

# Universal behavior of multiplicity differences in the quark-hadron phase transition

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(Received 11 July 1997; published 8 January 1998)

The scaling behavior of the factorial moments of the differences in multiplicities between well separated bins in heavy-ion collisions is proposed as a probe of the quark-hadron phase transition. The method takes into account some of the physical features of nuclear collisions that cause some difficulty in the application of the usual method. It is shown in the Ginzburg-Landau theory that a numerical value  $\gamma$  of the scaling exponent can be determined independent of the parameters in the problem. The universality of  $\gamma$  characterizes the quark-hadron phase transition, and can be tested directly by appropriately analyzed data. [S0556-2821(98)05103-0]

PACS number(s): 13.85.Hd, 24.60.Ky

## I. INTRODUCTION

The study of multiplicity fluctuations as a phenomenological manifestation of the quark-hadron phase transition (PT) has been pursued in recent years in the framework of the Ginzburg-Landau (GL) formalism for both second-order [1–3] and first-order [4–6] PTs. In both cases scaling behaviors of the factorial moments have been found and are characterized by scaling exponents  $\nu$ . The value of  $\nu$  for a second-order PT is independent of the details of the GL parameters. It therefore provides a distinctive signature for the existence of a quark-gluon plasma, if its transition to hadrons is of the second order [7].

The experimental verification of  $\nu$  has not been carried out so far in heavy-ion collisions although it has been checked to a high degree of accuracy in quantum optics [8]. There are a number of reasons for the difficulties, which will be described below. The aim of this paper is to devise a method to circumvent the obstacles that stand in the way of extracting the signal from the experimental data. In so doing we also broaden the scope of the analysis to include aspects of wavelets and correlations, in addition to incorporating some evolutionary properties of heavy-ion collisions that are particularly relevant to hadron production during PT.

Let us now examine the difficulties in analyzing multiplicity fluctuations in heavy-ion collisions. The factorial moments that have been suggested to quantify the fluctuations are defined by [9]

$$F_q = \frac{\langle n(n-1)\cdots(n-q+1) \rangle}{\langle n \rangle^q}, \quad (1)$$

where the averages are performed over a distribution  $P_n$  of the multiplicities  $n$  in a bin of size  $\delta$ . Note that  $n$  must be  $\geq q$  in order for an event to contribute to  $F_q$ , for which  $q$  is usually an integer ranging from 2 to 5. As  $\delta$  is decreased, the average multiplicity  $\langle n \rangle$  in a bin decreases, and may become  $\ll q$  in a hadronic collision. Thus  $F_q$  measures the tail end of the distribution  $P_n$ , where  $n \gg \langle n \rangle$ , i.e., large multiplicity fluctuations. Intermittency refers to the power-law dependence of  $F_q$  on  $\delta$  [9]:

$$F_q \propto \delta^{-\varphi_q}, \quad (2)$$

a behavior that has been found to be ubiquitous in hadronic and leptonic processes [10]. However, in nuclear collisions the situation is very different.

In the first place the average multiplicity in an event in nuclear collisions is so high that even in bins of small  $\delta$  the values of  $n$  are large compared to the order  $q$  in Eq. (1) that has been examined experimentally. Thus, unlike hadronic and leptonic processes, the existent nuclear data have not been analyzed to study large multiplicity fluctuations, for which  $q$  must be increased to values  $\gg \langle n \rangle$  in small bins. The moments  $F_q$  for  $q=3, 4$ , and 5 that have been determined are dominated by contributions from the lower-order correlations. This point has been emphasized by Sarcevic and collaborators [11,12], where the nuclear data on the cumulants  $K_q$  are shown to be consistent with zero for  $q \geq 3$ . Whether  $K_q$  acquire nonvanishing values at smaller  $\delta$  (so that  $\langle n \rangle \ll q$ ) is not known. Thus, until the experiments can be improved to render the analysis at very small  $\delta$  feasible, a new method must be devised to circumvent this difficulty of extracting dynamical information at medium values of  $q$ . Our strategy is to consider the distribution of the differences in multiplicities between bins and to examine the scaling behavior of its moments.

Another difficulty associated with the problem of linking multiplicity fluctuations to the dynamics of phase transition is that the particles are integrated over the entire duration when quarks turn into hadrons, while the system undergoes an expansion. In a second-order PT the fluctuations in hadronization can be large relative to the average, but that average is over a short period near  $T_c$  where hadronization is not robust. When integrated over the whole history of the nuclear collision process, such fluctuations may well be averaged out, leaving no discernible effect at the detector, which collects all the particles produced in an event. This problem is present even if there is no thermalization of the hadrons in the final state, which we shall assume in order to focus our investigation here. Our present method is to apply the GL theory in increments of time when hadrons are produced near  $T_c$  and to integrate the production process over the entire duration to yield the measurable multiplicity fluctuations. Our aim is to show that with appropriate care in

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treating the moments certain scaling behavior persists and characterizes the dynamics of phase transition.

