Formation of cosmic structures in a light gravitino-dominated universe

Elena Pierpaoli,[1] Stefano Borgani,[2] Antonio Masiero,[1,3] and Masahiro Yamaguchi[4,*]
[1]SISSA-International School for Advanced Studies, via Beirut 2-4, I-34013 Trieste, Italy
[2]INFN, Sezione di Perugia, Dipartimento di Fisica, Università di Perugia, via A. Pascoli, I-06100 Perugia, Italy
[3]Dipartimento di Fisica, Università di Perugia and INFN, Sezione di Perugia, via A. Pascoli, I-06100 Perugia, Italy
[4]Institute für Theoretische Physik, Physik Department, Technische Universität München, D-85747 Garching, Germany

(Received 2 September 1997; published 27 January 1998)

We analyze the formation of cosmic structures in models where dark matter is dominated by light gravitinos with a mass of 100 eV – 1 keV, as predicted by gauge-mediated supersymmetry (SUSY) breaking models. After evaluating the number of degrees of freedom at the gravitino decoupling (g_*), we compute the transfer function for matter fluctuations and show that gravitinos behave like warm dark matter (WDM) with a free-streaming scale comparable to the galaxy mass scale. We consider different low-density variants of the WDM model, both with and without a cosmological constant, and compare the predictions on the abundances of neutral hydrogen within high-redshift damped Ly-α systems and on the number density of local galaxy clusters with the corresponding observational constraints. We find that none of the models satisfy both constraints at the same time, unless a rather small Ω_0 value ($\lesssim 0.4$) and a rather large Hubble parameter ($\gtrsim 0.9$) is assumed. Furthermore, in a model with warm + hot dark matter, with the hot component provided by massive neutrinos, the strong suppression of fluctuation on scales of $\sim 1\, h^{-1}$Mpc precludes the formation of high-redshift objects, when the low-z cluster abundance is required. We conclude that all different variants of a light gravitino DM dominated model show strong difficulties for what concerns cosmic structure formation. This gives a severe cosmological constraint on the gauge-mediated SUSY breaking scheme. [S0556-2821(98)03204-4]

PACS number(s): 95.35.+d, 04.65.+e, 14.80.Ly

I. INTRODUCTION

Since the moment (early 1980s) that low-energy supersymmetry (SUSY) was invoked in gauge unified schemes to tackle the gauge hierarchy problem [1], it became apparent that it had also a major impact on several cosmological issues. By far the most studied consequence was the presence of a stable SUSY particle in all models where a discrete symmetry, known as R-parity, is imposed to prevent the occurrence of baryon and lepton renormalizable terms in the superpotential. Indeed, R-parity assigns a different quantum number to ordinary particles and their SUSY partners. Hence the lightest SUSY particle (LSP) is absolutely stable and constitutes, together with photons and neutrinos, a viable candidate for relic particles of the early Universe.

The two best candidates we have to play the role of LSP are the lightest neutralino (i.e. the lightest among the fermionic partners of the neutral gauge and Higgs fields) and the gravitino (the fermionic partner of the graviton in the gravity multiplet) [2]. Which of the two is the actual LSP strictly depends on the mechanism one envisages for the SUSY breaking, or, more precisely, for the transmission of the breaking of SUSY from some hidden sector to the observable sector of the theory (ordinary particles and their superpartners belong to this latter sector). If the ''messengers'' of the SUSY breaking are a of gravitational nature (as happens in the more ''orthodox'' supergravity models), then the lightest neutralino is likely to be the LSP. In these schemes the gravitino mass sets the scale of SUSY breaking in the observable sector and, hence, it is expected to be in the $10^2 - 10^3$ GeV range. On the other hand, it has been vigorously emphasized recently (after ten years of silence about this alternative) that gauge, instead of gravitational, interactions may be the vehicle for the transmission of the SUSY breaking information to the observable sector [3]. In these scenarios the scale of SUSY breaking is much lower than in the supergravity case and consequently, as we will see below, the gravitino mass is much lower than 10^2 GeV. Hence in this class of gauge mediated SUSY breaking (GMSB) models the gravitino is more likely to play the role of LSP with a mass which can range a lot, depending on the specific scale of SUSY breaking, say from a fraction of eV up to O(GeV).

From a cosmological point of view, the neutralino LSP scenario with a lightest neutralino in the tens of GeV range constitutes an ideal ground for a cold dark matter (CDM) proposal [4]. Indeed there exists a sufficiently vast area of the SUSY parameter space where such an LSP becomes non relativistic at a sufficiently early epochs so as to make its free-streaming mass much smaller than the typical galaxy mass scale ($\sim 10^{11} M_\odot$). The standard version of the CDM scenario, with $\Omega_0 = 1$ for the density parameter, $h = 0.5$ for the Hubble parameter[1] and $P(k) \propto k$ for the post-inflationary power spectrum of Gaussian adiabatic density fluctuations, is generally accepted to fail in reproducing several observational tests. On scales of few tens of h^{-1}Mpc it develops a wrong shape of the power spectrum [5]. Furthermore, once normalized to match the detected level of cosmic microwave background (CMB) temperature anisotropies [6], it produces too large fluctuations on scales $\lesssim 10\, h^{-1}$Mpc, with a subse-

*On leave of absence from Department of Physics, Tohoku University, Sendai 980-77, Japan.

[1]We take $H_0 = 100\ h$ km s^{-1} Mpc^{-1} for the Hubble constant.

0556-2821/98/57(4)/2089(12)/$15.00

quent overproduction of galaxy clusters [7].

This failure of the standard CDM model may be overcome in these SUSY models by finding a way to suppress fluctuations on $10\,h^{-1}$Mpc scales, without decreasing too much power on the $\sim 1\,h^{-1}$Mpc scale, which would delay too much the galaxy formation epoch. A first possibility is adding to the LSP CDM candidate some massive light neutrino [cold+hot DM model (CHDM)] to provide about 20–30% of the critical density [8]. This has just the effect of decreasing the fluctuation amplitude around the neutrino free-streaming scale, so as to change the power-spectrum in the right direction. A further possibility is assuming a density parameter substantially smaller than unity, either with or without a cosmological constant to provide spatial flatness [9]. A lower cosmic density gives rise to a larger horizon size at the matter-radiation equality epoch, so as to increase the large-to-small scale power ratio in the spectrum of cosmic density fluctuations.

If the gravitino is the LSP, one loses the traditional CDM candidate, being such a gravitino a more likely warm (WDM) candidate, its free-streaming mass scale being comparable to the galaxy mass scale [10,11]. This happens when its mass lies in the range [0.1–1] keV, which represents the situation that we will analyze in detail in this paper.

It is already known that just replacing the cold LSP with a warm one in the standard CDM scenario does not provide a viable scenario for the formation of cosmic structures [12]. Indeed, the effect of introducing the warm component is that of suppressing fluctuations only at the galaxy mass scale, while leaving the power spectrum unaffected on the cluster mass scales, where standard CDM fails.

Therefore, if we desire a GMSB scheme to provide the dominant DM content of the Universe, we need some prescription to improve the WDM scenario. To this purpose, we will analyze in the following what happens if we follow the same pattern as for improving CDM, namely either adding a hot neutrino component or lowering the density parameter. Our analysis will focus on the interesting class of GMSB schemes, although many of our conclusions may equally well apply to models with a generic WDM other than the light gravitino.

The purpose of our analysis is twofold. On one hand, given the success of suitable CHDM and low-density CDM models in accounting for several observational constraints (in particular providing a low level of density fluctuations at the $10h^{-1}$Mpc scale to avoid cluster overproduction, while having enough power at about $1\,h^{-1}$Mpc to form galaxies at an early enough epoch), we ask whether the agreement can be kept when a warm gravitino component replaces the cold candidate. On the other hand, from a more particle physics oriented point of view, we would like to make use of the cosmological constraints related to the DM issue to infer constraints on the GMSB models, in particular shedding some light on the range of the allowed (or at least cosmologically favoured) scales of SUSY breaking in this class of theories.

The paper is organized as follows. In Sec. II we present the general features of the GMSB models, focusing in particular on their predicted light gravitinos. We compare the two scenarios, gravity-mediated and gauge-mediated SUSY breaking, in relation to their LSP predictions and implications for DM. We provide the main tools for the computation of the relic gravitino abundance in GMSB models. Section III describes the scenarios for the formation of cosmic structures when the DM content is dominated by light gravitinos. Here we compute the corresponding power spectra of density fluctuations at the outset of recombination. Afterwards, we present the observational data that we will use to constrain this class of models, namely the abundance of neutral hydrogen within high-redshift damped Ly-α systems and the number density of local galaxy clusters. In Sec. IV we compare the model predictions for the formation of cosmic structures with the abovementioned data. The main conclusions of our analysis are summarized in the final Sec. V.

II. LIGHT GRAVITINOS IN SUSY

In a supersymmetric model [1], each ordinary particle is associated with a superpartner. We assign R-parity even to the ordinary particles and odd to their superpartners. In supergravity, that is a natural extension of the supersymmetric standard models to the framework of local supersymmetry, we have another R-odd particle, the gravitino, which is the superpartner of the graviton. The lightest of the R-odd particles, namely the lightest superparticle (LSP), is absolutely stable, under the assumption of the R-parity conservation, which was originally introduced in order to avoid too fast proton decays. The LSP is thus a dark matter candidate, if its expected relic abundances lie within a suitable range of values.

