Determination of decuplet baryon magnetic moments from QCD sum rules

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A complete set of QCD sum rules for the magnetic moments of decuplet baryons are derived using the external field method. They are analyzed thoroughly using a Monte Carlo based procedure. Valid sum rules are identified under the criteria of OPE convergence and ground state dominance and their predictions are obtained. The performances of these sum rules are further compared and a favorable sum rule is designated for each member. Correlations between the input and the output parameters are examined and large sensitivities to the quark condensate magnetic susceptibility χ are found. Using realistic estimates of the QCD input parameters, the uncertainties on the magnetic moments are found relatively large and they can be attributed mostly to the poorly known χ. It is shown that the accuracy can be improved to the 30% level, provided the uncertainties in the QCD input parameters can be determined to the 10% level. The computed magnetic moments are consistent with existing data. Comparisons with other calculations are made.
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I. INTRODUCTION

The QCD sum rule method [1] has proved to be a powerful tool in revealing the deep connection between hadron phenomenology and QCD vacuum structure via a few condensate parameters. The method has been successfully applied to a variety of problems to gain a field-theoretical understanding into the structure of hadrons. Calculations of the nucleon magnetic moments in the approach were first carried out in Refs. [2] and [3]. They were later refined and extended to the entire baryon octet in Refs. [4–7]. On the other hand, the magnetic moments of decuplet baryons were less well studied within the same approach. There were previous, unpublished reports in Ref. [8] on Δ^{++} and Ω^- magnetic moments. The magnetic form factor of Δ^{++} in the low Q^2 region was calculated based on a rather different technique [9]. In recent years, the magnetic moment of Ω^- has been measured with remarkable accuracy [10]: $\mu_{\Omega^-}=(-2.02\pm0.05)\mu_N$. The magnetic moment of Δ^{++} has also been extracted from pion bremsstrahlung [11]: $\mu_{\Delta^{++}}=(4.5\pm1.0)\mu_N$. In an earlier work [12], the magnetic moment of Δ^0 extracted from π^-p bremsstrahlung data was found to be consistent with $\mu_{\Delta^0}=0$. The experimental information provides new incentives for theoretical scrutiny of these observables.

In this work, we present a systematic, independent calculation of the magnetic moments for the entire decuplet family in the QCD sum rule approach. The goal is twofold. First, we want to find out if the approach can be successfully applied to these observables by carrying out an explicit calculation. Second, we want to achieve some realistic understanding of the uncertainties involved in such a determination by employing a Monte Carlo based analysis procedure. This would help us assess the limitations and find ways for improvements.

We will show that both goals are achieved in this work. The entire calculation is more challenging than the octet case due to the more complex spin structure of spin-3/2 particles. One has to overcome an enormous amount of algebra to arrive at the final results. But conceptually it presents no apparent difficulties. Particular attention is paid to the complete treatment of the phenomenological representation, which leads to the isolation of the tensor structures from which the QCD sum rules for the magnetic moments can be constructed. Flavor symmetry breakings in the strange quark are treated consistently across the decuplet family. The success also hinges upon a new analysis of the two-point functions [13], which provides more accurately determined current couplings for normalization. Part of the results on Δ^{++} and Ω^- have been communicated in a Letter [14].

Magnetic moments of decuplet baryons have also been studied in various other methods, including lattice QCD [15], chiral perturbation theory [16], Bethe-Salpeter formalism [17], non-relativistic quark model [18], relativistic quark models [19–23], chiral quark-soliton model [24], chiral bag model [25], cloudy bag model [26], Skyrme model [27]. A comparison will be made with some of the calculations and with existing data.

Section II deals with the derivation of the QCD sum rules. Section III discusses the Monte Carlo analysis procedure. Section IV gives the results and discussion. Section V contains the conclusions. The Appendix collects the QCD sum rules derived.

II. METHOD

Consider the time-ordered two-point correlation function in the QCD vacuum in the presence of a *constant* background electromagnetic field $F_{\mu\nu}$:

$$\Pi_{\alpha\beta}(p)=i\int d^4x e^{ip\cdot x}\langle 0|T\{\eta_\alpha(x)\,\overline{\eta_\beta}(0)\}|0\rangle_F, \qquad (1)$$

where η_α is the interpolating field for the propagating baryon. The subscript F means that the correlation function is to be evaluated with an electromagnetic interaction term added to the QCD Lagrangian:

$$\mathcal{L}_I = -A_\mu J^\mu, \tag{2}$$

where A_μ is the external electromagnetic potential and $J^\mu = e_q \bar{q} \gamma^\mu q$ the quark electromagnetic current.

Since the external field can be made arbitrarily small, one can expand the correlation function

$$\Pi_{\alpha\beta}(p) = \Pi^{(0)}_{\alpha\beta}(p) + \Pi^{(1)}_{\alpha\beta}(p) + \cdots . \tag{3}$$

Here $\Pi^{(0)}_{\alpha\beta}(p)$ is the correlation function in the absence of the field, and gives rise to the mass sum rules of the baryons. The magnetic moments will be extracted from the QCD sum rules obtained from the linear response function $\Pi^{(1)}_{\alpha\beta}(p)$.

The action of the external electromagnetic field is twofold: It couples directly to the quarks in the baryon interpolating fields, and it also polarizes the QCD vacuum. The latter can be described by introducing new parameters called vacuum susceptibilities.

The interpolating field is constructed from quark fields, and has the quantum numbers of the baryon under consideration. We use the following interpolating fields for the baryon decuplet family:

$$\eta_\alpha^{\Delta^{++}} = \epsilon^{abc}(u^{aT} C \gamma_\alpha u^b) u^c,$$

$$\eta_\alpha^{\Delta^{+}} = \sqrt{1/3}\epsilon^{abc}[2(u^{aT} C \gamma_\alpha d^b) u^c + (u^{aT} C \gamma_\alpha u^b) d^c],$$

$$\eta_\alpha^{\Delta^{0}} = \sqrt{1/3}\epsilon^{abc}[2(d^{aT} C \gamma_\alpha u^b) d^c + (d^{aT} C \gamma_\alpha d^b) u^c],$$

$$\eta_\alpha^{\Delta^{-}} = \epsilon^{abc}(d^{aT} C \gamma_\alpha d^b) d^c,$$

$$\eta_\alpha^{\Sigma^{*+}} = \sqrt{1/3}\epsilon^{abc}[2(u^{aT} C \gamma_\alpha s^b) u^c + (u^{aT} C \gamma_\alpha u^b) s^c],$$

$$\eta_\alpha^{\Sigma^{*0}} = \sqrt{2/3}\epsilon^{abc}[2(u^{aT} C \gamma_\alpha d^b) s^c + (d^{aT} C \gamma_\alpha s^b) u^c + (s^{aT} C \gamma_\alpha u^b) d^c],$$

$$\eta_\alpha^{\Sigma^{*-}} = \sqrt{1/3}\epsilon^{abc}[2(d^{aT} C \gamma_\alpha s^b) d^c + (d^{aT} C \gamma_\alpha d^b) s^c],$$

$$\eta_\alpha^{\Xi^{*0}} = \sqrt{1/3}\epsilon^{abc}[2(s^{aT} C \gamma_\alpha u^b) s^c + (s^{aT} C \gamma_\alpha s^b) u^c],$$

$$\eta_\alpha^{\Xi^{*-}} = \sqrt{1/3}\epsilon^{abc}[2(s^{aT} C \gamma_\alpha d^b) s^c + (s^{aT} C \gamma_\alpha s^b) d^c],$$

$$\eta_\alpha^{\Omega^{-}} = \epsilon^{abc}(s^{aT} C \gamma_\alpha s^b) s^c. \tag{4}$$

Here implicit function forms $\eta(x)$ and $q(x)$ $(q=u,d,s)$ are assumed. C is the charge conjugation operator. The superscript T means transpose. The indices a, b and c are color indices running from 1 to 3. The antisymmetric tensor ϵ^{abc} ensures the three quarks form a color singlet state. The normalization factors are chosen so that correlation functions of these interpolating fields coincide with each other under SU(3)-flavor symmetry [see Eqs. (20)–(23)].

The interpolating field excites (or annihilates) the ground state as well as the excited states of the baryon from the QCD vacuum. The ability of an interpolating field to annihilate the *ground state* baryon into the QCD vacuum is described by a phenomenological parameter λ_B (called current coupling or pole residue), defined by the overlap

$$\langle 0 | \eta_\alpha | B p s\rangle = \lambda_B u_\alpha(p,s), \tag{5}$$

where u_α is the Rarita-Schwinger spin vector [28].

A. Phenomenological representation

On the hadronic level, let us consider the linear response defined by

$$\Pi^{(1)}_{\alpha\beta}(p) = i \int d^4x e^{ip\cdot x} \langle 0 | \eta_\alpha(x) \left[-i \int d^4y A_\mu(y) J^\mu(y) \right] \overline{\eta_\beta}(0) | 0 \rangle. \tag{6}$$

After inserting two complete sets of physical intermediate states, it becomes

$$\Pi^{(1)}_{\alpha\beta}(p) = \int d^4x \int d^4y \frac{d^4k}{(2\pi)^4} \frac{d^4k'}{(2\pi)^4} \times \sum_{BB'} \sum_{ss'} \frac{-i}{k^2 - M_B^2 - i\epsilon} \frac{-i}{k'^2 - M_{B'}^2 - i\epsilon} \times e^{ip\cdot x} A_\mu(y) \langle 0 | \eta_\alpha(x) | ks\rangle \langle ks | J^\mu(y) | k's'\rangle \times \langle k's' | \overline{\eta_\beta}(0) | 0\rangle. \tag{7}$$

QCD sum rule calculations are most conveniently done in the fixed-point gauge. For electromagnetic fields, it is defined by $x_\mu A^\mu(x) = 0$. In this gauge, the electromagnetic potential is given by

$$A_\mu(y) = -\frac{1}{2} F_{\mu\nu} y^\nu. \tag{8}$$

The electromagnetic vertex of spin-3/2 baryons is defined by the current matrix element [15]

$$\langle ks | J^\mu(0) | k's'\rangle = \overline{u_\alpha}(k,s) \mathcal{O}^{\alpha\mu\beta}(P,q) u_\beta(k',s'). \tag{9}$$

The Lorentz covariant tensor

$$\mathcal{O}^{\alpha\mu\beta}(P,q) \equiv -g^{\alpha\beta}\left(a_1 \gamma^\mu + \frac{a_2}{2M_B} P^\mu \right) - \frac{q^\alpha q^\beta}{(2M_B)^2}\left(c_1 \gamma^\mu + \frac{c_2}{2M_B} P^\mu \right), \tag{10}$$

where $P = k + k'$ and $q = k - k'$, satisfies the standard requirements of invariance under time reversal, parity, G parity, and gauge transformations. The parameters a_1, a_2, c_1 and c_2 are independent covariant vertex functions. They are related to the multipole form factors by

$$G_{E0}(q^2) = (1 + \tfrac{2}{3}\tau)[a_1 + (1+\tau)a_2] - \tfrac{1}{3}\tau(1+\tau)[c_1 + (1+\tau)c_2],$$

$$G_{E2}(q^2) = [a_1 + (1+\tau)a_2] - \tfrac{1}{2}(1+\tau)[c_1 + (1+\tau)c_2],$$

$$G_{M1}(q^2) = (1 + \tfrac{4}{5}\tau)a_1 - \tfrac{2}{5}\tau(1+\tau)c_1,$$

$$G_{M3}(q^2) = a_1 - \tfrac{1}{2}(1+\tau)c_1, \tag{11}$$

where $\tau=-q^2/(2M_B)^2(\geqslant 0)$. They are referred to as charge (E0), electric quadrupole (E2), magnetic dipole (M1), and magnetic octupole (M3) form factors. The magnetic moment is related to the magnetic dipole form factor $G_{M1}(q^2)$ at zero momentum transfer. From Eq. (11), it is clear that

$$G_{M1}(0)=a_1\equiv\mu_B, \tag{12}$$

where the magnetic moment μ_B is in units of particle's natural magneton: $e\hbar/(2cM_B)$. So the goal is to isolate terms in Eq. (7) that involve only a_1.

The ground state contribution to Eq. (7) can be written as

$$\begin{aligned}\Pi^{(1)}_{\alpha\beta}(p)=&\frac{i}{2}\lambda_B^2F_{\mu\nu}\int d^4x\,\frac{d^4k}{(2\pi)^4}\,e^{i(p-k)\cdot x}\\&\times\frac{1}{k^2-M_B^2-i\epsilon}\sum_s u_\alpha(k,s)\overline{u}_\rho(k,s)\\&\times\frac{\partial}{\partial q^\nu}\left[\frac{1}{(k-q)^2-M_B^2-i\epsilon}\,O^{\rho\mu\lambda}(2k-q,q)\right.\\&\left.\left.\times\sum_{s'}u_\lambda(k-q,s')\overline{u}_\beta(k-q,s')\right]\right|_{q=0}.\end{aligned} \tag{13}$$

In arriving at Eq. (13), we have used a number of steps: the translation invariance on $\eta_\alpha(x)$ and $J^\mu(y)$, a change of variable from k' to q, the relation

$$\int d^4y e^{iq\cdot y}y^\nu=-i(2\pi)^4\frac{\partial}{\partial q^\nu}\delta^4(q), \tag{14}$$

integration by parts, and the Rarita-Swinger spin sum [28]

$$\sum_s u_\alpha(p,s)\overline{u}_\beta(p,s)=-(\hat{p}+M_B)\left(g_{\alpha\beta}-\frac{1}{3}\gamma_\alpha\gamma_\beta-\frac{2p_\alpha p_\beta}{3M_B^2}+\frac{p_\alpha\gamma_\beta-p_\beta\gamma_\alpha}{3M_B}\right), \tag{15}$$

with normalization $\overline{u}_\alpha u_\alpha=2M_B$. The caret notation denotes $\hat{p}=p^\alpha\gamma_\alpha$.