Our result reveals a new scaling exponent  $\gamma$ , different from the one,  $\nu$ , found in Refs. [1,2]. It is not a revision of  $\nu$ , but a new exponent, since different quantities are investigated. Independent of the theoretical considerations underlying this work, the proposed moments can be determined experimentally. Nuclear data should be analyzed in the way suggested, even if the PT is not an issue. If the scaling behavior is found, but the scaling exponent does not agree with the predicted  $\gamma$ , it would not only imply that there has not been a PT of the GL type, but also present a numerical objective for a successful hadronization model to attain for heavy-ion collisions.

## II. MULTIPLICITY DIFFERENCE CORRELATORS

To overcome the problem of high multiplicity per bin in heavy-ion collisions, we introduce the factorial moments that we shall refer to as multiplicity difference correlators (MDCs). They are a form of hybrid of factorial correlators [9] and wavelets [13]. MDCs are not as elaborate as either one of the two separately, but are simpler combinations of the two, possessing the virtues of both.

Let us start with a brief look at the application of wavelet analysis to multiparticle physics [13,14] and astrophysics [15]. Given any event, the structure of a spatial pattern, exemplified here by a one-dimensional function  $g(x)$ , can be analyzed by a multiresolution decomposition, using the basis functions  $\psi_{jk}^H(x)$  such that the wavelet coefficients are

$$W_{jk} = \int_0^L dx \psi_{jk}^H(x) g(x), \quad (3)$$

where  $L$  is the domain of  $g(x)$  being analyzed. This is like a Fourier transform, except that instead of the exponential factor  $\exp(in\pi x/L)$ , one uses the Haar wavelets

$$\psi_{jk}^H(x) = \psi^H(2^j x - k), \quad (4)$$

where

$$\psi^H(x) = \begin{cases} 1, & 0 \leq x < 1/2, \\ -1, & 1/2 \leq x < 1, \\ 0, & \text{elsewhere.} \end{cases} \quad (5)$$

The basis function  $\psi_{jk}^H(x)$  has a scale index  $j$  and a translation index  $k$ , which enable the wavelet analysis to identify the location and scale of an arbitrary fluctuation of  $g(x)$  in terms of  $W_{jk}$  [16].

While Eq. (3) and other transforms like it are very powerful and have various virtues, such as their invertibility, they offer more than what is needed at this point in our search for a measure of the multiplicity fluctuations in heavy-ion collisions that can convey the signature of phase transition. A complete set of  $W_{jk}$  records all the information in the spatial pattern in an event, whereas we look for an efficient way of capturing some general features averaged over all events.

The Haar wavelets, however, contain some features that we regard as important ingredients for an improvement of the usual factorial moments and correlators [9,10,17]. For a fixed set of the indices  $j$  and  $k$ , we have

$$\psi_{jk}^H(x) = \pm 1 \text{ for } x \in x_{\pm}, \quad (6)$$

where

$$x_+ = \left\{ x \mid k 2^{-j} \leq x < \left( k + \frac{1}{2} \right) 2^{-j} \right\},$$

$$x_- = \left\{ x \mid \left( k + \frac{1}{2} \right) 2^{-j} \leq x < (k+1) 2^{-j} \right\},$$

and so,

$$W_{jk} = n_+ - n_-, \quad n_{\pm} = \int_{x_{\pm}} dx g(x). \quad (7)$$

It is the difference between the contributions in the neighboring bins,  $x_+$  and  $x_-$ . In some wavelet analyses moments that involve sums over  $k$  are studied to reveal scaling behaviors in  $j$  [13–15]. That can be done for one event, sometimes referred to as horizontal analysis. For our purpose in what follows, we prefer to emphasize first the vertical analysis; i.e., we average over all events for fixed bins. For multiparticle production  $g(x)$  would be the particle density and  $n_{\pm}$  the multiplicities in the bins  $x_{\pm}$ . Studying the difference  $n_+ - n_-$  of neighboring multiplicities is a way to overcome the problem of high multiplicity per bin without abandoning the focus on multiplicity fluctuations.

