τ neutrino decays and big bang nucleosynthesis

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We investigate the nonradiative decay during nucleosynthesis of a massive τ neutrino with a mass of 0.1–1 MeV into an electron neutrino and a scalar or pseudoscalar particle ϕ. The full Boltzmann equation is used and shown to give markedly different results than the usual nonrelativistic formalism for relativistic or semirelativistic neutrino decays. Indeed, the region we investigate is where the formalism that has previously been applied to solving this problem is expected to break down. We also compare the nucleosynthesis predictions from this scenario with results from the standard model and with some of the available observational determinations of the primordial abundances. It is found that for relativistic or semirelativistic decays the helium abundance can be significantly lowered without changing other light element abundances. Since a problem with the standard model of big bang nucleosynthesis is that helium appears to be overproduced, a decay of the type we discuss can be a possible solution. [S0556-2821(98)05204-7]

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I. INTRODUCTION

A number of indications seem to point to neutrinos having a mass. First of all there is the solar neutrino problem which is by far the strongest evidence we have for neutrino mass [1]. Furthermore, there are also indications of a nonzero neutrino mass from atmospheric neutrino data [2] and, lastly, one group claims to have seen evidence for neutrino oscillations in a laboratory neutrino beam also indicating a nonzero neutrino mass [3]. Now, from laboratory experiments an upper limit to the tau neutrino mass can be obtained, which is presently $m \lesssim 24$ MeV [4]. From cosmology, we have the well known limit on stable low mass ($m \lesssim$ GeV) neutrinos [5]:

$$\Omega_\nu h^2 = \frac{g_\nu}{2} \frac{m_\nu}{93.03 \ \text{eV}}, \tag{1}$$

using a present photon temperature of 2.736 K. h is the dimensionless Hubble constant and Ω is the cosmological density parameter. $g_\nu = 2$ for one flavor of neutrino and antineutrino. Since observations demand that $\Omega_\nu h^2 \leq 1$ [5], we have a mass limit on any given stable neutrino.[1] Thus, any neutrino with a mass in the range 100 eV–24 MeV is necessarily unstable. There are, however, many possible modes of decay for a massive neutrino.

For example there is the predicted decay [7] $\nu_i \to \nu_j e^+ e^-$ if the mass is larger than $2m_e$ and the mixing angle between the two neutrinos is different from zero. A flavor changing neutral current can also lead to the decay $\nu_i \to \nu_j \bar{\nu}_j \nu_j$. There can also be other more exotic modes of decay, for example decay via emission of scalars or pseudoscalars. This decay mode is generic for example in the majoron models of neutrino mass [8].

The effect of such unstable tau neutrinos on nucleosynthesis have been investigated many times in the literature [9–15], the most recent investigations being those of Dodelson, Gyuk and Turner [13] and Kawasaki *et al.* [14,15]. Dodelson, Gyuk and Turner have performed a detailed study of several possible decay modes in the context of nonrelativistic decays, whereas Kawasaki *et al.* have performed a calculation using the full Boltzmann equation for the decay mode $\nu_\tau \to \nu_\mu \phi$ [16,17]. In all cases it is found that it is possible to change significantly the primordial abundances via decay of the tau neutrino.

In the present paper we focus on the decay

$$\nu_\tau \to \nu_e \phi, \tag{2}$$

where ν_τ is assumed to be a Majorana particle and ϕ is a light scalar or pseudoscalar particle. This differs from the decay $\nu_\tau \to \nu_\mu \phi$ in that it includes an electron neutrino in the final state. Since ν_e enters directly into the weak interactions that interconvert neutrons and protons this decay can potentially alter the outcome of nucleosynthesis drastically. Indeed the nonrelativistic results of Dodelson, Gyuk and Turner indicate that the primordial helium abundance, Y_P, can be changed radically, either increasing or decreasing Y_P depending on the mass and lifetime of the tau neutrino.

Now, in the past few years, evidence has been gathering that the standard picture of the way light nuclei are formed in the early Universe may be facing a crisis [18]. The main point is that helium is overproduced relative to the other light nuclei so that the standard theoretical predictions are only marginally consistent with the observational results [18]. Other measurements of the primordial helium abundance do yield somewhat higher values [19], and the unknown systematical errors both in observations and in chemical evolution calculations may, however, be larger than presently assumed so that it is perhaps premature to talk of a real ''crisis'' for big bang nucleosynthesis.