Direct evaluation of Eq. (13) leads to numerous tensor structures, not all of which are independent of each other. The dependences can be removed by ordering the gamma matrices in a specific order. Here we choose to order in $\hat{p}\gamma_\alpha\gamma_\mu\gamma_\nu\gamma_\beta$. After a lengthy calculation, 18 tensor structures which involve only a_1 are isolated. They can be organized as

$$\begin{aligned}\Pi^{(1)}_{\alpha\beta}(p)=&\mathrm{WE}_1(p^2)\hat{p}F^{\mu\nu}\sigma_{\mu\nu}g_{\alpha\beta}+\mathrm{WO}_1(p^2)F^{\mu\nu}\sigma_{\mu\nu}g_{\alpha\beta}+\mathrm{WE}_2(p^2)\hat{p}p_\alpha F^{\mu\nu}\sigma_{\mu\nu}p_\beta+\mathrm{WO}_2(p^2)p_\alpha F^{\mu\nu}\sigma_{\mu\nu}p_\beta\\&+\mathrm{WE}_3(p^2)\hat{p}\gamma_\alpha F^{\mu\nu}\sigma_{\mu\nu}\gamma_\beta+\mathrm{WO}_3(p^2)\gamma_\alpha F^{\mu\nu}\sigma_{\mu\nu}\gamma_\beta+\mathrm{WE}_4(p^2)p_\alpha F^{\mu\nu}\sigma_{\mu\nu}\gamma_\beta+\mathrm{WO}_4(p^2)\hat{p}p_\alpha F^{\mu\nu}\sigma_{\mu\nu}\gamma_\beta\\&+\mathrm{WE}_5(p^2)\gamma_\alpha F^{\mu\nu}\sigma_{\mu\nu}p_\beta+\mathrm{WO}_5(p^2)\hat{p}\gamma_\alpha F^{\mu\nu}\sigma_{\mu\nu}p_\beta+\mathrm{WE}_6(p^2)\hat{p}\gamma_\alpha F^{\mu\nu}(\gamma_\mu g_{\beta\nu}-\gamma_\nu g_{\beta\mu})\\&+\mathrm{WO}_6(p^2)\gamma_\alpha F^{\mu\nu}(\gamma_\mu g_{\beta\nu}-\gamma_\nu g_{\beta\mu})+\mathrm{WE}_7(p^2)\hat{p}F^{\mu\nu}(\gamma_\mu g_{\alpha\nu}-\gamma_\nu g_{\alpha\mu})\gamma_\beta+\mathrm{WO}_7(p^2)F^{\mu\nu}(\gamma_\mu g_{\alpha\nu}-\gamma_\nu g_{\alpha\mu})\gamma_\beta\\&+\mathrm{WE}_8(p^2)p_\alpha F^{\mu\nu}(\gamma_\mu g_{\beta\nu}-\gamma_\nu g_{\beta\mu})+\mathrm{WO}_8(p^2)\hat{p}p_\alpha F^{\mu\nu}(\gamma_\mu g_{\beta\nu}-\gamma_\nu g_{\beta\mu})+\mathrm{WE}_9(p^2)F^{\mu\nu}(\gamma_\mu g_{\alpha\nu}-\gamma_\nu g_{\alpha\mu})p_\beta\\&+\mathrm{WO}_9(p^2)\hat{p}F^{\mu\nu}(\gamma_\mu g_{\alpha\nu}-\gamma_\nu g_{\alpha\mu})p_\beta+\cdots\;.\end{aligned} \tag{16}$$

The tensor structures associated with WE_i have odd number of gamma matrices, while those associated with WO_i have even number of gamma matrices. Apart from a common factor $i\lambda_B^2\mu_B/(p^2-M_B^2)^2$, the invariant functions are given by

$$\mathrm{WE}_1=\frac{1}{2},\quad \mathrm{WO}_1=\frac{1}{2}M_B,$$

$$\mathrm{WE}_2=\frac{-1}{9M_B^2},\quad \mathrm{WO}_2=\frac{-1}{9M_B},$$

$$\mathrm{WE}_3=\frac{-7}{18},\quad \mathrm{WO}_3=\frac{-7}{18}M_B,$$

$$\mathrm{WE}_4=\frac{7}{18},\quad \mathrm{WO}_4=\frac{7}{18M_B},$$

$$\mathrm{WE}_5=\frac{-7}{18},\quad \mathrm{WO}_5=\frac{-7}{18M_B},$$

$$\mathrm{WE}_6=\frac{2}{3},\quad \mathrm{WO}_6=\frac{2}{3}M_B,$$

$$\mathrm{WE}_7=\frac{-2}{3},\quad \mathrm{WO}_7=\frac{-2}{3}M_B,$$

$$\mathrm{WE}_8=\frac{-2}{3},\quad \mathrm{WO}_8=\frac{-2}{3M_B},$$

$$\mathrm{WE}_9=\frac{-2}{3},\quad \mathrm{WO}_9=\frac{-2}{3M_B}. \tag{17}$$

In addition to the ground state contribution, there exist also excited state contributions. For a generic invariant function, the pole structure has the form

$$\frac{\lambda_B^2\mu_B}{(p^2-M_B^2)^2}+\sum_{B*}\frac{C_{B\leftrightarrow B*}}{(p^2-M_B^2)(p^2-M_{B*}^2)}+\cdots, \quad (18)$$

where $C_{B\leftrightarrow B*}$ are constants. The first term is the ground state double pole which contains the desired magnetic moment of the baryon, the second term represents the non-diagonal transitions between the ground state and the excited states caused by the external field, and the ellipsis represents pure excited state contributions. Upon Borel transform, one has

$$\frac{\lambda_B^2\mu_B}{M^2}e^{-M_B^2/M^2}+e^{-M_B^2/M^2}\times\left[\sum_{B*}\frac{C_{B\to B*}}{M_{B*}^2-M_B^2}(1-e^{-(M_{B*}^2-M_B^2)/M^2})\right]+\cdots. \quad (19)$$

We see that the transitions give rise to a contribution that is not exponentially suppressed relative to the ground state. This is a general feature of the external-field technique. The strength of such transitions at each structure is *a priori* unknown and is an additional source of contamination in the determination of μ_B not found in mass sum rules. The usual treatment of the transitions is to approximate the quantity in the square brackets by a constant, which is to be extracted from the sum rule along with the ground state property of interest. Inclusion of such contributions is necessary for the correct extraction of the magnetic moments. The pure excited state contributions are exponentially suppressed relative to the ground state and can be modeled in the usual way by introducing a continuum model and threshold parameter.

B. Calculation of the QCD side

On the quark level, one evaluates the correlation function in Eq. (1) using the operator product expansion (OPE). The calculation is most readily done in coordinate space. To arrive at the final sum rules, one needs a subsequent Fourier transform, followed by a Borel transform.

We decide to carry out four separate calculations for $\Omega^-(sss)$, $\Sigma^{*+}(uus)$, $\Xi^{*0}(uss)$, and $\Sigma^{*0}(uds)$. They have distinct strange quark content, which requires special treatment. The QCD sum rules for other members can be obtained by appropriate substitutions in those for these four members.

The master formula, which is obtained from contracting out the quark pairs in the correlation function, is given by, for Ω^-,

$$\langle 0|T\{\eta_\alpha^{\Omega^-}(x)\overline{\eta_\beta^{\Omega^-}}(0)\}|0\rangle_F=2\epsilon^{abc}\epsilon^{a'b'c'}\{S_s^{aa'}\mathrm{Tr}[\gamma_\beta C S_s^{bb'T}C\gamma_\alpha S_s^{cc'}]+2S_s^{aa'}\gamma_\beta C S_s^{bb'T}C\gamma_\alpha S_s^{cc'}\}, \quad (20)$$

for Σ^{*+},

$$\begin{aligned}\langle 0|T\{\eta_\alpha^{\Sigma^{*+}}(x)\overline{\eta_\beta^{\Sigma^{*+}}}(0)\}|0\rangle_F=\frac{2}{3}\epsilon^{abc}\epsilon^{a'b'c'}\{&S_u^{aa'}\mathrm{Tr}[\gamma_\beta C S_u^{bb'T}C\gamma_\alpha S_s^{cc'}]+S_u^{aa'}\mathrm{Tr}[\gamma_\beta C S_s^{bb'T}C\gamma_\alpha S_u^{cc'}]\\&+S_s^{aa'}\mathrm{Tr}[\gamma_\beta C S_u^{bb'T}C\gamma_\alpha S_u^{cc'}]+2S_u^{aa'}\gamma_\beta C S_u^{bb'T}C\gamma_\alpha S_s^{cc'}+2S_u^{aa'}\gamma_\beta C S_s^{bb'T}C\gamma_\alpha S_u^{cc'}\\&+2S_s^{aa'}\gamma_\beta C S_u^{bb'T}C\gamma_\alpha S_u^{cc'}\},\end{aligned} \quad (21)$$

for Ξ^{*0},

$$\begin{aligned}\langle 0|T\{\eta_\alpha^{\Xi^{*0}}(x)\overline{\eta_\beta^{\Xi^{*0}}}(0)\}|0\rangle_F=\frac{2}{3}\epsilon^{abc}\epsilon^{a'b'c'}\{&S_s^{aa'}\mathrm{Tr}[\gamma_\beta C S_s^{bb'T}C\gamma_\alpha S_u^{cc'}]+S_s^{aa'}\mathrm{Tr}[\gamma_\beta C S_u^{bb'T}C\gamma_\alpha S_s^{cc'}]\\&+S_u^{aa'}\mathrm{Tr}[\gamma_\beta C S_s^{bb'T}C\gamma_\alpha S_s^{cc'}]+2S_s^{aa'}\gamma_\beta C S_s^{bb'T}C\gamma_\alpha S_u^{cc'}+2S_s^{aa'}\gamma_\beta C S_u^{bb'T}C\gamma_\alpha S_s^{cc'}\\&+2S_u^{aa'}\gamma_\beta C S_s^{bb'T}C\gamma_\alpha S_s^{cc'}\},\end{aligned} \quad (22)$$

and, for Σ^{*0},

$$\begin{aligned}\langle 0|T\{\eta_\alpha^{\Sigma^{*0}}(x)\overline{\eta_\beta^{\Sigma^{*0}}}(0)\}|0\rangle_F=\frac{2}{3}\epsilon^{abc}\epsilon^{a'b'c'}\{&S_u^{aa'}\mathrm{Tr}[\gamma_\beta C S_d^{bb'T}C\gamma_\alpha S_s^{cc'}]+S_d^{aa'}\mathrm{Tr}[\gamma_\beta C S_s^{bb'T}C\gamma_\alpha S_u^{cc'}]\\&+S_s^{aa'}\mathrm{Tr}[\gamma_\beta C S_u^{bb'T}C\gamma_\alpha S_d^{cc'}]+S_u^{aa'}\gamma_\beta C S_d^{bb'T}C\gamma_\alpha S_s^{cc'}+S_d^{aa'}\gamma_\beta C S_s^{bb'T}C\gamma_\alpha S_u^{cc'}\\&+S_s^{aa'}\gamma_\beta C S_u^{bb'T}C\gamma_\alpha S_d^{cc'}+S_u^{aa'}\gamma_\beta C S_s^{bb'T}C\gamma_\alpha S_d^{cc'}+S_d^{aa'}\gamma_\beta C S_u^{bb'T}C\gamma_\alpha S_s^{cc'}\\&+S_s^{aa'}\gamma_\beta C S_d^{bb'T}C\gamma_\alpha S_u^{cc'}\}.\end{aligned} \quad (23)$$

In the above equations,

$$S_q^{ab}(x,0;F)\equiv\langle 0|T\{q^a(x)\overline{q^b}(0)\}|0\rangle_F, \quad q=u,d,s, \quad (24)$$

is the fully interacting quark propagator in the presence of the electromagnetic field. To first order in $F_{\mu\nu}$ and m_q (assume $m_u=m_d=0, m_s\neq 0$), and order x^4, it is given by [2,5,6]

$$\begin{aligned}
S_q^{ab}(x,0;Z) \equiv {} & \frac{i}{2\pi^2}\frac{\hat{x}}{x^4}\delta^{ab} - \frac{m_q}{4\pi^2 x^2}\delta^{ab} - \frac{1}{12}\langle\bar{q}q\rangle\delta^{ab} + \frac{im_q}{48}\langle\bar{q}q\rangle\hat{x}\delta^{ab} + \frac{1}{192}\langle\bar{q}g_c\sigma\cdot Gq\rangle x^2\delta^{ab} - \frac{im_q}{1152}\langle\bar{q}g_c\sigma\cdot Gq\rangle\hat{x}x^2\delta^{ab} \\
& - \frac{1}{3^3 2^{10}}\langle\bar{q}q\rangle\langle g_c^2 G^2\rangle x^4\delta^{ab} + \frac{i}{32\pi^2}(g_c G^n_{\alpha\beta})\frac{\hat{x}\sigma^{\alpha\beta}+\sigma^{\alpha\beta}\hat{x}}{x^2}\left(\frac{\lambda^n}{2}\right)^{ab} + \frac{1}{48}\frac{i}{32\pi^2}\langle g_c^2 G^2\rangle\frac{\hat{x}\sigma^{\alpha\beta}+\sigma^{\alpha\beta}\hat{x}}{x^2}\left(\frac{\lambda^n}{2}\right)^{ab} \\
& + \frac{1}{3^2 2^{10}}\langle\bar{q}q\rangle\langle g_c^2 G^2\rangle x^2\sigma^{\alpha\beta}\left(\frac{\lambda^n}{2}\right)^{ab} - \frac{1}{192}\langle\bar{q}g_c\sigma\cdot Gq\rangle\sigma^{\alpha\beta}\left(\frac{\lambda^n}{2}\right)^{ab} + \frac{im_q}{768}\langle\bar{q}g_c\sigma\cdot Gq\rangle(\hat{x}\sigma^{\alpha\beta}+\sigma^{\alpha\beta}\hat{x})\left(\frac{\lambda^n}{2}\right)^{ab} \\
& + \frac{ie_q}{32\pi^2}F_{\alpha\beta}\frac{\hat{x}\sigma^{\alpha\beta}+\sigma^{\alpha\beta}\hat{x}}{x^2}\delta^{ab} - \frac{e_q}{24}\chi\langle\bar{q}q\rangle F_{\alpha\beta}\sigma^{\alpha\beta}\delta^{ab} + \frac{ie_q m_q}{96}\chi\langle\bar{q}q\rangle F_{\alpha\beta}(\hat{x}\sigma^{\alpha\beta}+\sigma^{\alpha\beta}\hat{x})\delta^{ab} \\
& + \frac{e_q}{288}\langle\bar{q}q\rangle F_{\alpha\beta}(x^2\sigma^{\alpha\beta}-2x_\rho x^\beta\sigma^{\beta\alpha})\delta^{ab} + \frac{e_q}{576}\langle\bar{q}q\rangle F_{\alpha\beta}[x^2(\kappa+\xi)\sigma^{\alpha\beta}-x_\rho x^\beta(2\kappa-\xi)\sigma^{\beta\alpha}]\delta^{ab} - \frac{e_q}{16}\langle\bar{q}q\rangle \\
& \times\left(\kappa F_{\alpha\beta} - \frac{i}{4}\xi\epsilon_{\alpha\beta\mu\nu}F^{\mu\nu}\right)\left(\frac{\lambda^n}{2}\right)^{ab} + \text{higher order terms.}
\end{aligned} \tag{25}$$

We use the convention $\epsilon^{0123}=+1$ in this work. The vacuum susceptibilities are defined by

$$\begin{aligned}
\langle\bar{q}\sigma_{\mu\nu}q\rangle_F &\equiv e_q\chi\langle\bar{q}q\rangle F_{\mu\nu}, \\
\langle\bar{q}g_c G_{\mu\nu}q\rangle_F &\equiv e_q\kappa\langle\bar{q}q\rangle F_{\mu\nu}, \\
\langle\bar{q}g_c\epsilon_{\mu\nu\rho\lambda}G^{\rho\lambda}\gamma_5 q\rangle_F &\equiv ie_q\xi\langle\bar{q}q\rangle F_{\mu\nu}.
\end{aligned} \tag{26}$$

Note that χ has the dimension of GeV^{-2}, while κ and ξ are dimensionless.

