Trace-anomaly-induced effective action for 2D and 4D dilaton coupled scalars

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The spherically symmetric reduction of higher-dimensional Einstein-scalar theory leads to a lower-dimensional dilatonic gravity with a dilaton coupled scalar [for example, from a four-dimensional (4D) to a 2D system]. We calculate the trace anomaly and anomaly-induced effective action for 2D and 4D dilaton coupled scalars. The large-N effective action for 2D quantum dilaton-scalar gravity is also found. These 2D results may be applied to the analysis of 4D spherical collapse. The role of new, dilaton-dependent terms in the trace anomaly for 2D black holes and Hawking radiation is investigated in some specific models of dilatonic gravity which represent a modification of the Callan-Giddings-Harvey-Strominger model. The conformal sector for 4D dilatonic gravity is constructed. The quantum back reaction of dilaton coupled matter is briefly discussed (it may lead to an inflationary universe with a nontrivial dilaton). [S0556-2821(98)02804-5]

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I. INTRODUCTION

There are motivations to study two-dimensional (2D) dilatonic gravity models. First of all, it is often easier to study 2D models than their 4D analogues. Especially, it may happen on the quantum level. For example, for a renormalizable classically solvable dilaton gravity coupled with minimal scalar matter, the problem of Hawking radiation and the back reaction of matter to a 2D black hole may be well understood in the $1/N$ expansion [1]. Modifications of the Callan-Giddings-Harvey-Strominger (CGHS) model and Hawking radiation have been investigated in Refs. [2–4] and many other works (for a review, see [5]). Second, some 2D dilatonic gravities are string inspired ones. They may serve as a laboratory for better understanding string theory itself. Third, if one starts from the 4D Einstein-scalar or 4D Einstein-Maxwell-scalar theory, then using a spherically symmetric reduction anzatz [6] one obtains the action for one of the 2D dilatonic gravity models with scalars. For example, applying such an anzatz (13) to 4D Einstein-Maxwell-minimal-scalar theory and integrating the angular modes, one gets

$$S=-\frac{1}{16\pi G}\int d^2x\sqrt{-g}e^{-2\phi}[R+2(\nabla\phi)^2+2e^{2\phi}-2Q^2e^{4\phi}]+\frac{1}{2}\int d^2x\sqrt{-g}e^{-2\phi}(\nabla\chi)^2. \tag{1}$$

Hence, 4D spherically symmetric collapse may be understood in terms of 2D dilatonic gravity.

Different from the CGHS model and its modifications, we have the scalar field nonminimally coupled with a dilaton. Then a generalization of the CGHS model and the study of Hawking radiation [7] in a generalized model in the large-N approximation (then one has to consider N scalars in the above model) requires the calculation of the trace anomaly for the dilaton coupled scalar.

Such a trace anomaly has been recently found for the above model in Ref. [8] and in the case of an arbitrary dilaton-scalar coupling function $f(\phi)$ in [9]. The correspondent trace-anomaly-induced effective action has also been calculated [8,9]. (Actually, the trace anomaly is proportional to the b_2 coefficient of the Schwinger-De Witt expansion for which a general dilaton coupled scalar was found some time ago, see Ref. [10].)

The natural next step is to discuss the quantum gravity contributions to such an action and its applications to 2D black holes and Hawking radiation. The question of the 4D generalization is also of interest.

The present paper is devoted to the study of this circle of questions. In the next section we discuss the general model of dilatonic gravity [11,12]. The trace anomaly and anomaly-induced effective action are calculated for N dilaton coupled two-dimensional scalars in the general form and in the large-N limit. (The contribution of the quantum dilaton is also included.) In the case when dilaton-scalar gravity is quantized, the one-loop divergent effective action is used to find the large-N nonlocal finite action. This action almost coincides with the anomaly-induced large-N action, as it should.

Section III is devoted to the investigation of 2D black holes in the modified CGHS model (we consider dilaton coupled scalars). The new, dilaton-dependent terms in the trace anomaly make the problem much more complicated than for the minimal case. In particular, the theory is not classically solvable anymore. A new black hole solution for purely induced theory is found. For some known black hole cases, Hawking radiation and black hole entropy are briefly discussed.

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In Sec. IV we calculate the trace anomaly and the induced effective action for the dilaton coupled 4D scalar. The motivation to do so is similar to the 2D case. Let us start from higher-dimensional Einstein-scalar theory. After a spherically symmetric reduction anzatz one is left with lower-dimensional (say 4D) dilatonic gravity with a dilaton coupled scalar.

Section V is devoted to the formulation of the conformal sector for 4D dilatonic gravity. The classical solutions of such a theory describe quantum cosmology (with the back reaction of matter). One of the solutions for purely induced theory may correspond to the inflationary Universe. In the Conclusion we give a summary and a list of the problems for future research.

II. ONE-LOOP EFFECTIVE ACTION IN THE LARGE-N APPROXIMATION

We will start with dilaton gravity of the most general form [11,12] interacting with scalar matter:

$$S=-\int d^2x\sqrt{-g}\left\{\frac{1}{2}Z(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+C(\phi)R+V(\phi,\chi)-\frac{1}{2}f(\phi)g^{\mu\nu}\sum_{i=1}^{N}\partial_\mu\chi_i\partial_\nu\chi_i\right\}. \tag{2}$$

It includes a dilaton field ϕ, N real dilaton coupled scalars χ_i, and dilaton-dependent couplings Z, C, f. The potential V is a function of ϕ and χ_i.