Once we consider multiplicity differences, there is no reason why we should restrict the two bins to only the neighboring ones. Let  $\Delta$  be the distance between two bins, each of size  $\delta$ , and let  $n_1$  and  $n_2$  be the multiplicities in those bins in a given event. Define  $m$  to be the multiplicity difference,  $m = n_1 - n_2$ . We shall be interested in the distribution  $Q_m(\Delta, \delta)$  after sampling over many events at fixed  $\Delta$  and  $\delta$ ; i.e.,  $Q_m$  is the probability of detecting multiplicity difference  $m$  between the two bins. The factorial moments we shall study are the MDCs:

$$\mathcal{F}_q = \frac{f_q}{f_1^q}, \quad f_q = \sum_{m=q}^{\infty} m(m-1) \cdots (m-q+1) Q_m. \quad (8)$$

They are not the same as the Białas-Peschanski (BP) correlators, which are normalized products of factorial moments of multiplicities in two bins, averaged over all events [9],

$$F_{q_1 q_2} = \frac{\langle n_1(n_1-1) \cdots (n_1-q_1+1) n_2(n_2-1) \cdots (n_2-q_2+1) \rangle}{\langle n_1 \rangle^{q_1} \langle n_2 \rangle^{q_2}}, \quad (9)$$

but the two are similar to the extent that  $n_1$  and  $n_2$  are the multiplicities in the two bins separated by  $\Delta$ . Here  $F_{q_1 q_2}$  have been found to depend on  $\Delta$ , but not on  $\delta$  [10], but  $\mathcal{F}_q$  as defined in Eq. (8) will depend on both.  $\mathcal{F}_q$  differ from  $F_q$  defined in Eq. (1) in that they are the moments of the multiplicity difference distribution (MDD)  $Q_m(\Delta, \delta)$ , which is a generalization of the usual multiplicity distribution in a way that incorporates the virtues of both wavelets and correlators.

Before we enter into the theoretical description of the MDD  $Q_m$  in the following sections, we note that the experimental determination of  $\mathcal{F}_q$  for hadronic and nuclear collisions should be performed independent of theory. Their dependences on  $\Delta$  and  $\delta$  will pose a challenge to any model of such collisions. Our concern in this paper is to make a theoretical prediction of what should be observed when there is a quark-hadron PT. But if there is no PT, the properties of  $\mathcal{F}_q$  will remain as valuable features of multiparticle production that a good model of soft interaction must explain.

### III. STATISTICAL AND DYNAMICAL FLUCTUATIONS

Although our aim is to study the nature of the fluctuations due to quark-hadron PTs in heavy-ion collisions, we begin with a formulation that is more generally valid for any hadronic or nuclear collisions. The multiplicities in the two bins discussed in the preceding section fluctuate both statistically and dynamically. Focusing on just the statistical part first and using, as usual, the Poisson distribution  $\mathcal{P}_{n_i}$  for it, we have the statistical MDD

$$P_m(s_1, s_2) = \sum_{n_1, n_2} \mathcal{P}_{n_1}(s_1) \mathcal{P}_{n_2}(s_2) \delta_{m-n_1+n_2}, \quad (10)$$

where  $s_1$  and  $s_2$  are the average multiplicities in the two bins and

$$\mathcal{P}_{n_i}(s_i) = \frac{1}{n_i!} s_i^{n_i} e^{-s_i}. \quad (11)$$

Note that, unlike the usual multiplicity,  $m$  can be both positive and negative. In fact, the sums in Eq. (10) can be analytically performed, yielding

$$P_m(s_1, s_2) = (s_1/s_2)^{m/2} I_m(2\sqrt{s_1 s_2}) e^{-s_1 - s_2}, \quad (12)$$

where  $I_m$  is the modified Bessel function.

In the following we shall not assign any intrinsic properties to the two bins and consider only the absolute difference between their multiplicities: i.e.,

$$m = |n_1 - n_2|. \quad (13)$$

Since  $I_m$  is symmetric under  $m \leftrightarrow -m$ , we have, from Eq. (12) with  $m \geq 0$ ,

$$P_m(s_1, s_2) = \cosh\left(\frac{m}{2} \ln \frac{s_1}{s_2}\right) I_m(2\sqrt{s_1 s_2}) e^{-s_1 - s_2} (2 - \delta_{m0}). \quad (14)$$

There is a discontinuity at  $m=0$  because for all  $m>0$  there is reflection of  $-m$  in Eq. (12) to  $+m$ . Note that  $P_m$  is properly normalized to  $\sum_{m=0}^{\infty} P_m = 1$ .