Our approach will not be so much to discuss the specific limits from nucleosynthesis since these are still quite uncer-

[1]Note that this relation changes slightly if the heating of neutrinos from e^+e^- annihilation is included [6].

tain as it will be a discussion of the differences between our way of solving the Boltzmann equations and those previously used. Nevertheless, in light of the possibility that some new element is missing from the standard nucleosynthesis calculations we think that it is important to try and find methods of changing the nucleosynthesis predictions by including plausible new physics in the calculations. One possible way of doing this is to include a massive and unstable tau neutrino.

In order to obtain good fits to the observational data it is, as just mentioned, necessary to lower the helium abundance somewhat compared to the other light nuclei. This can be achieved by having relatively low mass tau neutrinos decay while they are still relativistic or semirelativistic. However, this is exactly the region where the nonrelativistic formalism breaks down because it assumes a delta function momentum distribution of the decay products and neglects inverse decays. It is therefore of significant interest to investigate this decay using the full Boltzmann equation in order to calculate abundances in this parameter region.

In the present paper we calculate the expected primordial abundances for a tau neutrino mass in the range 0.1–1 MeV. In Sec. II we describe the necessary formalism needed for this calculation. In Sec. III we discuss our numerical results. Section IV contains a description of our nucleosynthesis calculations compared to the observational data and finally Sec. V contains a summary and discussion.

II. NECESSARY FORMALISM

The fundamental way to describe the evolution of different particle species in the early Universe is to use the Boltzmann equation

$$L[f]=\sum C_i[f], \tag{3}$$

where the sum is over different possible collisional terms for the given particle, such as decay, scattering and pair annihilation. In our case, we include the standard weak interactions of neutrinos with each other and with electrons and positrons. Furthermore we include a decay term. We shall assume, however, that the scalar particles are collisionless except for the decays and inverse decays. That is, they have no self interactions and no other interactions with neutrinos. This may or may not be a good assumption, depending on the various coupling constants. It greatly simplifies the calculations, however. Now, the various terms in the Boltzmann equation can be written as follows

$$L[f]=\frac{\partial f}{\partial t}-\frac{dR}{dt}\frac{1}{R}p\frac{\partial f}{\partial p}. \tag{4}$$

Since there are only 2-particle interactions like $1+2\to 3+4$, $C_{\rm weak}$ can be written as

$$C_{\rm weak}[f]=\frac{1}{2E_1}\int d^3\tilde{p}_2 d^3\tilde{p}_3 d^3\tilde{p}_4 \times\Lambda(f_1,f_2,f_3,f_4)S\sum|M|^2_{12\to 34} \times\delta^4(p_1+p_2-p_3-p_4)(2\pi)^4, \tag{5}$$

where $\Lambda(f_1,f_2,f_3,f_4)=(1-f_1)(1-f_2)f_3f_4-(1-f_3)(1-f_4)f_1f_2$ is the phase space factor, including Pauli blocking of the final states, and $d^3\tilde{p}=d^3p/[(2\pi)^3 2E]$. S is a symmetrization factor of 1/2! for each pair of identical particles in initial or final states [20], and $\Sigma|M|^2$ is the weak interaction matrix element squared and spin summed. The matrix elements for the relevant processes have been compiled for example by Hannestad and Madsen [21]. p_i is the four-momentum of particle i.

Since we are only looking at Majorana neutrinos the decay terms are quite simple. Since there is almost no net lepton number in the early Universe the Majorana neutrino is effectively an unpolarized species. However, this means that there can be no preferred direction in the rest frame of the parent particle. Therefore the decay is necessarily isotropic in this reference frame. In this case the decay terms can be written as [22]

$$C_{\rm dec}[f_{\nu_\tau}]=-\frac{m^2_{\nu_\tau}}{\tau m_0 E_{\nu_\tau}p_{\nu_\tau}}\int_{E^-_\phi}^{E^+_\phi}dE_\phi\Lambda(f_{\nu_\tau},f_{\nu_e},f_\phi) \tag{6}$$

