Kerr spinning particle, strings, and superparticle models

A. Burinskii*
Gravity Research Group, IBRAE Academy of Sciences, B. Tulskaya 52, Moscow 113191, Russia
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A combined model of the Kerr spinning particle and superparticle is considered. The structure of Kerr geometry is presented in a complex form as being created by a complex source. A natural supergeneralization of this construction is obtained corresponding to a complex ''supersource.'' Performing a supershift to the Kerr and Kerr-Sen solutions we obtain metrics of supergravity black holes with a nonlinear realization of broken supersymmetry. [S0556-2821(98)00204-5]

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I. INTRODUCTION

It was mentioned about 30 years ago that Kerr geometry displays some remarkable features suggesting certain relationships with the spinning elementary particles. In particular, the gyromagnetic ratio of the Kerr-Newman solution is the same as that of the Dirac electron. This fact stimulated treatment of the models of spinning particles based on Kerr-Newman geometry [1–6]. Some stringlike structures were obtained in Kerr geometry. The first one is connected with a singular ring of the Kerr solution [6,7]. Two others are linked with a complex representation of Kerr geometry (initiated by Lind and Newman [8]) in which the Kerr-Newman solution is considered as a retarded-time field generated by a mysterious complex source propagating along a complex world line. It was mentioned [9] that the complex world line is really a world sheet or a special type of string. The stringy boundary conditions for this complex world line are connected with a third stringy structure of Kerr geometry—an orbifold [9].

A new and a very important period was started by Witten [10] who pointed out the role of black holes in string theory and also especially with the paper by Sen [11] who gave a generalization of the Kerr solution to low energy string theory.

It was shown [7] that near the Kerr singular ring the Kerr-Sen solution acquires a metric similar to the field around a heterotic string. Recently, much attention has also been paid to multidimensional Kerr solutions and to a treatment of black holes as fundamental string states [11,12], leading to a conclusion, suggested from different points of view, that some black holes should be treated as elementary particles [13].

On the other hand, after obtaining supersymmetry, great attention has been paid to the models of spinning particles based on the Grassmann anticommuting parameters (Volkov and Akulov [14], Casalbuoni [15], Brink and Schwarz [16], and others [17,18], which has also found an important application in superstring theory.

In this paper we consider one very natural way to combine the Kerr spinning particle and superparticle models in such a manner that the superparticle plays the role of a ''source'' of the supergeneralized Kerr geometry. Our treatment is based on the formalism by Debney, Kerr, and Schild [2] adopted to the above-mentioned complex representation of Kerr geometry.

The main idea of this work is extremely simple: to replace the mysterious complex source of Kerr geometry by a complex supersource which can be obtained by an extra supershift.

Tugai and Zheltukhin [18] have recently shown that the application of the supershift to Coulomb solution in a flat space gives rise to a Maxwell supermultiplet of fields. On the other hand it was shown by Appel as early as in 1887 [19] that a complex shift yields a ringlike singularity and specific Kerr's twofoldedness of space. Therefore, on the basis of these examples one can mention that the methods of a complex shift and supershift have much in common mathematically, though they lead to very different physical consequences. In this paper examples of the simultaneous application of both above transformations to the Kerr and to the Kerr-Sen solutions are given.[1] As a result we derive the metrics of rotating super black holes with broken four-dimensional supersymmetry generated by a superparticle source.

II. COMPLEX STRUCTURE OF THE KERR SOLUTION

Starting from the Kerr-Schild form of metric $g_{ik}=\eta_{ik}+2hk_ik_k$; where $\eta_{ik}=\mathrm{diag}(-1,1,1,1)$ is the auxiliary Minkowski metric in Cartesian coordinates (t,x,y,z), one can see that the main peculiarities of the Kerr solution are connected with a form of the harmonic scalar function h and vector field k of principal null directions (PN congruence). The function h is the Appel potential

$$h=m\,\mathrm{Re}(1/\tilde{r}\,), \tag{1}$$

*Email address: grg@ibrae.ac.ru

[1]In spite of a quite long story of supersymmetry the number of known nontrivial supersolutions in electrodynamics and supergravity is very small. Nontrivial supergravity solutions cannot be obtained by a supergauge transformation from the corresponding known solutions of Einstein's gravity. The only nontrivial super-black-hole (BH) solution known to us is a supergeneralization of the Reissner-Nordström solution given by Aichelburg and Güven [20].