If  $s_1$  and  $s_2$  are large, then  $\mathcal{P}_{n_i}(s_i)$  may be well approximated by Gaussian distributions, and  $P_m$  would also be Gaussian with a width proportional to  $(s_1 s_2)^{1/4}$ . It is because of this reduced width that we consider MDDs: As discussed in Sec. I, we need smaller values of  $m$  to render lower-order moments effective in measuring the fluctuations. If  $s_1$  and  $s_2$  are small, then  $P_m$  becomes Poissonian also. For that reason we shall consider factorial moments of  $P_m$ , since the statistical fluctuations can thereby be filtered out [9]. In both experimental analysis and theoretical consideration, the bin width  $\delta$  is to be varied so that  $s_i$  will range over both large and small values; hence, no approximation of  $P_m$  will be made.

We now introduce the dynamical component of the fluctuations. Denoting it by  $D(s_1, s_2, \Delta, \delta)$  we have, for the observable MDD,

$$Q_m(\Delta, \delta) = \int ds_1 ds_2 P_m(s_1, s_2) D(s_1, s_2, \Delta, \delta). \quad (15)$$

In essence this is a double Poisson transform of the dynamical  $D$ , which is a generalization of the formalism for photon counting in quantum optics, later adapted by BP for particle production [9]. It is this distribution  $Q_m$  that we have proposed to study by use of the MDC  $\mathcal{F}_q$ , defined in Eq. (8). Our aim is to examine the scaling properties of  $\mathcal{F}_q$  and extract universal features that are characteristic of the dynamics of the problem.

There are many directions in which one can pursue from here. For the dynamical distribution  $D$  one can consider the mathematical  $\alpha$  model [18] or the more physical models such as the FRITIOF model [19], VENUS [20], and ECCO [21]. Alternatively, one may want to emphasize the large-scale space-time structure by considering the small- $\Delta$  behavior in an interferometry type of analysis [22]. For us in this paper we want to consider the opposite, namely, the large- $\Delta$  behavior where the global size of the particle-emitting volume is unimportant. The usual short-range correlation in rapidity in low- $p_T$  multiparticle production [10] would also be not important, if  $\Delta$  is sufficiently large, but not large enough to cause kinematical constraint. Our purpose is to go to a region where fluctuations due to the dynamics of PT are the only ones that need to be taken into account.

More specifically, we shall identify  $D(s_1, s_2, \Delta, \delta)$  with the Boltzmann factor  $\exp(-F)$  in the Ginzburg-Landau theory, in which  $s_1$  and  $s_2$  will be functions of the order parameter. In fact, for two identical bins at large  $\Delta$  apart in two regions of the expanding system that have similar spatial and temporal properties, we may set  $s_1 = s_2$ , ignore  $\Delta$ , and rewrite Eq. (15) as

$$Q_m(\delta) = \int ds P_m(s) D(s, \delta). \quad (16)$$

There remains a complication arising from the property that PT occur over an extended period of time and that the detected hadrons are the result of an integration over that period. That is the subject we turn to in the next section.

#### IV. HADRONIZATION IN GINZBURG-LANDAU THEORY

The conventional view of the physical system in a heavy-ion collision at very high energy is that a cylinder of locally thermalized partons expands as a fluid, mainly in the longitudinal direction, but also in the radial direction at a slower rate. If the colliding nuclei are massive enough and the incident energy high enough, the temperature in the interior of the cylinder may be higher than the critical temperature  $T_c$  for quark-hadron PT, which we shall assume to be second order. Because of the transverse expansion, the temperature  $T$  decreases with increasing radius, at least initially and for the most part of the lifetime of the system. Thus hadronization takes place mainly on the surface of the cylinder where  $T \approx T_c$ . Being a second-order PT, there is no mixed phase where quarks and hadrons coexist. We assume that there is no thermalized hadronic phase surrounding the partonic cylinder, and so the hadrons formed on the surface move in free flow to the detector. With these simplifying assumptions, which are not unrealistic at extremely high collision energies, we can then focus on the issue of relating the hadronization process on the surface to the hadron multiplicity collected by the detector.

Consider first just one bin, which occupies  $\delta\eta\delta\varphi$  in pseudorapidity  $\eta$  and azimuthal angle  $\varphi$ . We define  $\delta^2$  to be its area. Such an area selected by an analyst of the data corresponds to a similar area on the surface of the cylinder. Let the hadronization time be  $t_h$ , which is of order 1 fm/c: It is the average time of the formation of one hadron on the surface. During that time, we use the GL description of PT to specify the probability  $D(s, \delta)$  that  $s$  hadrons are created in the area  $\delta^2$ . The GL free energy, being time independent, does not track the time evolution of the system. It is also not equipped to describe the spatial variations on the surface; nor need it be. As a mean field theory, it is concerned with the probability for PT near  $T_c$  as a function of the order parameter  $\phi$ . Let the two-dimensional coordinates on the surface be labeled by  $z$ ; then, the GL free energy is [23]

