\lambda} \log \frac{\lambda}{M}, \quad (18)$$

whose definition arises from the condition $g_{vv} = 0$, which is also consistent with the usual definition $(\nabla\phi)^2 = 0$ in two-dimensional dilaton gravity. The classical field equations are easily obtained from the action (1):

$$2e^{-2\phi}[\nabla_a \nabla_b \phi + g_{ab}\{(\nabla\phi)^2 - \nabla^2\phi - \lambda^2\}]$$
$$= \tfrac{1}{2}[\nabla_a f \nabla_b f - \tfrac{1}{2} g_{ab} (\nabla f)^2], \quad (19)$$

$$R + 4\lambda^2 + 4\nabla^2\phi - 4(\nabla\phi)^2 = 0, \quad (20)$$

$$\nabla^2 f = 0. \quad (21)$$

Since we are interested in physics only in the vicinity of the apparent horizon, it is sufficient to verify that the Vaidya metric (16) and the dilaton field (17) are consistent with the field equations (19)–(21) near the apparent horizon. After some manipulation, the field equations are required to satisfy

$$\partial_r f = \partial_v \partial_r f \approx 0, \quad (22)$$

$$\partial_v M \approx \tfrac{1}{2}(\partial_v f)^2, \quad (23)$$

where we shall use $\approx$ to express the equalities holding approximately near the apparent horizons from now on. From Eqs. (22) and (23), we find the general solution

$$f(v) \approx \pm \int^v dv \sqrt{2\partial_v M}. \quad (24)$$

Consequently, it has been checked that the solutions (16) and (17) are at least classically consistent with the field equations near the apparent horizon as long as Eq. (24) is satisfied. Incidentally, Eq. (24) represents the physical fact that the increase of the black hole mass, $\partial_v M > 0$, is classically allowed, but the loss of it, $\partial_v M < 0$, i.e., Hawking radiation, is classically forbidden and can occur only through the quantum tunneling effects owing to $f(v)$ being a real scalar field.

IV. QUANTUM GRAVITY NEAR THE APPARENT HORIZON

We now consider the dynamical black hole (16) and the linear dilaton (17). Our main concern in this section is to construct a quantum theory of two-dimensional dilaton gravity holding near the apparent horizon.

Let us begin by introducing the coordinates

$$x^a = (x^0, x^1) = (v - r, r). \quad (25)$$

Next we set up the gauge conditions such that the gauge symmetries associated with the two-dimensional reparametrization invariances are completely fixed,

$$g_{ab} = \begin{pmatrix} -\alpha^2 + \frac{\beta^2}{\gamma} & \beta \\ \beta & \gamma \end{pmatrix}$$
$$= \begin{pmatrix} -\left(1 - \frac{M}{\lambda} e^{-2\lambda r}\right) & \frac{M}{\lambda} e^{-2\lambda r} \\ \frac{M}{\lambda} e^{-2\lambda r} & 1 + \frac{M}{\lambda} e^{-2\lambda r} \end{pmatrix}, \quad (26)$$

where the black hole mass M is a function of the two-dimensional coordinates x^a. Notice that we have chosen these gauge conditions to correspond to the Vaidya metric built in the previous section. Of course, at this stage, we cannot restrict the mass function to be a function depending on only the v coordinate from an argument of symmetries. Near the apparent horizons (18), Eq. (26) yields

$$\alpha \approx \frac{1}{\sqrt{2}}, \quad \beta \approx 1, \quad \gamma = \frac{1}{\alpha^2} \approx 2. \quad (27)$$

Note that the dynamical degrees of freedom representing the "graviton" γ are effectively fixed in Eq. (27). At this point, let us make physically plausible assumptions [4,5] near the apparent horizon,

$$f \approx f(v), \quad M \approx M(v), \quad \phi \approx -\lambda r. \quad (28)$$

As shown in Sec. III, these assumptions are consistent with the field equations, but their quantum mechanical meaning is not clear at present. Given the assumptions (28), near the apparent horizon the canonical conjugate momenta (8) become

$$p_f \approx \partial_v f,$$

$$p_\phi \approx -2M - \frac{1}{\lambda}\,\partial_v M, \tag{29}$$

$$p_\gamma \approx M.$$

Then after a bit lengthy calculation, one arrives at the remarkable relation that the Hamiltonian constraint $H_0 = 0$ becomes proportional to the supermomentum constraint $H_1 = 0$:

$$\sqrt{2}H_0 \approx 2H_1 \approx p_f^2 + 2\lambda p_\phi + 4\lambda M. \tag{30}$$

This relation can be understood from an observation that the time translation generated by the Hamiltonian constraint is frozen on the apparent horizon due to gravitational time dilation in the present coordinate system [6].