As a starting point, we review some properties of the gravitino. Imposing the vanishing cosmological constant in the Einstein supergravity Lagrangian, one finds that the gravitino mass is related to the SUSY breaking scale Λ_{SUSY} as follows:

$$m_{\tilde{G}}=\frac{1}{\sqrt{3}}\frac{\Lambda_{SUSY}^2}{M_{Pl}}, \tag{1}$$

where M_{Pl} is the reduced Planck mass $\sim 2.4\times 10^{18}$ GeV. On the other hand, the soft SUSY breaking masses for the superparticles are given as

$$m_{soft}\sim\frac{\Lambda_{SUSY}^2}{M}, \tag{2}$$

where M effectively represents the mass scale of the interactions that transmit the breakdown of SUSY in the hidden sector to the observable sector, the latter including particles of the SUSY standard model. We call M the messenger mass scale. In the conventional scenario of the gravity-mediated SUSY breaking, the transmission is due to gravitational interactions. In this case, the messenger mass scale is $M \sim M_{Pl}$, so that the gravitino mass will be comparable to the other soft masses. In order to have the soft masses at the electro-weak scale the SUSY breaking scale should be at an intermediate scale $\sim\sqrt{m_W M_{Pl}}$.

On the other hand, one can consider the case where the SUSY breaking is transmitted by gauge interaction. The idea of the gauge mediation [13] is older than the gravity-mediation, and has recently been revived with fruitful results [3]. In this case the gauge interaction can set the messenger mass scale much lower than the Planck mass. Since the soft masses are fixed at the electro-weak scale, the SUSY breaking scale can be much smaller than the intermediate scale of $\sqrt{m_W M_{Pl}}$. Correspondingly the gravitino can be much lighter than the other superparticles. Now a crucial question is: how light is the gravitino? The answer should depend on the details of the messenger of the SUSY breaking. In most of the gauge-mediated models, there are three independent sectors. They are the hidden sector, the messenger sector and the observable sector. The interaction between the last two sectors is the standard-model gauge interaction, so its strength is fixed. But the interaction between the first two is model dependent, so is the messenger mass scale. For example, in the original models of gauge-mediation [3] it was shown [14] that Λ_{SUSY} cannot be smaller than 10^7 GeV, the corresponding gravitino mass being $\sim 10^2$ keV. However, a lighter gravitino should be possible from viewpoints of both model building and phenomenology. In the SUSY gauge-mediated approach the soft masses arise at the loop level (to avoid the supertrace constraint [15]). Hence a lower bound on Λ_{SUSY} is provided by the relation

$$\Lambda_{SUSY} \gtrsim \frac{16\pi^2}{g^2} m_{soft} \tag{3}$$

where g is some gauge coupling constant. For instance, recently Izawa *et al.* [16] have constructed a model where $m_{soft} \lesssim 0.1 g^2/16\pi^2 \Lambda_{SUSY}$. In this case, Λ_{SUSY} can be as small as $O(10^5)$ GeV for $m_{soft} = O(10^2)$ GeV. In view of the above consideration, in this paper we consider the following gravitino mass range[2]

$$1 \quad \text{eV} \lesssim m_{\tilde{G}} \lesssim \text{a few TeV}. \tag{4}$$

It is noteworthy that interaction of the longitudinal component (spin 1/2 component) of the gravitino is fixed by the low-energy theorem. Namely the would-be Goldstino has a derivative coupling to the supercurrent with $1/\Lambda_{SUSY}^2 = 1/\sqrt{3} m_{\tilde{G}} M_{Pl}$ suppression. After integration by parts and the use of equations of motion,[3] we obtain the following effective Lagrangian [18]:

$$\mathcal{L}_{eff} = \frac{m_\lambda}{8\sqrt{6} m_{\tilde{G}} M_{Pl}} \overline{\tilde{G}} [\gamma_\mu, \gamma_\nu] \lambda F_{\mu\nu} + \frac{m_\chi^2 - m_\phi^2}{\sqrt{3} m_{\tilde{G}} M_{Pl}} \overline{\tilde{G}} \chi_L \phi^* + \text{H.c.}, \tag{5}$$

where $\tilde{G}$ represents the longitudinal component of the gravitino (the Goldstino) and m_λ, m_χ and m_ϕ are the masses of a gaugino λ, a chiral fermion χ and its superpartner ϕ, respectively. The point is that as the gravitino mass gets smaller the interaction becomes stronger. What happens physically is that a lighter gravitino corresponds to a lighter messenger scale, and therefore the Goldstino which is in the hidden sector has a stronger interaction to the fields in the observable sector. This point is crucial when we discuss the cosmology of the light gravitino.

A. Two scenarios: neutralino LSP and gravitino LSP

Among the superparticles which appear in the supersymmetric standard models, a neutralino tends to be the lightest one and, therefore, it is stable. The neutralino LSP with mass of the order of 100 GeV turns out to be a good candidate for the cold dark matter (CDM) [2]. In gravity-mediated models with $m_{\tilde{G}} \sim 10^2$–10^3 GeV, we face the traditional gravitino cosmological problems [19]. Namely, unless gravitinos are strongly diluted at inflation and they are not regenerated in the reheating phase ($T_{reh} \lesssim 10^8$ GeV), they would spoil the canonical picture of big-bang nucleosynthesis (BBN).

On the other hand, if the gravitino is lighter than the neutralino, the latter is no longer stable, and decays to the gravitino. It was pointed out [20] that its decays would also destroy the BBN if its life time is sufficiently large. A limit on the life time depends on the abundances of the neutralinos before decay. We quote here a conservative bound of 10^6 sec as an upper bound for the life time of the neutralino from the BBN constraint.

In this case the gravitino will be the stable LSP. Suppose that the spin 1/2 components of gravitinos were in thermal equilibrium at an early epoch.[4] As temperature went down, the processes which kept the gravitinos in equilibrium became ineffective and they decoupled from the thermal bath. After that, the number of gravitinos per comoving volume was frozen out. This freeze-out took place while the gravitinos were relativistic. Following a standard procedure [21], one can calculate the relic density of the gravitinos [22]

$$\Omega_{\tilde{G}} h^2 = 0.282 \quad \text{eV}^{-1} m_{\tilde{G}} Y_\infty = 1.17 \left(\frac{100}{g_*}\right) \left(\frac{m_{\tilde{G}}}{10^3 \quad \text{eV}}\right), \tag{6}$$

where $\Omega_{\tilde{G}}$ is the contribution of the (thermal) gravitinos to the density parameter, h is the Hubble parameter in units of 100 km/s/Mpc, and g_* stands for the effective degrees of freedom of relativistic particles when the freeze-out of the gravitinos takes place. Note that $g_* = 106.75$ for the full set of particle contents of the minimal standard model and $g_* = 228.75$ for those of the minimal supersymmetric standard model. Thus one expects that g_* at the freeze-out will fall somewhere in between the two numbers. The computation of g_* is a crucial point of our analysis and we will come back to it later on. For later convenience, we introduce the yield, Y_∞, of the gravitinos, defined by

$$Y_\infty = \left(\frac{n_{\tilde{G}}}{s}\right)_\infty = \frac{0.617}{g_*}, \tag{7}$$

[2] In the framework of no-scale models, one may consider a somewhat larger range of the gravitino masses [17].

[3] Here we present the formulas for massless gauge bosons. One needs to modify the formulas when the gauge bosons get massive due to symmetry breakdown. In our numerical computation in Sec. II B this correction is taken into account.

[4] We assume that the Universe underwent inflationary era, so that the spin 3/2 components of the gravitinos were not thermalized after that.

where $n_{\tilde{G}}$ is the number density of the gravitinos and s is the entropy density. The subscript ∞ means that the ratio is evaluated at a sufficiently late time (i.e., low temperature) at which it is constant.

We will first briefly discuss the case when the relic abundance of the gravitinos calculated in this way exceeds the closure limit, $\Omega_{\tilde{G}} \gtrsim 1$. This corresponds to the gravitino mass region $m_{\tilde{G}} \gtrsim 1$ keV $(g_*/100)h^2$. In this case, as was discussed in Refs. [20,14], entropy production is needed to dilute the gravitino abundance in order not to overclose the Universe. To avoid an excessive reproduction of the gravitinos after the entropy production, its reheating temperature must be low; its upper bound varies from 10^3 to 10^8 GeV, depending on the gravitino mass. The lower the gravitino mass is, the lower the reheating temperature should be. If the reheating temperature happens to saturate the upper bound quoted above, the gravitinos will dominate the energy density of the Universe, and play the role of DM.

On the other hand, the low reheating temperature required by the closure limit leads to the question of how to generate the baryon asymmetry of the Universe. Since the reheating temperature can be still higher than the electro-weak scale, baryogenesis during the electro-weak phase transition may work for some region of the parameter space [23]. Another possibility is to use the Affleck-Dine mechanism, which was explored in detail in Ref. [14] in the framework of the gauge-mediated SUSY breaking.