$$F[\phi] = \int_{\delta^2} dz [a|\phi(z)|^2 + b|\phi(z)|^4 + c|\partial\phi/\partial z|^2], \quad (17)$$

where  $a$ ,  $b$ , and  $c$  are GL parameters that depend on  $T$ . For hadronization within the bin of size  $\delta^2$  it is only necessary to integrate  $z$  over that area. For  $T \lesssim T_c$  the GL theory requires that  $a < 0$  and  $b > 0$ . We have found in [1,3] that for small bins the third term in Eq. (17) does not have any significant effect on the multiplicity fluctuations, and so we shall set  $c = 0$ , as it has been done in all previous work [1–8]. Furthermore, we make the approximation that  $\phi(z)$  is constant in  $\delta^2$  so that

$$F[\phi] = \delta^2(a|\phi|^2 + b|\phi|^4). \quad (18)$$

Assuming that this is valid for any local area on the cylindrical surface, the same free energy is to be used for both bins of  $P_m$  in Eq. (15), resulting in the same average multiplicity  $s_1$  and  $s_2$ . We now must specify the relationship between  $s = s_1 = s_2$  and  $|\phi|^2$ .

As discussed extensively in [2], the square of the order parameter  $|\phi|^2$  is the hadron density,  $\rho$ , which, in the ab-

sence of fluctuations around the minimum of Eq. (18) at  $|\phi_0|^2$ , is zero for  $T > T_c$ , but positive for  $T < T_c$ . Thus the average multiplicity in a bin is

$$\bar{n}_0 = \delta^2 |\phi|^2. \quad (19)$$

It should be borne in mind, however, that this is the average multiplicity during the hadronization time  $t_h$ , when the system is momentarily regarded as static and the GL consideration is applied to describe the formation of hadrons from quarks at  $T \lesssim T_c$ . It is not the average multiplicity in  $\delta^2$  registered by the detector, since the experimental measurement integrates the hadronization process over the entire lifetime  $\mathcal{T}$  of the whole parton system, during which partons are continuously converted to hadrons. Thus the measured average multiplicity in  $\delta^2$  is

$$s = \delta^2 \int_0^{\mathcal{T}} dt |\phi(t)|^2, \quad (20)$$

where  $\phi(t)$  is formally the time-dependent order parameter, which has to be varied in a functional-integral description of  $Q_m(\delta)$ . Indeed,  $s$  in Eq. (20) is the average multiplicity of  $P_m(s)$  in Eq. (16) but  $\bar{n}_0$  in Eq. (19) is the relevant average multiplicity for the GL description during  $t_h$ . Thus, to adapt the formalism of the previous sections to the PT process in heavy-ion collisions, it is necessary to modify Eq. (16) into a functional integral

$$Q_m(\delta, \tau) = Z^{-1} \int \mathcal{D}\phi P_m(\delta^2 \tau |\phi|^2) \times \exp[-\delta^2(a|\phi|^2 + b|\phi|^4)], \quad (21)$$

where  $\mathcal{D}\phi = \pi d|\phi|^2$ ,  $Z = \int \mathcal{D}\phi \exp[-F(\phi)]$ , and the integral in Eq. (20) has been discretized into  $\tau = \mathcal{T}/t_h$  segments. In each segment  $\phi$  is spatially and temporally constant in  $\delta^2$  and is itself integrated over the whole complex plane. For collisions of large nuclei,  $\tau$  may be a large number. Herein lies the crux of the problem: The PT at  $T \lesssim T_c$  gives sparingly few hadrons within  $\delta^2$  in any time interval around  $t_h$ , but the detected number in a bin at the end of the collision process is roughly  $\tau$  times as many. Our aim to find some universal feature of the problem that is essentially independent of  $a$ ,  $b$ ,  $\delta$ , and  $\tau$ .

#### V. SCALING BEHAVIOR

We now proceed to determine the multiplicity difference correlator MDC, which in the present case of large  $\Delta$  has no dependence on  $\Delta$  and is just the normalized factorial moments  $\mathcal{F}_q$  of the MDD  $Q_m$  defined in Eq. (8). As we have mentioned in general terms earlier, an important reason for studying  $Q_m$  instead of the usual multiplicity distribution  $P_n$  in a single bin is the largeness of  $\delta^2 \tau |\phi|^2$  in Eq. (21), when  $\tau$  is large. In that case the bin multiplicity  $n$  is high, and so factorial moments of low orders are ineffective in extracting genuine correlations. The multiplicity difference  $m$  is on the average proportional to  $\sqrt{\tau}$ , thus enabling  $\mathcal{F}_q$  to be more effective at low  $q$ .