The calculation proceeds by substituting the quark propagator into the master formulas, keeping terms to first order in the external field and in the strange quark mass. Terms up to dimension 8 are considered. The various combinations can be represented by diagrams. Figure 1 shows the basic diagrams considered for the decuplet baryon magnetic moments. Figure 2 shows the diagrams considered for the strange quark mass corrections. Note that each diagram is only generic. All possible color permutations are understood. Numerous tensor structures emerge from the calculations. Upon ordering the gamma matrices in the same order as in the phenomenological side, 18 invariant functions are obtained at the corresponding tensor structures. By equating them with those in Eq. (16), QCD sum rules are constructed. These invariant functions can be classified by the chirality of the vacuum condensates they contain. Eight of them, denoted by WE_i, involve only dimension-even condensates; thus we call the corresponding sum rules chiral even. The other eight, denoted by WO_i, involve only dimension-odd condensates, and we call the corresponding sum rules chiral odd. Note that previous works such as Refs. [2,4] use the chirality of the tensor structures to refer to the sum rules. The two are opposite.

To keep the presentation smooth, the complete set of sum rules (a total of 160 for the decuplet family) obtained in this work are given in the Appendix in a highly condensed form. As it turns out, the validity of a particular sum rule depends on the input parameter set. Sum rules that are valid for one set may become invalid for another, and *vice versa*. For this reason, it is useful to present all of the sum rules. Another benefit is that it provides a basis for other authors to check the calculation. Sufficient detail is given in this work for that purpose.

The various symbols in the sum rules are explained in the following. The condensate parameters are denoted by

$$a=-(2\pi)^2\langle\bar{u}u\rangle, \quad b=\langle g_c^2G^2\rangle,$$
$$\langle\bar{u}g_c\sigma\cdot Gu\rangle=-m_0^2\langle\bar{u}u\rangle. \tag{27}$$

The rescaled current coupling

$$\tilde{\lambda}_B=(2\pi)^2\lambda_B. \tag{28}$$

The quark charge factors e_q are given in units of electric charge

$$e_u=2/3, \quad e_d=-1/3, \quad e_s=-1/3. \tag{29}$$

Note that we choose to keep the quark charge factors explicit in the sum rules. The advantage is that it can facilitate the study of quark effective magnetic moments. The parameters f and ϕ account for the flavor symmetry breaking of the strange quark in the condensates and susceptibilities:

$$f=\frac{\langle\bar{s}s\rangle}{\langle\bar{u}u\rangle}=\frac{\langle\bar{s}g_c\sigma\cdot Gs\rangle}{\langle\bar{u}g_c\sigma\cdot Gu\rangle}, \quad \phi=\frac{\chi_s}{\chi}=\frac{\kappa_s}{\kappa}=\frac{\xi_s}{\xi}. \tag{30}$$

The four-quark condensate is parametrized by the factorization approximation

$$\langle\bar{u}u\bar{u}u\rangle=\kappa_v\langle\bar{u}u\rangle^2, \tag{31}$$

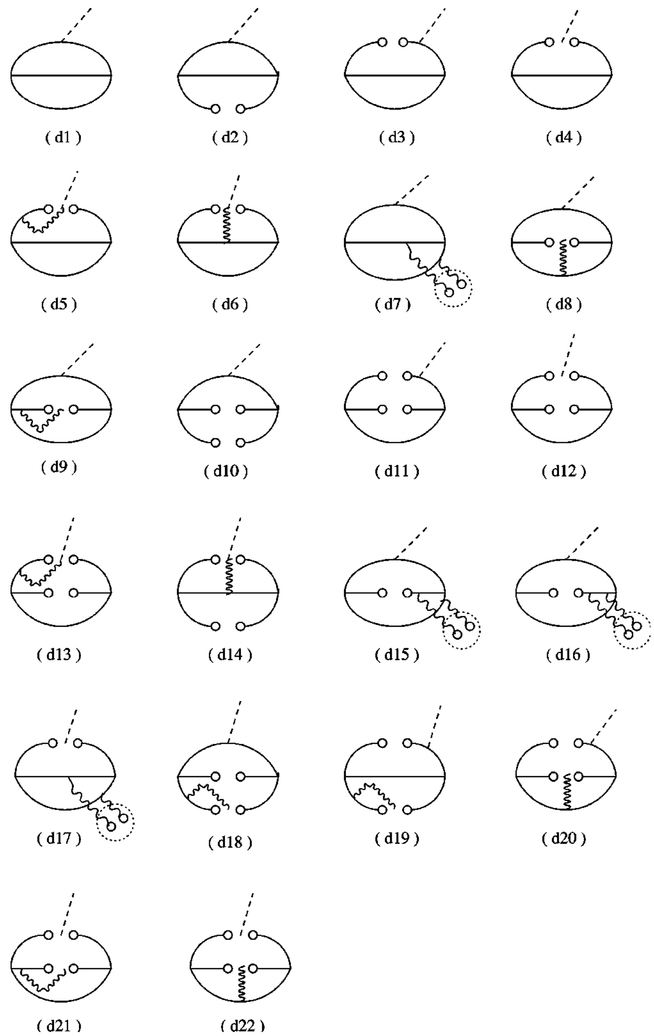

FIG. 1. Diagrams considered for the decuplet baryon magnetic moments.

and we will investigate its possible violation via the parameter κ_v. The anomalous dimension corrections of the currents and various operators are taken into account in the leading logarithmic approximation via the factor

$$L^{\gamma}=\left[\frac{\alpha_s(\mu^2)}{\alpha_s(M^2)}\right]^{\gamma}=\left[\frac{\ln(M^2/\Lambda_{QCD}^2)}{\ln(\mu^2/\Lambda_{QCD}^2)}\right]^{\gamma}, \qquad (32)$$

where $\mu=500$ MeV is the renormalization scale and Λ_{QCD} is the QCD scale parameter. As usual, the excited state contributions are modeled using terms on the OPE side surviving $M^2\rightarrow\infty$ under the assumption of duality, and are represented by the factors

$$E_n(x)=1-e^{-x}\sum_n \frac{x^n}{n!}, \quad x=w_B^2/M_B^2, \qquad (33)$$

where w_B is an effective continuum threshold. Note that w_B is in principle different for different sum rules and we will treat it as a free parameter in the the analysis.

The coefficients for other members of the decuplet family can be obtained by appropriate replacements of quark contents. They are

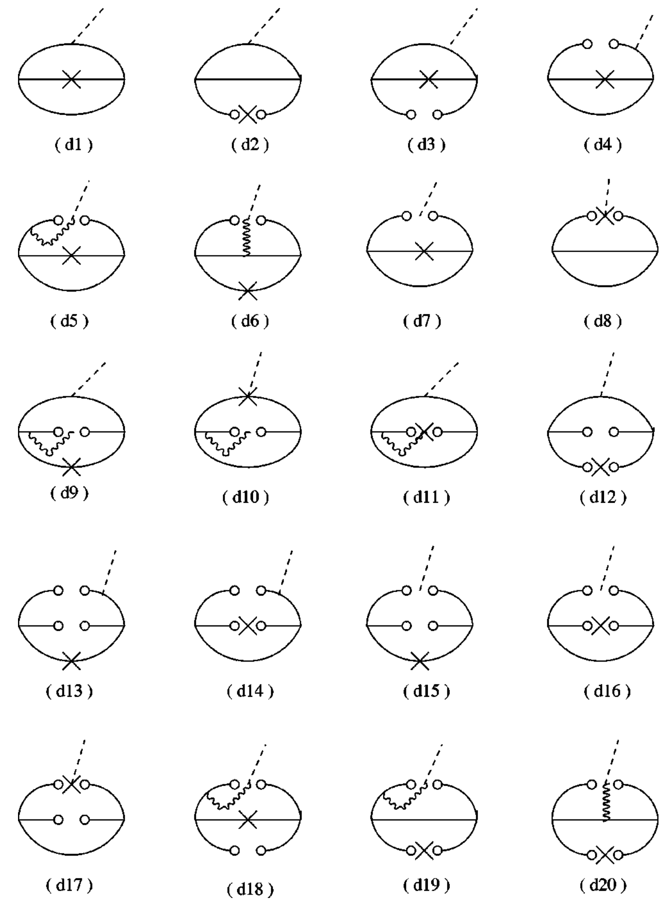

FIG. 2. Diagrams considered for the strange quark mass corrections to the decuplet baryon magnetic moments.

(1) for Δ^{++}, replace s quark by u quark in Ω^-,
(2) for Δ^+, replace s quark by d quark in Σ^{*+},
(3) for Δ^0, replace s quark by d quark in Ξ^{*0},
(4) for Δ^-, replace s quark by d quark in Ω^-,
(5) for Σ^{*-}, replace u quark by d quark in Σ^{*+},
(6) for Ξ^{*-}, replace u quark by d quark in Ξ^{*0}.

Here the conversions between u and d quarks are achieved by simply switching their charge factors e_u and e_d. The conversions from s quark to u or d quarks involve setting $m_s=0$, $f=\phi=1$, in addition to the switching of charge factors.

Furthermore, in the course of collecting the coefficients for the four selected members Σ^{*+}, Σ^{*0}, Ξ^{*0}, Ω^-, we discovered some relations among them that allow one to write down one set of c_i staring from another. The relations are given as follows.

(1) From Σ^{*+} to Σ^{*0}: simply replace every occurrence of e_u by $(e_u+e_d)/2$.

(2) From Ξ^{*0} to Ω^- involves converting the u quark to s quark. This is achieved by collapsing each coefficient into a single term that has the maximum number of e_s, f, ϕ in that coefficient. The numerical factor of it is the sum of the numerical factors in front each of the terms in the coefficient. For example, $(2e_s+e_u)$ goes to $3e_s$, $(2e_sf\phi+e_u)$ goes to $3e_sf\phi$, $(2e_sf-3e_s-3e_uf+e_u)$ goes to $-3e_sf$, etc.

These relations were also used as consistency checks of the calculation.

From the above discussions, we see that it is possible to write down the coefficients for all other members of the decuplet family starting just from those for Σ^{*+} and Ξ^{*0}. In the sum rule from WE_1 in Eq. (A1), the complete sets of c_i are given for the four selected members Σ^{*+}, Σ^{*0}, Ξ^{*0} and Ω^-. They are intended as examples for the reader to get familiar with the relations. The rest of the sum rules are presented with c_i only given for Σ^{*+} and Ξ^{*0}.

Finally, let us point out some exact relations among the OPE sides of the sum rules:

$$\mathrm{OPE}_{\Delta^+}=\tfrac{1}{2}\mathrm{OPE}_{\Delta^{++}}, \quad (34)$$

$$\mathrm{OPE}_{\Delta^0}=0, \quad (35)$$

$$\mathrm{OPE}_{\Delta^-}=-\mathrm{OPE}_{\Delta^+}, \quad (36)$$

$$\mathrm{OPE}_{\Sigma^{*0}}=\tfrac{1}{2}(\mathrm{OPE}_{\Sigma^{*+}}+\mathrm{OPE}_{\Sigma^{*-}}). \quad (37)$$

These results are consequences of symmetries in the correlation functions. As an example, let us examine Eq. (34). For a given diagram, the master formula for Δ^{++} can be written as $2(e_u C_1+e_u C_2)=\frac{4}{3}(C_1+C_2)$ where C_1 has the trace dependence, while C_2 not. On the other hand, the master formula for Δ^+ can be written as $\frac{2}{3}[(2e_u+e_d)C_1+(2e_u+e_d)C_2]=\frac{2}{3}(C_1+C_2)$, hence the factor of 2. The key here is that (a) each term is proportional to a quark charge factor; (b) SU(2) flavor symmetry in u and d quarks; (c) it is the same C_1 and C_2 that appear in both cases. The argument can be generalized to any diagrams, only with C_1 and C_2 different from diagram to diagram. Thus the factor of 2 will survive, regardless of the number of diagrams considered. One can argue for the rest of the relations by the same token. The above results have been explicitly verified using the calculated coefficients in the sum rules. They also provided a set of highly non-trivial checks of the calculation. A number of hard-to-detect errors have been eliminated this way.

Now let us consider the phenomenological side of Eq. (37). Since the continuum is modeled using terms on the OPE side, the continuum contributions also differ by a factor of 2. Assuming the transitions, which are modeled by a constant, also differ by a factor of 2, then Eq. (34) can be extended to the magnetic moments. This assumption was confirmed by numerical analysis. The same is true for Eq. (35) and Eq. (36). The situation for Eq. (37) is a little different. The convergence properties may change when two OPE series are added up. Numerical analysis confirmed that fewer sum rules are valid for Σ^{*0} than for Σ^{*+} and Σ^{*-}.