When the gravitino mass is smaller than $(g_*/100)h^2$ keV, the thermal relic density of the gravitinos $\Omega_{\tilde{G}}$ is smaller than one. This is the region that we will study in detail in this paper. As we discussed previously, models providing this range for the gravitino mass can be devised. It is also interesting to mention that a possible explanation of the $ee\gamma\gamma$ event [24] at the Collider Detector at Fermilab (CDF) by the light gravitino scenario [25] requires this range of gravitino mass; otherwise the neutralino would not decay into a photon and a gravitino inside the detector. A particularly interesting parameter region for cosmology is the region in which $0.1 \lesssim \Omega_{\tilde{G}} \lesssim 1$ is realized, and thus the gravitino mass density constitutes a significant portion of the density of the whole Universe. A DM particle with mass within the sub-keV to keV range is known as warm dark matter [26,10,11]. Differently from CDM, it is characterized by having a sizable free streaming length until matter-radiation equality, roughly of the order of Mpc, but still much smaller than that of the hot dark matter (HDM), like a few eV neutrino. We will discuss scenarios of cosmic structure formation within a WDM dominated universe in the following sections.

If, instead, the gravitino mass is as small as to give $\Omega_{\tilde{G}} \ll 0.1$, then it becomes cosmologically irrelevant and an alternative DM candidate is required.

B. Computation of g_*

Before moving to the discussion of cosmic structure formation, we would like to come back to the question of g_*, the effective degree of freedom of relativistic particles at the freeze-out of gravitinos. Of particular interest is the region where $m_{\tilde{G}} \lesssim 1$ keV so that gravitinos of thermal origin dominate the energy density of the Universe. The crucial relevance of g_* lies in the fact that, for a specified value of $\Omega_{\tilde{G}}h^2$, it fixes the corresponding gravitino mass and, therefore, the free-streaming scale.

The production and destruction rates of the gravitinos due to scattering processes are proportional to the fifth power of the temperature and, therefore, their abundance rapidly drops down as the temperature decreases. Thus, decay and inverse decay processes are more important for a light gravitino whose freeze-out occurs at a rather low temperature [20]. In the following we will focus on these processes.

The relevant Boltzmann equation can be casted in the form

$$\dot{n}_{\tilde{G}} + 3Hn_{\tilde{G}} = C, \tag{8}$$

where $n_{\tilde{G}}$ is the gravitino number density and H is the expansion rate of the Universe. As a collision term, we consider contributions from two body decay (and inverse decay) processes

$$C = \sum_{a,b} \Gamma(a \to b\tilde{G}) \left\langle \frac{m_a}{E_a} \right\rangle n_a \left(1 - \frac{n_{\tilde{G}}}{n_{\tilde{G}}^{eq}} \right). \tag{9}$$

Here $\Gamma(a \to b\tilde{G})$ is the partial width of the species a to b and $\tilde{G}$, $\langle m_a/E_a \rangle$ stands for the thermal average of the Lorentz boost factor, with m_a and E_a being mass and energy of a, n_a is its number density and finally the superscript "eq" indicates the equilibrium value of a given quantity. After some algebra, the above Boltzmann equation can be rewritten as

$$Y' - \frac{s'}{3s}RY = -\frac{s'}{3s}RY^{eq}, \tag{10}$$

$$R = \frac{\sum \Gamma(a \to b\tilde{G}) \langle m_a/E_a \rangle n_a / n_{\tilde{G}}^{eq}}{H}, \tag{11}$$

where Y is the yield of the gravitinos as defined by Eq. (7), and the apex symbol denotes derivative with respect to the temperature. Equation (10) can be solved to give

$$Y(T) = Y^{eq}(T) + \int_T^{T_0} dT' \times \exp\left(-\int_T^{T'} dT'' R(T'') s'/3s \right) Y^{eq\,\prime}(T'). \tag{12}$$

Here the temperature T_0 is taken to be sufficiently high so that the gravitino is still in thermal equilibrium.

In order to understand the meaning of Eq. (12), let us consider the case where $R(T)$ changes abruptly at a temperature T_f such as $R(T) = \infty$ for $T > T_f$ and 0 for $T < T_f$. In this case we can approximate $\exp(-\int_T^{T'} dT'' R(T'') s'/3s)$ with a step function $\theta(T_f - T')$, so that

$$Y(T) = Y^{eq}(T) + \int_T^{T_0} dT' \, \theta(T_f - T') Y^{eq\,\prime}(T') = Y^{eq}(T_f), \tag{13}$$

thus reproducing the usual result. In the present case, however, $R(T)$ gradually decreases as a species becomes nonrelativistic. Therefore, we need to integrate Eq. (12) numeri-

TABLE I. Value of effective degrees of freedom of relativistic particles, at the gravitino freeze-out, g_*, as a function of the gravitino mass $m_{\tilde{G}}$ and the $\mathrm{U}(1)_Y$ gaugino mass M_1. In the case (a) the right-handed slepton mass is $m_{\tilde{l}_R}=M_1$, and in (b) $m_{\tilde{l}_R}=2M_1$.

(a) $m_{\tilde{l}_R}=M_1$		$m_{\tilde{G}}$ (eV) 10	50	100	200	500	1000
M_1 (GeV)	50	87	93	101	110	122	136
	100	87	89	93	111	114	124
	150	87	89	92	97	109	119
	200	88	90	93	97	105	115

(b) $m_{\tilde{l}_R}=2M_1$		$m_{\tilde{G}}$ (eV) 10	50	100	200	500	1000
M_1 (GeV)	50	87	91	95	102	116	128
	100	87	90	93	98	107	116
	150	88	91	93	97	104	111
	200	88	92	94	98	103	108

cally to evaluate $Y(T)$ accurately. For sufficiently low T, the yield $Y(T)$ approaches its constant value Y_∞, from which we obtain g_* using Eq. (7).

Results are presented in Table I. We show the value g_* for a range of model parameters. In this computation, we assumed a typical sparticle mass spectrum in a simple class of gauge-mediated models [27]. Explicitly, we take for the gauginos

$$M_1=\frac{5}{3}\frac{\alpha_1}{4\pi}\Lambda_G, \quad M_2=\frac{\alpha_2}{4\pi}\Lambda_G, \quad M_3=\frac{\alpha_3}{4\pi}\Lambda_G, \tag{14}$$

and for the sfermion masses

$$m^2=2\left[C_3\left(\frac{\alpha_3}{4\pi}\right)^2+C_2\left(\frac{\alpha_2}{4\pi}\right)^2+\frac{5}{3}\left(\frac{Y}{2}\right)^2\left(\frac{\alpha_1}{4\pi}\right)^2\right]\Lambda_S^2. \tag{15}$$

In the above expressions α_i is a gauge coupling constant in the standard model, Y is a hypercharge of $\mathrm{U}_Y(1)$, while $C_3=4/3$ for a $\mathrm{SU}(3)_C$ triplet, $C_2=3/4$ for a $\mathrm{SU}(2)_L$ doublet, and 0 otherwise. Λ_G, Λ_S are introduced to parameterize the transmission of SUSY breaking from the messenger sector to the observable sector.[5] We provide g_* values for two cases: (a) the right-handed slepton mass $m_{\tilde{l}_R}$ equals to the bino mass M_1, i.e. $m_{\tilde{l}_R}=M_1$, and (b) $m_{\tilde{l}_R}=2M_1$. In both cases, we find that g_* is around 100 for a wide range of the parameter space. For a given $\Omega_{\tilde{G}}$, a lower value of g_* implies a lighter gravitino, making structure formations at small scales more difficult, as we will discuss in the following sections. The fact that g_* tends to lie in the lower side should be kept in mind, though we will explore a somewhat wider range for g_*.

[5]To avoid further complication, we set a light Higgs boson mass to be the Z^0 mass, and masses of heavier Higgs and higgsinos to be the same as the left-handed slepton mass. Furthermore we did not include D- or F-term contributions to the scalar masses. Also we ignored the mixing in the mass matrix of the neutralino and the chargino sector.

III. LIGHT GRAVITINOS AND COSMIC STRUCTURE FORMATION

A. Computation of the transfer functions

The fundamental quantity that allows to make predictions about the formation of cosmological structures, once the underlying Friedmann background is fixed, is the transfer function $T(k)$, which convey all the informations about the evolution of a density fluctuation mode at the wavenumber k through the matter-radiation equality and recombinations epochs. In the following we will discuss how the transfer functions for the models under consideration are computed. As for models containing only the warm gravitinos (WDM) we will consider the $\Omega_0\leq 1$ cases, both with (ΛCDM) and without (OCDM) a cosmological constant term, $\Omega_\Lambda=1-\Omega_0$, to restore the spatial flatness. Furthermore, we will consider also the class of $\Omega_0=1$ mixed models, whose DM content consists both of warm gravitinos and one species of hot neutrinos, having mass $m_\nu\simeq 91\,\Omega_\nu h^2$ eV (Ω_ν is the neutrino contribution to the density parameter).

Here we will only sketch our implementation of the Boltzmann code to compute $T(k)$ and we refer to the relevant literature ([28]; [29]) for more technical details.

The transfer function is defined as

$$T(k)=\frac{\sum_{i=1}^{N_s}\Omega_i\delta_{i,z=0}}{\sum_{i=1}^{N_s}\Omega_i\delta_{i,z=z_i}}, \tag{16}$$

where N_s is the number of different massive species in the model, δ_i is the energy overdensity of the ith component and z_i a suitable initial redshift such that the smallest considered scale is much larger than the horizon scale at z_i.