which may be expressed in oblate spheroidal coordinates r,θ as

$$\tilde{r}=r+ia\cos\theta. \tag{2}$$

It has a ringlike singularity $r=\cos\theta=0$ which is a branch line of Kerr geometry. The space is covered by two sheets corresponding to the positive and negative values of r. The function $\tilde{r}$ may also be represented as a complex radial distance

$$\tilde{r}=\sqrt{(x_a-x_{0a})(x^a-x_0^a)},\quad a=1,2,3, \tag{3}$$

from the complex point $x_0=(0,0,ia)$. It involves a complex interpretation of the Kerr solution, initiated by Lind and Newman [8], in which Kerr geometry is represented as a retarded-time field generated by a ''complex point source'' which propagates in complex Minkowski space CM^4 along a complex ''world line'' $x_0^i(\tau)$ $(i=0,1,2,3)$, parametrized by a complex time parameter $\tau=t+i\sigma=x_0^0(\tau)$. This interpretation is also suggested by the analysis of the field of principal null directions k which is geodesic and shear free.

An important role in this construction is played by complex light cones, whose apexes lie on the complex ''world line'' $x_0(\tau)$. The complex light cone

$$K=\{x:x=x_0^i(\tau)+\psi_R^\alpha\sigma^i_{\alpha\dot{\alpha}}\overline{\psi_L^{\dot{\alpha}}}\} \tag{4}$$

may be split into two families of null planes: ''right'' ($\psi_R=$const; $\overline{\psi_L}$ variable) and ''left'' ($\overline{\psi_L}=$const; ψ_R variable). The rays of the PN congruence $k(x)$ of Kerr geometry are the tracks of these complex null planes (right or left) on the real slice of Minkowski space [6,9,21]. PN congruence propagates from a ''negative'' sheet of three-space onto ''positive'' one crossing the disk spanned by the Kerr singular ring. In the null coordinates $u=(z+t)/\sqrt{2}$, $v=(z-t)/\sqrt{2}$, $\xi=(x+iy)/\sqrt{2}$, $\overline{\xi}=(x-iy)/\sqrt{2}$ we have

$$k=k_i dx^i=P^{-1}(du+\overline{Y}d\xi+Yd\overline{\xi}-Y\overline{Y}dv), \tag{5}$$

where $Y(x)$ is a complex projective spinor field $Y=\overline{\psi^{\dot{2}}}$, $\overline{\psi^{\dot{1}}}=1$.[2]

The condition for a complex light cone to have a real slice is

$$[x-x_0(\tau)]^2=0, \tag{6}$$

where x is a real point. In the rest frame and with gauge $x_0^0=\tau$ this equation may be split as a complex retarded-time equation

$$t-\tau=\tilde{r}=-(x_i-x_{0i})\dot{x}_0^i. \tag{7}$$

It fixes the relation $\mathrm{Im}\tau=\sigma=a\cos(\theta)$ between the imaginary part of the complex time and a family of the null rays with polar direction θ,ϕ.

The Kerr theorem [2,6,21] allows one to describe the geodesic and shear-free PN congruences in twistor terms via the function $Y(x)$ which is a solution of the equation[3] $F(Y,\lambda_1,\lambda_2)=0$, F being an analytical function. The complex radial distance $\tilde{r}$ may be expressed as $\tilde{r}=-dF/dY$. Singular regions are defined as caustics of the congruence satisfying the system of equations $F=0$, $dF/dY=0$.

For the Kerr congruence the function F can be expressed via parameters of the complex world line $x_0(\tau)$ [6,9,21]

$$F\equiv(\lambda_1-\lambda_1^0)K\lambda_2^0-(\lambda_2-\lambda_2^0)K\lambda_1^0, \tag{8}$$

where $K=[\partial_\tau x_0^i(\tau)]\partial_i$, and λ_1^0,λ_2^0 are values of the twistor coordinates on the world line $x_0(\tau)$. The resulting function F is quadratic in Y and the solution $Y(x)$ may be given in explicit form.

Therefore, the Kerr solution may be represented as a retarded-time field created by a mysterious ''complex point source'' propagating in the auxiliary complex Minkowski space CM^4.