III. MONTE CARLO ANALYSIS

To analyze the sum rules, we use a Monte Carlo based procedure recently developed in Ref. [29]. The basic steps are as follows. First, the uncertainties in the QCD input parameters are assigned. Then, randomly selected, Gaussianly distributed sets for these uncertainties are generated, from which an uncertainty distribution in the OPE, $\sigma^2_{OPE}(M_j)$ where M_j are evenly distributed points in the desired Borel window, can be constructed. Next, a χ^2 minimization is applied to the sum rule by adjusting the phenomenological fit parameters. Note that the uncertainties in the OPE are not uniform throughout the Borel window. They are larger at the lower end where uncertainties in the higher-dimensional condensates dominate. Thus, it is crucial that the appropriate weight is used in the calculation of χ^2. For the OPE obtained from the kth set of QCD parameters, the χ^2 per degree of freedom is

$$\frac{\chi_k^2}{N_{DF}}=\frac{1}{n_B-n_p}\times\sum_{j=1}^{n_B}\frac{[\Pi_k^{OPE}(M_j)-\Pi_k^{phen}(M_j;\lambda_k,m_k,w_k)]^2}{\sigma^2_{OPE}(M_j)}, \quad (38)$$

where n_p is the number of phenomenological search parameters, and Π^{phen} denotes the phenomenological representation. In practice, $n_B=51$ points were used along the Borel axis. The procedure is repeated for many QCD parameter sets, resulting in distributions for phenomenological fit parameters, from which errors are derived. Usually, 200 such configurations are sufficient for getting stable results. We generally select 1000 sets which help resolve more subtle correlations among the QCD parameters and the phenomenological fit parameters.

The Borel window over which the two sides of a sum rule are matched is determined by the following two criteria. First, *OPE convergence*: the highest-dimension operators contribute no more than 10% to the QCD side. Second, *ground-state dominance*: excited state contributions should not exceed more than 50% of the phenomenological side. The first criterion effectively establishes a lower limit, the second an upper limit. Those sum rules which do not have a Borel window under these criteria are considered invalid.

A. QCD input parameters

The QCD input parameters and their uncertainty assignments are given as follows. The condensates are taken as

$$a=0.52\pm0.05\ \mathrm{GeV}^3,\quad b=1.2\pm0.6\ \mathrm{GeV}^4,$$

$$m_0^2=0.72\pm0.08\ \mathrm{GeV}^2. \quad (39)$$

For the factorization violation parameter, we use

$$\kappa_v=2\pm1 \quad \text{and}\ 1\leq\kappa_v\leq4. \quad (40)$$

The QCD scale parameter is restricted to $\Lambda_{QCD}=0.15\pm0.04$ GeV. The vacuum susceptibilities have been estimated in studies of nucleon magnetic moments [2–4], but the values vary in a wide range depending on the method used. Here we take some median values with 50% uncertainties:

$$\chi=-6.0\pm3.0\ \mathrm{GeV}^{-2}\quad \text{and}\ 0\ \mathrm{GeV}^{-2}\leq\chi\leq-10\ \mathrm{GeV}^{-2}, \quad (41)$$

and

$$\kappa=0.75\pm0.38,\quad \xi=-1.5\pm0.75. \quad (42)$$

TABLE I. Monte Carlo analysis of the QCD sum rules for the magnetic moment of Δ^{++} and Ω^{-}. The six columns correspond to, from left to right, the sum rule that has a valid Borel region, the Borel region determined by the 10%–50% criteria, the percentage contribution (Cont) of the excited states and transitions to the phenomenological side at the lower end of the Borel region (it increases to 50% at the upper end), the continuum threshold, the transition strength, the magnetic moment in nuclear magnetons. The uncertainties in each sum rule were obtained from consideration of 1000 QCD parameter sets.

Sum rule	Region (GeV)	Cont (%)	w (GeV)	A (GeV^{-2})	μ_B (μ_N)
Δ^{++}: WE_1	0.70–1.53	1.7	1.65	-0.28 ± 0.52	7.76 ± 2.67
WE_3	1.04–1.42	19	1.65	0.20 ± 0.20	3.06 ± 1.14
WE_4	0.675–1.56	5	1.65	-0.35 ± 0.37	3.34 ± 1.44
WE_5	0.765–1.47	8.5	1.65	0.53 ± 0.81	3.56 ± 3.49
Ω^{-}: WE_1	0.592–1.70	2.3	2.30	-0.12 ± 0.11	-2.66 ± 0.88
WE_2	0.872–1.53	20	2.30	-0.26 ± 0.20	-5.31 ± 3.66
WE_3	0.885–1.68	8.5	2.30	-0.09 ± 0.04	-1.24 ± 0.51
WE_4	0.60–1.72	2	2.30	-0.03 ± 0.05	-1.24 ± 0.24
WE_5	0.747–1.66	7.4	2.30	-0.14 ± 0.14	-1.32 ± 1.08
WE_6	0.59–2.32	0.86	2.30	-0.01 ± 0.02	-1.14 ± 0.40
WE_8	0.69–2.60	3.3	2.30	0.03 ± 0.03	-0.65 ± 1.22
WO_1	0.663–1.26	12	2.30	-0.32 ± 0.42	-0.65 ± 1.22
WO_2	1.06–1.43	31	2.30	-0.62 ± 0.18	-4.94 ± 5.58
WO_4	0.836–2.22	7.4	2.30	-0.03 ± 0.01	-0.70 ± 0.24

Note that χ is almost an order of magnitude larger than κ and ξ, and is the most important of the three. The strange quark parameters are placed at [5,13]

$$m_s=0.15\pm0.02\ \text{GeV},\quad f=0.83\pm0.05,\quad \phi=0.60\pm0.05. \tag{43}$$

These uncertainties are assigned conservatively and in accordance with the state of the art in the literature. While some may argue that some values are better known, others may find that the errors are underestimated. In any event, one will learn how the uncertainties in the QCD parameters are

TABLE II. Same as Table I, but for Σ^{*+}, Σ^{*0} and Σ^{*-}. The presence of a second row in a specific sum rule indicates that the continuum threshold was successfully searched.

Sum rule	Region (GeV)	Cont (%)	w (GeV)	A (GeV^{-2})	μ_B (μ_N)
Σ^{*+}: WE_1	0.853–1.445	11	1.80	0.28 ± 0.18	2.96 ± 1.41
WE_3	0.996–1.39	23	1.80	0.23 ± 0.08	1.49 ± 0.79
WE_4	0.622–1.61	1	1.80	-0.05 ± 0.10	1.74 ± 0.42
WE_5	0.715–1.45	10	1.80	0.34 ± 0.35	1.82 ± 1.94
	0.715–1.45	6	2.65 ± 5.96	0.26 ± 0.49	1.71 ± 1.96
WE_6	0.575–1.96	0.2	1.80	-0.01 ± 0.08	2.10 ± 0.79
	0.575–1.96	0.9	1.56 ± 0.11	-0.06 ± 0.04	2.00 ± 0.68
WE_8	0.79–2.36	13	1.80	-0.17 ± 0.08	1.09 ± 0.71
	0.79–2.36	15	1.52 ± 5.39	-0.21 ± 0.08	1.08 ± 0.67
WO_4	0.89–1.46	23	1.80	0.07 ± 0.06	0.39 ± 0.48
Σ^{*0}: WE_5	0.577–1.95	2.8	1.80	0.01 ± 0.01	0.19 ± 0.13
WE_8	0.639–1.70	9.4	1.80	0.03 ± 0.01	-0.18 ± 0.06
WO_2	0.846–1.38	18	1.80	0.11 ± 0.09	1.00 ± 0.96
WO_8	0.662–1.66	5	1.80	-0.01 ± 0.01	-0.30 ± 0.18
WO_9	0.627–1.73	3.4	1.80	-0.01 ± 0.01	-0.33 ± 0.19
Σ^{*-}: WE_1	0.662–1.54	1	1.80	-0.05 ± 0.19	-3.34 ± 1.33
WE_3	0.926–1.42	16	1.80	-0.17 ± 0.08	-1.42 ± 0.71
WE_4	0.602–1.61	1.3	1.80	0.06 ± 0.10	-1.70 ± 0.38
WE_5	0.735–1.37	13	1.80	-0.33 ± 0.36	-1.40 ± 1.74
WE_6	0.588–1.97	0.2	1.80	0.01 ± 0.07	-1.72 ± 0.63
WE_8	0.71–2.51	9.4	1.80	0.15 ± 0.07	-1.22 ± 0.65
WO_1	0.618–1.05	10	1.80	-0.34 ± 0.77	-0.66 ± 1.45
WO_4	0.89–1.57	19	1.80	-0.08 ± 0.05	-0.54 ± 0.39

TABLE III. Same as Table I, but for Ξ^{*0} and Ξ^{*-}.

Sum rule	Region (GeV)	Cont (%)	w (GeV)	A (GeV^{-x})	μ_B (μ_N)
Ξ^{*0}: WE_8	0.636–1.55	13	2.00	0.08 ± 0.03	-0.35 ± 0.12
WO_2	0.977–1.25	13	2.00	0.11 ± 0.13	2.25 ± 1.92
WO_8	0.654–1.85	3.7	2.00	-0.02 ± 0.02	-0.62 ± 0.34
WO_9	0.621–1.91	2.8	2.00	-0.02 ± 0.02	-0.69 ± 0.35
Ξ^{*-}: WE_1	0.628–1.61	1.3	2.00	-0.07 ± 0.14	-2.88 ± 1.02
WE_2	0.898–1.12	37	2.00	-0.30 ± 0.83	-3.65 ± 5.68
WE_3	0.906–1.53	12	2.00	-0.12 ± 0.05	-1.25 ± 0.55
WE_4	0.6–1.66	0.3	2.00	0.0004 ± 0.07	-1.38 ± 0.27
WE_5	0.74–1.50	10	2.00	-0.22 ± 0.21	-1.27 ± 1.30
WE_6	0.59–2.11	0.5	2.00	-0.006 ± 0.04	-1.38 ± 0.48
WE_8	0.70–2.54	6.3	2.00	0.07 ± 0.05	-0.88 ± 0.47
WO_1	0.641–1.12	13	2.00	-0.35 ± 0.57	-0.58 ± 1.26
WO_4	0.863–1.86	12	2.00	-0.05 ± 0.02	-0.60 ± 0.29

mapped into uncertainties in the phenomenological fit parameters. In the numerical analysis below, we will also examine how the spectral parameters depend on different uncertainty assignments in these input parameters.

B. Search procedure

To extract the magnetic moments, a two-stage fit was performed. First, the corresponding chiral-odd mass sum rule, as obtained previously in Ref. [13], was fitted to get the mass M_B, the coupling $\tilde{\lambda}_B^2$ and the continuum threshold w_1. Then, M_B and $\tilde{\lambda}_B^2$ were used in the magnetic moment sum rule for a three-parameter fit: the transition strength A, the continuum threshold w_2, and the magnetic moment μ_B. Note that w_1 and w_2 are not necessarily the same. We impose a physical constraint on both w_1 and w_2, requiring that they be larger than the mass, and discard QCD parameter sets that do not satisfy this condition. In the actual analysis of the sum rules, however, we found that a full search was not always successful. In such cases, the search algorithm consistently returned w_2 either zero or smaller than M_B. This signals insufficient information in the OPE to completely resolve the spectral parameters. To proceed, we fixed w_2 at w_1, which is a commonly adopted choice in the literature, and searched for A and μ_B. The two-stage fit incorporates the uncertainties from the two-point functions in a correlated fashion into the three-point functions, and represents a more realistic scenario.

To illustrate how well a sum rule works, we first cast it into the subtracted form

$$\Pi_S=\tilde{\lambda}_B^2\mu_B e^{-M_B^2/M^2}, \tag{44}$$

and then plot the logarithm of the absolute value of the two sides against the inverse of M^2. In this way, the right-hand side will appear as a straight line whose slope is $-M_B^2$ and whose intercept with the y axis gives some measure of the coupling strength and the magnetic moment. The linearity (or deviation from it) of the left-hand side gives an indication of OPE convergence, and information on the continuum model and the transitions. The two sides are expected to match for a good sum rule. This way of matching the sum rules is similar to looking for a ''plateau'' as a function of Borel mass in the conventional analysis, but has the advantage of not restricting the analysis regime in Borel space to the valid regimes common to *both* two-point and three-point correlation functions.

IV. RESULTS AND DISCUSSIONS

We have analyzed all of the sum rules for the entire decuplet family. We confirmed the three relations among magnetic moments as extended from Eqs. (34) to (36). So we will only present results for seven members. Valid sum rules were identified using the criteria discussed earlier. The results are given in three tables: Tables I–III. The corresponding overlap plots are given in seven figures: Figs. 3–9. These plots show how well a sum rule performs in the entire Borel region. Such information is absent in the tables. From the results, the following observations are in order.

In general, more chiral-even sum rules are valid than chiral-odd ones. This is consistent with previous findings for

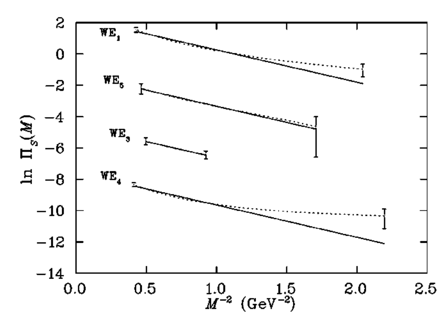

FIG. 3. Overlap plots of the valid QCD sum rules for the Δ^{++} magnetic moment. Each sum rule is searched independently. The solid line corresponds to the ground state contribution, the dotted line the rest of the contributions (OPE minus continuum minus transition). The error bars are only shown at the two ends for clarity. From top down, the sum rules are arranged by magnitudes of μ_B extracted from them. For better viewing, the curves for each sum rule are shifted downward by 3 units relatively to the previous one.

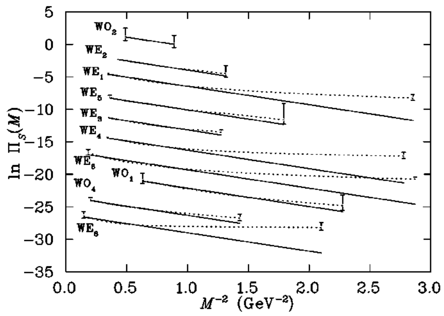

FIG. 4. Similar to Fig. 3, but for Ω^-.