We evaluate the transfer function for the models of interest in two steps: firstly we solve the equations for the fluctuation evolution of all the species involved in the models (namely the baryons, the radiation, the massless and massive neutrinos, and the gravitinos) for a number of k values; secondly, we find a suitable analytic expression which is able to provide a good fitting to the transfer functions for the whole class of considered models, by varying a minimal set of parameters.

As for the fluctuation evolution, the goal is to find the final amplitude δ_i for the different species, given the initial one. This goal is achieved in different ways for different components. For baryons only two differential equations must be solved: one regarding their overdensity and one for their velocity; for relativistic particles it is necessary to solve a hierarchy of coupled differential equations for the coefficients of the harmonic expansion of the perturbation in order to well describe the free-streaming behavior.

For massive free-streaming particles, different free-streaming behaviors can be expected depending on which fraction of particles has to be considered still relativistic at a certain epoch. For these species it is therefore necessary to follow the fluctuation evolution separately for particles having different momenta. A representative set of different values of the momentum is chosen, and the density fluctuation evolution is evaluated for each value of this set. The overall δ_i is therefore found by integrating the zeroth order harmonic coefficients over the momentum, with weights chosen on the basis of the distribution function. This is the reason why, unlike for CDM, for massive free-streaming components the shape of the spectral distribution function affects the shape of the final transfer function.

In our case, both gravitinos and massive neutrinos have an initial thermal distribution, so the equations describing their evolution are qualitatively the same for both the components. What makes the difference between the two is the redshift at which they become nonrelativistic, being higher for the warm $\tilde{G}$ than for hot ν. As a consequence, such two particle populations will be characterized by different free-streaming scales.

All the calculations were performed in the synchronous gauge. For a detailed description on how a thermal free-streaming component is treated in the syncronous gauge, see Mah and Bertschinger [28]. From a numerical point of view, we find that a higher degree of accuracy is needed when dealing with WDM-dominated models if compared to the CDM-dominated ones. The reason is that all the δ_i are coupled by means of the potential; whose evolution equation, in turn, depends upon all the the different overdensities, each of them contributing with a weight Ω_i. If the overdensity of the most abundant component is not well evaluated, the error propagates via the potential to all the other components, and over time. In the case of standard MDM, CDM plays this role, it stabilizes the value of the potential so that a lower accuracy in the integrals over the momenta of the hot component is allowed.

In the models considered hereafter, gravitinos and massive neutrinos are the most abundant components, and their overdensities are evaluated by mean of integrals. It is therefore necessary to choose the integration method that, at the same time, (i) provides the best accuracy, and (ii) minimizes the number of values of the momentum over which the integration is performed, so as to keep the number of differential equations to be solved as small as possible.

Within the class of Gauss integration methods [30], we verified that, keeping fixed the number of integration points in momentum space, Gauss-Legendre integration performs better than Gauss-Laguerre, especially for high values of k. Furthermore, we found that using Gauss-Legendre integration, 20 integration points are adequate to obtain stable results.

We computed the transfer function up to $k_{max} \simeq 1\ \mathrm{Mpc}^{-1}$ (for $\Omega_0=1$ and $h=0.5$), with higher k values requiring too high an accuracy to be reached within a reasonable computational time. We will show in the following that such a k_{max} value is larger than the free-streaming wave number, k_{fs}. Therefore, we expect that the behavior of the transfer function at $k>k_{max}$ has a marginal influence on the hierarchical clustering regime at $k<k_{fs}$, we are interested in.

In order to provide an analytical fitting to the transfer functions for the class of purely WDM models, we resorted to the expression provided by Bardeen et al. [31]:

$$T_{WDM}(k)=T_{CDM}(k)\ \exp\left(-\frac{kR_{fs}}{2}-\frac{(kR_{fs})^2}{2}\right), \quad (17)$$

where

$$T_{CDM}=\frac{\ln(1+2.34q)}{2.34q}[1+3.89q+(16.1q)^2+(5.46q)^3+(6.71q)^4]^{-1/4} \quad (18)$$

is the transfer function for CDM models. Here, $q=k/\Gamma h$ and the expression for the shape parameter, $\Gamma=\Omega_0 h \exp(-\Omega_B-\sqrt{2h}\Omega_B/\Omega_0)$ accounts for the presence of a non-negligible baryon fraction Ω_B [32].

Therefore, by fitting the transfer function, as computed by the Boltzmann code, with Eqs. (17) and (18) one obtains the value for the free-streaming scale, R_{fs}. More in detail, our procedure to estimate R_{fs} proceeds as follows.

(a) We run the Boltzmann code assuming $\Omega_0=1$ and taking $g_*=100$ and 200; the first value is rather representative of realistic cases, while the larger g_* corresponds to a very cold $\tilde{G}$ population.

(b) The free-streaming scale for the $\Omega_0<1$ cases is then computed by resorting to the scaling relation $R_{fs}\propto m_{\tilde{G}}^{-1} \propto \Omega_{\tilde{G}}^{-1}$ [cf. Eq. (6)], where $\Omega_{\tilde{G}}=\Omega_0-\Omega_B$.

As a result, we find that

$$R_{fs}=0.51(\Omega_{\tilde{G}}h^2)^{-1}\left(\frac{g_*}{100}\right)^{-4/3}\mathrm{Mpc} \quad (19)$$

always provides an accurate fitting of the exponential suppression of fluctuations on small scales. We note that our value for R_{fs} is larger by a factor ~ 2.5 than that given by Kawasaki et al. [11]. This difference mainly comes from the fact that our value is directly obtained by fitting the exactly computed transfer function, while their value comes from the usual relation between R_{fs} and z_{nr} [see, e.g., Eq. (9.88) in the Kolb and Turner book [21]], the redshift at which gravitinos becomes nonrelativistic, that represents an approximation to the R_{fs} value. We also confirm the warning by Bardeen *et al.* [31], who pointed out that the exponential cutoff in Eq. (17) marginally underestimates the transfer function on intermediate scales, $0.1 \lesssim k \lesssim 0.5(\Omega_0 h^2)^{-1}\ \mathrm{Mpc}^{-1}$. However, we did not attempt here to look for a more accurate fitting expression, since *(a)* the effect is always quite small ($\lesssim 5$–10%) and *(b)* we will mainly concentrate our analysis on the small scales relevant to galaxy and galaxy cluster formation.

We plot in Fig. 1 the $T_{WDM}(k)$ shape for $\Omega_0=1$ for different g_* values (left panel) and for two $\Omega_0<1$ cases (right panel), also comparing with the corresponding CDM cases. It is apparent the power suppression on small scales, which depends both on g_* and on the parameters of the Friedmann background [cf. Eq. (19)].

As for the warm + hot DM (WHDM) case, transfer functions have been computed for $\Omega_\nu=0.1$, 0.2, 0.3, 0.4 and 0.5 in the case of only one massive neutrino (cf. Ref. [8] for the effect of introducing more than one massive ν), taking $g_*=100$ and 200 and always assuming $\Omega_0=1$. The analytical fitting is provided by Eq. (17), where the CDM transfer func tion is replaced by the CHDM one, as provided by Pogosyan and Starobinski [33]. Taking $\Omega_{\tilde{G}}=1-\Omega_\nu-\Omega_B$, we find that Eq. (19) always provides an accurate fitting to the exponential cutoff in the transfer function. The shapes of $T_{WHDM}(k)$

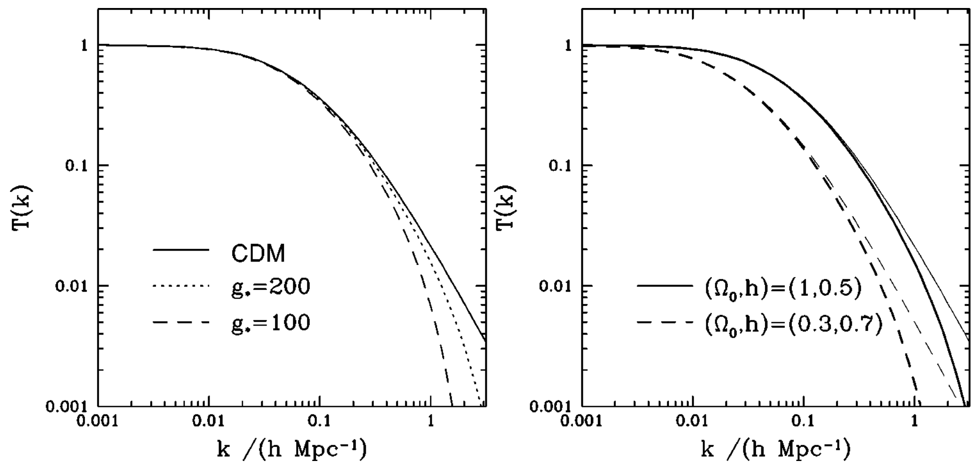

FIG. 1. The shape of the transfer functions for WDM gravitino models. Left panel: the effect of varying g_* for $\Omega_0=1$ and $h=0.5$; solid, dotted and dashed curves correspond to the CDM case, to $g_*=200$ and $g_*=100$, respectively. Right panel: the effect of varying the Friedmann background; heavy and light curves correspond to the CDM and WDM with $g_*=200$ cases, respectively.

are plotted in Fig. 2, showing both the effect of changing g_* at fixed Ω_ν (left panel) and the effect of changing Ω_ν at fixed g_* (right panel).

