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Gluon and charm content of the η' meson and instantons

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Motivated by recent CLEO measurements of the $B\to\eta' K$ decay, we evaluate the gluon and charm content of the η' meson using the interacting instanton liquid model of the QCD vacuum. Our main result is $\langle 0|g^3 f^{abc} G^a_{\mu\nu}\tilde{G}^b_{\nu\alpha}G^c_{\alpha\mu}|\eta'\rangle = -(2.3–3.3)\ \mathrm{GeV}^2\times\langle 0|g^2 G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}|\eta'\rangle$. It is very large due to the strong field of small-size instantons. We show that it provides quantitative explanations of the CLEO data on the $B\to\eta' K$ decay rate (as well as the inclusive process $B\to\eta' + X$), via a virtual Cabibbo-unsuppressed decay into a $\overline{c}c$ pair which then becomes η'. If so, a significant charm component may be present in other hadrons also: We briefly discuss the contribution of the charmed quark to the *polarized* deep-inelastic scattering on a proton. [S0556-2821(98)06103-7]

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Instantons of a small size ($\rho = 0.2–0.4$ fm) have been known for long time to be a very important component of the QCD vacuum [1]. In general, their fields are responsible for a scale of 1 GeV which restricts perturbative QCD from below and effective hadronic Lagrangians from above. Because of fermionic zero modes, they play an especially important role for light (u,d,s) quark physics (for a recent review see [2]). It was nevertheless believed that they are irrelevant for charm-related physics: Indeed, the instanton-induced spin-dependent and -independent potentials between heavy quarks are small compared to the standard confining-plus-perturbative one. However, as we show in this paper, the situation is reversed for *virtual* $\overline{c}c$ pairs: They can only appear due to the strongest gluonic fluctuations in a vacuum, and those are instantons. (In fact, the gluonic fields in the center of relevant instantons is so large that one may even question whether gG/m_c^2 is a good expansion parameter.)

A way to see this is to look at the charm component in hadrons with different quantum numbers. The object of this paper, η', has been long known to play a very special role in QCD: Separated by a large gap from other pseudoscalars [the Weinberg's U(1) problem [3,4]] it serves as a screening mass for the topological charge (see the recent detailed discussion in [5]). Thus testing whether the high dimension gluonic operator does or does not couple strongly to the η' we are actually testing whether the strongest vacuum fluctuations do or do not possess topological charge. No effect of such a magnitude should exist, e.g., for vector mesons: Indeed, the empirical Zweig rule is very strict in vector channels, allowing only tiny flavor mixing.

Recently, the CLEO Collaboration has reported [6] measurements of inclusive and exclusive production of the η' in B decays:

$$B(B\to\eta' + X; 2.2\ \mathrm{GeV} < E_{\eta'} < 2.7\ \mathrm{GeV}) = (7.5\pm 1.5\pm 1.1)\times 10^{-4}, \quad (1)$$

$$B(B\to\eta' + K) = (7.8^{+2.7}_{-2.2}\pm 1.0)\times 10^{-5}. \quad (2)$$

Simple estimates [7] show that these data are in severe contradiction with the standard b-quark decay into light quarks: Cabibbo suppression V_{ub} leads to decay rates two orders of magnitude smaller than the data (both inclusive and exclusive ones). An alternative mechanism, suggested in [7], is based on the Cabibbo-favored $b\to c\overline{c}s$ process, followed by a transition of virtual $\overline{c}c$ into the η'. The latter transition may be possible, provided there exists a large intrinsic charm component of the η'. Its quantitative measure can be expressed through the matrix element

$$\langle 0|\overline{c}\gamma_\mu\gamma_5 c|\eta'(q)\rangle \equiv i f^{(c)}_{\eta'} q_\mu, \quad (3)$$

and one needs $f^{(c)}_{\eta'} \approx 140$ MeV in order to explain the CLEO data; see [7]. This value is surprisingly large, being only a few times smaller than the analogously normalized residue $\langle 0|\overline{c}\gamma_\mu\gamma_5 c|\eta_c(q)\rangle = i f_{\eta_c} q_\mu$ with $f_{\eta_c}\simeq 400$ MeV known experimentally from the $\eta_c\to\gamma\gamma$ decay. It is also comparable to a similarly defined coupling of η' to the axial current of light quarks: Note, however, that instantons in fact lead to a *repulsive* interaction between them, and thus the light quark wave function of η' should be depleted at the origin.