III. GEOMETRY GENERATED BY THE SUPER WORLD LINE.

Now we would like to generalize this complex retarded-time construction to the case of at complex ''supersource'' propagating along a super world line

$$X_0^i(\tau)=x_0^i(\tau)-i\theta\sigma^i\overline{\zeta}+i\zeta\sigma^i\overline{\theta},\ \zeta^\alpha(\tau),\ \overline{\zeta^{\dot{\alpha}}}(\tau). \tag{9}$$

Similarly to the above ''real slice'' we introduce a ''B slice'' as a ''body'' of superspace [20], where the nilpotent part of x^i is equal to zero. The ''real slice'' is a real subset of the ''B slice.'' The real slice condition (6) takes now the form $s^2=[x_i-X_{0i}(\tau)][x^i-X_0^i(\tau)]=0$. Selecting the nilpotent parts of this equation we obtain the above real slice condition (6) and the B-slice conditions

$$[x^i-x_0^i(\tau)](\theta\sigma_i\overline{\zeta}-\zeta\sigma_i\overline{\theta})=0, \tag{10}$$

$$(\theta\sigma\overline{\zeta}-\zeta\sigma\overline{\theta})^2=0. \tag{11}$$

Equation (10) may be rewritten using Eq. (4) in the form

$$(\theta^\alpha\sigma_{i\alpha\dot{\alpha}}\overline{\zeta^{\dot{\alpha}}}-\zeta^\alpha\sigma_{i\alpha\dot{\alpha}}\overline{\theta^{\dot{\alpha}}})\psi^\beta\sigma^i_{\beta\dot{\beta}}\overline{\psi^{\dot{\beta}}}=0, \tag{12}$$

which yields

$$\overline{\psi\theta}=0,\ \overline{\psi\zeta}=0, \tag{13}$$

which in turn is a condition of proportionality of the commuting spinors $\overline{\psi(x)}$ and anticommuting spinors $\overline{\theta}$ and $\overline{\zeta}$, providing the left null superplanes to reach the B slice. Taking into account that $\overline{\psi^{\dot{2}}}=Y(x)$, $\overline{\psi^{\dot{1}}}=1$ we obtain

$$\overline{\theta^{\dot{2}}}=Y(x)\,\overline{\theta^{\dot{1}}},\quad \overline{\theta^{\dot{\alpha}}}=\overline{\theta^{\dot{1}}}\overline{\psi^{\dot{\alpha}}},\quad \overline{\zeta^{\dot{2}}}=Y(x)\,\overline{\zeta^{\dot{1}}},\quad \overline{\zeta^{\dot{\alpha}}}=\overline{\zeta^{\dot{1}}}\overline{\psi^{\dot{\alpha}}}. \tag{14}$$

[2]Here we use the spinor notation of Ref. [22].

[3]The three parameters Y, $\lambda_1=u+Y\overline{\xi}$, $\lambda_2=\xi-Yv$ are projective twistor coordinates.

It also gives that $\overline{\theta\,\theta}=\overline{\zeta\,\zeta}=0$, and Eq. (11) is satisfied automatically.

Therefore, the B-slice condition fixes a correspondence between the coordinates θ, $\overline{\zeta}$ and twistor null planes forming the Kerr congruence, and consequently the coordinate $\overline{\zeta}$ of the super world line must be engaged partially to provide a B slice and parametrize the ''left'' complex planes and the null rays of the Kerr congruence. The conjugate sector also gives $\theta^{\alpha}=\overline{Y(x)}\theta^1$; however, the coordinate of the super world line ζ remains independent, and can be left as an arbitrary function of time.[4] Therefore, the roles of the chiral and antichiral Grassmann coordinates of the super world line are to be divided.