First of all we will be interested in the study of the one-loop divergent effective action for the above theory. We consider two different cases. Let in the theory (2) the dilaton ϕ and scalars χ_i are the quantum fields, while the gravitational field is the external field. Then one can apply the background field method [13] and calculate the one-loop effective action [10] (see Eq. (41) of Ref. [10]):

$$\begin{aligned}\Gamma_{\rm div}=&-\frac{1}{2\epsilon}\int d^2x\sqrt{-g}\left\{\left(\frac{C''}{Z}-\frac{N+1}{6}\right)R+\frac{V''}{Z}-\frac{N}{f}\frac{\partial^2 V}{\partial\chi^2}\right.\\&+\left(\frac{f'^2}{2fZ}-\frac{f''}{2Z}\right)(\nabla^\lambda\chi_i)(\nabla_\lambda\chi_i)+\left(\frac{Nf''}{2f}-\frac{Nf'^2}{4f^2}-\frac{Z'^2}{4Z^2}\right)\\&\left.\times(\nabla^\lambda\phi)(\nabla_\lambda\phi)+\left(\frac{Nf'}{2f}-\frac{Z'}{2Z}\right)\Delta\phi\right\},\end{aligned} \tag{3}$$

where $\epsilon=2\pi(n-2)$ and we use dimensional regularization.

The remarkable fact about the system (2) with $C=V=0$ and the gravitational field being the classical one is that the system is a conformally invariant system. Then on the quantum level the conformal (or trace) anomaly T is given by

$$\Gamma_{\rm div}=\frac{1}{n-2}\int d^2x\sqrt{-g}b_2,\quad T=b_2. \tag{4}$$

From here one gets (see also [9])

$$\begin{aligned}T=\frac{1}{24\pi}&\left\{(N+1)R-3\left(\frac{f'^2}{2fZ}-\frac{f''}{2Z}\right)(\nabla^\lambda\chi_i)(\nabla_\lambda\chi_i)\right.\\&-3\left(\frac{Nf''}{f}-\frac{Nf'^2}{2f^2}-\frac{Z'^2}{2Z^2}\right)(\nabla^\lambda\phi)(\nabla_\lambda\phi)\\&\left.-3\left(\frac{Nf'}{f}-\frac{Z'}{Z}\right)\Delta\phi\right\},\end{aligned} \tag{5}$$

while for a purely scalar field (dilaton is classical) all terms with Z in Eq. (5) disappear [9]. For the special case $N=1$, $f(\phi)=e^{-2\phi}$ and no quantum dilaton,

$$T=\frac{1}{24\pi}\{R-6(\nabla^\lambda\phi)(\nabla_\lambda\phi)+6\Delta\phi\}. \tag{6}$$

This trace anomaly was recently calculated in Ref. [8] using the zeta-regularization method. The coefficient of the third term in Eq. (6) disagrees with the results of Ref. [8]. The reasons for this disagreement were discussed in Ref. [9] (we are not using zeta regularization, but dimensional regularization).

Making the conformal transformation of metric $g_{\mu\nu}\to e^{2\sigma}g_{\mu\nu}$ in the trace anomaly, using relation

$$T=\frac{1}{\sqrt{g}}\frac{\delta}{\delta\sigma}W[\sigma], \tag{7}$$

one can find anomaly-induced effective action $W[\sigma]$. In the covariant, nonlocal form it may be represented as the following [9]:

$$\begin{aligned}W=-\frac{1}{2}\int d^2x\sqrt{-g}&\left[\frac{c}{2}R\frac{1}{\Delta}R+F_1(\phi)(\nabla^\lambda\chi_i)(\nabla_\lambda\chi_i)\frac{1}{\Delta}R\right.\\&\left.+\left(F_2(\phi)-\frac{\partial F_3(\phi)}{\partial\phi}\right)\nabla^\lambda\phi\nabla_\lambda\phi\frac{1}{\Delta}R+R\int F_3(\phi)d\phi\right],\end{aligned} \tag{8}$$

where

$$c=\frac{N+1}{24\pi},\quad F_1(\phi)=-\frac{1}{8\pi}\left(\frac{f'^2}{fZ}-\frac{f''}{Z}\right),$$
$$F_2(\phi)=-\frac{1}{8\pi}\left(\frac{Nf''}{f}-\frac{Nf'^2}{2f^2}-\frac{Z'^2}{2Z^2}\right),$$
$$F_3(\phi)=-\frac{1}{8\pi}\left(\frac{Nf'}{f}-\frac{Z'}{Z}\right). \tag{9}$$

Note that for $f=e^{-2\phi}$, $Z=0$ [i.e., one has to omit all Z-dependent terms in Eq. (9)] the effective action (8) was calculated in Ref. [8]. Note also that actually it is very easy to get the large-N limit of the effective action (8). To do so one only has to omit Z-dependent terms in $F_2(\phi)$, $F_3(\phi)$ and the second term in c. In addition, if the dilaton is purely classical one should put $F_1=0$.

Let us consider now the theory with the action (2) as quantum dilaton-matter gravity where all fields $g_{\mu\nu}$, ϕ, and

χ_i are quantized ones. Using the background field method [13], $g_{\mu\nu}\to g_{\mu\nu}+h_{\mu\nu}$, $\phi\to\phi+\varphi$, where $h_{\mu\nu}$, φ are quantum fields, and the minimal gauge [11]

$$S_{\mathrm{gf}}=-\frac{1}{2}\int C_{\mu\nu}\chi^{\mu}\chi^{\nu}, \tag{10}$$

where $\chi^{\mu}=-\nabla_{\nu}\overline{h^{\mu\nu}}+(C'/C)\nabla^{\mu}\varphi$, $C_{\mu\nu}=-C\sqrt{g}g_{\mu\nu}$, $\overline{h}_{\mu\nu}=h_{\mu\nu}-\frac{1}{2}g_{\mu\nu}h$, the one-loop effective action maybe found. For pure dilaton gravity it was obtained in Ref. [11] and later in Refs. [10,14].