Because the c quark is heavy, it may only exist in the η' in a virtual loop, and its contribution can be evaluated in terms of gluonic fields. Taking the divergence of the axial current in Eq. (3) one gets

$$f^{(c)}_{\eta'}=\frac{1}{m^2_{\eta'}}\langle 0|2m_c\overline{c\,i}\gamma_5 c+\frac{\alpha_s}{4\pi}G_{\mu\nu}\widetilde{G}_{\mu\nu}|\eta'\rangle, \qquad (4)$$

which can be further simplified by the operator product expansion in inverse powers of the c-quark mass,

$$2m_c\overline{c\,i}\gamma_5 c=-\frac{\alpha_s}{4\pi}G_{\mu\nu}\widetilde{G}_{\mu\nu} -\frac{1}{16\pi^2 m_c^2}g^3 f^{abc}G^a_{\mu\nu}\widetilde{G}^b_{\nu\alpha}G^c_{\alpha\mu}+O(G^4/m_c^4). \qquad (5)$$

[See the Appendix in [7] for a detailed derivation of this result. Further terms in expansion (5) are neglected in what follows.] Thus the problem is reduced to the matrix element of a particular dimension-6 pseudoscalar gluonic operator:

$$f^{(c)}_{\eta'}=-\frac{1}{16\pi^2 m^2_{\eta'}}\frac{1}{m_c^2}\langle 0|g^3 f^{abc}G^a_{\mu\nu}\widetilde{G}^b_{\nu\alpha}G^c_{\alpha\mu}|\eta'\rangle. \qquad (6)$$

The magnitude of the matrix element (3) was related [7] to the *vacuum* expectation value of similar operators:

$$f^{(c)}_{\eta'}\simeq-\frac{3}{4\pi^2 b}\frac{1}{m_c^2}\frac{\langle g^3G^3\rangle_{YM}}{\left\langle 0\left|\frac{\alpha_s}{4\pi}G_{\mu\nu}\widetilde{G}_{\mu\nu}\right|\eta'\right\rangle}, \qquad (7)$$

where $\langle G^3\rangle$ should be evaluated in pure gluodynamics, not QCD. Unfortunately, only an indirect order-of-magnitude estimate for this latter quantity was given in [7], leading to a rather wide range $f^{(c)}_{\eta'}=50$–180 MeV.

We have performed a direct calculation of this quantity using the interacting instanton liquid model (IILM). In its present form, this model takes into account instantons coupling to light quarks to *all orders* in a 't Hooft effective interaction, which was shown to be crucial for η' physics. It has correctly reproduced multiple mesonic, baryonic, or glueball correlation functions, and also has increasing direct support from lattice studies of instantons (see [2]).

The calculation is based on a numerical evaluation of the following two-point Euclidean correlation functions:

$$K_{22}(x)=\langle 0|g^2G^a_{\mu\nu}\widetilde{G}^a_{\mu\nu}(x),\quad g^2G^a_{\mu\nu}\widetilde{G}^a_{\mu\nu}(0)|0\rangle, \qquad (8)$$

$$K_{23}(x)=\langle 0|g^2G^a_{\mu\nu}\widetilde{G}^a_{\mu\nu}(x),\quad g^3f^{abc}G^a_{\mu\nu}\widetilde{G}^b_{\nu\lambda}G^c_{\lambda\mu}(0)|0\rangle, \qquad (9)$$

$$K_{33}(x)=\langle 0|g^3f^{abc}G^a_{\mu\nu}\widetilde{G}^b_{\nu\lambda}G^c_{\lambda\mu}(x), \quad g^3f^{abc}G^a_{\mu\nu}\widetilde{G}^b_{\nu\lambda}G^c_{\lambda\mu}(0)|0\rangle. \qquad (10)$$