We are now ready to carry out the canonical quantization of the model. Following Dirac's quantization procedure of the first-class constraints [13], residual symmetry (30) is imposed on the state,

$$\left(-\frac{\partial^2}{\partial f^2} - 2i\lambda\,\frac{\partial}{\partial\phi} + 4\lambda M\right)\Psi = 0, \tag{31}$$

which is nothing but the Wheeler-DeWitt equation. A special solution can be found to be

$$\Psi = (Be^{\sqrt{A}f(v)} + Ce^{-\sqrt{A}f(v)})\exp\left(i\,\frac{A-4\lambda M}{2\lambda}\,\phi\right), \tag{32}$$

where A, B, and C are integration constants. Without losing generality, we shall choose the boundary condition $B = 0$.

V. HAWKING RADIATION

We now turn our attention to an application of quantum gravity near the apparent horizon for an understanding Hawking radiation in two-dimensional dilaton gravity. A similar analysis was carried out in four- [4,5] and three- [6] dimensional black holes. The main motivation behind the present work is to clarify to what extent quantum gravity holding near the apparent horizon reflects the physical properties of quantum black holes since we have a better grasp of the quantum mechanical features of a black hole in two-dimensional dilaton gravity compared to four- and three-dimensional gravities.

To begin with, let us define the expectation value $\langle\mathcal{O}\rangle$ of an operator $\mathcal{O}$ by

$$\langle\mathcal{O}\rangle = \frac{1}{\int df |\Psi|^2}\int df\;\Psi^\dagger\mathcal{O}\Psi. \tag{33}$$

Under this definition, it is straightforward to evaluate the expectation value of the change rate in black hole mass,

$$\langle\partial_v M\rangle = -\frac{A}{2}, \tag{34}$$

where the constraint (30) [or p_ϕ in Eq. (29)] and the physical state (32) were used. Moreover, in a similar manner one can calculate that in the radius in the apparent horizon

$$\langle\partial_v r_{\mathrm{AH}}\rangle = -\frac{A}{4\lambda\langle M\rangle}. \tag{35}$$

To represent Hawking radiation, we have to select the integration constant A to be a positive constant, for example, k_1^2; then, Eqs. (32), (34), and (35) reduce to

$$\Psi = C\,\exp\left(-|k_1|f(v) + i\,\frac{A-4\lambda M}{2\lambda}\,\phi\right), \tag{36}$$

$$\langle\partial_v M\rangle = -\frac{k_1^2}{2}, \tag{37}$$

$$\langle\partial_v r_{\mathrm{AH}}\rangle = -\frac{k_1^2}{4\lambda\langle M\rangle}. \tag{38}$$

To see explicitly that this is in fact Hawking radiation carried by the matter field f, it is useful to argue the expectation value of the energy-momentum tensor of the matter field, which is defined as

$$\langle T^f{}_{ab}\rangle = \left\langle\frac{1}{\sqrt{-g}}\,\frac{\delta S_f}{\delta g^{ab}}\right\rangle = -\frac{1}{2}\left\langle\nabla_a f\nabla_b f - \frac{1}{2}\,g_{ab}(\nabla f)^2\right\rangle, \tag{39}$$

where S_f is the matter part in the action (1). Then T_{vv}, in which we are interested, is calculated to be

$$\langle T^f{}_{vv}\rangle = \frac{k_1^2}{2}, \tag{40}$$

which is precisely equal to the opposite sign of Eq. (37) and thus means that the matter flux is equivalent to Hawking radiation, as expected. Alternatively, if we choose A to be a negative constant, e.g., $-k_2^2$, we gain a physical situation where external neutral matter flows into a black hole across the horizon.