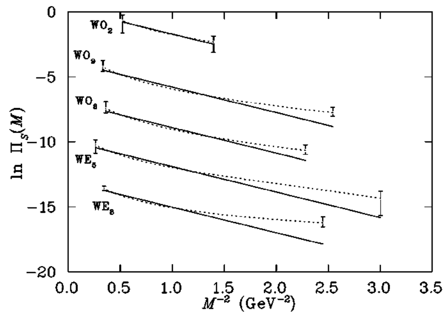

FIG. 6. Similar to Fig. 3, but for Σ^{*0}.

the octet baryon magnetic moments. It was argued in Ref. [2] that the interval of dimensions (not counting the dimension of $F_{\mu\nu}$) in the chiral-even sum rules (0–8) is larger than that in the chiral-odd sum rules (1–7). Indeed, more chiral-even sum rules (WE_2, WE_4, WE_5, WE_6, WE_8, WE_9) have power corrections up to $1/M^4$ than chiral-odd ones (WO_2, WO_4, WO_8, WO_9). Because of the additional terms in the OPE series, these sum rules are expected to be more reliable than the other sum rules. The situation here is almost opposite to that for the two-point functions [13]. It was pointed out in Ref. [30] that chiral-odd sum rules are more reliable than chiral-even sum rules for baryon two-point functions. The reason could be traced to the fact that even and odd parity excited states contribute with different signs. In the three-point functions, however, the statement is no longer valid due to the appearance of transitions and vacuum susceptibilities. Therefore, caution should be used when applying the chirality argument to determine the reliability of a sum rule in three-point functions. In addition, numerical analysis showed that the sum rules from WE_1, WE_3, WO_1 are valid for the standard input parameter set, despite the absence of $1/M^4$ terms. We have varied the central values of the input parameters and discovered that sum rules that were valid for one set of input parameters became invalid for another, and *vice versa*. Thus the situation with three-point functions is more complicated. Our experience is that each sum rule should be examined individually in order to find out its reliability.

It turns out that for most of the valid sum rules, a full search was unsuccessful, except for three sum rules: WE_5, WE_6 and WE_8 for Σ^{*+}. Of the three, only WE_6 returned a continuum threshold with reasonable error. The other two returned it with large errors. The large errors indicate that the sum rules are not very stable: They contain barely enough information to completely resolve the spectral parameters. The important point is that the results with the continuum threshold searched or not are almost the same. This suggests that fixing it to that of the corresponding two-point function seems a good approximation.

It is gratifying to observe that the valid sum rules for most members give consistent predictions for the magnetic moments in terms of the sign, except for Σ^{*0} and Ξ^{*0} whose magnitudes are small. The magnitudes for the magnetic moments are consistent within errors for the most part, with only a few exceptions. The performances of the sum rules are quite different within each member. This is best displayed in the overlap plots. In some sum rules, the overlap is poor, as evidenced by the deviation from linearity (dotted lines). It signals poor OPE convergence in these sum rules. As expected, the deviation is more severe in the lower end of the Borel region where nonperturbative physics dominates. These sum rules will more likely suffer from uncertainties associated with the selection of the Borel window. As a result, the spectral parameters extracted from them are less reliable. One way to alleviate the problem is to increase the lower end of the Borel window to values where the overlap is good, even to extend the upper end to ensure the existence of a window. This was not attempted in this work because we feel the results obtained this way are somewhat misleading. The reason is that the sum rules in these windows will be dominated mostly by perturbative physics. It is common knowledge that if one goes deep enough into the Borel space, one can always find a match in a QCD sum rule. But such practice is against the philosophy of the QCD sum rule approach, which relies upon power corrections to resolve the spectral properties. Therefore, some standard is necessary to emphasize such physics, and we feel the 10%–50% criteria adopted here are a reasonable choice.

Based on the quality of the overlap, the broadness of the Borel window and its reach into the lower end, the size of the

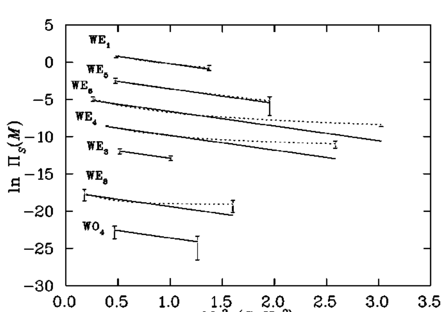

FIG. 5. Similar to Fig. 3, but for Σ^{*+}.

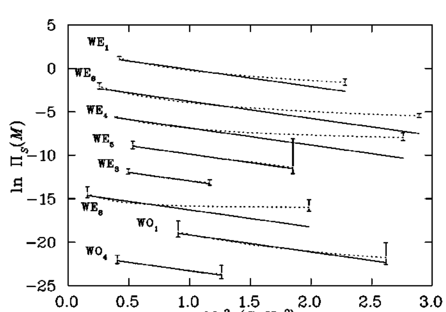

FIG. 7. Similar to Fig. 3, but for Σ^{*-}.

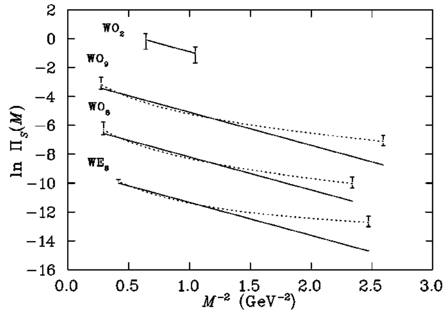

FIG. 8. Similar to Fig. 3, but for Ξ^{*0}.

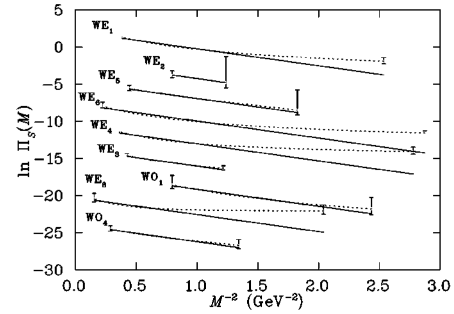

FIG. 9. Similar to Fig. 3, but for Ξ^{*-}.

continuum contribution, and the standard QCD input parameter set, we designate one sum rule for each member as the most favorable. They are WE_5 for Δ^{++}, WE_5 for Σ^{*+}, WO_8 for Σ^{*0}, WE_5 for Σ^{*-}, WO_8 for Ξ^{*0}, WE_5 for Ξ^{*-}, and WE_5 for Ω^-. The selection is undoubtedly subjective. The reader may find a different set that has equal or comparable performance. We want to stress that such a selection depends on the QCD input parameters. It is possible that the ones selected here become invalid for a different set of input parameters, in which case a new set should be selected.

Relatively large errors were found in the valid sum rules using the standard QCD input parameter set: from 50% to 100% in the magnetic moments. But in most cases, the sign and order of magnitudes are unambiguously predicted when compared to the measured values. The situation is similar to a previous finding on g_A [31]. To gain some idea about how the uncertainties depend on the input, we also analyzed the sum rules by adjusting the error estimates individually. We found large sensitivities to the quark condensate magnetic susceptibility χ. In fact, most of the errors came from the uncertainties in χ. We also tried with reduced error estimates on all the QCD input parameters: 10% relative errors uniformly. It leads to about 30% accuracy on the magnetic moments in the favorable sum rules. Further improvement of the accuracy by reducing the errors in the input is beyond the capability of these sum rules as the χ^2/N_{DF} becomes unacceptably large, signaling an internal inconsistency of the sum rules. For that purpose, one would have to resort to finding sum rules that have better convergence properties and depend less critically on the poorly known χ.

To get a different perspective on how the spectral parameters depend on the input parameters, we study correlations among the parameters by way of scatter plots. In the Monte Carlo analysis, all the parameters are correlated. Therefore, one can study the correlations between any two parameters by looking at their scatter plots. Such plots are useful in revealing how a particular sum rule resolves the spectral properties. We have examined numerous such plots. Here we focus on the favorable sum rules as selected earlier. To conserve space, we only give two examples. Figure 10 shows the scatter plot for correlations between Ω^- magnetic moment and the QCD input parameters for the sum rule from WE_5. Figure 11 shows a similar plot for Σ^{*0} and the sum rule from WO_8. Perhaps the most interesting feature is the strong correlations with χ in both sum rules. This is the reason for the large sensitivities to this parameter as alluded to earlier. Precise determination of χ is crucial for keeping the uncertainties in the spectral parameters under control.

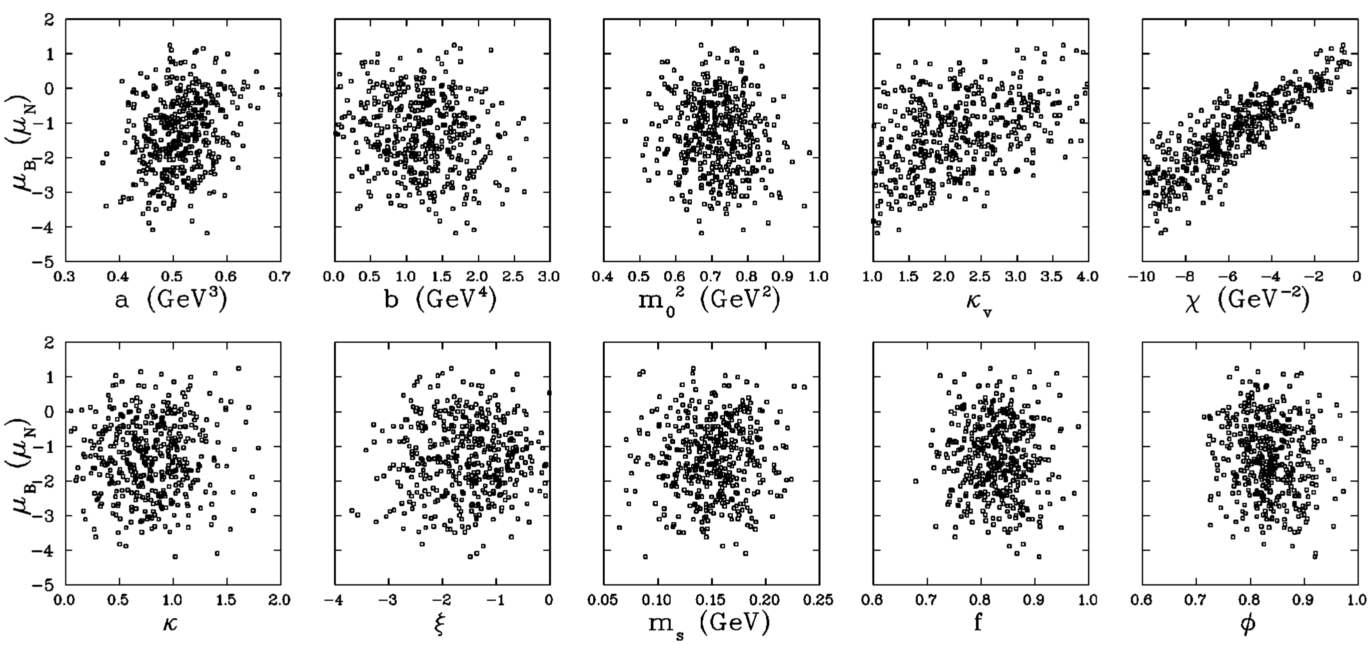

FIG. 10. Scatter plots showing correlations between the magnetic moment of Ω^- and the standard QCD input parameters for the sum rule from WE_5. The result is drawn from 430 QCD parameters sets.

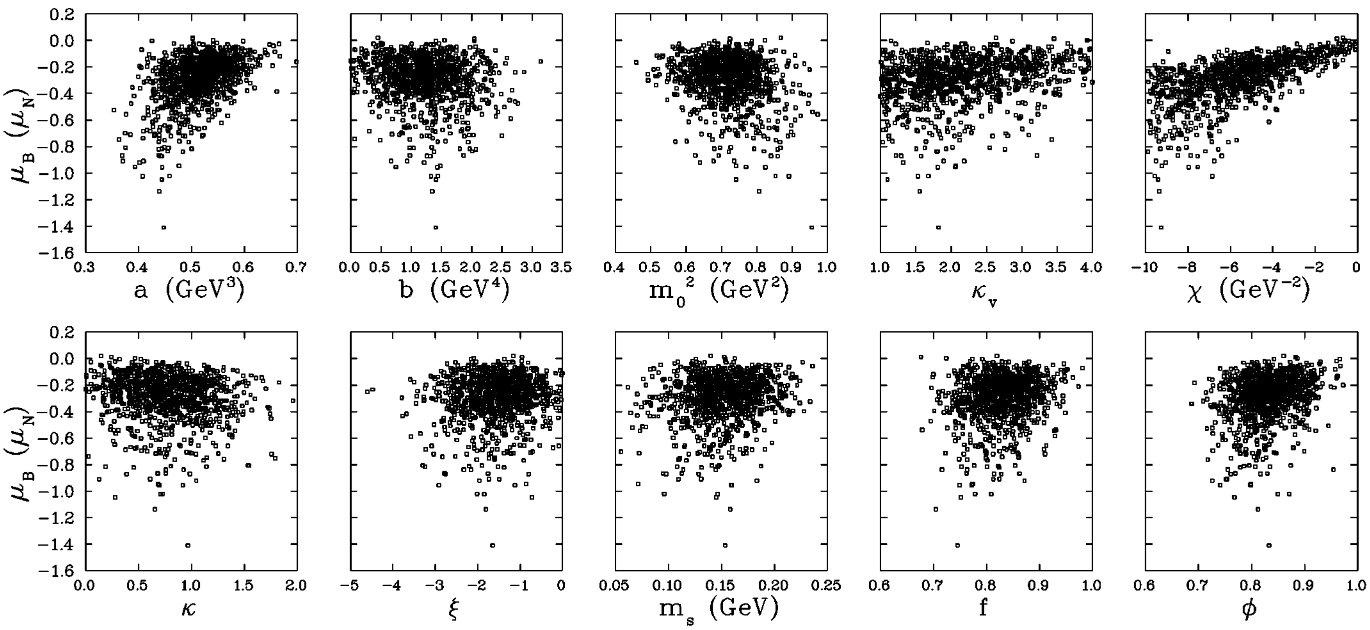

FIG. 11. Similar to Fig. 10, but for Σ^{*0} and the sum rule from WO_8. The result is drawn from 1000 QCD parameters sets.