$$C_{\rm dec}[f_{\nu_e}]=\frac{g_{\nu_\tau}}{g_{\nu_e}}\frac{m^2_{\nu_\tau}}{\tau m_0 E_{\nu_e}p_{\nu_e}}\int_{E^-_{\nu_\tau}}^{E^+_{\nu_\tau}}dE_{\nu_\tau}\Lambda(f_{\nu_\tau},f_{\nu_e},f_\phi) \tag{7}$$

$$C_{\rm dec}[f_\phi]=\frac{g_{\nu_\tau}}{g_\phi}\frac{m^2_{\nu_\tau}}{\tau m_0 E_\phi p_\phi}\int_{E^-_{\nu_\tau}}^{E^+_{\nu_\tau}}dE_{\nu_\tau}\Lambda(f_{\nu_\tau},f_{\nu_e},f_\phi), \tag{8}$$

where $\Lambda(f_{\nu_\tau},f_{\nu_e},f_\phi)=f_{\nu_\tau}(1-f_{\nu_e})(1+f_\phi)-f_{\nu_e}f_\phi(1-f_{\nu_\tau})$, $m_0^2=m^2_{\nu_\tau}-2(m^2_\phi+m^2_{\nu_e})+(m^2_\phi-m^2_{\nu_e})^2/m^2_{\nu_\tau}$. τ is the lifetime of the heavy neutrino and g is the statistical weight of a given particle. We use $g_{\nu_\tau}=g_{\nu_e}=2$ and $g_\phi=1$, corresponding to $\phi=\overline{\phi}$. This assumption is not significant to the present investigation. Furthermore we shall assume that the masses of ν_e and ϕ are effectively zero during nucleosynthesis.

The integration limits are

$$E^\pm_{\nu_\tau}(E_i)=\frac{m_0 m_{\nu_\tau}}{2m_i^2}[E_i(1+4(m_i/m_0)^2)^{1/2}\pm(E_i^2-m_i^2)^{1/2}] \tag{9}$$

and

$$E^\pm_i(E_{\nu_\tau})=\frac{m_0}{2m_H}[E_{\nu_\tau}(1+4(m_i/m_0)^2)^{1/2}\pm p_{\nu_\tau}] \tag{10}$$

where the index $i=\nu_e,\phi$.

Apart from the Boltzmann equation one needs equations to relate the evolution of time, the cosmic expansion rate and

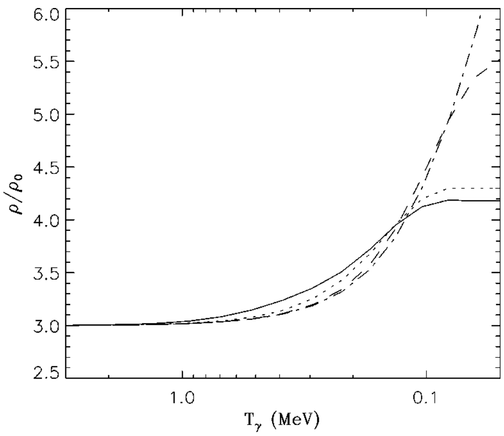

FIG. 1. The energy density of the different neutrinos and the scalar particle in units of the energy density of a standard massless neutrino for a tau neutrino mass of 0.5 MeV. The full line is for $\tau=1$ s, the dotted for $\tau=10$ s, the dashed for $\tau=100$ s and the dot-dashed for $\tau=1000$ s.

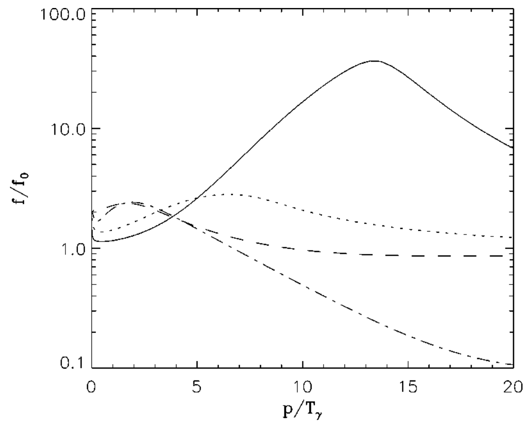

FIG. 2. The electron neutrino distribution at asymptotically low temperature (after complete decay) in units of the distribution of a standard massless neutrino. The tau neutrino mass is 0.5 MeV. The full line is for $\tau=1$ s, the dotted for $\tau=10$ s, the dashed for $\tau=100$ s and the dot-dashed for $\tau=1000$ s.