For dilaton-matter gravity with the classical action (2) the complete result was obtained in Ref. [10] [(see Eqs. (31), (32)]:

$$\Gamma_{\mathrm{div}}=-\frac{1}{2\epsilon}\int d^2x\sqrt{-g}\left\{\frac{24-N}{6}R+\frac{2}{C}V+\frac{2}{C'}V'-\frac{V_{,ii}}{f}\right.$$
$$+\left(\frac{C''}{C}-\frac{3C'^2}{C^2}-\frac{C''Z}{C'^2}+\frac{Nf''}{2f}-\frac{Nf'^2}{4f^2}\right)(\nabla^{\lambda}\phi)(\nabla_{\lambda}\phi)$$
$$\left.+\left(\frac{C'}{C}-\frac{Z}{C'}+\frac{Nf'}{2f}\right)\Delta\phi\right\}. \tag{11}$$

Using Eq. (11) one can find the one-loop effective action for any specific model.

For example, let us take

$$Z(\phi)=4e^{-2\phi},\quad C(\phi)=e^{-2\phi},$$
$$V(\phi,\chi)=2,\quad f(\phi)=e^{-2\phi} \tag{12}$$

in the action (2). Then the theory (2) with dilatonic couplings (12) can be obtained (for $N=1$) by using the spherically symmetric reduction anzatz [6]

$$ds^2=g_{\mu\nu}dx^{\mu}dx^{\nu}+e^{-2\phi}d\Omega^2 \tag{13}$$

from 4D Einstein-scalar or 4D Einstein-Maxwell-scalar theory (in the last case, $V=2-2Q^2e^{2\phi}$). Hence, in such a case the action (2) with dilatonic couplings (12) may describe the radial modes of the extremal dilatonic black holes in four dimensions [15]. In other words, 2D dilatonic black holes may also describe 4D spherically symmetric collapse.

Using general expression (11) we may write the one-loop effective action for the theory (12) (keeping N an arbitrary integer)

$$\Gamma_{\mathrm{div}}=-\frac{1}{2\epsilon}\int d^2x\sqrt{-g}\left\{\frac{24-N}{6}R+4e^{2\phi}\right.$$
$$\left.+(N-12)(\nabla^{\lambda}\phi)(\nabla_{\lambda}\phi)-N\Delta\phi\right\}. \tag{14}$$

This theory is the one-loop renormalizable one. Note that recently a very interesting attempt to calculate the one-loop effective action (including the local and nonlocal finite terms) for the model (12) was done in Ref. [16]. Unfortunately, the result [16] includes a number of mistakes. In particular, the divergent part of the one-loop effective action in the same minimal gauge (10) of Ref. [11] disagrees with the expression (14) which coincides (at least, in the gravitational sector) with the independent results of Refs. [11,10,14] in all cases where such a comparison may be done. Hence, the result of Ref. [16] contradicts those of Refs. [11,10,14] and does not have the correct on-shell limit [17]. One of the causes of this mistake is that for the calculation of the effective action in the model (12) another dilatonic gravity classically equivalent to Eq. (12) (after conformal transformation and dilaton rescaling) is used. However, it was proved in Ref. [17] (with an explicit example) that classically equivalent 2D dilatonic gravities (in the sense of conformal transformation) are not quantum equivalent off shell. They lead to different divergent one-loop effective actions which coincide only on shell.

Let us turn again to the general model (2). In the large-N limit from Eq. (11) we get

$$\Gamma_{\mathrm{div}}=-\frac{1}{2\epsilon}\int d^2x\sqrt{-g}\left\{-\frac{N}{6}R-\frac{N}{f}\frac{\partial^2V}{\partial\chi\partial\chi}\right.$$
$$\left.+\left(\frac{Nf''}{2f}-\frac{Nf'^2}{4f^2}\right)(\nabla^{\lambda}\phi)(\nabla_{\lambda}\phi)+\frac{Nf'}{2f}\Delta\phi\right\}. \tag{15}$$

Actually, the expression (15) is given by matter, matter-graviton, and matter-dilaton loops. It may be considered as the source for the effective trace anomaly, as in Eq. (5). Integrating such a trace anomaly over σ in the same way as in Eq. (7), we get

$$W=-\frac{N}{2\pi}\int d^2x\sqrt{-g}\left[\frac{1}{48}R\frac{1}{\Delta}R-\frac{1}{8}\ln fR-\frac{1}{16\pi}\frac{f'^2}{f^2}(\nabla^{\lambda}\phi)\right.$$
$$\left.\times(\nabla_{\lambda}\phi)\frac{1}{\Delta}R+\frac{1}{2Nf}\sum_{i=1}^{N}\frac{\partial^2V}{\partial\chi_i\partial\chi_i}e^{(1/\Delta)R}\right]. \tag{16}$$

The expression (16) gives the large-N limit of the effective action in quantum dilatonic gravity (2). Note the appearance of a new nonlocal term related with the scalar potential V (if it is present in the theory). Notice that V breaks the conformal invariance of the scalar field. That is why it should not be included to the system (2) when only scalars are quantized. Then a few more terms of the same structure as the last one in Eq. (16) may be expected in the strict calculation of the one-loop finite effective action in dilaton-scalar gravity.