Other charged members [all use sum rules (SR's) from WE_5] display qualitatively the same patterns for parameters other than χ and the factorization violation parameter κ_v. For χ, positively charged members (Δ^{++} and Σ^{*+}) show negative correlations. The opposite is true for negatively charged members (Ω^-, Σ^{*-} and Ξ^{*-}): They show positive correlations with χ. The patterns for κ_v essentially follow those for χ, although the correlations are weaker. The correlation patterns for Ξ^{*0} are qualitatively the same as those for Σ^{*0}.

Table IV shows a comparison of the magnetic moments from various calculations and existing experimental data. The results with 10% errors from the QCD sum rule method are used in the comparison. Note that the central values are slightly different from those in Tables I–III where conservative uncertainties were used. The reason is that the resultant distributions vary with input errors and are not Gaussian in this case. In such event the median and the average of the asymmetric errors are quoted. The QCDSR results are consistent with data, although the central value for Ω^- is slightly underestimated. We would like to point out that it is possible to reproduce the central value for Ω^- (using it as input) by fine-tuning of the susceptibility χ alone, given the sensitivity to this parameter and the large freedom at the present time on its value. However, we feel that such an attempt is not very meaningful given the accuracy of the method. A more meaningful practice would be to reanalyze the octet baryon magnetic moments by the same method as employed here, obtain a best fit on the the susceptibilities using their accurately measured values, and then use them to predict the decuplet magnetic moments. It would yield valuable information on these important quantities and on the consistency of the approach. From the table, it is fair to say that the QCDSR approach is at least competitive with other calculations. The results came about from a rather different perspective: the nonperturbative structure of the QCD vacuum. The results from various calculations roughly agree, except for the charge-neutral resonances Δ^0, Σ^{*0}, and Ξ^{*0} for which both the sign and the magnitude vary. It would be helpful to have experimental information on the other members of the decuplet, although such measurements appear difficult.

TABLE IV. Comparisons of decuplet baryon magnetic moments from various calculations: this work (QCDSR), lattice QCD (Latt) [15], chiral perturbation theory (χPT) [16], light-cone relativistic quark model (RQM) [19], non-relativistic quark model (NQM) [18], chiral quark-soliton model (χQSM) [24]. All results are in units of nuclear magnetons.

Baryon	Expt.	QCDSR	Latt	χPT	RQM	NQM	χQSM
Δ^{++}	4.5 ± 1.0	4.13 ± 1.30	4.91 ± 0.61	4.0 ± 0.4	4.76	5.56	4.73
Δ^{+}		2.07 ± 0.65	2.46 ± 0.31	2.1 ± 0.2	2.38	2.73	2.19
Δ^{0}	≈0	0.00	0.00	-0.17 ± 0.04	0.00	-0.09	-0.35
Δ^{-}		-2.07 ± 0.65	-2.46 ± 0.31	-2.25 ± 0.25	-2.38	-2.92	-2.90
Σ^{*+}		2.13 ± 0.82	2.55 ± 0.26	2.0 ± 0.2	1.82	3.09	2.52
Σ^{*0}		-0.32 ± 0.15	0.27 ± 0.05	-0.07 ± 0.02	-0.27	0.27	-0.08
Σ^{*-}		-1.66 ± 0.73	-2.02 ± 0.18	-2.2 ± 0.2	-2.36	-2.56	-2.69
Ξ^{*0}		-0.69 ± 0.29	0.46 ± 0.07	0.1 ± 0.04	-0.60	0.63	0.19
Ξ^{*-}		-1.51 ± 0.52	-1.68 ± 0.12	-2.0 ± 0.2	-2.41	-2.2	-2.48
Ω^{-}	-2.024 ± 0.056	-1.49 ± 0.45	-1.40 ± 0.10	input	-2.48	-1.84	-2.27

V. CONCLUSION

It has been demonstrated in this work that the magnetic moments of decuplet baryons can be successfully computed in the QCD sum rule approach. A complete set of QCD sum rules are derived using the external field technique. They are analyzed extensively with a comprehensive Monte Carlo based procedure which, in our opinion, provides the most realistic estimates of the uncertainties present in the approach.

Valid sum rules are identified using criteria established by OPE convergence and ground-state dominance. For each member, usually several sum rules are valid, but not all of them perform equally well. This was best displayed by the overlap plots. Some have large deviations in the lower end of the Borel window, signaling insufficient convergence in the OPE. These sum rules are less reliable. Based on overall performance, a favorable sum rule was selected for each member. They are WE_5 for charged members, WO_8 for charge-neutral members. We also found the following relations between the magnetic moments: $\mu_{\Delta^+}=\frac{1}{2}\mu_{\Delta^{++}}$, $\mu_{\Delta^0}=0$, and $\mu_{\Delta^-}=-\mu_{\Delta^+}$, and approximately $\mu_{\Sigma^{*0}}=\frac{1}{2}(\mu_{\Sigma^{*r+}}+\mu_{\Sigma^{*-}})$.

Using conservative estimates of the QCD input parameters, the uncertainties in the extracted magnetic moments are found relatively large as compared to the two-point functions. We found that the results are sensitive to the quark condensate magnetic susceptibility χ. In fact, most of the uncertainties could be attributed to χ. A better estimate of this parameter is clearly needed. By varying the uncertainty estimates in the input parameters, we found that a 30% accuracy can be achieved with the designated sum rules if the QCD input parameters could be determined to the 10% accuracy level.

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APPENDIX: QCD SUM RULES FOR MAGNETIC MOMENTS OF DECUPLET BARYONS

Here we give the complete set of QCD sum rules derived in this work. For each member, there are 18 sum rules, 9 chiral even, 9 chiral odd. It turns out that the sum rules from WE_6 and WE_7 are degenerate, and so are those from WO_4 and WO_5. So the number of independent sum rules is 16 for each member. The total number for the entire decuplet family is 160. They are given in the following in a highly compact form. The explanation on how to obtain a sum rule for a particular member is discussed in the main text.

The sum rule from WE_1:

$$c_1L^{4/27}E_1M^4+c_2m_s\chi aL^{-12/27}E_0M^2+c_3bL^{4/27}+c_4\chi a^2L^{12/27}+(c_5+c_6)m_saL^{4/27}+(c_7+c_8)a^2L^{28/27}\frac{1}{M^2}$$
$$+c_9\chi m_0^2a^2L^{-2/27}\frac{1}{M^2}+c_{10}m_sm_0^2aL^{-10/27}\frac{1}{M^2}=\frac{1}{2}\tilde{\lambda}_B^2\left(\frac{\mu_B}{M^2}+A\right)e^{-M_B^2/M^2}, \tag{A1}$$

where the coefficients for Σ^{*+} are

$$c_1=\tfrac{1}{8}(e_s+2e_u),\quad c_2=\tfrac{-7}{18}(e_sf\phi+2e_u),$$
$$c_3=\tfrac{-1}{72}(e_s+2e_u),\quad c_4=\tfrac{-1}{9}(e_sf\phi+e_uf+e_u),$$
$$c_5=\tfrac{1}{18}(-2e_sf+9e_s+9e_uf+5e_u),\quad c_6=\tfrac{-1}{18}(e_sf\phi+2e_u)(7\kappa+\xi),$$
$$c_7=\tfrac{1}{27}(-e_sf+3e_s+5e_uf-e_u)\kappa_v,\quad c_8=\tfrac{-1}{54}(e_sf\phi+e_uf+e_u)(7\kappa+\xi),$$
$$c_9=\tfrac{7}{216}(e_sf\phi+e_uf+e_u),\quad c_{10}=\tfrac{-5}{72}(e_s+e_uf+e_u), \tag{A2}$$

for Σ^{*0},

$$c_1=\tfrac{1}{8}(e_s+e_u+e_d),\quad c_2=\tfrac{-7}{18}(e_sf\phi+e_u+e_d),$$
$$c_3=-1/72(e_s+e_u+e_d),\quad c_4=\tfrac{-1}{9}(e_sf\phi+(e_u+e_d)(f+1)/2),$$
$$c_5=\tfrac{1}{18}(-2e_sf+9e_s+(e_u+e_d)(9f+5)/2),\quad c_6=\tfrac{-1}{18}(e_sf\phi+e_u+e_d)(7\kappa+\xi),$$
$$c_7=\tfrac{1}{27}(-e_sf+3e_s+(e_u+e_d)(5f-1)/2)\kappa_v,\quad c_8=\tfrac{-1}{54}(e_sf\phi+(e_u+e_d)(f+1)/2)(7\kappa+\xi),$$
$$c_9=\tfrac{7}{216}(e_sf\phi+(e_u+e_d)(f+1)/2),\quad c_{10}=\tfrac{-5}{72}(e_s+(e_u+e_d)(f+1)/2), \tag{A3}$$

for Ξ^{*0},

$$c_1=\tfrac{1}{8}(2e_s+e_u),\quad c_2=\tfrac{-7}{18}(2e_sf\phi+e_u),$$
$$c_3=\tfrac{-1}{72}(2e_s+e_u),\quad c_4=\tfrac{-1}{9}f(e_sf\phi+e_s\phi+e_u),$$
$$c_5=\tfrac{1}{18}(5e_sf+9e_s+9e_uf-2e_u),\quad c_6=\tfrac{-1}{18}(2e_sf\phi+e_u)(7\kappa+\xi),$$
$$c_7=\tfrac{1}{27}f(-e_sf+5e_s+3e_uf-e_u)\kappa_v,\quad c_8=\tfrac{-1}{54}f(e_sf\phi+e_s\phi+e_u)(7\kappa+\xi),$$
$$c_9=\tfrac{7}{216}f(e_sf\phi+e_s\phi+e_u),\quad c_{10}=\tfrac{-5}{72}(e_sf+e_uf+e_s),\tag{A4}$$

for Ω^-,

$$c_1=\tfrac{3}{8}e_s,\quad c_2=\tfrac{-7}{6}e_sf\phi,$$
$$c_3=\tfrac{-1}{24}e_s,\quad c_4=\tfrac{-1}{3}e_sf^2\phi,$$
$$c_5=\tfrac{7}{6}e_sf,\quad c_6=\tfrac{-1}{6}e_sf\phi(7\kappa+\xi),$$
$$c_7=\tfrac{2}{9}e_sf^2\kappa_v,\quad c_8=\tfrac{-1}{17}e_sf^2\phi(7\kappa+\xi),$$
$$c_9=\tfrac{7}{72}e_sf^2\phi,\quad c_{10}=\tfrac{-5}{24}e_sf.\tag{A5}$$

The sum rule from WE_2:

$$c_1L^{4/27}E_0M^2+c_2m_s\chi aL^{-12/27}+c_3bL^{4/27}\frac{1}{M^2}+c_4m_saL^{4/27}\frac{1}{M^2}+c_5m_sm_0^2aL^{-10/27}\frac{1}{M^4}=\frac{-1}{9}\tilde{\lambda}_B^2\left(\frac{\mu_B}{M_B^2M^2}+A\right)e^{-M_B^2/M^2},\tag{A6}$$

where the coefficients for Σ^{*+} are

$$c_1=\tfrac{-1}{12}(e_s+2e_u),\quad c_2=\tfrac{-1}{9}(e_sf\phi+2e_u),\quad c_3=\tfrac{-1}{48}(e_s+2e_u),\quad c_4=\tfrac{1}{3}(e_s+e_uf+e_u),$$
$$c_5=\tfrac{1}{18}(e_s+e_uf+e_u),\tag{A7}$$

for Ξ^{*0},

$$c_1=\tfrac{-1}{12}(2e_s+e_u),\quad c_2=\tfrac{-1}{9}(2e_sf\phi+e_u),\quad c_3=\tfrac{-1}{48}(2e_s+e_u),\quad c_4=\tfrac{1}{3}(e_sf+e_uf+e_s),$$
$$c_5=\tfrac{1}{18}(e_sf+e_uf+e_s).\tag{A8}$$

The sum rule from WE_3:

$$c_1L^{4/27}E_1M^4+c_2m_s\chi aL^{-12/27}E_0M^2+c_3bL^{4/27}+c_4\chi a^2L^{12/27}+(c_5+c_6)m_saL^{4/27}+(c_7+c_8)a^2L^{28/27}\frac{1}{M^2}$$
$$+c_9\chi m_0^2a^2L^{-2/27}\frac{1}{M^2}+c_{10}m_sm_0^2aL^{-10/27}\frac{1}{M^2}=\frac{-7}{18}\tilde{\lambda}_B^2\left(\frac{\mu_B}{M^2}+A\right)e^{-M_B^2/M^2},\tag{A9}$$

where the coefficients for Σ^{*+} are

$$c_1=\tfrac{-1}{24}(e_s+2e_u),\quad c_2=\tfrac{5}{24}(e_sf\phi+2e_u),$$
$$c_3=\tfrac{1}{576}(e_s+2e_u),\quad c_4=\tfrac{1}{18}(e_sf\phi+e_uf_s+e_u),$$
$$c_5=\tfrac{1}{36}(2e_sf-3e_s-3e_uf+e_u),\quad c_6=\tfrac{1}{36}(e_sf\phi+2e_u)(4\kappa+\xi),$$
$$c_7=\tfrac{1}{54}(e_sf-3e_s/2-2e_uf+e_u)\kappa_v,\quad c_8=\tfrac{1}{108}(e_sf\phi+e_uf+e_u)(4\kappa+\xi),$$
$$c_9=-7/432(e_sf\phi+e_uf+e_u),\quad c_{10}=\tfrac{1}{48}(e_s+e_uf+e_u),\tag{A10}$$

for Ξ^{*0},

$$c_1=-\tfrac{1}{24}(2e_s+e_u),\quad c_2=\tfrac{5}{24}(2e_sf\phi+e_u),$$

$$c_3=\tfrac{1}{576}(2e_s+e_u),\quad c_4=\tfrac{1}{18}f(e_sf\phi+e_s\phi+e_u),$$

$$c_5=\tfrac{1}{36}(e_sf-3e_s-3e_uf+2e_u),\quad c_6=\tfrac{1}{36}(2e_sf\phi+e_u)(4\kappa+\xi),$$

$$c_7=\tfrac{1}{54}f(e_sf-2e_s-3e_uf/2+e_u)\kappa_v,\quad c_8=\tfrac{1}{108}f(e_sf\phi+e_s\phi+e_u)(4\kappa+\xi),$$

$$c_9=-\tfrac{7}{432}f(e_sf\phi+e_s\phi+e_u),\quad c_{10}=\tfrac{1}{48}(e_sf+e_uf+e_s). \tag{A11}$$