Studies of $K_{22}(x)$ have been made previously [8], where it was demonstrated that in the ''unquenched'' ensemble of instantons with dynamical quarks the nonperturbative part changes sign at distances $x>0.6$ fm, displaying a ''Debye cloud'' of compensating topological charge. It is identified with the η' contribution, and leads to an estimate

$$\langle 0|g^2G^a_{\mu\nu}\widetilde{G}^a_{\mu\nu}|\eta'\rangle=\frac{16\pi^2}{\sqrt{3}}f_{\eta'}m^2_{\eta'}\approx 7\ \text{GeV}^3, \qquad (11)$$

which agrees reasonably well with other estimates in the literature. In this formula we have expressed the matrix element (11) in terms of the standard parameter $f_{\eta'}\approx 85$ MeV which is defined as follows:

$$\langle 0|\frac{1}{\sqrt{3}}\sum_{i=u,d,s}\overline{q}_i\gamma_\mu\gamma_5 q_i|\eta'\rangle=if_{\eta'}q_\mu .$$

Using an anomaly in the chiral limit, $m_u=m_d=m_s=0$, we arrive at Eq. (11).

We have calculated the correlators mentioned by numerical simulation, using as the ensemble 16 instantons and 16 anti-instantons, put into a box 4×2^3 fm^4, with dynamical quarks [9]. Unfortunately, the propagation of the gluons in the background nonperturbative fields of instantons was not studied in such detail for light quarks, and so far we do not have a gluon propagator program which could be used for all distances. At small x purely perturbative results [e.g., $K^{pert}_{22}(x)=384g^4/\pi^4x^8$] dominate, while the nonperturbative fields can be included via the operator product expansion (see, e.g., [10,1]). At large x we would argue below that (at least with dynamical quarks) the nonperturbative fields dominate.

The quantity $f^{(c)}_{\eta'}$, Eq. (6), can be obtained from the correlation functions (8), (9), (10) [11]:

$$\left|\frac{f^{(c)}_{\eta'}\sqrt{3}m_c^2}{f_{\eta'}}\right|=\left|\frac{K_{23}(x\to\infty)}{K_{22}(x\to\infty)}\right|=\sqrt{\frac{K_{33}(x\to\infty)}{K_{22}(x\to\infty)}}. \qquad (12)$$

It is expected that at large distances the contribution to two other correlators would also be dominated by the nonperturbative field of the instantons. If so, one has a simple estimate for the ratio of matrix elements:

$$\left|\frac{\langle 0|g^3f^{abc}G^a_{\mu\nu}\widetilde{G}^b_{\nu\alpha}G^c_{\alpha\mu}|\eta'\rangle}{\langle 0|g^2G^a_{\mu\nu}\widetilde{G}^a_{\mu\nu}|\eta'\rangle}\right|=\frac{12}{5}\left\langle\frac{1}{\rho^2}\right\rangle\approx 1\text{–}1.5\ \text{GeV}^2. \qquad (13)$$

Two numbers given here correspond to averaging over the instanton size distribution for two variants of the instanton–anti-instanton interaction, the so-called ''streamline'' and ''ratio ansatz'' ones, and indicate the systematics involved. The latter (giving a smaller average size and larger number above) should be considered preferable, because it better agrees with the size distribution directly obtained from lattice gauge field configurations; see discussion in [2]. (Recent measurements [12] using the refined ''inverse blocking'' method have found somewhat smaller instantons than others, but those seem to belong to correlated instanton–anti-instanton pairs, which would not contribute to the compensating Debye cloud we look for.)

In our measurements of K_{23},K_{33} both ratios entering Eq. (12) were found to stabilize at large enough $x>0.8$ fm at the *same* numerical value. We take it as an indication that η' contribution does in fact dominate, although we were not able to see that all correlators fall off with the right mass

[13]. Numerical values of the ratios are about 1.5–2.2 GeV^2, for the two ensembles mentioned. These numbers are somewhat larger than in Eq. (13) because the second operator in the correlator makes it more biased toward smaller instantons.