the photon temperature. These quantities can be calculated by use of the energy conservation equation

$$d(\rho R^3)/dt+pd(R^3)/dt=0 \tag{11}$$

and the Friedmann equation

$$H^2=8\pi G\rho/3. \tag{12}$$

R is the cosmological scale factor, H is the Hubble parameter and ρ is the total energy density of all particles present.

III. NUMERICAL RESULTS

We have solved the Boltzmann equation for the evolution of distribution functions together with the energy conservation equation, Eq. (11), and the Friedmann equation, Eq. (12). Specifically we have solved for masses of 0.1–1 MeV and lifetimes larger than 0.1 s.

In Fig. 1 we show the evolution of energy density in neutrinos and the pseudoscalar particle for a tau neutrino mass of 0.5 MeV. The energy density evolves quite differently in the different cases. Since the energy density in a nonrelativistic species only decreases as R^{-3} compared to R^{-4} for relativistic particles the rest mass energy of the tau neutrino will dominate completely at late times if it is stable. If it decays the rest mass energy is transferred into relativistic energy so that the total energy density no longer increases relative to that of a single standard massless neutrino species. This difference is clearly seen between different tau neutrino lifetimes.

In Fig. 2 we show the spectral distribution of the electron neutrino for a tau neutrino mass of 0.5 MeV and different lifetimes. To understand this plot better we can define a ''relativity parameter,'' μ, for the decay

$$\mu_{\nu_\tau}\equiv\frac{m_{\nu_\tau}^2\tau_{\nu_\tau}}{9\ \mathrm{MeV}^2\ \mathrm{s}}. \tag{13}$$

A particle shifts from relativistic to nonrelativistic at a temperature of roughly $T\simeq m/3$. When the Universe is radiation dominated

$$\frac{t}{1\ \mathrm{s}}\simeq\left(\frac{T}{1\ \mathrm{MeV}}\right)^{-2}. \tag{14}$$

Therefore, if the decay is relativistic,

$$\tau<t(T=m/3)\simeq\frac{9m^{-2}}{\mathrm{MeV}^{-2}}\ \mathrm{s}. \tag{15}$$

Thus, if $\mu_i<1$ the decay is relativistic, whereas if $\mu_i>1$ it is nonrelativistic. For lifetimes of 1, 10, 100 and 1000 s the relativity parameters are respectively 0.028, 0.28, 2.78 and 27.8. For nonrelativistic decays the decay neutrino distribution assumes a rather narrow shape coming from the delta function energy distribution. For very short lifetimes the decay installs an equilibrium between ν_e, ν_τ and ϕ because of rapid inverse decays. This can lead to a significant depletion of high momentum electron neutrinos as also noted by Madsen [12] who treated this case of very short lifetimes using equilibrium thermodynamics.

IV. NUCLEOSYNTHESIS EFFECTS

In order to estimate the effect on nucleosynthesis, we have employed the nucleosynthesis code of Kawano [23], modified in order to incorporate a decaying neutrino. This includes taking into account the changing energy density as well as the change in electron neutrino distribution.

A decaying tau neutrino can affect nucleosynthesis in several different ways. First, the cosmic energy density ρ is changed. Since the cosmic expansion rate is given directly in terms of this energy density via the Friedmann equation, Eq. (12), it is also changed. It is a well known fact that increasing the energy density leads to an earlier freeze-out of the n-p conversion and therefore produces more helium [5], whereas decreasing the energy density decreases the helium fraction. This is the effect discussed by Kawasaki *et al.* [15], namely that an MeV neutrino decaying into sterile daughter products