The retarded time equation (7) takes the form $t-T=\widetilde{R}=\widetilde{r}+\eta$, where $\widetilde{R}=-(x_i-X_{0i})\dot{X}_0^i$ is a superdistance. The ''body part'' of this equation satisfies the above relation (7); $T=\tau-\eta$ is a supertime containing the nilpotent term

$$\eta=i\theta\sigma^0\overline{\zeta}-i\zeta(\tau)\sigma^0\overline{\theta}. \tag{15}$$

In the stationary case $\dot{x}_0^i=(1,0,0,0)$, $\dot{\zeta}=0$ we have, on the B slice,

$$\widetilde{R}=r+ia\cos\theta+i\theta\sigma^0\overline{\zeta}-i\zeta(\tau)\sigma^0\overline{\theta}. \tag{16}$$

The corresponding supergeneralization of the Kerr theorem may be achieved by substitution of the super world line $X_0(\tau)$ instead of $x_0(\tau)$ in the function F. As a result one can obtain a superfield $Y(x)$ which on the B slice takes the usual form since all the nilpotent terms disappear. From the Kerr theorem one obtains the general expression for superdistance out of a B slice,

$$\widetilde{R}=-d\hat{F}/dY=\widetilde{r}-i[x^i-x_0^i(\tau)]\dot{\zeta}(\tau)\sigma_i\overline{\theta}$$
$$-i[\dot{x}_0^i(\tau)+i\dot{\zeta}(\tau)\sigma^i\overline{\theta}](\theta\sigma_i\overline{\zeta}-\zeta\sigma_i\overline{\theta}),$$

which may be useful when applying the (anti)chiral differential operators $D_{\alpha},\overline{D}_{\dot{\alpha}}$ [18,22].

IV. SUPERSHIFT OF THE KERR SOLUTION

One can note that the Kerr solution is a particular solution of supergravity with vanishing spin-3/2 field, and that in the stationary case $\dot{X}_0=(1,0,0,0)$, the $\dot{\zeta}=0$ solution with a supersource (9) can be obtained from the Kerr solution by a supershift

$$x'^i=x^i+i\theta\sigma^i\overline{\zeta}-i\zeta\sigma^i\overline{\theta},\quad \theta'=\theta+\zeta,\quad \overline{\theta'}=\overline{\theta}+\overline{\zeta}, \tag{17}$$

which is a ''trivial'' supergauge transformation in supergravity. However, the subsequent imposition of a B slice constraint is a nonlinear operation breaking four-dimensional supersymmetry [14,23,22]. As a result the arising spin-3/2 field cannot be gauged away.

[4]The coordinates θ^1, $\overline{\theta^{\dot{1}}}$, and $\overline{\zeta^{\dot{1}}}$ are independent too.

Starting from tetrad form of the Kerr solution $ds^2=2e^1e^2+2e^3e^4$, where

$$e^1=d\xi-Ydv=\partial_{\overline{Y}}\overline{\psi}\sigma_i\overline{\psi}dx^i/\sqrt{2},\quad e^2=d\overline{\xi}-\overline{Y}dv$$
$$=\partial_Y\psi\sigma_i\overline{\psi}dx^i/\sqrt{2}, \tag{18}$$

$$e^3=du+\overline{Y}d\xi+Yd\overline{\xi}-Y\overline{Y}dv=\psi\sigma_i\overline{\psi}dx^i/\sqrt{2}, \tag{19}$$

$$e^4=-\partial_{\overline{Y}}\partial_Y e^3-he^3, \tag{20}$$

and using the expressions

$$dx'^i=dx^i+i(\theta^1\overline{\zeta^{\dot{1}}})(\partial_{\overline{Y}}\overline{\psi}\sigma^i\overline{\zeta})d\overline{Y}+i(\theta^1\overline{\zeta^{\dot{1}}})(\psi\sigma^i\partial_Y\overline{\psi})dY$$
$$+i\,\overline{\theta^{\dot{1}}}(\zeta\sigma^i\partial_Y\overline{\psi})dY, \tag{21}$$

obtained from the coordinate transformations (17) under constraints (14), and also substitution $\widetilde{R}\to\widetilde{r}$, one obtains the following tetrad:

$$e'^1=e^1+(A-C^1\overline{\theta^{\dot{1}}})dY,\quad e'^2=e^2+Ad\overline{Y}, \tag{22}$$

$$e'^3=e^3-C^3\overline{\theta^{\dot{1}}}dY,\quad e'^4=dv+\widetilde{h}e'^3, \tag{23}$$

where $dY=\widetilde{R}^{-1}(Pe^1-P_{\overline{Y}}e^3)$, and

$$A=i\sqrt{2}(\theta^1\overline{\zeta^{\dot{1}}}),\quad C^a=ie_i^a(\zeta\sigma^i\partial_Y\overline{\psi}), \tag{24}$$

$$\widetilde{h}=m(\mathrm{Re}\widetilde{R}^{-1})/P^3,\quad P=\sqrt{2}^{-1}(1+Y\overline{Y}). \tag{25}$$

As a result we obtain the metric of a super black hole $ds^2=e'^1e'^2+e'^3e'^4$ with broken four-dimensional supersymmetry. For parameters of spinning particles it corresponds to a specific state of a ''black hole'' without horizons and very far from extreme.