Setting  $s_1 = s_2 = s$  in Eq. (14), with

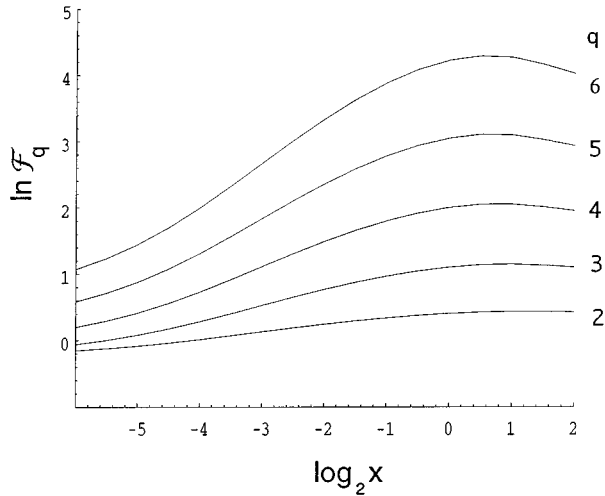


FIG. 1. Factorial moments  $\mathcal{F}_q$  versus  $x$  in a log-log plot for various values of  $q$  at  $\tau=10$ .

$$s = \delta^2 \tau |\phi|^2, \quad (22)$$

we have the distribution  $P_m$  in Eq. (21),

$$P_m(s) = (2 - \delta_{m0}) I_m(2s) e^{-2s}, \quad (23)$$

where  $m \geq 0$ . Let us simplify Eq. (21) by using the variable  $u^2 = \delta^2 b |\phi|^4$ , getting

$$Q_m(\tau, x) = Z^{-1} \int_0^\infty du P_m(\tau x u) e^{xu - u^2}, \quad (24)$$

where

$$Z = \int_0^\infty du e^{xu - u^2}, \quad x = |a| \delta / \sqrt{b}. \quad (25)$$

For notational economy  $\tau$  has been redefined as

$$\tau = \frac{\mathcal{T}}{t_h |a|}. \quad (26)$$

It is therefore the  $\tau$  defined earlier in units of  $|a|$ . It makes physical sense because  $|a|$  is a measure of how readily hadronization can take place in GL theory. The minimum of  $F[\phi]$  in Eq. (18) is at  $|\phi_0|^2 = |a|/2b$ , for  $a < 0$ , and so there would be virtually no hadron condensates apart from fluctuations, if  $|a| \rightarrow 0$ , resulting in  $\tau \rightarrow \infty$ . Thus the expanding parton system must drive the surface temperature to below  $T_c$ , making  $a$  sufficiently negative and  $|\phi_0|^2$  large enough to produce hadrons at a rate just such as to carry away the necessary energy to maintain the hydrodynamical flow with  $T < T_c$  at the surface. The beauty of the GL approach is that all the complications of the hydrodynamics of the problem are hidden in a few parameters, which would be in the final answer [like  $\tau$  and  $x$  in Eq. (24)] unless we can find observables that are insensitive to them.

To find such observables we now calculate the normalized factorial moments  $\mathcal{F}_q$  of  $Q_m$  using Eq. (8). We first fix  $\tau$  and examine  $\mathcal{F}_q$  as a function of  $x$ , which is proportional to  $\delta$ . In Fig. 1 we show how  $\ln \mathcal{F}_q$  depends on  $\log_2 x$  for  $\tau=10$  and

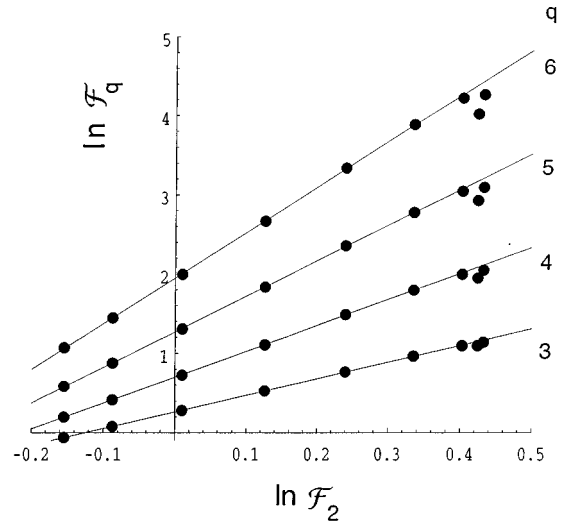


FIG. 2. Factorial moments  $\mathcal{F}_q$  versus  $\mathcal{F}_2$  in a log-log plot for various values of  $q$  at  $\tau=10$ . Dots are calculated values at  $x=2^{i/2}$  for  $-12 \leq i \leq 4$ . Straight lines are fits of the linear portions of the dots.