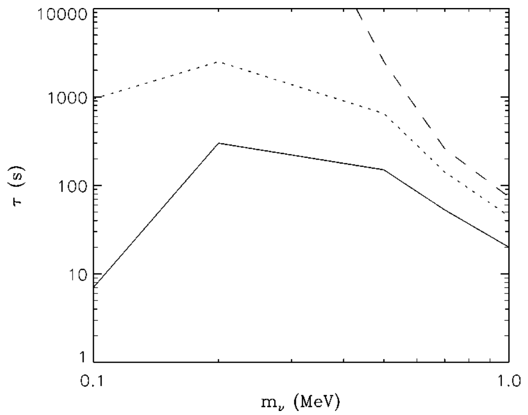

FIG. 3. Helium abundance contours as a function of tau neutrino mass and lifetime for a baryon-to-photon ratio, η, of 3×10^{-10}. The full line is $Y_P=0.20$, the dotted is $Y_P=0.22$ and the dashed is $Y_P=0.24$. The value in the standard model for this η is $Y_P=0.2389$.

while still relativistic or semirelativistic can actually decrease the cosmic energy density thereby decreasing the helium abundance.

However, there is also another another effect stemming from the change in electron neutrino temperature. Since the electron neutrino enters directly into the n-p processes this can be called a ''first order'' effect and is potentially much more important than the ''second order'' effect of changing the energy density. This effect of change in the electron neutrino temperature has already been discussed by several authors in the context of nonrelativistic decays [9,13].

If the decay is nonrelativistic, the energy of a produced electron neutrino is $m/2$. If this energy is significantly above the energy threshold for the two processes

$$p+\overline{\nu_e}\rightarrow n+e^+ \tag{16}$$

and

$$p+\overline{\nu_e}+e^-\rightarrow n, \tag{17}$$

the decaying tau neutrinos will act to produce more He [13,9]. The reason is that the absorption cross section at high energies is the same on neutrons and protons. Since there are many more protons present than neutrons, more neutrons will be produced. In the end this leads to a higher helium fraction. However, if the mass of the decaying neutrino is below this threshold the produced electron neutrinos will stimulate the conversion of neutrons into protons thereby actually decreasing the He abundance. This effect then competes with the rest mass effect which increases Y_P.

If the decay is relativistic the electron neutrinos are produced at roughly thermal energies. Effectively this amounts to increasing the electron neutrino temperature. This in turn leads to a decrease in helium production. If the decay takes place at high temperatures it is because beta equilibrium is kept for a longer time, whereas if the decay takes place at temperatures below the threshold for proton to neutron conversion it still leads to lower helium abundance because an increase in the electron neutrino temperature stimulates the conversion of neutrons to protons over the inverse reaction.

In Fig. 3 we show contour lines for the helium abundance as function of neutrino mass and lifetime. It is seen that helium can be significantly suppressed relative to the standard case if the lifetime is short enough and increased if the mass and lifetime are both high. If the mass and lifetime are both sufficiently high the helium abundance is instead increased. Notice also that even for small masses of the order 0.1 MeV the helium abundance can be changed significantly compared to the standard value for rather a large range of lifetimes.

Our Fig. 3 should be compared with for example the results of Terasawa and Sato [9] or Dodelson, Gyuk and Turner [13] obtained using nonrelativistic theory. The most straightforward comparison is with Figs. 2b and 3b in Ref. [9]. For long lifetimes the difference is quite small as would be expected since this is the nonrelativistic limit. However, for short lifetimes the difference is significant. For very short lifetimes, our calculated He abundance is larger than that of Terasawa and Sato. The reason is that if one uses the full Boltzmann equation in this case, decays and inverse decays will bring the particle distributions into equilibrium as discussed in Sec. III. Thus, if one keeps on going to shorter and shorter lifetimes nothing new happens since it is already decay in equilibrium. Therefore our curve for He flattens out instead of decreasing for very short lifetimes. For somewhat longer lifetimes our He abundance is on the other hand smaller than that found by Terasawa and Sato. The reason here is that the decay produces a peak of very low momentum electron neutrinos and that these states are not upscattered because the weak interactions have already frozen out. In the end this produces a somewhat colder electron neutrino distribution than would have been obtained using nonrelativistic theory and therefore predicts less helium. This low momentum peak can be seen in Fig. 2 for the example of a 0.5 MeV τ neutrino. For nonrelativistic decays this peak disappears because low momemtum states are not energetically accessible. In essence our predicted curve for the He abundance is therefore much flatter at short or intermediate lifetimes than what one would obtain using the nonrelativistic formalism. For very long lifetimes our calculation fits fairly well with that obtained by Terasawa and Sato as could be expected.