This derivation of a super Kerr metric is similar to the first derivation of the Kerr-Newman solution by complex shift from the Reissner-Nordström metric given by Newman and collaborators [24]. The first use of a complex shift in scalar electrodynamics is traced back to Appel who discovered the potential $e\mathrm{Re}(1/\widetilde{r})$ characterized by a typical Kerr singularity and twofoldedness of space. The first use of a supershift in electrodynamics was considered in the recent work by Tugai and Zheltukhin [18]. As a result a supermultiplet of Maxwell fields was generated from the Coulomb solution.

Therefore, at the moment there are several known applications of the method in consideration. For example, the simplest interesting new solutions can be obtained by simultaneously performing the complex shift and supershift to the Coulomb solution in flat space. Similarly, a supergeneralization of the Kerr-Newman solution leading to a supermultiplet of Maxwell fields on the Kerr background may be obtained, as well as a supergeneralization of the Kerr-Sen solution.

V. SUPERSHIFT OF THE KERR-SEN SOLUTION TO DILATON-AXION GRAVITY

The Kerr-Sen solution, a generalization of the Kerr solution to low energy string theory [11], may be written in the form [7]

$$ds^2_{dil}=2e^{-2(\Phi-\Phi_0)}\tilde{e}^1\tilde{e}^2+2\tilde{e}^3\tilde{e}^4, \tag{26}$$

where

$$\tilde{e}^1=(PZ)^{-1}dY, \quad \tilde{e}^2=(P\overline{Z})^{-1}d\overline{Y}, \tag{27}$$

$$\tilde{e}^3=P^{-1}e^3, \tag{28}$$

$$\tilde{e}^4=dr+iaP^{-2}(\overline{Y}dY-Yd\overline{Y})+(H_{dil}-1/2)e^3, \tag{29}$$

and[5]

$$H_{dil}=Mr/\Sigma_{dil}, \quad \Sigma_{dil}=e^{-2(\Phi-\Phi_0)}(Z\overline{Z})^{-1}, \tag{30}$$

$$e^{-2(\Phi-\Phi_0)}=1+(Q^2/2M)(Z+\overline{Z}), \quad Z^{-1}\equiv\tilde{r}. \tag{31}$$

The field of principal null directions is $\tilde{e}^3$. Following Eq. (6.1) of Ref. [2] this tetrad is related to the Kerr-Schild tetrad (18),(19),(20) as follows:

$$\tilde{e}^1=e^1-P^{-1}P_{\overline{Y}}e^3, \quad \tilde{e}^2=e^2-P^{-1}P_Ye^3, \tag{32}$$

$$\tilde{e}^3=P^{-1}e^3, \tag{33}$$

$$\tilde{e}^4=Pe^4_{dil}+P_Ye^1+P_{\overline{Y}}e^2-P_YP_{\overline{Y}}P^{-1}e^3. \tag{34}$$

Therefore the Kerr-Sen metric (26) may be reexpressed in a form containing the Kerr-Schild tetrad e^a, dilaton factor $e^{-2(\Phi-\Phi_0)}$, and a deformed function

$$H_{dil}=he^{2(\Phi-\Phi_0)}, \tag{35}$$

instead of the function h in the tetrad vector e^4 given by Eq. (20).

It was shown in Ref. [7] that the field of principal null directions e^3 survives in the Kerr-Sen solution and retains the property of being geodesic and shear free. It means that the Kerr theorem is applicable to this solution too, as well as the above geometrical construction if the tetrad is expressed in Cartesians coordinates x,y,z,t. The corresponding ''supershifted'' solution is obtained by the substitution $\tilde{R}\rightarrow\tilde{r}$ in the expression for the dilaton factor and by using the ''supershifted'' Kerr-Schild tetrad (22),(23),(24) in the expressions (32),(33),(34).