The action (8) should be used to take into account the back reaction of the quantum dilaton-matter system to classical dilatonic gravity. On the same time the action (16) should be added to the classical dilatonic matter-gravity action if one would like to take into account the back reaction of quantum dilaton-matter gravity (in the large-N limit). The nonlocal actions (8),(16) open the way to new generalizations of models such as the CGHS model [1], where one can find new black hole solutions and (or) new terms in the Hawking radiation. In the next section, we are going to discuss some simple properties of the above effective actions in connection with 2D black holes.

III. 2D BLACK HOLES AND HAWKING RADIATION

We start with a system where the dilaton gravity of special form [1] couples with the dilaton coupled scalar fields:

$$S_0=\frac{1}{2\pi}\int d^2x\sqrt{-g}\left\{e^{-2\phi}[R+4g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+4\lambda^2]-\frac{1}{2}f(\phi)\sum_{i=1}^{N}g^{\mu\nu}\partial_\mu\chi_i\partial_\nu\chi_i\right\}. \quad (17)$$

We would like to consider the modifications of the CGHS model due to the back reaction of dilaton coupled scalars. Hence, we calculate the effective action (8) for dilaton coupled scalars in the large-N approximation (the gravitational field is the classical field):

$$W=-\frac{1}{2}\int d^2x\sqrt{-g}\left[\frac{N}{48\pi}R\frac{1}{\Delta}R-\frac{1}{8\pi}\left(\frac{f'^2}{f}-f''\right)(\nabla^\lambda\chi_i)\times(\nabla_\lambda\chi_i)\frac{1}{\Delta}R-\frac{N}{16\pi}\frac{f'^2}{f^2}\nabla^\lambda\phi\nabla_\lambda\phi\frac{1}{\Delta}R-\frac{N}{8\pi}\ln fR\right]. \quad (18)$$

Hence the complete action of our theory is given by

$$S=S_0+W. \quad (19)$$

We treat this theory as a classical system. The background scalar field is considered to be zero so we omit all scalar terms in the above expression.

In the conformal gauge

$$g_{\pm\mp}=-\frac{1}{2}e^{2\rho},\quad g_{\pm\pm}=0, \quad (20)$$

the equations of motion are obtained by the variation over $g^{\pm\pm}$, $g^{\pm\mp}$, and ϕ:

$$\begin{aligned}0=T_{\pm\pm}=&e^{-2\phi}[4\partial_\pm\rho\partial_\pm\phi-2(\partial_\pm\phi)^2]\\&+\frac{N}{12}(\partial_\pm^2\rho-\partial_\pm\rho\partial_\pm\rho)\\&+\frac{N}{8}\left\{(\partial_\pm\tilde{\phi}\partial_\pm\tilde{\phi})\rho+\frac{1}{2}\frac{\partial_\pm}{\partial_\mp}(\partial_\pm\tilde{\phi}\partial_\mp\tilde{\phi})\right\}\\&+\frac{N}{8}\{-2\partial_\pm\rho\partial_\pm\tilde{\phi}+\partial_\pm^2\tilde{\phi}\}+t^\pm(x^\pm),\end{aligned} \quad (21)$$

$$\begin{aligned}0=T_{\pm\mp}=&e^{-2\phi}(2\partial_+\partial_-\phi-4\partial_+\phi\partial_-\phi-\lambda^2e^{2\rho})\\&-\frac{N}{12}\partial_+\partial_-\rho-\frac{N}{8}\partial_+\tilde{\phi}\partial_-\tilde{\phi}-\frac{N}{4}\partial_+\partial_-\tilde{\phi},\end{aligned} \quad (22)$$

$$\begin{aligned}0=&e^{-2\phi}(-4\partial_+\partial_-\phi+4\partial_+\phi\partial_-\phi+2\partial_+\partial_-\rho\\&+\lambda^2e^{2\rho})-\frac{Nf'}{f}\left\{\frac{1}{16}\partial_+(\rho\partial_-\tilde{\phi})\right.\\&\left.+\frac{1}{16}\partial_-(\rho\partial_+\tilde{\phi})-\frac{1}{8}\partial_+\partial_-\rho\right\}.\end{aligned} \quad (23)$$

Here

$$\tilde{\phi}=\ln f \quad (24)$$

and $t(x^\pm)$ is a function which is determined by the boundary condition.

First we consider the large-N limit, where the classical part can be ignored. Then the field equations become more simple:

$$\begin{aligned}0=&\frac{1}{N}T_{\pm\pm}\\=&\frac{1}{12}(\partial_\pm^2\rho-\partial_\pm\rho\partial_\pm\rho)\\&+\frac{1}{8}\left\{(\partial_\pm\tilde{\phi}\partial_\pm\tilde{\phi})\rho+\frac{1}{2}\frac{\partial_\pm}{\partial_\mp}(\partial_\pm\tilde{\phi}\partial_\mp\tilde{\phi})\right\}\\&+\frac{1}{8}\{-2\partial_\pm\rho\partial_\pm\tilde{\phi}+\partial_\pm^2\tilde{\phi}\}+t^\pm(x^\pm),\end{aligned} \quad (25)$$

$$0=\frac{1}{N}T_{\pm\mp}=-\frac{1}{12}\partial_+\partial_-\rho-\frac{1}{8}\partial_+\tilde{\phi}\partial_-\tilde{\phi}-\frac{1}{4}\partial_+\partial_-\tilde{\phi}, \quad (26)$$

$$0=\frac{1}{16}\partial_+(\rho\partial_-\tilde{\phi})+\frac{1}{16}\partial_-(\rho\partial_+\tilde{\phi})-\frac{1}{8}\partial_+\partial_-\rho. \quad (27)$$