According to Figs. 1 and 2, it turns out that the effect of replacing the CDM component with light gravitinos of mass given by Eq. (6) is that of eliminating the hierarchical clustering below some free-streaming mass scale. In order to provide an estimate of the free-streaming mass scale, we resort to the almost Gaussian cutoff at large k, to define it as

$$M_{fs}=(2\pi R_{fs}^2)^{3/2}\bar{\rho}\simeq 0.55\left(\frac{g_*}{100}\right)^{-4}(\Omega_{\tilde{G}}h^2)^{-3}\Omega_0 h^2 M_{12}, \tag{20}$$

where $\bar{\rho}$ is the average cosmic density and $M_{12}=10^{12}M_\odot$. Therefore, Eq. (20) provides the limiting mass for the development of hierarchical clustering: structures of smaller masses form after structure of mass larger than M_{fs}, as a product of their fragmentation. As a consequence, we expect that a crucial constraint for the whole class of WDM-dominated models will come from the abundance of high-redshift cosmic structures.

Having fixed the expression for the transfer function, we define the power spectrum of the density fluctuations as $P(k)=AT^2(k)k^{n_{pr}}$, where n_{pr} is the primordial (post-inflationary) spectral index. The amplitude A is determined by following the recipe by Bunn and White [34] to normalize both low–density flat and open models to the 4-year COBE data. In the following, we will not consider the case of non-negligible contribution of tensor mode fluctuations to the CMB anisotropies. Indeed, such an effect would lead to a smaller spectrum amplitude, with a subsequent delay of the galaxy formation epoch that, as we will see, represents a major problem for WDM-dominated models.

We plot in Fig. 3 the rms mass fluctuation σ_M for the same models whose $T(k)$ have been plotted in Fig. 1. This quantity is defined as

$$\sigma_M^2=\frac{1}{2\pi^2}\int_0^\infty dk\ k^2\ P(k)\ W^2(kR_M), \tag{21}$$

where the length scale associate to the mass scale M, $R_M=(4\pi\bar{\rho}/3)^{-1}M^{1/3}$, is the radius of the top-hat sphere whose Fourier representation is given by $W(x)=3(\sin x - x\cos x)/x^3$.

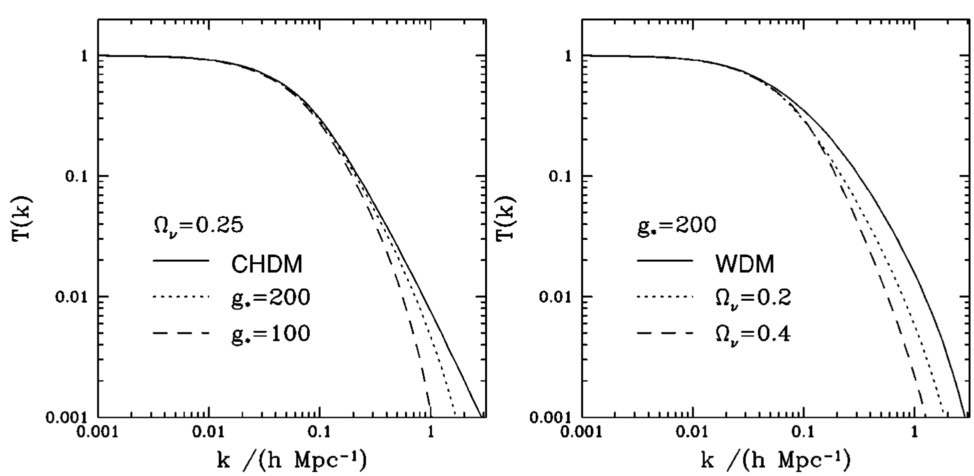

FIG. 2. The shape of the transfer function for the warm + hot DM models. Left panel: the effect of varying g_* at a fixed value of $\Omega_\nu=0.25$. Right panel: the effect of varying Ω_ν at a fixed $g_*=200$.

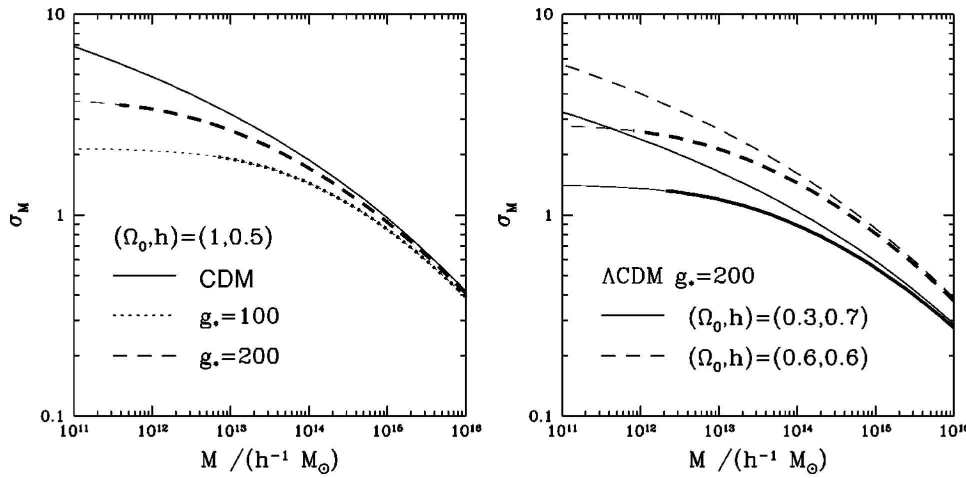

FIG. 3. The mass-scale dependence of the r.m.s. density fluctuations within a top-hat sphere. Left and right panels are for the same models as reported in Fig. 1. Heavy and light curves are for WDM and CDM cases. As for the WDM curves, the value of M at which they become lighter corresponds to the value of the free-streaming mass, defined according to Eq. (20).

For each model, the corresponding free-streaming mass scale corresponds to the transition from heavy to light curves in Fig. 3, while the completely light curves represent the corresponding CDM cases. It is apparent that such a scale is always at least of the order of a large galaxy halo. The flattening of σ_M at small masses represents the imprint of nonhierarchical clustering. On the other hand, it turns out that the behavior on the scales of galaxy clusters, $\sim 10^{15} h^{-1} M_\odot$, is rather similar as for the CDM-dominated case. In the following we will use the abundance of local galaxy clusters and of high-redshift protogalaxies, through data about damped Ly-α systems, to constrain the whole class of WDM-dominated models. Constraints on larger scales, like bulk-flows data [35], are much more similar to the CDM case.

B. Observational constraints

1. High-redshift objects

The first constraint that we consider comes from the abundance of neutral hydrogen (HI) contained within damped Ly-α systems (DLAS; see Ref. [36] for a review about DLASs). DLAS are observed as wide absorption through in quasar spectra, due to a high HI column density ($\gtrsim 10^{20}$ cm^{-2}). Since at $z \gtrsim 3$ the fractional density of neutral hydrogen associated with DLASs, Ω_{HI}, is comparable to that associated to visible matter in local galaxies, it has been argued that DLASs trace a population of collapsed protogalactic objects. In this context, a crucial question is to understand whether the observed Ω_{HI} provides a fair representation of the collapsed gas fraction at a given redshift. Effects such as gas consumption into stars, amplification biases due to gravitational lensing of background quasistellar objects (QSOs) [37] and dust obscuration [38] could well alter final results. However, such effects are believed to play a role at low redshift ($z \sim 1-2$), while they are expected to be less relevant at the highest redshifts at which DLAS data are available. For this reason, we will consider as the most constraining datum the value of Ω_{HI} reported by Storrie-Lombardi et al. [39] at redshift $z \simeq 4.25$ and will assume that all the HI gas at that redshift is involved in the absorbers.

Several authors recognized DLASs as a powerful test for DM models using both linear theory and numerical simulations [40]. The recent availability of high-resolution spectra for several DLAS systems, allowed Prochaska and Wolfe [41] to use the internal kinematics of such systems to severely constrain a CDM model.

In order to connect model predictions to observations, we consider the fraction of DM which at redshift z is collapsed into structures of mass M,

$$\Omega_{coll} = \mathrm{erfc}\left[\frac{\delta_c(z)}{\sqrt{2}\sigma_M(z)}\right]. \tag{22}$$

Accordingly, $\Omega_{HI} = \Omega_B \Omega_{coll}$. Here, $\sigma_M(z)$ is the r.m.s. fluctuation at the mass scale M at redshift z within a top-hat sphere. Furthermore, $\delta_c(z)$ is the critical density contrast whose value predicted by the model for the collapse of a spherical top-hat fluctuation in a critical density universe, $\delta_c = 1.69$ independent of the redshift, has been confirmed by N-body simulations [42]. In our analysis we used the expressions for $\delta_c(z)$ provided in Ref. [43] for both low-density flat and open universes. We note, however, that at the redshift z=4.25, that we are considering, the resulting δ_c value is always very close to 1.69.