The sum rule from WE_4:

$$c_1L^{4/27}E_1M^4+c_2m_s\chi aL^{-12/27}E_0M^2+c_3bL^{4/27}+(c_4+c_5)m_saL^{4/27}+(c_6+c_7)a^2L^{28/27}\frac{1}{M^2}+c_8m_0^2a^2L^{14/27}\frac{1}{M^4}$$
$$=\frac{7}{18}\tilde{\lambda}_B^2\left(\frac{\mu_B}{M^2}+A\right)e^{-M_B^2/M^2}, \tag{A12}$$

where the coefficients for Σ^{*+} are

$$c_1=\tfrac{1}{24}(e_s+2e_u),\quad c_2=\tfrac{-5}{36}(e_sf\phi+2e_u),$$

$$c_3=\tfrac{-1}{96}-1/96(e_s+2e_u),\quad c_4=\tfrac{1}{18}(e_sf+6e_s+6e_uf+8e_u),$$

$$c_5=\tfrac{-1}{72}(e_sf\phi+2e_u)(10\kappa+\xi),\quad c_6=\tfrac{1}{27}(e_sf+3e_s/2+4e_uf+e_u)\kappa_v,$$

$$c_7=\tfrac{-1}{108}(e_sf\phi+e_uf+e_u)(4\kappa+\xi),\quad c_8=\tfrac{-7}{648}(e_sf+e_uf+e_u), \tag{A13}$$

for Ξ^{*0},

$$c_1=\tfrac{1}{24}(2e_s+e_u),\quad c_2=\tfrac{-5}{36}(2e_sf\phi+e_u),$$

$$c_3=\tfrac{-1}{96}(2e_s+e_u),\quad c_4=\tfrac{1}{18}(8e_sf+6e_s+6e_uf+e_u),$$

$$c_5=\tfrac{-1}{72}(2e_sf\phi+e_u)(10\kappa+\xi),\quad c_6=\tfrac{1}{27}f(e_sf+4e_s+3e_uf/2+e_u)\kappa_v,$$

$$c_7=\tfrac{1}{108}f(e_sf\phi+e_s\phi+e_u)(4\kappa+\xi),\quad c_8=\tfrac{-7}{648}f(e_sf+e_s+e_u). \tag{A14}$$

The sum rule from WE_5:

$$c_1L^{4/27}E_1M^4+c_2m_s\chi aL^{-12/27}E_0M^2+c_3bL^{4/27}+(c_4+c_5)m_saL^{4/27}+c_6\chi a^2L^{12/27}+(c_7+c_8)a^2L^{28/27}\frac{1}{M^2}$$
$$+c_9\chi m_0^2a^2L^{-2/27}\frac{1}{M^2}+c_{10}m_sm_0^2aL^{-10/27}\frac{1}{M^2}+c_{11}m_0^2a^2L^{14/27}\frac{1}{M^4}=\frac{-7}{18}\tilde{\lambda}_B^2\left(\frac{\mu_B}{M^2}+A\right)e^{-M_B^2/M^2}, \tag{A15}$$

where the coefficients for Σ^{*+} are

$$c_1=\tfrac{-1}{24}(e_s+2e_u),\quad c_2=\tfrac{5}{18}(e_sf\phi+2e_u),$$

$$c_3=\tfrac{-1}{144}(e_s+2e_u),\quad c_4=\tfrac{1}{6}(e_sf+e_s+e_uf+3e_u),$$

$$c_5=\tfrac{1}{24}(e_sf\phi+2e_u)(2\kappa+\xi),\quad c_6=\tfrac{1}{9}(e_sf\phi+e_uf+e_u),$$

$$c_7=\tfrac{2}{27}(e_sf+e_uf+e_u)\kappa_v,\quad c_8=\tfrac{1}{108}(e_sf\phi+e_uf+e_u)(4\kappa+\xi),$$

$$c_9 = \tfrac{-7}{216}\,(e_s f\phi + e_u f + e_u), \quad c_{10} = \tfrac{1}{24}\,(e_s + e_u f + e_u),$$

$$c_{11} = \tfrac{7}{648}\,(e_s f + e_u f + e_u), \tag{A16}$$

for Ξ^{*0},

$$c_1 = \tfrac{-}{24}\,(2e_s + e_u), \quad c_2 = \tfrac{5}{18}\,(2e_s f\phi + e_u),$$

$$c_3 = \tfrac{-1}{144}\,(2e_s + e_u), \quad c_4 = \tfrac{1}{6}\,(3e_s f + e_s + e_u f + e_u),$$

$$c_5 = \tfrac{1}{24}\,(2e_s f\phi + e_u)(2\kappa + \xi), \quad c_6 = \tfrac{1}{9}\,f(e_s f\phi + e_s\phi + e_u),$$

$$c_7 = \tfrac{2}{27}\,f(e_s f + e_s + e_u)\kappa_v\,, \quad c_8 = \tfrac{1}{108}\,f(e_s f\phi + e_s\phi + e_u)(4\kappa + \xi),$$

$$c_9 = \tfrac{-7}{216}\,f(e_s f\phi + e_s\phi + e_u), \quad c_{10} = \tfrac{1}{24}\,(e_s f + e_u f + e_s),$$

$$c_{11} = \tfrac{-7}{648}\,f(e_s f + e_s + e_u). \tag{A17}$$

The sum rule from WE_6:

$$c_1 m_s \chi a L^{-12/27} E_0 M^2 + (c_2 + c_3) m_s a L^{4/27} + (c_4 + c_5) a^2 L^{28/27} \frac{1}{M^2} + c_6 m_s m_0^2 a L^{-10/27} \frac{1}{M^2} + c_7 m_0^2 a^2 L^{14/27} \frac{1}{M^4}$$

$$= \frac{2}{3}\,\tilde{\lambda}_B^2 \left(\frac{\mu_B}{M^2} + A \right) e^{-M_B^2/M^2}, \tag{A18}$$

where the coefficients for Σ^{*+} are

$$c_1 = \tfrac{-2}{3}\,(e_s f\phi + 2e_u), \quad c_2 = \tfrac{1}{18}\,(e_s f + 2e_u),$$

$$c_3 = \tfrac{1}{72}\,(e_s f\phi + 2e_u)(2\kappa - \xi), \quad c_4 = \tfrac{1}{27}\,(e_s f + e_u f + e_u)\kappa_v\,,$$

$$c_5 = \tfrac{1}{108}\,(e_s f\phi + e_u f + e_u)(2\kappa - \xi), \quad c_6 = \tfrac{1}{36}\,(e_s + e_u f + e_u),$$

$$c_7 = \tfrac{-7}{648}\,(e_s f + e_u f + e_u), \tag{A19}$$

for Ξ^{*0},

$$c_1 = \tfrac{-2}{3}\,(2e_s f\phi + e_u), \quad c_2 = \tfrac{1}{18}\,(2e_s f + e_u),$$

$$c_3 = \tfrac{1}{72}\,(2e_s f\phi + e_u)(2\kappa - \xi), \quad c_4 = \tfrac{1}{27}\,f(e_s f + e_s + e_u)\kappa_v\,,$$

$$c_5 = \tfrac{1}{108}\,f(e_s f\phi + e_s\phi + e_u)(2\kappa - \xi), \quad c_6 = \tfrac{1}{36}\,(e_s f + e_u f + e_s),$$

$$c_7 = \tfrac{-7}{648}\,f(e_s f + e_s + e_u). \tag{A20}$$

The sum rule from WE_7 is identical to that from WE_6 after multiplying an overall sign on both sides.
The sum rule from WE_8:

$$c_1 m_s \chi a L^{-12/27} E_0 M^2 + c_2 b L^{4/27} + c_3 \chi a^2 L^{12/27} + (c_4 + c_5) m_s a L^{4/27} + (c_6 + c_7) a^2 L^{28/27} \frac{1}{M^2} + c_8 \chi m_0^2 a^2 L^{-2/27} \frac{1}{M^2}$$

$$+ c_9 m_s m_0^2 a L^{-10/27} \frac{1}{M^2} + c_{10} m_0^2 a^2 L^{14/27} \frac{1}{M^4} = \frac{-2}{3}\,\tilde{\lambda}_B^2 \left(\frac{\mu_B}{M^2} + A \right) e^{-M_B^2/M^2}, \tag{A21}$$

where the coefficients for Σ^{*+} are

$$c_1 = \tfrac{1}{3}\,(e_s f\phi + 2e_u), \quad c_2 = \tfrac{5}{144}\,(e_s + 2e_u),$$

$$c_3 = \tfrac{-2}{9}(e_s f\phi + e_u f + e_u), \quad c_4 = \tfrac{-1}{9}\,(5e_s f + 12e_s + 12e_u f + 22e_u),$$

$$c_5 = \tfrac{1}{36}\,(e_s f\phi + 2e_u)(14\kappa - \xi), \quad c_6 = \tfrac{-1}{27}\,(8e_s f + 6e_s + 20e_u f + 8e_u)\kappa_v\,,$$

$$c_7 = \tfrac{1}{54}\,(e_s f\phi + e_u f + e_u)(4\kappa + \xi), \quad c_8 = \tfrac{7}{108}\,(e_s f\phi + e_u f + e_u),$$

$$c_9=\frac{1}{36}(e_s+e_uf+e_u),\quad c_{10}=\frac{7}{108}(e_sf+e_uf+e_u), \tag{A22}$$

for Ξ^{*0},

$$c_1=\frac{1}{3}(2e_sf\phi+e_u),\quad c_2=\frac{5}{144}(2e_s+e_u),$$

$$c_3=\frac{-2}{9}f(e_sf\phi+e_s\phi+e_u),\quad c_4=\frac{-1}{9}(22e_sf+12e_s+12e_uf+5e_u),$$

$$c_5=\frac{1}{36}(2e_sf\phi+e_u)(14\kappa-\xi),\quad c_6=\frac{-1}{27}f(8e_sf+20e_s+6e_uf+8e_u)\kappa_v,$$

$$c_7=\frac{1}{54}f(e_sf\phi+e_s\phi+e_u)(4\kappa+\xi),\quad c_8=\frac{7}{108}f(e_sf\phi+e_s\phi+e_u),$$

$$c_9=\frac{1}{36}(e_sf+e_uf+e_s),\quad c_{10}=\frac{7}{108}f(e_sf+e_s+e_u). \tag{A23}$$

The sum rule from WE_9 has the same form as that from WE_8, only with different c_i:

$$c_1=(e_sf\phi+2e_u),\quad c_2=\frac{-5}{144}(e_s+2e_u),$$

$$c_3=\frac{2}{9}(e_sf\phi+e_uf+e_u),\quad c_4=\frac{1}{9}(4e_sf+12e_s+12e_uf+20e_u),$$

$$c_5=\frac{1}{18}(e_sf\phi+2e_u)(-8\kappa+\xi),\quad c_6=\frac{1}{9}(2e_sf+2e_s+6e_uf+2e_u)\kappa_v,$$

$$c_7=\frac{-1}{9}(e_sf\phi+e_uf+e_u)\kappa,\quad c_8=\frac{-7}{108}(e_sf\phi+e_uf+e_u),$$

$$c_9=\frac{-1}{12}(e_s+e_uf+e_u),\quad c_{10}=\frac{-7}{162}(e_sf+e_uf+e_u), \tag{A24}$$

for Ξ^{*0},

$$c_1=(2e_sf\phi+e_u),\quad c_2=\frac{-5}{144}(2e_s+e_u),$$

$$c_3=\frac{2}{9}f(e_sf\phi+e_s\phi+e_u),\quad c_4=\frac{1}{9}(20e_sf+12e_s+12e_uf+4e_u),$$

$$c_5=\frac{1}{18}(2e_sf\phi+e_u)(-8\kappa+\xi),\quad c_6=\frac{1}{9}f(2e_sf+6e_s+2e_uf+2e_u)\kappa_v,$$

$$c_7=\frac{-1}{9}f(e_sf\phi+e_s\phi+e_u)\kappa,\quad c_8=\frac{-7}{108}f(e_sf\phi+e_s\phi+e_u),$$

$$c_9=\frac{-1}{12}(e_sf+e_uf+e_s),\quad c_{10}=\frac{-7}{162}f(e_sf+e_s+e_u). \tag{A25}$$

The sum rule from WO_1:

$$c_1\chi aE_1M^4+c_2m_sL^{16/27}E_1M^4+(c_3+c_4)aL^{16/27}E_0M^2+c_5m_0^2aL^{2/27}+c_6\chi ab+c_7m_s\chi a^2+c_8abL^{16/27}\frac{1}{M^2}$$
$$+(c_9+c_{10})m_sa^2L^{16/27}\frac{1}{M^2}=\frac{1}{2}\tilde{\lambda}_B^2\left(\frac{\mu_BM_B}{M^2}+A\right)e^{-M_B^2/M^2}, \tag{A26}$$

where the coefficients for Σ^{*+} are

$$c_1=\frac{-1}{6}(e_sf\phi+2e_u),\quad c_2=\frac{1}{6}(e_s+2e_u),$$

$$c_3=\frac{-5}{27}(e_sf+2e_u),\quad c_4=\frac{5}{432}(e_sf\phi+2e_u)(-8\kappa+7\xi),$$

$$c_5=\frac{1}{12}(e_s+e_uf+e_u),\quad c_6=\frac{1}{96}(e_sf\phi+2e_u),$$

$$c_7=\frac{-2}{9}-(e_sf\phi+e_uf+e_u),\quad c_8=\frac{-1}{216}(e_s+e_uf+e_u),$$

$$c_9=\frac{1}{27}(-2e_sf+3e_s+4e_uf-2e_u)\kappa_v,\quad c_{10}=\frac{-5}{432}(e_sf\phi+e_uf+e_u)(8\kappa+11\xi), \tag{A27}$$

for Ξ^{*0},

$$c_1=\frac{-1}{6}(2e_sf\phi+e_u),\quad c_2=\frac{1}{6}(2e_s+e_u),$$

$$c_3=\frac{-5}{27}(2e_sf+e_u),\quad c_4=\frac{5}{432}(2e_sf\phi+e_u)(-8\kappa+7\xi),$$

$$c_5=\frac{1}{12}(e_sf+e_uf+e_s),\quad c_6=\frac{1}{96}(2e_sf\phi+e_u),$$

$$c_7 = \tfrac{-2}{9} f(e_s f\phi + e_s\phi + e_u), \quad c_8 = \tfrac{-1}{216}(e_s f + e_u f + e_s),$$

$$c_9 = \tfrac{1}{27} f(-2e_s f + 4e_s + 3e_u f - 2e_u)\kappa_v, \quad c_{10} = \tfrac{-5}{432} f(e_s f\phi + e_s\phi + e_u)(8\kappa + 11\xi). \tag{A28}$$