The function $t^\pm(x^\pm)$ can be absorbed into the choice of the coordinate and we can choose

$$t^\pm(x^\pm)=0. \quad (28)$$

Combining Eqs. (25) and (26), we obtain

$$-\frac{1}{3}(\partial_\pm\rho)^2+\frac{1}{2}\rho(\partial_\pm\tilde{\phi})^2-\partial_\pm\rho\partial_\pm\tilde{\phi}=0, \quad (29)$$

i.e.,

$$\partial_\pm\tilde{\phi}=\frac{1+\sqrt{1+(2/3)\rho}}{\rho}\partial_\pm\rho$$

or

$$\frac{1-\sqrt{1+(2/3)\rho}}{\rho}\partial_\pm\rho. \quad (30)$$

This tells us that

$$\tilde{\phi}=\int d\rho\frac{1\pm\sqrt{1+(2/3)\rho}}{\rho}. \quad (31)$$

Substituting Eq. (31) into Eq. (27), we obtain

$$\partial_+\partial_-\left\{\left(1+\frac{2}{3}\rho\right)^{3/2}\right\}=0, \quad (32)$$

i.e.,

$$\rho=\frac{3}{2}\{-1+[\rho^+(x^+)+\rho^-(x^-)]^{2/3}\}. \quad (33)$$

Here $\rho^{\pm}$ is an arbitrary function of $x^{\pm}=t\pm x$. We can straightforwardly confirm that the solutions (31) and (33) satisfy Eq. (26). The scalar curvature is given by

$$R=8e^{-2\rho}\partial_+\partial_-\rho$$
$$=-\frac{8}{3}\frac{e^{-3\{-1+[\rho^+(x^+)+\rho^-(x^-)]^{2/3}\}}}{[\rho^+(x^+)+\rho^-(x^-)]^{4/3}}\rho^{+\prime}(x^+)\rho^{-\prime}(x^-). \tag{34}$$

Note that when $\rho^+(x^+)+\rho^-(x^-)=0$, there is a curvature singularity. Especially if we choose

$$\rho^+(x^+)=\frac{r_0}{x^+},\quad \rho^-(x^-)=-\frac{x^-}{r_0}, \tag{35}$$

there are curvature singularities at $x^+x^-=r_0^2$ and a horizon at $x^+=0$ or $x^-=0$. The asymptotic flat regions are given by $x^+\to+\infty$ $(x^-<0)$ or $x^-\to-\infty$ $(x^+>0)$. Therefore we can regard $x^{\pm}$ as corresponding to the Kruskal coordinates in four dimensions.

In order to discuss the Hawking radiation (which is usually related with the trace anomaly [18]), it is necessary to find the exact vacuum not only at the classical level but even at the quantum level. In the following, we determine the function $\tilde{\phi}=\ln f(\phi)$ in Eq. (17) so that the linear dilaton vacuum

$$\rho=\phi=-\frac{1}{2}(\ln x^+ +\ln x^- +\ln\lambda^2) \tag{36}$$

is an exact solution. Substituting Eq. (36) into Eq. (22), we find

$$T_{\pm\pm}=\frac{N}{16}[(\phi')^2+\phi'']\frac{\lambda^2}{x^+x^-}=0. \tag{37}$$

This shows that

$$\tilde{\phi}=2\ln(\phi+c). \tag{38}$$

Substituting Eq. (38) into Eq. (23) we find that the constant of the integration should vanish: $c=0$, i.e.,

$$f(\phi)=\phi^2. \tag{39}$$

The solution (39) when substituted into Eq. (36) satisfies Eq. (21). If we divide the energy-momentum tensor into classical and quantum parts, the Hawking radiation is given by substituting the classical solution into the quantum part (the part proportional to N). When we substitute the shock wave solution[1]

$$\rho=\begin{cases}-\frac{1}{2}\ln\left(1+\frac{a}{\lambda}e^{\lambda\sigma^-}\right), & \sigma<\sigma_0,\\ -\frac{1}{2}\ln\left(1+\frac{a}{\lambda}e^{\lambda(\sigma^--\sigma^++\sigma_0^+)}\right), & \sigma^+>\sigma_0,\end{cases} \tag{40}$$

$$\phi=\begin{cases}-\frac{\lambda}{2}\sigma^+-\frac{1}{2}\ln\left(e^{-\lambda\sigma^-}+\frac{a}{\lambda}\right), & \sigma^+<\sigma_0,\\ -\frac{1}{2}\ln\left(\frac{a}{\lambda}e^{\lambda\sigma_0}+e^{\lambda(\sigma^+-\sigma^-)}\right) & \sigma^+>\sigma_0,\end{cases} \tag{41}$$

we find that ϕ-dependent terms in the quantum part of the energy momentum tensor vanish when $|\sigma^+|\to\infty$. This means that the behavior in the asymptotic region, especially the Hawking radiation, is identical with that of the CGHS model [1].