$2 \leq q \leq 6$ . No simple behavior can be ascribed to the rising and falling of the curves. It should be noticed that the increase of  $\mathcal{F}_q$  with increasing  $\delta$  is opposite to the usual behavior of the single bin factorial moments  $\mathcal{F}_q$  [1,3,7], which increase with decreasing  $\delta$ . Thus  $\mathcal{F}_q$  do not have the intermittency behavior of BP [9].

The similarity of the dependences of  $\mathcal{F}_q$  on  $x$  for the various  $q$  values shown in Fig. 1 suggests that we should examine, as in [1], the dependences of  $\mathcal{F}_q$  on  $\mathcal{F}_2$  for  $3 \leq q \leq 6$ . That is shown in Fig. 2, where the dots are the values of  $\mathcal{F}_q$  for  $\log_2 x = i/2$  with  $i$  being integers in the range  $-12 \leq i \leq 4$ . The straight lines are fits of the linear portions. Clearly,  $\mathcal{F}_q$  exhibit the power-law behavior

$$\mathcal{F}_q \propto \mathcal{F}_2^{\beta_q}, \quad (27)$$

which we shall call  $F$  scaling. It is a scaling behavior that is independent of  $\delta$  and  $b$ . The dependence on  $a$  is so far unknown, since the calculation is done at a fixed  $\tau$ .

From the slopes of the straight lines in Fig. 2, we show in Fig. 3 the dependence of  $\beta_q$  on  $q$ . It can be well fitted by the formula

$$\beta_q = (q-1)^\gamma, \quad (28)$$

where  $\gamma = 1.099$ . We use the symbol  $\gamma$  here for the scaling exponent, instead of  $\nu$ , which we have used previously for a similarly defined quantity [as in Eq. (28)] for  $F_q$ . For comparison, we recall that [1]

$$\nu = 1.304. \quad (29)$$

Thus the scaling exponent  $\gamma$  for MDCs is significantly lower. A lower value of that exponent means larger fluctuations.

So far, the result is for  $\tau=10$  only. To see the dependences on  $\tau$ , we have repeated the calculation for a range of  $\tau$  values. Scaling behavior as in Eq. (27) has been found in

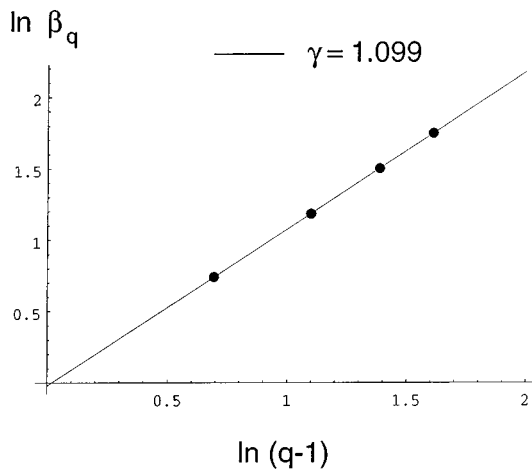


FIG. 3. Dots are the slopes of the straight lines in Fig. 2, plotted against  $q-1$  in a log-log plot. The straight line is the best fit of the dots, whose slope is  $\gamma$ , defined in Eq. (28).

each case, and Eq. (28) is also well satisfied. Figure 4 shows the dependence of  $\gamma$  on  $\tau$ . Evidently, it is nearly constant for  $3 < \tau < 30$  with the value

$$\gamma = 1.09 \pm 0.02. \quad (30)$$

Thus the result of this study is embodied in just one number,  $\gamma$ . It is independent of  $a$ ,  $b$ ,  $\delta$ , and  $\tau$ , provided that  $a$  is negative to allow hadronization.

The universality of  $\gamma$  is remarkable and should be checked experimentally. If a signature of a quark-hadron PT depends on the details of the heavy-ion collisions, such as nuclear sizes, collision energy, transverse energy, etc., even after they have passed the thresholds for the creation of quark-gluon plasma, such a signature is likely to be sensitive to the theoretical model used. Here  $\gamma$  is independent of such details; it depends only on the validity of the GL description of PT for the present problem. The MDD  $Q_m$  can readily be measured experimentally, and the moments  $\mathcal{F}_q$  directly determined as functions of bin size. If the scaling behavior (27), supplemented Eq. (28), is satisfied with  $\gamma \approx 1.1$ , then we may interpret the system as having undergone a second-order PT describable by GL theory.