In Fig. 4 we show the abundance of D, ^{3}He and ^{7}Li for a specific example of m = 0.5 MeV. We see that the abundances of these elements only change by relatively small amounts even for great variations in neutrino lifetime. Thus, the main effect of the decay is to lower the helium abundance while leaving the other abundances more or less unchanged.

The calculated abundances for different masses and lifetimes are compared with observational limits. Unfortunately there is a great deal of controversy connected with these. However, since our main emphasis is on the differences between our approach to solving the Boltzmann equations and the nonrelativistic approximations previously used, and not so much on the specific nucleosynthesis limits to tau neutrino mass and lifetime we will not go into too much detail regarding this point. For ^{4}He we use the value calculated by Hata *et al.* [18] of

$$Y_P=0.232\pm0.002\pm0.005. \tag{18}$$

For deuterium the situation is somewhat complicated. From

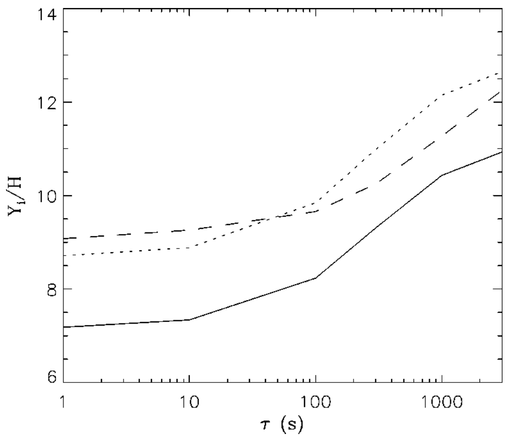

FIG. 4. The abundance of D, ^{3}He and ^{7}Li as a function of tau neutrino lifetime. The curves have been calculated for $m=0.5$ MeV and $\eta=3\times10^{-10}$. The full line shows $(D/H)/10^{-5}$ the dashed shows $[(D+{}^3He)/H]/10^{-5}$ and the dot-dashed shows $({}^7Li/H)/10^{-11}$.

measurements in the local interstellar medium one can obtain a deuterium abundance of [18]

$$D/H \simeq 1.6\times10^{-5}, \quad (19)$$

which can be viewed as a lower limit to the primordial abundance. However, some recent results from quasi stellar objects (QSO) absorption systems seem to indicate a primordial value much higher than this [24]

$$D/H \simeq 1.9-2.5\times10^{-4}. \quad (20)$$

Other similar observations yield much lower values, closer to the local one [25]. In light of the controversy of using deuterium results from these measurements, we use the locally obtainable lower limit in the present paper. From evolution arguments one can also obtain an upper limit to the primordial D+ ^{3}He abundance of [26]

$$(D+{}^3He)/H \lesssim 1.1\times10^{-4}. \quad (21)$$

Finally for the abundance of ^{7}Li we use a bound of

$$^7Li/H = 1.4\pm0.3^{+1.8}_{-0.4}\times10^{-10} \quad (22)$$

obtained by Copi, Schramm and Turner [26].

Altogether these are the observational values which the theoretical predictions should be able to reproduce. In the standard model the theoretical predictions are only marginally consistent with observations because helium is overproduced compared to the other light nuclei. In our scenario this problem is resolved by having the tau neutrino decay during nucleosynthesis into an electron neutrino final state.

In Fig. 5 we show the allowed region of lifetime versus mass for the tau neutrino using the above constraints. In all the mass interval from 0.1–1 MeV it is possible to obtain a fit to the observed abundances. Note however, that for masses in the high end of this region a fit can only be obtained in a very narrow lifetime interval. This is because of the very steep dependence of Y_P on the lifetime in this region. For lower masses a good fit can be obtained in a broad region of lifetimes.