We note that the Press and Schechter approach [44], on which Eq. (22) is based, holds only in the case of hierarchical clustering. In our case of WDM models, hierarchical clustering only takes place on mass scales larger than M_{fs}. On smaller scales, the lack of fluctuations causes the flattening of σ_M. Therefore, by estimating σ_M at arbitrarily small masses, one obtains the r.m.s. fluctuations at the free-streaming mass scale. In our approach, we will give up the dependence on mass scale M, which amounts to assume that DLASs are assumed to be hosted within protostructures of mass of about M_{fs}; protostructures of smaller mass, instead, are produced later by fragmentation of larger lumps.

As for the observational value of Ω_{HI}, Storrie-Lombardi et al. [39] provided for $\Omega_0 = 1$, $\Omega_{HI} = (1.1 \pm 0.2) \times 10^{-3}\ h^{-1}$ at $z = 4.25$. In the light of all the above uncertainties in directly relating Ω_{HI} to Ω_{coll}, we prefer to maintain a conservative approach here and to consider a model as ruled out if it predicts Ω_{HI} to be less than the observational 1σ lower limit. At this level of comparison we do not consider as reliable to put constraints to model producing too high a Ω_{HI} value.

Furthermore, we should also rescale appropriately the value by Storrie-Lombardi to include the more general $\Omega_0 < 1$ cases. Therefore, the limiting value that we consider is

$$\Omega_{HI}=0.0009h^{-1}f(\Omega_0,\Omega_\Lambda,z=4.25), \tag{23}$$

where

$$f(\Omega_0,\Omega_\Lambda,z)=\left(\frac{1+\Omega_0 z}{1+z}\right)^{1/2}; \quad \Omega_\Lambda=0$$

$$f(\Omega_0,\Omega_\Lambda,z)=\frac{[(1+z)^3\Omega_0+\Omega_\Lambda]^{1/2}}{(1+z)^{3/2}};$$

$$\Omega_\Lambda=1-\Omega_0. \tag{24}$$

2. The cluster abundance

As for the cluster abundance, it has been recognized to be a sensitive constraint on the amplitude of the power spectrum [7]. Based on the Press and Schechter approach [44], it is easy to recognize that the number density of clusters with mass exceeding a given value is exponentially sensitive to the r.m.s. fluctuation on the cluster mass scale. Fitting the local x-ray cluster temperature function with the Press-Schechter approach [44] led several authors to obtain rather stringent relationships between σ_8, the r.m.s. fluctuation value within a top-hat sphere of $8\,h^{-1}$Mpc radius, and Ω_0 [45]. In the following we will resort to the constraint by Viana and Liddle, who provided the most conservative and, probably, realistic estimate of errors, mostly contributed by cosmic variance effects on the local cluster population:

$$\sigma_8\Omega_0^{\alpha(\Omega_0)}=0.60^{+0.22}_{-0.16}$$

$$\alpha(\Omega_0)=0.36+0.31\Omega_0-0.28\Omega_0^2; \quad \Omega_\Lambda=0$$

$$\alpha(\Omega_0)=0.59-0.16\Omega_0+0.06\Omega_0^2; \quad \Omega_\Lambda=1-\Omega_0, \tag{25}$$

with uncertainties corresponding to the 95% confidence level.

IV. DISCUSSION

As for the purely WDM models, we plot in Fig. 4 the constraints on the (Ω_0,h) plane, for $g_*=150$, from DLAS and cluster abundance. Only scale-free primordial spectra (i.e., $n_{pr}=1$) are considered here. Left and right panels correspond to the low-density flat (ΛWDM) and open (OWDM) cases, respectively. The solid line delimiting the coarsely shaded area indicates the limit for the region of the parameter space which is allowed by the observed Ω_{HI} in DLASs: model lying below such curves should be considered as ruled out, since they produce a too small Ω_{HI} value at $z=4.25$. The cluster abundance constraint by Eq. (25) is represented by the finely shaded region. The dashed curves connect models having the same age of the Universe: t_0=9, 11, 13, 15, and 17 Gyrs from upper to lower curves.

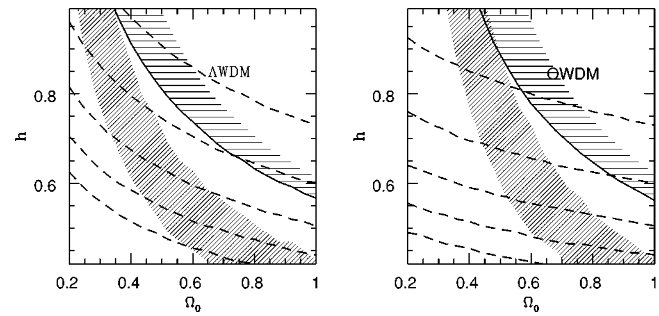

FIG. 4. Observational constraints for COBE-normalized WDM models, with $g_*=150$, on the (Ω_0,h) parameter space, for flat low-density (ΛWDM) and open (OWDM) models. The finely shaded area corresponds to the 95% C.L. region allowed by the cluster abundance, as estimated by Viana and Liddle [45] (see text). The heavy solid curve delimiting the coarsely shaded area represents the limit of the region allowed by the data about the Ω_{HI} in DLAS at $z=4.25$, as given by Storrie-Lombardi et al. [39] (see text); models lying below such curves are excluded. Horizontal dashed curves connect models having the same age of the Universe: t_0=9,11,13,15,17 Gyrs from upper to lower curves.

As a main result, we note that there is almost no overlapping between the regions allowed by the two observational constraints: for fixed values of the Hubble parameter, cluster abundance tends to select relatively smaller Ω_0 in order to satisfy the low-normalization request of Eq. (25). On the other hand, the DLAS constraint favor higher density parameters, which has the effect of both decreasing the free-streaming scale and to increase the small-scale power even in the absence of any free-streaming. Judging from this plot, one would conclude that the whole class of gravitino-dominated WDM models would be ruled out by combining constraints on the cluster and on the galaxy mass scale. A residual possibility seems to exist to reach a concordance for $\Omega_0 \lesssim 0.4$ ($\Omega_0 \lesssim 0.5$) and a high Hubble parameter, $h \gtrsim 1$ ($h \gtrsim 0.9$) for OWDM (ΛWDM) models. However, two main problems arise in this case: *(a)* all the current determinations of the Hubble constant indicates $0.5<h<0.8$ [46]; *(b)* the resulting age of the Universe would be definitely too small, especially for OWDM models, even on the light of the new recalibration of globular cluster ages, based on the recent data from the Hypparcos satellite [47].

We also checked the possibility of considering nonscale-free primordial spectra ($n_{pr}\neq 1$), although results are not explicitly presented here. We verified that assuming either blue ($n_{pr}>1$) or red ($n_{pr}<1$) spectra does not improve the situation. In the first case, power is added on small scales, with the result that smaller Ω_0 are allowed by DLASs. However, the price to be paid is a rapid increase of the cluster abundance, that also pushes toward smaller Ω_0 the finely shaded area. As for red spectra, the opposite situation occurs: the reduction of small-scale power leads both constraints to favor relatively larger Ω_0 values, with no overlapping with the two allowed regions of the (Ω_0,h) plane ever attained.

As a matter of fact, the situation becomes even worse when considering $\Omega_0=1$ WHDM models. Results for this class of models are reported in Fig. 5 on the (Ω_ν,n_{pr}) plane. Left and right panels are for $h=0.5$ and 0.6, respectively;

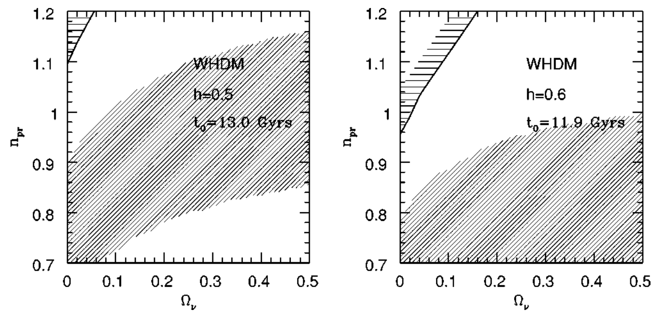

FIG. 5. Observational constraints for Cosmic Background Explorer (COBE) normalized WHDM models, with $g_*=150$, on the (Ω_ν, n_{pr}) plane, for $h=0.5$ (left panel) and $h=0.6$ (right panels). A vanishing tensor mode contribution to CMB temperature anisotropies is assumed for $n_{pr}<1$ models. The two shaded areas have the same meaning as in Fig. 4.

smaller h values are disfavoured by H_0 determinations, while larger values are constrained by the age of the Universe. In both cases the regions allowed by DLAS and cluster abundance are largely disjoined, especially as higher Ω_ν are considered. Indeed, increasing the neutrino fraction has the effect of further reducing the power on small scales, thus further suppressing the high-redshift galaxy formation.

Based on such results we should conclude that none of the variants of the WDM gravitino-dominated scenario is able to account at the same time for the relatively small abundance of clusters at low redshift and for the relatively high Ω_{HI} in collapsed structures at high redshift. It is worth reminding that this result has been obtained with the rather conservative choice of $g_*=150$. As we have shown in the previous section, more realistic value of g_* should be even smaller, thus putting WDM-dominated model in an even worse shape.