The sum rule from WO_2:

$$c_1\chi a E_0 M^2 + (c_2 + c_3) a L^{16/27} + c_4 m_s \chi a^2 + c_5 \chi a b \frac{1}{M^2} + (c_6 + c_7) m_s a^2 L^{16/27} \frac{1}{M^4} = \frac{-1}{9}\tilde{\lambda}_B^2 \left(\frac{\mu_B}{M_B M^2} + A \right) e^{-M_B^2/M^2}, \tag{A29}$$

where the coefficients for Σ^{*+} are

$$c_1 = \tfrac{1}{9}(e_s f\phi + 2e_u), \quad c_2 = \tfrac{5}{27}(e_s f + 2e_u),$$

$$c_3 = \tfrac{1}{54}(e_s f\phi + 2e_u)(5\kappa + 2\xi), \quad c_4 = \tfrac{-2}{9}(e_s f\phi + e_u f + e_u),$$

$$c_5 = \tfrac{1}{216}(e_s f\phi + 2e_u), \quad c_6 = \tfrac{-2}{9}(e_s f + e_u f + e_u)\kappa_v,$$

$$c_7 = \tfrac{-1}{9}(e_s f\phi + e_u f + e_u)\kappa, \tag{A30}$$

for Ξ^{*0},

$$c_1 = \tfrac{1}{9}(2e_s f\phi + e_u), \quad c_2 = \tfrac{5}{27}(2e_s f + e_u),$$

$$c_3 = \tfrac{1}{54}(2e_s f\phi + e_u)(5\kappa + 2\xi), \quad c_4 = \tfrac{-2}{9} f(e_s f\phi + e_s\phi + e_u),$$

$$c_5 = \tfrac{1}{216}(2e_s f\phi + e_u), \quad c_6 = \tfrac{-2}{9} f(e_s f + e_s + e_u)\kappa_v,$$

$$c_7 = \tfrac{-1}{9} f(e_s f\phi + e_s\phi + e_u)\kappa. \tag{A31}$$

The sum rule from WO_3:

$$c_1\chi a E_1 M^4 + (c_2 + c_3) a L^{16/27} E_0 M^2 + c_4 m_0^2 a L^{2/27} + c_5 \chi a b + c_6 m_s \chi a^2 + c_7 a b L^{16/27} \frac{1}{M^2} + (c_8 + c_9) m_s a^2 L^{16/27} \frac{1}{M^2}$$
$$= \frac{-7}{18}\tilde{\lambda}_B^2 \left(\frac{\mu_B M_B}{M^2} + A \right) e^{-M_B^2/M^2}, \tag{A32}$$

where the coefficients for Σ^{*+} are

$$c_1 = \tfrac{5}{72}(e_s f\phi + 2e_u), \quad c_2 = \tfrac{1}{72}(7e_s f + 6e_s + 6e_u f + 20e_u),$$

$$c_3 = \tfrac{1}{288}(e_s f\phi + 2e_u)(2\kappa - 11\xi), \quad c_4 = \tfrac{-1}{12}(e_s + e_u f + e_u),$$

$$c_5 = \tfrac{-11}{1728}(e_s f\phi + 2e_u), \quad c_6 = \tfrac{1}{12}(e_s f\phi + e_u f + e_u),$$

$$c_7 = \tfrac{5}{1728}(e_s + e_u f + e_u), \quad c_8 = \tfrac{-1}{36}(e_s f + 3e_s + 7e_u f + e_u)\kappa_v,$$

$$c_9 = \tfrac{1}{432}(e_s f\phi + e_u f + e_u)(12\kappa + 7\xi), \tag{A33}$$

for Ξ^{*0},

$$c_1 = \tfrac{5}{72}(2e_s f\phi + e_u), \quad c_2 = \tfrac{1}{72}(20e_s f + 6e_s + 6e_u f + 7e_u),$$

$$c_3 = \tfrac{1}{288}(2e_s f\phi + e_u)(2\kappa - 11\xi), \quad c_4 = \tfrac{-1}{12}(e_s f + e_u f + e_s),$$

$$c_5 = \tfrac{-11}{1728}(2e_s f\phi + e_u), \quad c_6 = \tfrac{-1}{12} f(e_s f\phi + e_s\phi + e_u),$$

$$c_7 = \tfrac{5}{1728}(e_s f + e_u f + e_s), \quad c_8 = \tfrac{-1}{36} f(e_s f + 7e_s + 3e_u f + e_u)\kappa_v,$$

$$c_9 = \tfrac{1}{432} f(e_s f\phi + e_s\phi + e_u)(12\kappa + 7\xi). \tag{A34}$$

The sum rule from WO_4:

$$c_1 m_s L^{-8/27} E_0 M^2 + (c_2+c_3) a L^{16/27} + c_4 m_s \chi a^2 + c_5 m_0^2 a L^{2/27} \frac{1}{M^2} + c_6 \chi a b \frac{1}{M^2} + c_7 a b L^{16/27} \frac{1}{M^4} + (c_8+c_9) m_s a^2 L^{16/27} \frac{1}{M^4}$$

$$= \frac{7}{18} \tilde{\lambda}_B^2 \left(\frac{\mu_B}{M_B\, M^2} + A \right) e^{-M_B^2/M^2}, \tag{A35}$$

where the coefficients for Σ^{*+} are

$$c_1 = \tfrac{1}{6}(e_s + 2e_u), \quad c_2 = \tfrac{1}{6}(e_s + e_u f + e_u),$$

$$c_3 = \tfrac{-1}{144}(e_s f \phi + 2e_u)(12\kappa + \xi), \quad c_4 = \tfrac{1}{9}(e_s f \phi + e_u f + e_u),$$

$$c_5 = \tfrac{-1}{12}(e_s + e_u f + e_u), \quad c_6 = \tfrac{-1}{288}(e_s f \phi + 2e_u),$$

$$c_7 = \tfrac{1}{432}(e_s + e_u f + e_u), \quad c_8 = \tfrac{-1}{18}(e_s + 2e_u f\ )\kappa_v,$$

$$c_9 = \tfrac{1}{432}(e_s f \phi + e_u f + e_u)(12\kappa + \xi), \tag{A36}$$

for Ξ^{*0},

$$c_1 = \tfrac{1}{6}(2e_s + e_u), \quad c_2 = -\tfrac{1}{6}(e_s f + e_u f + e_s),$$

$$c_3 = \tfrac{-1}{144}(2e_s f \phi + e_u)(12\kappa + \xi), \quad c_4 = \tfrac{1}{9} f(e_s f \phi + e_s \phi + e_u),$$

$$c_5 = \tfrac{-1}{12}(e_s f + e_u f + e_s), \quad c_6 = \tfrac{-1}{288}(2e_s f \phi + e_u),$$

$$c_7 = \tfrac{1}{432}(e_s f + e_u f + e_s), \quad c_8 = \tfrac{-1}{18} f(2e_s + e_u f\ )\kappa_v,$$

$$c_9 = \tfrac{1}{432} f(e_s f \phi + e_s \phi + e_u)(12\kappa + \xi). \tag{A37}$$

The sum rule from WO_5 is identical to that from WO_4 after multiplying an overall sign on both sides.
The sum rule from WO_6:

$$c_1 \chi a E_1 M^4 + c_2 m_s L^{-8/27} E_1 M^4 + (c_3+c_4) a L^{16/27} E_0 M^2 + c_5 m_0^2 a L^{2/27} + c_6 \chi a b + c_7 a b L^{16/27} \frac{1}{M^2} + (c_8+c_9) m_s a^2 L^{16/27} \frac{1}{M^2}$$

$$= \frac{2}{3} \tilde{\lambda}_B^2 \left(\frac{\mu_B M_B}{M^2} + A \right) e^{-M_B^2/M^2}, \tag{A38}$$

where the coefficients for Σ^{*+} are

$$c_1 = \tfrac{-1}{9}(e_s f \phi + 2e_u), \quad c_2 = \tfrac{-1}{3}(e_s + 2e_u),$$

$$c_3 = \tfrac{-1}{54}(11e_s f + 18e_s + 18e_u f + 40e_u), \quad c_4 = \tfrac{1}{216}(e_s f \phi + 2e_u)(14\kappa + 23\xi),$$

$$c_5 = \tfrac{1}{6}(e_s + e_u f + e_u), \quad c_6 = \tfrac{5}{432}(e_s f \phi + 2e_u),$$

$$c_7 = \tfrac{-1}{432}(e_s + e_u f + e_u), \quad c_8 = \tfrac{1}{27}(5e_s f + 3e_s + 11e_u f + 5e_u)\kappa_v,$$

$$c_9 = \tfrac{1}{54}(e_s f \phi + e_u f + e_u)(2\kappa - 3\xi), \tag{A39}$$

for Ξ^{*0},

$$c_1 = \tfrac{-1}{9}(2e_s f \phi + e_u), \quad c_2 = \tfrac{-1}{3}(2e_s + e_u),$$

$$c_3 = \tfrac{-1}{54}(40e_s f + 18e_s + 18e_u f + 11e_u), \quad c_4 = \tfrac{1}{216}(2e_s f \phi + e_u)(14\kappa + 23\xi),$$

$$c_5 = \tfrac{1}{6}(e_s f + e_u f + e_s), \quad c_6 = \tfrac{5}{432}(2e_s f \phi + e_u),$$

$$c_7 = -1/432(e_s f + e_u f + e_s), \quad c_8 = \tfrac{1}{27} f(5e_s f + 11e_s + 3e_u f + 5e_u)\kappa_v,$$

$$c_9 = \tfrac{1}{54} f(e_s f \phi + e_s \phi + e_u)(2\kappa - 3\xi). \tag{A40}$$

The sum rule from WO_7 has the same form as that from WO_6 after multiplying an overall sign on both sides. They only differ in c_4 and c_9 for Σ^{*+}:

$$c_4=\tfrac{1}{216}(e_s f\phi+2e_u)(14\kappa+29\xi),\quad c_9=\tfrac{1}{10}(e_s f\phi+e_u f+e_u)(-4\kappa+5\xi), \tag{A41}$$

for Ξ^{*0},

$$c_4=\tfrac{1}{216}(2e_s f\phi+e_u)(14\kappa+29\xi),\quad c_9=\tfrac{1}{108}f(e_s f\phi+e_s\phi+e_u)(-4\kappa+5\xi). \tag{A42}$$

The sum rule from WO_8:

$$c_1\chi aE_0M^2+c_2m_sL^{-8/27}E_0M^2+(c_3+c_4)aL^{16/27}+c_5m_0^2aL^{2/27}\frac{1}{M^2}+c_6\chi ab\frac{1}{M^2}+c_7abL^{16/27}\frac{1}{M^4}+(c_8+c_9)m_sa^2L^{16/27}\frac{1}{M^4}$$
$$=\frac{-2}{3}\tilde{\lambda}_B^2\left(\frac{\mu_B}{M_B\,M^2}+A\right)e^{-M_B^2/M^2}, \tag{A43}$$

where the coefficients for Σ^{*+} are

$$c_1=\tfrac{-1}{9}(e_s f\phi+2e_u),\quad c_2=\tfrac{-1}{3}(e_s+2e_u),$$
$$c_3=\tfrac{-1}{27}(5e_s f+9e_s+9e_u f+19e_u),\quad c_4=\tfrac{1}{108}(e_s f\phi+2e_u)(8\kappa-\xi),$$
$$c_5=\tfrac{1}{6}(e_s+e_u f+e_u),\quad c_6=\tfrac{1}{432}(e_s f\phi+2e_u),$$
$$c_7=\tfrac{-1}{216}(e_s+e_u f+e_u),\quad c_8=\tfrac{1}{9}(2e_s f+e_s+4e_u f+2e_u)\kappa_v,$$
$$c_9=\tfrac{1}{108}(e_s f\phi+e_u f+e_u)(6\kappa-\xi), \tag{A44}$$

for Ξ^{*0},

$$c_1=\tfrac{-1}{9}(2e_s f\phi+e_u),\quad c_2=\tfrac{-1}{3}(2e_s+e_u),$$
$$c_3=\tfrac{-1}{27}(19e_s f+9e_s+9e_u f+5e_u),\quad c_4=\tfrac{1}{108}(2e_s f\phi+e_u)(8\kappa-\xi),$$
$$c_5=\tfrac{1}{6}(e_s f+e_u f+e_s),\quad c_6=\tfrac{1}{432}(2e_s f\phi+e_u),$$
$$c_7=\tfrac{-1}{216}(e_s f+e_u f+e_s),\quad c_8=\tfrac{1}{9}f(2e_s f+4e_s+e_u f+2e_u)\kappa_v,$$
$$c_9=\tfrac{1}{108}f(e_s f\phi+e_s\phi+e_u)(6\kappa-\xi). \tag{A45}$$

The sum rule from WO_9 has the same form as that from WO_8. They only differ in c_4 and c_9 for Σ^{*+}:

$$c_4=\tfrac{1}{27}(e_s f\phi+2e_u)(2\kappa-\xi),\quad c_9=\tfrac{1}{18}(e_s f\phi+e_u f+e_u)\kappa, \tag{A46}$$

for Ξ^{*0},

$$c_4=\tfrac{1}{27}(2e_s f\phi+e_u)(2\kappa-\xi),\quad c_9=\tfrac{1}{18}f(e_s f\phi+e_s\phi+e_u)\kappa. \tag{A47}$$

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