Summarizing, we find that metric is given by

$$ds^2_{dil}=2e^{-2(\Phi-\Phi_0)}e'^1e'^2+2e'^3e'^4_{dil}+2[1-e^{-2(\Phi-\Phi_0)}]$$
$$\times(P_Ye'^1+P_{\overline{Y}}e'^2-P_YP_{\overline{Y}}P^{-1}e'^3)e'^3/P, \tag{36}$$

where

$$e'^1=e^1+(A-C^1\dot{\overline{\theta^1}})dY, \quad e'^2=e^2+Ad\overline{Y}, \tag{37}$$

$$e'^3=e^3-C^3\dot{\overline{\theta^1}}dY, \quad e'^4_{dil}=dv+H_{dil}e'^3, \tag{38}$$

and where e^a are given by Eqs. (18),(19),(20), and also

$$dY=\tilde{R}^{-1}(Pe^1-P_{\overline{Y}}e^3), \tag{39}$$

$$A=i\sqrt{2}(\theta^1\dot{\overline{\zeta^1}}), \quad C^a=ie^a_i(\zeta\sigma^i\partial_Y\overline{\psi}), \tag{40}$$

$$H_{dil}=e^{2(\Phi-\Phi_0)}M\mathrm{Re}\tilde{R}^{-1}/P^3; \quad P=(1+Y\overline{Y})/\sqrt{2}. \tag{41}$$

$$e^{-2(\Phi-\Phi_0)}=1+(Q^2/M)\mathrm{Re}\tilde{R}^{-1}. \tag{42}$$

VI. CONCLUSION

As we pointed out in the Introduction, Kerr geometry contains stringlike structures and one of them is the Kerr singular ring. The gravitational field near this ring is similar [7] to the field around a heterotic string [11]. In super Kerr geometry we find some extra suggestions of this relationship. In the presented super-black-hole metrics the four-dimensional supersymmetry is broken because of the nonlinear realization of supersymmetry caused by B-slice constraints. However, there survives (2,0) supersymmetry based on the complex time parameter τ and anticommuting superpartners $\dot{\overline{\theta^1}}$ and θ^1. It is known from the analysis of the Kerr theorem [9,21] that only an analytic dependence in the even function $x_0(\tau)$ is admissible. On the other hand, during the above consideration we did not meet the demands for the Grassmann parameter $\zeta(\tau)$ to be analytic in τ. It means that the arising (2,0) superfields can depend on τ and $\overline{\tau}$, leading to both right and left modes in the fermionic sector that must induce traveling waves along the Kerr singular ring.

It has also to be noted that for the known parameters of spinning particles the angular momentum is very high, regarding the mass parameter, and the corresponding black holes are to be in a specific state ''. . . which is neither 'black' and nor 'hole' . . . '' [25]. In this case the ringlike singularity is naked, and space is branched on two sheets, $r>0$ and $r<0$, respectively. There appears a problem of the real source of the Kerr solution in addition to the mysterious complex supersource considered above.

To avoid this twofoldedness the ''negative'' sheet of space is truncated and a matter source is placed on the disk $r=0$ spanned by the Kerr singular ring. Such disklike or membranelike sources of the Kerr solution were considered in Einstein's gravity [3,5,26,27], as well as in low energy string theory [7,12]. However, the analysis shows [3,5] that very exotic properties of material are necessary to provide a continuity of metric by crossing the disk. It was obtained that the Kerr disk has to be in a rigid relativistic rotation and built of material having a pseudovacuum character, zero energy density in a corotating coordinate system.[6] There is no classical matter possessing this property, and some attention was also paid to a possible role of quantum effects [26].

Since supersymmetry takes an intermediate position between the classical and quantum regions, one could expect

[5]The definition of H_{dil} in Ref. [7] is different and contains an extra factor of two.

[6]A more complete list of references to the problem of the Kerr source can be found in Ref. [9].

that in some cases it could provide the necessary pseudovacuum character of matter, especially taking into account the remarkable cancellation contributions of fermionic and bosonic fields. It gives rise to a hope that the old problem of the source of the Kerr solution could find its resolution in a composite source built of a supermultiplet of matter fields. Obtaining the corresponding real sources compatible with Kerr supergeometry is a very important problem in future investigations.

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