Proceeding to the final result, we now consider QCD radiative corrections. The experimental number mentioned above is defined at the scale $\mu_1^2 \approx m_c^2$, which is different from that obtained in the instanton calculation. In the latter case the charge and fields are normalized at $\mu_2^2 \approx gG$ where G is the typical gauge field at the points which contribute the most to the correlators. The two scales are not too far apart numerically, $\mu_2^2 \approx 0.5$–1 GeV^2, but the anomalous dimension of this operator [14] is large, leading to the correction

$$f_{\eta'}^{(c)}(\mu_1 \simeq m_c) = \left[\frac{\alpha_s(\mu_1)}{\alpha_s(\mu_2)}\right]^{-18/2b} f_{\eta'}^{(c)}(\mu_2) \simeq 1.5 f_{\eta'}^{(c)}(\mu_2). \tag{14}$$

Here we use $m_c(\mu_1 \simeq m_c) \simeq 1.25$ GeV for the numerical estimates. This concludes our derivation of the parameter

$$\left|\frac{f_{\eta'}^{(c)}}{f_{\eta'}}\right| \simeq 0.85\text{–}1.22, \tag{15}$$

where the second value is preferable; see above. (We present our final result as the ratio $f_{\eta'}^{(c)}/f_{\eta'}$ instead of the absolute value of $f_{\eta'}^{(c)}$ because most systematic errors are gone for the ratio.) The final uncertainty in Eq. (15) comes from the systematic errors of the instanton model, which can be judged from comparison of the instanton size distribution or the scalar glueball size to the corresponding lattice results. It will certainly be soon reduced by ongoing works. Finally, we compare it with the ''experimental'' value needed to explain the CLEO measurements (3), and conclude that our result obtained in the instanton liquid model agrees with it, within the uncertainties.

The next logical question to ask is whether the connection between strong instanton fields and charm leads to phenomena unrelated to η'. One intriguing direction to study is η_c hadronic decays: Their deviations from a simple perturbative pattern (which works well for J/ψ) are well known; see, e.g., [18]. Let us also mention a recent intriguing observation by Bjorken [19] that its three leading hadronic decay channels ($\eta'\pi\pi, \eta\pi\pi, KK\pi$) fit well to a pattern following from the instanton-induced 't Hooft effective Lagrangian.

Another question is whether the ''intrinsic charm'' (see, e.g., [20]) of other hadrons is due to the same mechanism. Especially close to the problem considered is the charm contribution [15] to the spin of the nucleon. The relevant matrix element is of charm axial vector current, as above,

$$\langle N|\overline{c}\gamma_\mu\gamma_5 c|N\rangle = g_A^{(c)}\overline{N}\gamma_\mu\gamma_5 N, \tag{16}$$

and it could be generated, e.g., by the η' ''cloud'' of the nucleon. Assuming now the η' dominance in this matrix element [16] one could get the following Goldberger-Treiman type relation [15] $g_A^{(c)} = (1/2M_N) g_{\eta' NN} f_{\eta'}^{(c)}$. Although the value of $g_{\eta' NN}$ is unknown, and its phenomenological estimates vary significantly, $g_{\eta' NN} = 3$–7 [17], one gets from this estimate a surprisingly large contribution

$$\langle N|\overline{c}\gamma_\mu\gamma_5 c|N\rangle = (0.2\text{–}0.5)\overline{N}\gamma_\mu\gamma_5 N, \tag{17}$$

comparable to the light quark one. Calculations of $g_A^{(c)}$ and $g_{\eta' NN}$ in the instanton model are in progress: Their lattice determination would be more than welcomed. Ultimately, the contribution of the charmed quarks in polarized deep-inelastic scattering may be tested experimentally, by tagging the charmed quark jets (e.g., by the COMPASS experiment at CERN).

In summary, it is by now widely known that the Zweig rule is badly broken in all scalar and pseudoscalar channels, and that the (rather large) mass of the η' is in fact due to light-quark–gluon mixing. Furthermore, these phenomena are generally attributed to instantons. In this work we have found that similar phenomena for larger-dimension (multi-gluon) operators are much stronger. The reason for that is the inhomogeneous vacuum, with a very strong field inside small-size instantons. We have found a very significant fraction of charm in η'. Perhaps the same mechanism may help to solve other puzzles, especially related to η_c decays and deep inelastic scattering (DIS) on the polarized nucleon.

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