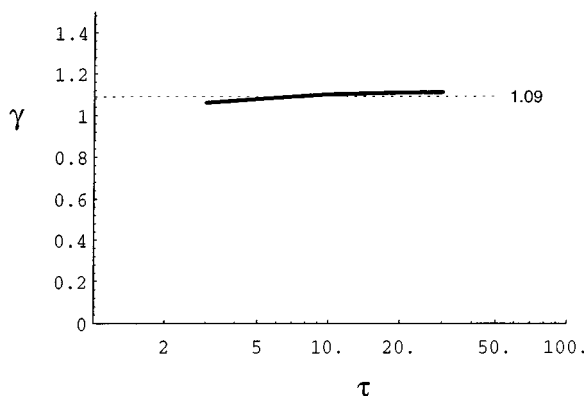


FIG. 4. Solid line shows the dependence of  $\gamma$  on  $\tau$ ; dotted line is at  $\gamma = 1.09$  to guide the eye.

In the absence of PT we may regard the two bins separated by a large  $\Delta$  to be totally uncorrelated. In that case  $Q_m(\delta)$  can be identified with  $P_m[s(\delta)]$ , where, if we vary  $s$  in the range  $1 < s < 100$ , the corresponding  $\mathcal{F}_q$  without PT can be calculated directly. Again  $F$  scaling is found, satisfying Eqs. (27) and (28), but this time with

$$\gamma = 1.33 \pm 0.02 \quad (\text{no PT}). \quad (31)$$

This value is sufficiently separated from that of Eq. (30) derived for PT so that phenomenological distinguishability between the two cases should be quite feasible.

## VI. CONCLUSION

In this work we have solved a number of problems that have obstructed the study of multiplicity fluctuations in heavy-ion collisions as a means of finding signatures of quark-hadron phase transition. One problem is the large multiplicities even in small bins, for which the usual factorial moments  $F_q$  fail to reveal distinctive features for  $3 \leq q \leq 6$ , since events with large fluctuations are submerged by generic events. That is mainly an experimental problem where the analysis of the data cannot be pushed to the regions  $\langle n \rangle_\delta < q$ . Another problem of a more theoretical nature is that the application of the Ginzburg-Landau theory of PT needs special tailoring for a system whose lifetime is finite, but long compared to the transition time for individual hadrons, and whose observables are integrated over that time. We have overcome both of these problems by showing the effectiveness of studying the fluctuations of multiplicity differences.

We have started with wavelet analysis and found that it generates more information than can easily be filtered to yield a succinct signature of PT. However, we extracted a simple feature of the wavelets and considered the MDD  $Q_m$  involving two bins separated by a distance  $\Delta$ . Although  $\Delta$  can be any value, we have considered only large  $\Delta$  in order to apply the simplest description of PT by GL theory. The time integration problem of the detectable multiplicities is handled at the expense of an extra parameter  $\tau$ , which cannot be specified without a hydrodynamical model lying outside the scope of this treatment. The goal has then been to find a measure of PT that is independent of the unknown parameters in the problem.

That goal was achieved by the discovery that the factorial moments  $\mathcal{F}_q$  of  $Q_m$  satisfy a scaling behavior that is characterized by a number  $\gamma \approx 1.1$ . It is independent of the details of the dynamics, except that a PT occurs in a way describable by GL theory. Thus  $\gamma$  is a universal constant for the problem. We call  $\gamma$  a scaling exponent, but it has absolutely no connection with the critical exponents of the conventional critical phenomena. There is no need to know the temperature, which is not measurable, or the critical  $T_c$ . The moments  $\mathcal{F}_q$  can directly be determined from the data and can therefore be checked for  $F$  scaling, independent of any theoretical input. Whatever experimental value the exponent  $\gamma$  turns out to be would be of great interest, since a value different from 1.1 would demand an explanation. If it approaches 1.1 as energy or nuclear size is increased, it would suggest an approach to the condition required for the formation of quark-gluon plasma.

Experimentally, it should be easy to vary  $\Delta$  and check not

only how  $\mathcal{F}_q$  themselves depend on  $\Delta$ , but also whether and how the scaling behavior is affected. The  $\Delta$  dependence has not been investigated here. Theoretically, there are many other challenges that also await undertaking. A verification of  $\gamma=1.1$  will undoubtedly stimulate an upsurge of interest in the study of multiplicity fluctuations in particle and nuclear collisions.

#### ACKNOWLEDGMENTS

Discussions and communications with Z. Cao, L. Z. Fang, and P. Lipa on wavelets during the early phase of this work have been beneficial. This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG03-96ER40972.

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