We now investigate the Bousso-Hawking choice [8]

$$f(\phi)=e^{-2\phi}(\tilde{\phi}=-2\phi). \tag{42}$$

Then the quantum part of the energy-momentum tensor has the following form:

$$T^q_{\pm\pm}=\frac{N}{12}(\partial_\pm^2\rho-\partial_\pm\rho\partial_\pm\rho)$$
$$+\frac{N}{2}\left\{(\partial_\pm\phi\partial_\pm\phi)\rho+\frac{1}{2}\frac{\partial_\pm}{\partial_\mp}(\partial_\pm\phi\partial_\mp\phi)\right\}$$
$$-\frac{N}{4}\{-2\partial_\pm\rho\partial_\pm\phi+\partial_\pm^2\phi\}+t(x^\pm) \tag{43}$$

$$T^q_{\pm\mp}=-\frac{N}{12}\partial_+\partial_-\rho-\frac{N}{2}\partial_+\phi\partial_-\phi+\frac{N}{2}\partial_+\partial_-\phi. \tag{44}$$

Substituting the classical shock wave solution (40), we find, when $\sigma^+<\sigma$,

$$T^q_{+-}=\frac{N\lambda^2}{8}\frac{1}{[1+(a/\lambda)e^{\lambda\sigma^-}]},$$

$$T^q_{++}=\frac{N\lambda^2}{16}\ln\left(1+\frac{a}{\lambda}e^{\lambda\sigma^-}\right)+t^+(\sigma^+),$$

$$T^q_{--}=-\frac{N\lambda^2}{48}\left\{1-\frac{1}{[1+(a/\lambda)e^{\lambda\sigma^-}]^2}\right\}$$
$$-\frac{N\lambda^2}{16}\frac{\ln[1+(a/\lambda)e^{\lambda\sigma^-}]}{[1+(a/\lambda)e^{\lambda\sigma^-}]^2}+\frac{N}{16}\frac{a\lambda e^{\lambda\sigma^-}\sigma^+}{[1+(a/\lambda)e^{\lambda\sigma^-}]^2}$$
$$+t^-(\sigma^-). \tag{45}$$

This tells us that there is incoming energy from the past null infinity ($\sigma^+\to-\infty$ or $\sigma^-\to-\infty$). However, the explicit estimation is problematic. The problem is caused by the fact that the dilaton vacuum is not the exact vacuum. Especially the last term in T^q_{--}, which is linear with respect to σ^+, tells us that we cannot use the dilaton vacuum as a classical approximation. The linear term also makes it impossible to determine $t^-(\sigma^-)$ by the boundary condition at $\sigma^+\to-\infty$

[1]The scalar field χ_i in Eq. (17) cannot make the shock wave. Here we suppose that the dilaton gravity also couples with another, minimal scalar field which appeared in the original CGHS model [1].

although Hawking radiation is essentially given by $t^{-}(\sigma^{-})$ as we will see in the following.

When $\sigma^{+}>\sigma_0$, we find

$$T^q_{+-}=\frac{N\lambda^2}{12}\frac{1}{[1+(a/\lambda)e^{\lambda(\sigma^{-}-\sigma^{+}+\sigma_0)}]^2} -\frac{N\lambda^2}{6}\frac{1}{[1+(a/\lambda)e^{\lambda(\sigma^{-}-\sigma^{+}+\sigma_0)}]},$$

$$T^q_{\pm\pm}=-\frac{N\lambda^2}{48}\left\{1+\frac{1}{[1+(a/\lambda)e^{\lambda(\sigma^{-}-\sigma^{+}+\sigma_0)}]^2}\right\} -\frac{N\lambda^2}{16}\frac{\ln[1+(a/\lambda)e^{\lambda(\sigma^{-}-\sigma^{+}+\sigma_0)}]-1}{[1+(a/\lambda)e^{\lambda(\sigma^{-}-\sigma^{+}+\sigma_0)}]^2}+t^{\pm}(\sigma^{\pm}). \tag{46}$$

Then when $\sigma^{+}\to+\infty$, the energy momentum tensor behaves as

$$T^q_{+-}\to-\frac{N\lambda^2}{12},$$

$$T^q_{\pm\pm}\to\frac{N\lambda^2}{48}+t^{\pm}(\sigma^{\pm}). \tag{47}$$

This expresses Hawking radiation but we cannot determine the unknown function $t^{-}(\sigma^{-})$.

Hence, we found that there may be new contributions to Hawking radiation from the dilaton-dependent terms in the trace anomaly. However, in order to make their accurate estimation one has to construct new solvable models of 2D black holes with an arbitrary f and (or) other choices for dilatonic couplings Z, C, and V in the general model of dilatonic gravity.

Finally, we evaluate the contribution to the black hole entropy from W in Eq. (18). The contribution from the first classical term may be investigated by standard methods [19]. Following this procedure, the contribution from the dilaton-dependent terms

$$-\frac{1}{2}\int d^2x\sqrt{-g}\left[-\frac{N}{16\pi}\frac{f'^2}{f^2}(\nabla^{\lambda}\phi)(\nabla_{\lambda}\phi)\frac{1}{\Delta}R-\frac{N}{8\pi}\ln fR\right] \tag{48}$$

can be evaluated as follows:

$$-\frac{N}{16}\int d^2x\sqrt{-g}\left(\frac{f'^2}{f^2}(\nabla^{\lambda}\phi)(\nabla_{\lambda}\phi)\psi\right)-\frac{N}{4\pi}\ln f(\phi_0). \tag{49}$$

Here ϕ_0 is the value of the classical solution for the dilaton field at the horizon,

$$\phi_0=-\frac{1}{2}\ln\left(\frac{M}{\lambda}\right), \tag{50}$$

and ψ is defined by

$$\Delta\psi=\delta(r) \tag{51}$$

with the boundary condition, where

$$\psi\to\ln r,\quad \text{when } r\to 0. \tag{52}$$

Here we choose the coordinate system where the metric of the black hole is given by

$$ds^2=dr^2+\sinh^2\sqrt{\frac{M}{\lambda}}dt^2 \tag{53}$$

when Wick rotated to the Euclidean signature. Hence, at least on a qualitative level we see the appearence of new terms in quantum corrections to the black hole entropy.