Several comments about Eqs. (36)–(38) are now in order. First, Eq. (36) shows that the physical state has an exponentially dampinglike form in the classically forbidden region, implying a quantum tunneling process. This behavior seems to match our interpretation of the present situation as Hawking radiation. Second, from Eq. (37), the Hawking radiation rate is independent of the black hole mass in contrast with the case of the four-dimensional Schwarzschild black hole where it is shown that $\langle\partial_v M\rangle \propto -1/\langle M^2\rangle$ [4,5] inferred by Hawking in his semiclassical approach [1]. This surprising result, however, has been already found in studies of the two-dimensional dilaton and other two-dimensional gravities [14]. This is because in two dimensions there exists a beautiful relation between the trace anomaly and Hawking radiation [15]. Namely, although for N species of massless scalar field the trace of the energy-momentum tensor vanishes classically, quantum mechanically there is the trace anomaly

$$\langle g^{ab}T^{f}{}_{ab}\rangle = \frac{N}{24}R. \tag{41}$$

(Up to now we have taken account of the case $N=1$.) As a result, in the coordinates where the metric is asymptotically constant on the null infinities $\mathcal{I}^{\pm}_{R}$, the Hawking radiation rate is asymptotically independent of the black hole mass and approaches the constant value $N\lambda^2/48$ [8]. One might ask what would happen if one replaces a single scalar matter f with N species of scalar f_i ($i=1,2,\dots,N$) in our formalism as considered in the original work of CGHS. In this case, the physical state (32) is replaced by

$$\Psi = \prod_{i=1}^{N} (B_i e^{\sqrt{A_i} f_i(v)} + C_i e^{-\sqrt{A_i} f_i(v)}) \exp\left(i \frac{A - 4\lambda M}{2\lambda} \phi \right), \tag{42}$$

with

$$\sum_{i=1}^{N} A_i = A, \tag{43}$$

and the result with respect to Hawking radiation (34) remains unchanged. If we consider completely identical N scalar matters such that $A = N\bar{A}$ ($A_1 = A_2 = \cdots = A_N \equiv \bar{A}$), Hawking radiation rate becomes, to scale with N,

$$\langle \partial_v M \rangle = -\frac{N\bar{A}}{2}, \tag{44}$$

which is equal to the result derived by CGHS up to a numerical constant.

Finally, Eq. (38) shows that as a black hole emits Hawking radiation and loses mass, the apparent horizon recedes toward the singularity. This is a very plausible picture from physical considerations. In the limit $\langle M \rangle \to 0$, this equation indicates that $\langle \partial_v r_{\mathrm{AH}} \rangle \to -\infty$, suggesting that the apparent horizon disappears and the curvature singularity might be visible to external observers (violation of weak cosmic censorship). However, in order to get a definite answer to this problem, it seems that we need a more improved and sophisticated model as discussed in the next section.

Before closing this section, it is valuable to inquire what would happen near the origin $r=0$. To keep the role of the x^0 coordinate as time, we here assume the inequality $M < \lambda$. Interesting enough, it turns out that one can also establish quantum gravity in this vicinity in a perfectly similar way to the case of the apparent horizon. For example, analogous equations to Eqs. (27), (29), (30), (32), and (34) can be deduced as follows:

$$\alpha \approx \frac{1}{\sqrt{1+\frac{M}{\lambda}}}, \quad \beta \approx \frac{M}{\lambda}, \quad \gamma = \frac{1}{\alpha^2} \approx 1 + \frac{M}{\lambda}, \tag{45}$$

$$p_f \approx \partial_v f, \quad p_\phi \approx -\frac{2}{\gamma\lambda}(\partial_v M + 2M^2), \quad p_\gamma \approx \frac{2M}{\gamma}, \tag{46}$$

$$\sqrt{\gamma} H_0 \approx \gamma H_1 \approx p_f^2 + \lambda \gamma p_\phi + 4M^2, \tag{47}$$

$$\Psi = (B e^{\sqrt{A} f(v)} + C e^{-\sqrt{A} f(v)}) \exp\left(i \frac{A - 4M^2}{\lambda \gamma} \phi \right), \tag{48}$$

$$\langle \partial_v M \rangle = -\frac{A}{2}. \tag{49}$$

These results, in particular, Eq. (49), are obviously consistent with results obtained by means of quantum gravity holding near the apparent horizon. One interesting problem in the future would be to find the physical state holding in the region between the curvature singularity and the apparent horizon by connecting the two states (32) and (49) consistently.