Which are the consequences of such results on the low-energy SUSY breaking models that we described in Sec. II? Of course, a first possibility is that gravitinos were so light as to be irrelevant from the point of view of cosmic structure formation. For instance, the current understanding of high-energy physics phenomenology would surely allow for $m_{\tilde{G}} \sim 1$ eV. In this case, $\Omega_{\tilde{G}}$ would be negligible. Of course, since $\tilde{G}$ represents the LSP, the source for a cold DM component should be found in this case outside the spectrum of SUSY particles (e.g., axions).[6]

On the other hand, if a scenario with $m_{\tilde{G}} \sim 100$ eV will turn out to be preferred, a nonnegligible $\Omega_{\tilde{G}}$ cannot be escaped. In this case, three possible alternative scenarios can be devised. The first one is to allow for cold + warm DM. However, since gravitinos have a much smaller free–streaming scale than neutrinos with $m_\nu \sim 5$ eV, this scenario would suffer from the same pitfalls of the standard CDM one, unless one takes $\Omega_0<1$. The second possibility would be to have a substantially larger g_*, so that gravitinos behave much like CDM. However, as we have seen in Sec. II B, it is not clear how a substantially larger g_* can be attained within plausible SUSY models. The third possibility would be to abandon the assumption of Gaussian fluctuations in favor of texture seeded galaxy formation [49], which would ease the formation of high redshift objects. However, also this possibility has been recently shown to suffer from serious troubles in producing a viable power spectrum of density fluctuations [50], which make texture-based models as virtually ruled out.

One may argue that the gravitino abundances will be diluted to a cosmologically negligible level by late-time entropy production. On the other hand, as the low value of g_* suggests, the reheat temperature after the entropy production should be lower than the electroweak scale to avoid the rethermalization of the gravitinos, which severely constraints possible ways to generate the baryon asymmetry of the Universe.

V. CONCLUSIONS

In this paper we analyzed the cosmological consequences of assuming dark matter to be dominated by light gravitinos with a mass in the range $\simeq 100$ eV – 1 keV, as predicted by gauge-mediated SUSY breaking (GMSB) models. We pointed out that gravitinos with such a mass behave like warm dark matter (WDM), since their free-streaming mass scale is comparable to the typical galaxy mass scale.

After estimating the number of degrees of freedom of relativistic species at the gravitino decoupling, g_*, we resorted to a Boltzmann code to compute the appropriate WDM transfer functions. These are used as the starting point to compare gravitino-dominated model predictions to observational data about the abundance of HI within high-redshift damped Ly–α systems and about the abundance of local galaxy clusters.

The main results of our analysis can be summarized as follows.

(a) Low-density WDM models with both flat (ΛCDM) and open (OCDM) geometry cannot satisfy the two observational constraints at the same time, unless a rather small Ω_0 value ($\lesssim 0.4$) and a rather large Hubble parameter ($\gtrsim 0.9$) are assumed. However, such requests would conflict with measurements of the Hubble constant and with current constraints about the age of the Universe.

(b) As for warm + hot (WHDM) models, we find that they have an even harder time. The combined free-streaming of both neutrinos and gravitinos generates a strong suppression of fluctuations at $\sim 1\, h^{-1}$Mpc scale. This makes extremely difficult to form high-redshift ($z\sim 4$) protogalactic objects if we require the model to match the low-z cluster abundance.

Based on such results we claim that no variant of a light gravitino DM dominated model is viable from the point of view of cosmic structure formation. Therefore, in the framework of GMSB models, this amounts to require the gravitino to be light enough ($m_{\tilde{G}} \lesssim 50$ eV) so as to be cosmologically irrelevant (unless entropy production with a sufficiently low-reheat temperature dilutes the gravitino abundances). In this case, however, one would lose the LSP candidate for implementing a CDM-dominated scenario.

As a concluding remark, we should point out that, from the point of view of the particle physics model building, we still lack an exhaustive construction of realistic GMSB schemes, in particular as far as the details of the messenger

[6]See however a recent proposal that a sneutrino in the messenger sector can be a CDM candidate [48].

sector are concerned. In this respect we hope that our analysis may constitute a useful guideline for the intense work which is going on in the GMSB option.

ACKNOWLEDGMENTS

S.B. wishes to acknowledge SISSA for his hospitality during several phases of preparation of this work. M.Y. also acknowledges the hospitality of SISSA where he stayed at a final stage of this work. The work of M.Y. was partially supported by a grant from Deutsche Forschungsgemeinschaft SFB-375-95, the European Commission programs ERBFMRX-CT96-0045 and CT96-0090, and the Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture of Japan No. 09640333.

APPENDIX

In this appendix, we summarize the decay widths to gravitino which are needed in the calculation of g_* in Sec. II B. We denote the gluino by $\widetilde{g}$ (with mass M_3), the W-inos $[\mathrm{U}(2)_L$ gauginos$]$ by $\widetilde{W}^{\pm}$, $\widetilde{W}^0$ (with mass M_2), and the b-ino $[U(1)_Y$ gaugino$]$ by $\widetilde{B}$ (with mass M_1). We ignored possible mixing between the gauginos and Higgsinos.

The decay widths involving the gauginos are

$$\Gamma(\widetilde{g}\to g+\widetilde{G})=\frac{1}{48\pi}\frac{M_3^5}{m_{\widetilde{G}}^2 M_{Pl}^2} \tag{A1}$$

$$\Gamma(\widetilde{W}^{\pm}\to W^{\pm}+\widetilde{G})=\frac{1}{48\pi}\frac{M_2^5}{m_{\widetilde{G}}^2 M_{Pl}^2}\left(1-\frac{m_W^2}{M_2^2}\right)^4 \quad (M_2>m_W) \tag{A2}$$

$$\Gamma(W^{\pm}\to \widetilde{W}^{\pm}+\widetilde{G})=\frac{1}{72\pi}\frac{m_W^5}{m_{\widetilde{G}}^2 M_{Pl}^2}\left(1-\frac{M_2^2}{m_W^2}\right)^4 \quad (M_2<m_W) \tag{A3}$$

$$\Gamma(\widetilde{B}\to \gamma+\widetilde{G})=\frac{\cos^2\theta_W}{48\pi}\frac{M_1^5}{m_{\widetilde{G}}^2 M_{Pl}^2} \tag{A4}$$

$$\Gamma(\widetilde{W}^0\to \gamma+\widetilde{G})=\frac{\sin^2\theta_W}{48\pi}\frac{M_2^5}{m_{\widetilde{G}}^2 M_{Pl}^2} \tag{A5}$$

$$\Gamma(\widetilde{B}\to Z+\widetilde{G})=\frac{\sin^2\theta_W}{48\pi}\frac{M_1^5}{m_{\widetilde{G}}^2 M_{Pl}^2}\left(1-\frac{m_Z^2}{M_1^2}\right)^4 \quad (M_1>m_Z) \tag{A6}$$

$$\Gamma(Z\to \widetilde{B}+\widetilde{G})=\frac{\sin^2\theta_W}{72\pi}\frac{m_Z^5}{m_{\widetilde{G}}^2 M_{Pl}^2}\left(1-\frac{M_1^2}{m_Z^2}\right)^4 \quad (M_1<m_Z) \tag{A7}$$

$$\Gamma(\widetilde{W}^0\to Z+\widetilde{G})=\frac{\cos^2\theta_W}{48\pi}\frac{M_2^5}{m_{\widetilde{G}}^2 M_{Pl}^2}\left(1-\frac{m_Z^2}{M_2^2}\right)^4 \quad (M_2>m_Z) \tag{A8}$$

$$\Gamma(Z\to \widetilde{W}^0+\widetilde{G})=\frac{\cos^2\theta_W}{72\pi}\frac{m_Z^5}{m_{\widetilde{G}}^2 M_{Pl}^2}\left(1-\frac{M_2^2}{m_Z^2}\right)^4 \quad (M_2<m_Z), \tag{A9}$$

where m_Z, m_W are Z- and W-gauge boson masses, respectively, and θ_W represents the electroweak mixing angle.

The decay width of a slepton with mass $m_{\widetilde{l}}$ to gravitino is given as

$$\Gamma(\widetilde{l}\to l+\widetilde{G})=\frac{1}{48\pi}\frac{m_{\widetilde{l}}^5}{m_{\widetilde{G}}^2 M_{Pl}^2}. \tag{A10}$$

A similar expression is obtained for the decay width of a squark.

[1] For a review, see H.P. Nilles, Phys. Rep. **110**, 1 (1984).

[2] G. Jungman, M. Kamionkowski, and K. Griest, Phys. Rep. **267**, 195 (1996), and references therein.

[3] M. Dine, A. Nelson, Y. Nir, and Y. Shirman, Phys. Rev. D **53**, 2658 (1996); M. Dine and A.E. Nelson, *ibid.* **48**, 1277 (1993); M. Dine, A.E. Nelson, and Y. Shirman, *ibid.* **51**, 1362 (1995).

[4] H. Pagels and J.R. Primack, Phys. Rev. Lett. **48**, 223 (1982); G.R. Blumenthal, S.M. Faber, J.R. Primack, and M.J. Rees, Nature (London) **311**, 517 (1984).

[5] J.A. Peacock and S.J. Dodds, Mon. Not. R. Astron. Soc. **267**, 1020 (1994).