IV. TRACE ANOMALY AND INDUCED EFFECTIVE ACTION FOR 4D DILATON COUPLED SCALAR

It could be interesting to generalize the results of Sec. II for the 4D case. The purpose of the present section will be to calculate the nonlocal effective action for the 4D dilaton coupled conformal scalar. Let us consider the theory with the following Lagrangian in curved spacetime (we work in Minkowski signature):

$$L=\varphi f(\phi)(\Box-\xi R)\varphi, \tag{54}$$

where φ is quantum scalar field, $\Box=g^{\mu\nu}\nabla_{\mu}\nabla_{\nu}$, ϕ is an external field (dilaton), and $f(\phi)$ is an arbitrary function.

It is very easy to check that for the conformal transformation

$$g_{\mu\nu}\to e^{2\sigma}g_{\mu\nu},$$

$$R\to e^{-2\sigma}[R-6\Box\sigma-6(\nabla_{\mu}\sigma)(\nabla^{\mu}\sigma)] \tag{55}$$

the theory with Lagrangian (54) is conformally invariant for $\xi=\frac{1}{6}$.

Let us calculate the divergent part of the effective action for the theory (54):

$$\Gamma_{\rm div}=-\frac{i}{2}{\rm Tr}\,\ln\left\{f(\phi)\left[\Box-\xi R+\frac{\Box f(\phi)}{2f(\phi)}+\frac{[\nabla^{\mu}f(\phi)]}{f(\phi)}\nabla_{\mu}\right]\right\} =\frac{1}{(n-4)}\int d^4x\sqrt{-g}b_4, \tag{56}$$

where b_4 is the b_4 coefficient of the Schwinger–De Witt expansion. The methods of its calculation are well known (see, for example, Sec. 3.6 of Ref. [13]). Applying these methods, after some algebra we get

$$(4\pi)^2b_4=\frac{1}{2}\left(\frac{1}{6}-\xi\right)^2R^2+\frac{1}{4}\frac{(\nabla f)^2}{f^2}\left(\frac{1}{6}-\xi\right)R +\frac{1}{32}\frac{[(\nabla f)(\nabla f)]^2}{f^4}+\frac{1}{2}\left(\frac{1}{6}-\xi\right)\Box R +\frac{1}{24}\Box\left(\frac{(\nabla f)(\nabla f)}{f^2}\right)+\frac{1}{180}(R^2_{\mu\nu\alpha\beta}-R^2_{\mu\nu}+\Box R). \tag{57}$$

Note that the $f(\phi)$ multiplier in Eq. (56) does not give the contribution to b_4.

For $\xi=\frac{1}{6}$, we get the trace anomaly

$$\langle T_\mu^\mu\rangle=b_4. \tag{58}$$

Hence the trace anomaly for the dilaton coupled 4D scalar is given by

$$T=\frac{1}{(4\pi)^2}\left\{\frac{1}{32}\frac{[(\nabla f)(\nabla f)]^2}{f^4}+\frac{1}{24}\Box\left(\frac{(\nabla f)(\nabla f)}{f^2}\right)+\frac{1}{180}(R^2_{\mu\nu\alpha\beta}-R^2_{\mu\nu}+\Box R)\right\}. \tag{59}$$

Here, the last term is the well-known conformal anomaly (for a review, see [20]) for the conformally invariant scalar. The first two terms in Eq. (59) are the dilaton contributions to the conformal anomaly.

Let us write Eq. (59) in a slightly different form:

$$T=\left\{b\left(F+\frac{2}{3}\Box R\right)+b'G+b''\Box R+a_1\frac{[(\nabla f)(\nabla f)]^2}{f^4}+a_2\Box\left(\frac{(\nabla f)(\nabla f)}{f^2}\right)\right\}, \tag{60}$$

where F is the square of the Weyl tensor in four dimensions, and G is Gauss-Bonnet invariant. For the scalar field, it follows from Eq. (59) that

$$b=\frac{1}{120(4\pi)^2},\quad b'=-\frac{1}{360(4\pi)^2},$$

$$a_1=\frac{1}{32(4\pi)^2},\quad a_2=\frac{1}{24(4\pi)^2}, \tag{61}$$

and in principle b'' is an arbitrary parameter (it may be changed by the finite renormalization of the local counterterm).

The nonlocal effective action induced by the conformal anomaly (without the dilaton) was calculated some time ago [21]. Using the equation

$$T=\frac{1}{\sqrt{-g}}\frac{\delta}{\delta\sigma}W(\sigma) \tag{62}$$

and integrating it, one can restore the nonlocal effective action W induced by the conformal anomaly

$$\begin{aligned}W=&b\int d^4x\sqrt{-g}F\sigma+b'\int d^4x\sqrt{-g}\left\{\sigma\left[2\Box^2+4R^{\mu\nu}\nabla_\mu\nabla_\nu-\frac{4}{3}R\Box+\frac{2}{3}(\nabla^\mu R)\nabla_\mu\right]\sigma+\left(G-\frac{2}{3}\Box R\right)\sigma\right\}-\frac{1}{12}\left(b''+\frac{2}{3}(b+b')\right)\int d^4x\sqrt{-g}[R-6\Box\sigma-6(\nabla\sigma)(\nabla\sigma)]^2\\&+\int d^4x\sqrt{-g}\left\{a_1\frac{[(\nabla f)(\nabla f)]^2}{f^4}\sigma+a_2\Box\left(\frac{(\nabla f)(\nabla f)}{f^2}\right)\sigma+a_2\frac{(\nabla f)(\nabla f)}{f^2}[(\nabla\sigma)(\nabla\sigma)]\right\}.\end{aligned} \tag{63}$$