VI. DISCUSSION

In this paper, we have investigated Hawking radiation in terms of quantum gravity holding near the apparent horizon. In particular, it was found that the Hawking radiation rate is independent of the black hole mass, and it scales with the number of massless scalar fields when there are N identical matters as shown in the other analysis of two-dimensional dilaton gravity.

As mentioned in the Introduction, one of the main motivations in this paper is to compare the results obtained in quantum gravity holding near the apparent horizon with those in fully quantized two-dimensional dilaton gravity where we know an exact solution of quantum black holes; thus, in this section we would like to make comments on their relation more closely. As a model of fully quantized dilaton gravity, we shall consider the study in Ref. [16]. However, it is fair to mention that we have not yet reached a complete understanding and a consensus of opinion as to which model really describes a fully quantum behavior of a black hole even in the context of two-dimensional dilaton gravity.

First of all, in Ref. [16] an exact operator quantization is performed for a model of two-dimensional dilaton gravity for the specific case of 24 massless matter scalars since the model can be canonically mapped to a free conformal field theory and hence can be solved analytically. On the other hand, our formalism in this paper can cope with any number of massless scalars, which would be an advantage of our work.

Second, let us turn our attention to physical states. In an exact operator quantization formalism [16] the physical states can be built up by Del Guidice–Di Vecchia–Fubini (DDF) states [17] and each physical state corresponds to a possible choice of the universe, all the events which occur in each universe being involved in a selected state. Thus in order to extract information on spacetime geometry it is necessary to choose by intuition an appropriate physical state to match a physical situation which we have in mind. Actually, the authors of Ref. [16] have explicitly shown that one of the physical states corresponds to the state describing quantum geometry in which the infalling matter energy distribution yields a black hole with or without a naked singularity. Speaking of our formalism, the physical state is almost unique, i.e., Eq. (32). This is because in the vicinity of the horizon we have looked for the physical state expressing black hole through the gauge fixing conditions (26) from the outset while an exact quantization is performed in the conformal gauge [16]. At present, we have no idea as to what precise connection there is about physical states in both formalisms. However, it seems to be valuable to point out that the matter part in both physical states has a qualitatively similar structure, which might be one of the key ingredients in understanding Hawking radiation within the quantum mechanical framework. At this point, note that the energies carried in by matter fields have the same behavior; namely, the total energy flux is divergent owing to a constant energy density, but it is certain that we need more studies to gain a definite answer about this result.

Finally, we would like to close this section by commenting on other features and future problems in our formalism. We have shown that the value of the coefficient of the Hawking flux cannot be determined in the present formalism which treats only the region in the vicinity of the horizon since it is fixed by imposing boundary conditions at null past infinities $\mathcal{I}_{\mathcal{R}}^-$ and $\mathcal{I}_{\mathcal{L}}^-$ [8,14]. Nevertheless, it is remarkable that the present formalism describes qualitative features of Hawking radiation without paying much attention to the interior and the exterior regions of a black hole. Actually, quantum gravity near the horizon has provided a nice description of Hawking radiation in a three-dimensional de Sitter black hole [18] where there is no asymptotically flat regions so that the usual technique cannot be applied to this case [6]. Next, as mentioned before, it was known that in two dimensions Hawking radiation stems from the trace anomaly [15]. Since the present formalism gives the qualitatively same picture as the semiclassical approach, it is certain that our formalism respects the contribution from the trace anomaly properly. However, it was conjectured in [14] and shown in [16] that the functional measures over the gravitational field, the dilaton, and the ghosts give rise to nontrivial contributions which are different from the form of the trace anomaly, and eventually yield the back reaction of Hawking radiation on the geometry. In this respect, unfortunately, it seems that our formalism largely ignores this issue. But this problem is shared in the more general ADM or the Wheeler-DeWitt formalisms in higher dimensions where we have no idea how to evaluate the functional measures. It is likely that one should set up the Wheeler-DeWitt equation after estimating the contributions from the functional measures carefully, although we do not know how to accomplish this procedure except in two dimensions. Although we have a lot of things to overcome in the future, we believe that an improvement of the formalism at hand would give us a useful analytical method for understanding various properties of quantum black holes and might to a certain extent realize the membrane paradigm of a black hole [19–21].

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