[6] K.M. Górski, A.J. Banday, C.L. Bennett, G. Hinshaw, A. Kogut, G.F. Smoot, and E.L. Wright, Astrophys. J. **464**, L11 (1996).

[7] S.D.M. White, G. Efstathiou, and C.S. Frenk, Mon. Not. R. Astron. Soc. **262**, 1023 (1993).

[8] J.R. Primack, in *Proceeding of the Princeton 20th Century Anniversary Conference, Critical Dialogues in Cosmology*, edited by N. Turok (World Scientific, Singapore, 1997).

[9] A.R. Liddle, D.H. Lyth, D. Roberts, and P.T.P. Viana, Mon. Not. R. Astron. Soc. **278**, 644 (1996); A.R. Liddle, D.H. Lyth, P.T.P. Viana, and M. White, *ibid.* **282**, 281 (1996).

[10] S. Borgani, A. Masiero, and M. Yamaguchi, Phys. Lett. B **386**, 189 (1996).

[11] M. Kawasaki, N. Sugiyama, and T. Yanagida, Mod. Phys. Lett. A **12**, 1275 (1997).

[12] S. Colombi, S. Dodelson, and L.M. Widrow, Astrophys. J. **458**, 1 (1996).

[13] M. Dine, W. Fischler, and M. Srednicki, Nucl. Phys. **B189**, 575 (1981); S. Dimopoulos and S. Raby, *ibid.* **B192**, 353 (1981); M. Dine and W. Fischler, Phys. Lett. **110B**, 227 (1982); M. Dine and M. Srednicki, Nucl. Phys. **B202**, 238 (1982); L. Alvarez-Gaume, M. Claudson, and M. Wise, *ibid.* **B207**, 96 (1982); C. Nappi and B. Ovrut, Phys. Lett. **113B**, 175 (1982).

[14] A. de Gouva, T. Moroi, and H. Murayama, Phys. Rev. D **56**, 1281 (1997).

[15] S. Ferrara, L. Girardello, and F. Palumbo, Phys. Rev. D **20**, 403 (1979).
[16] K.-I. Izawa, Y. Nomura, K. Tobe, and T. Yanagida, Phys. Rev. D **56**, 2886 (1997); Y. Nomura and K. Tobe, Report No. hep-ph/9708377.
[17] E. Cremmer, S. Ferrara, C. Kounnas, and D.V. Nanopoulos, Phys. Lett. **133B**, 61 (1983); J. Ellis, C. Kounnas, and D.V. Nanopoulos, Nucl. Phys. **B241**, 406 (1984); **B247**, 373 (1984); J. Ellis, A. Lahanas, D.V. Nanopoulos, and K. Tamvakis, Phys. Lett. **134B**, 439 (1984); J. Ellis, K. Enqvist, and D.V. Nanopoulos, *ibid.* **147B**, 99 (1984).
[18] P. Fayet, Phys. Lett. **84B**, 421 (1979); Phys. Lett. B **175**, 471 (1986).
[19] J. Ellis, J.E. Kim, and D.V. Nanopoulos, Phys. Lett. **145B**, 181 (1984); R. Juszkiewicz, J. Silk, and Z. Stebbins, *ibid.* **158B**, 463 (1985); J. Ellis, D.V. Nanopoulos, and S. Sarkar, Nucl. Phys. **B259**, 175 (1985); V.S. Berezinsky, Phys. Lett. B **261**, 71 (1991); M. Kawasaki and T. Moroi, Prog. Theor. Phys. **93**, 879 (1995).
[20] T. Moroi, H. Murayama, and M. Yamaguchi, Phys. Lett. B **303**, 289 (1993).
[21] E.W. Kolb and M.S. Turner, *The Early Universe* (Addison, Wesley, Chicago, IL, 1990).
[22] Pagels and Primack [4].
[23] M. Carena, M. Quiros, A. Riotto, I. Vilja, and C.E.M. Wagner, Nucl. Phys. **B503**, 387 (1997).
[24] S. Park, in *10th Topical Workshop on Proton-Antiproton Collider Physics*, edited by R. Raja and J. Yoh (AIP Press, New York, 1995).
[25] S. Dimopoulos, M. Dine, S. Raby, and S. Thomas, Phys. Rev. Lett. **76**, 3494 (1996); S. Ambrosanio, G.L. Kane, G.D. Kribs, S.P. Martin, and S. Mrenna, *ibid.* **76**, 3498 (1996); Phys. Rev. D **54**, 5395 (1996); S. Dimopoulos, S. Thomas, and J.D. Wells, *ibid.* **54**, 3283 (1996); K.S. Babu, C. Kolda, and F. Wilczek, Phys. Rev. Lett. **77**, 3070 (1996); J. Lopez and D.V. Nanopoulos, Mod. Phys. Lett. A **11**, 2473 (1996); Phys. Rev. D **55**, 4450 (1997).
[26] P.J.E. Peebles, Astrophys. J. **258**, 415 (1982); J.R. Bond, A.S. Szalay, and M.S. Turner, Phys. Rev. Lett. **48**, 1636 (1982); K.A. Olive and M.S. Turner, Phys. Rev. D **25**, 213 (1982).
[27] G. Dvali, G.F. Giudice, and A. Pomarol, Nucl. Phys. **B478**, 31 (1996).
[28] C.P. Ma and E. Bertschinger, Astrophys. J. **455**, 7 (1995).
[29] A.R. Liddle and D.H. Lyth, Phys. Rep. **231**, 1 (1993).
[30] W. Press and S. Teukolsky, ''Numerical Recipes in Fortran'' (Cambridge University Press, Cambridge, England, 1992), p. 140.
[31] J.M. Bardeen, J.R. Bond, N. Kaiser, and A.S. Szalay, Astrophys. J. **304**, 15 (1986).
[32] N. Sugiyama, Astrophys. J., Suppl. Ser. **100**, 281 (1995).
[33] D. Yu Pogosyan and A.A. Starobinski, Astrophys. J. **447**, 465 (1995).
[34] E.F. Bunn and M. White, Astrophys. J. **480**, 6 (1997).
[35] A. Dekel, Annu. Rev. Astron. Astrophys. **32**, 371 (1994); M.A. Strauss and J.A. Willick, Phys. Rep. **261**, 271 (1995); L.N. da Costa, S. Borgani, W. Freudling, R. Giovanelli, M.P. Haynes, J. Salzer, and G. Wegner (in preparation).
[36] A. Wolfe, in *Relativistic Astrophysics and Particle Cosmology*, edited by C.W. Ackerlof and M.A. Srednicki (New York Acad. Sci., New York, 1993).
[37] M. Bartelman and A. Loeb, Astrophys. J. **457**, 529 (1996).
[38] S.M. Fall and Y.C. Pei, *QSO Absorbtion Lines* (Springer-Verlag, Berlin, 1996).
[39] L.J. Storrie-Lombardi, R.G. McMahon, M.J. Irwin, and C. Hazard, in ''Proceeding of the ESO Workshop on QSO Absorbtion Lines,'' Report No. astro-ph/9503089 (1995).
[40] H.J. Mo and J. Miralda–Escudé, Astrophys. J. Lett. **430**, L25 (1994); G. Kauffmann and S. Charlot, *ibid.* **430**, L97 (1994); C.P. Ma and E. Bertschinger, *ibid.* **434**, L5 (1994); A. Klypin, S. Borgani, J. Holtzman, and J.R. Primack, Astrophys. J. **444**, 1 (1995); S. Borgani, F. Lucchin, S. Matarrese, and L. Moscardini, Mon. Not. R. Astron. Soc. **280**, 749 (1996); J.P. Gardner, N. Katz, L. Hernquist, and D.H. Weinberg, Report No. astro-ph/9609072 (1996); M.G. Haenhelt, M. Steinmetz, and M. Rauch, Report No. astro-ph/9706201 (1997).
[41] J.X. Prochaska and A.M. Wolfe, Report No. astro-ph/9704196 (1997).
[42] C. Lacey and S. Cole, Mon. Not. R. Astron. Soc. **271**, 676 (1994).
[43] T. Kitayama and Y. Suto, Astrophys. J. **469**, 480 (1996).
[44] W.H. Press and P.L. Schechter, Astrophys. J. **187**, 425 (1974).
[45] P.T.P. Viana and A.R. Liddle, Mon. Not. R. Astron. Soc. **281**, 323 (1996); V.R. Eke, S. Cole, and C.S. Frenk, *ibid.* **282**, 263 (1996); U.L. Pen, Report No. astro-ph/9610147 (1996).
[46] W.L. Freedman, in *Proceeding of the Princeton 20th Century Anniversary Conference, Critical Dialogues in Cosmology*, edited by N. Turok (World Scientific, Singapore, 1997).
[47] R.G. Gratton, F. Fusi–Pecci, E. Carretta, G. Clementini, C. Corsi, and M. Lattanzi, Report No. astro-ph/9704150 (1997).
[48] S. Dimopoulos, G.F. Giudice, and A. Pomarol, Phys. Lett. B **389**, 37 (1996); T. Han and R. Hempfling, *ibid.* hep-ph/9708264.
[49] U.L. Pen, U. Seljak, and N. Turok, Phys. Rev. Lett. **79**, 1611 (1997).
[50] R. Cen, Report No. astro-ph/9707240.