Here the σ-independent term is dropped and the last terms represent the contributions from the dilaton-dependent terms in the trace anomaly. Similarly one can calculate the trace anomaly and induced effective action for other theories such as the dilaton coupled spinor or the dilaton coupled Weyl gravity with Lagrangian

$$L=F(\phi)C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta}, \tag{64}$$

or dilaton coupled vector $L=-\frac{1}{4}g(\phi)F_{\mu\nu}F^{\mu\nu}$. Notice that in the 2D case such a vector field is not a conformally invariant one.

V. CONFORMAL SECTOR OF DILATON GRAVITY AND QUANTUM COSMOLOGY

Let us consider now the classical theory of dilatonic gravity:

$$L_{\rm cl}=Z(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+C(\phi)R+V(\phi), \tag{65}$$

where Z, C, V are the arbitrary dilatonic functions. For a specific choice of these functions the theory (65) represents the low-energy string effective action or Brans-Dicke gravity. So it may be considered as string-motivated classical gravity.

Adding the induced action to the action (65) (where part of the linear σ terms are dropped), we get in the case of conformally flat fiducial metric $g_{\mu\nu}=e^{2\sigma}\eta_{\mu\nu}$,

$$\begin{aligned}S=W+S_{\rm cl}=\int d^4x\Big\{&2b'(\Box\sigma)^2-[3b''+2(b+b')]\\&\times[\Box\sigma+(\partial_\mu\sigma)^2]^2+a_1\frac{[(\nabla f)(\nabla f)]^2}{f^4}\sigma\\&+a_2\Box\left(\frac{(\nabla f)(\nabla f)}{f^2}\right)\sigma\\&+a_2\frac{(\nabla f)(\nabla f)}{f^2}g^{\mu\nu}(\nabla_\mu\sigma)(\nabla_\nu\sigma)\\&+e^{2\sigma}Z(\phi)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+e^{2\sigma}C(\phi)[-6\Box\sigma\\&-6(\partial_\mu\sigma)(\partial^\mu\sigma)]+e^{4\sigma}V(\phi)\Big\}.\end{aligned} \tag{66}$$

The action (66) describes the conformal sector of 4D dilatonic gravity. It is a direct generalization of the conformal sector of 4D gravity which was introduced and studied in Refs. [22].

A very interesting problem for future research is to study the quantum structure of the theory with the action (66), its properties, the existence of fixed points, etc. In particular, one can expect, as it happened with its analogue for $\phi=$const [22], that it may provide the solution of the cosmological constant problem. The gravitational dressing of matter β functions in such a theory may lead to the interesting consequences for standard model and grand unified theories [23].

The classical solutions of the theory (66) should define the cosmology of an early universe with a back reaction of the conformal dilaton coupled matter. However, it is not easy to search for solutions of the theory (66). [Of course, one can work again in the large-N expansion which justifies the neglect of the classical term in Eq. (66).] So we will start using dilaton coupled Weyl gravity as the classical gravity. Adding to the action of such a theory the induced effective action we omit the linear σ terms. (That may be justified by adding to the theory of the dilaton- and gravity-dependent sources for σ.) Working on a conformally flat metric $g_{\mu\nu}=e^{2\sigma}\eta_{\mu\nu}$ [where classical action (64) is equal to zero], we may find the following classical solutions:

$$\sigma=\alpha \ln H_1 \eta, \quad f=\beta \ln H_2 \eta, \tag{67}$$

where H_1, H_2, α, and β are some constants. Their explicit values are defined by the complicated algebraic system of two equations

$$\frac{\delta W}{\delta\sigma}=0, \quad \frac{\delta W}{\delta f}=0. \tag{68}$$

Note that for the same a_1, a_2 coefficients in W (63) one can change the coefficients b, b', and b'' by adding the conformal matter minimally interacting with the dilaton. Hence the solutions (67) define the whole class of metrics. In particular, for $\alpha=-1$, we get the solution which corresponds to the inflationary universe of Starobinsky type [24], however, now with a nontrivial dilaton. One can investigate other types of solutions for induced effective actions, for example, black hole type solutions.

VI. SUMMARY

In summary, the trace anomaly and induced action for the dilaton coupled scalar in 2D and 4D were found. The large-N effective action for quantum dilaton-scalar gravity was also evaluated. The appearence of new, dilaton-dependent terms in the effective action was shown. Some preliminary results on the role of these terms for 2D black holes and Hawking radiation were reported. The conformal sector of 4D dilatonic gravity was constructed and quantum cosmology was discussed.

Our results bring attention to a number of problems. Let us mention some of them.

(1) The construction of classically solvable dilaton gravities with dilaton coupled scalars, the search for new black holes in such models, and the calculation of new corrections to Hawking radiation and black hole entropy.

(2) The trace anomaly for the 4D dilaton coupled vector, spinor, and graviton, and the study of quantum cosmology with a back reaction of such fields.

(3) The study of one-loop renormalizability of the theory (64).

(4) The investigation of quantum structure for the conformal sector of dilatonic gravity.

(5) Generalizations of the C theorem with an account of dilaton-dependent terms.

We hope to return to the study of some of these problems in the near future.

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