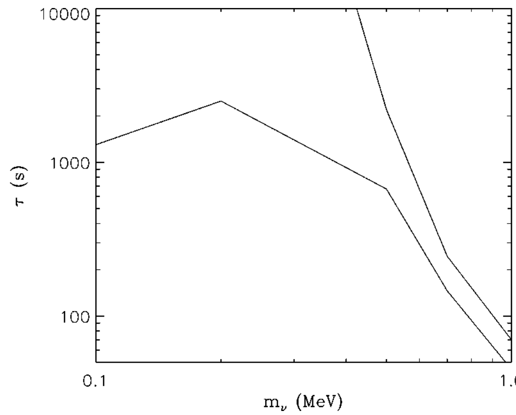

FIG. 5. Allowed region of tau neutrino mass and lifetime. The allowed region is between the two full lines.

Another important fact is that since the helium abundance is lowered without disturbing greatly the other abundances, the upper and lower bound on the baryon-to-photon ratio, η, is now given essentially only by the limits coming from D, ^{3}He and ^{7}Li. This also means that a relatively high value for η can be accommodated, about 6×10^{-10}, coming from requiring that ^{7}Li should not be overproduced.

In Fig. 6 we show the allowed region of η_{10} as a function of tau neutrino lifetime for three different masses. We have also plotted the upper and lower limits to η_{10} from the standard calculation.

V. CONCLUSION

We have studied the decay of a relatively low mass tau neutrino into an electron neutrino and a scalar or pseudoscalar particle using the full Boltzmann equation. It was found that the primordial helium abundance, Y_P, can change drastically compared to the standard value. This is in concordance with the findings of previous authors who used a nonrelativistic treatment [9,13]. Our actual numerical values differ significantly from those previously obtained by use of nonrelativistic formalism, but the general trend is the same, namely that low mass neutrinos decaying while rela-

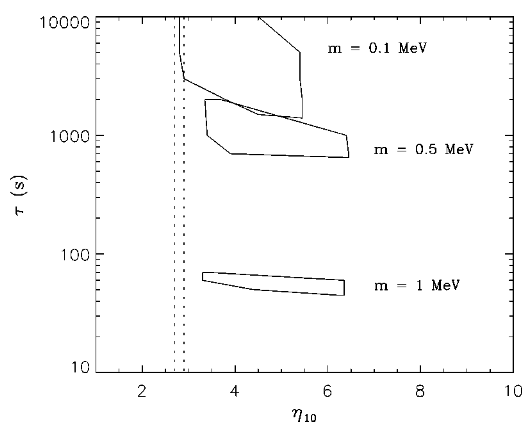

FIG. 6. Allowed regions of $\eta_{10}\equiv10^{10}\times\eta$ for different tau neutrino masses and lifetimes. The regions inside the full lines are allowed regions. The vertical dotted lines show the consistency interval for η_{10} in the standard model for our chosen observational constraints.

tivistic or semirelativistic lower the helium abundance.

The decay we have studied differs completely from the $\nu_\tau \to \nu_\mu \phi$ decay studied by Kawasaki *et al.* [14,15] because the electron neutrinos directly affect the weak reaction rates that interconvert neutrons and protons. Only if much more reliable estimates of the primordial abundances are developed will it be possible to discern between the two different decay modes.

Our aim has mainly been to discuss the differences between using the full Boltzmann formalism and using the nonrelativistic approximation in doing these calculations. We have not done very detailed statistical analysis in order to obtain strict nucleosynthesis limits.

However, it was shown that a good fit to the observed primordial abundances can be achieved for a large range of different masses and lifetimes. Given the possibly large unknown systematical errors in the observations and chemical evolution models it is perhaps too early to talk of a real crisis for big bang nucleosynthesis. However, once the observational bounds become more strict there might very well turn out to be such a crisis.

In light of this possible discrepancy between observed and predicted abundances we still feel that it is important to explore possible ways to change the light element abundances via plausible introduction of new physics. The tau neutrino decay into an electron neutrino final state is just such a possibility.

Perhaps one should also finally note that even if the helium abundance turns out to have been significantly underestimated a tau neutrino decay of the type we have discussed can still make nucleosynthesis predictions fit the observations, but for completely different values of mass and lifetime.

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