$\tau \to \eta(\eta')2\pi\nu$, $3\pi\nu$ and the WZW anomaly

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It is found that the branching ratio of $\tau \to \eta' 2\pi\nu$ is two orders of magnitude smaller than that of $\tau \to \eta 2\pi\nu$. The branching ratio of $\tau \to \eta\pi\pi\pi\nu$ is calculated. The theoretical result agrees with the data. It is the first time that the anomalous Wess-Zumino-Witten vertex ηaa is tested. An a_1 resonance is predicted in the final state of the three pions. A prediction of the branching ratio of $\tau \to \eta'\pi\pi\pi\nu$ is presented. [S0556-2821(98)00403-2]

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All the hadrons produced in τ hadronic decays are mesons made of light quarks, therefore, the τ mesonic decays provide a test ground for all meson theories. Chiral symmetry plays an essential role in studying τ mesonic decays [1]. The Wess-Zumino-Witten (WZW) anomalous action [2] is general and model independent. It is an important part of the meson theory. A test of the WZW action is very significant in the physics of strong interactions. Various anomalous WZW vertices were used to calculate the decay widths of mesons [3,4]. The τ mesonic decays provide a comprehensive test ground for the WZW anomaly. The abnormal vertices $\eta\rho\rho$ and $\omega\rho\pi$ derived from the WZW anomalous action were studied [5] in τ mesonic decays. In Ref. [6] we studied more abnormal τ mesonic decays. It is pointed out in Ref. [7] that the η production in τ decay is associated with an anomaly. Recently the CLEO Collaboration reported a measurement of the branching ratio of $\tau \to \eta(3h^-)\nu$ [8]:

$$B[\tau^- \to \nu_\tau(3h^-)\eta] = (4.1 \pm 0.7 \pm 0.7) \times 10^{-4}. \tag{1}$$

In Ref. [7] an abnormal axial-vector current of η and pions has been constructed to calculate the branching ratio of $\tau \to \eta 3\pi\nu$. The theoretical prediction is

$$1.2 \times 10^{-6}$$

which is less than the experimental value by more than two orders of magnitude. The decay $\tau \to \eta 3\pi\nu$ is caused by the axial-vector current and, as pointed out in Ref. [7], by the anomalous meson vertex. The result of Ref. [7] indicates that resonances should be taken into account. As shown in the studies [1,5,6], resonances play an essential role in τ mesonic decays. In this paper we are going to find all anomalous WZW terms which contribute to this decay mode. In constructing the anomalous vertices, the resonances a_1 and ρ need to be taken into account. In doing so, an effective chiral theory of mesons is required.

In Ref. [4] an effective chiral theory of three nonets of pseudoscalar, vector, and axial-vector mesons is proposed. This theory has been used to study meson physics at low energies. So far, it has been phenomenologically successful. The chiral symmetry breaking scale Λ is determined to be 1.6 GeV [4]. Therefore, this theory is suitable for studying τ mesonic decays. Our studies of τ mesonic decays based on this effective theory are presented in Ref. [6]. The theoretical results agree with the data reasonably well. The Wess-Zumino-Witten action is derived from the leading terms of the imaginary part of the effective Lagrangian [4]. In this paper the Lagrangian of Ref. [4] is used to derive all vertices needed. The expression of the axial-vector current ($\Delta s = 0$) of mesons presented in Ref. [6] is taken. The calculations done in this paper are at the tree level which is supported by the argument of large N_C expansion [4]. It is necessary to point out that in the studies of this paper there is no new parameter.

The anomalous vertices can be found in the WZW action [2,3]. The formalism of the WZW action is model independent. However, the meson fields in the WZW action need to be normalized to physical fields by the normal part of the Lagrangian (see Refs. [3,4]). In Ref. [4] a method is developed to derive the anomalous vertices which are found to have the same expressions as the ones obtained from the WZW action. In this way the meson fields are normalized to physical mesons. In this paper we use the method presented in Ref. [4] to derive all anomalous vertices needed.

We first present the study of $\tau \to \eta(\eta')\pi\pi\nu$ which has been studied by many authors [5]. As pointed out in Ref. [5], both the contact term and the anomalous vertex $\eta\rho\rho$ contribute to $\tau \to \eta\pi\pi\nu$. Using the Lagrangian of Ref. [4], these vertices are determined without any new parameters. The anomalous WZW vertices derived in Ref. [4] are

$$\mathcal{L}_{\eta\rho\rho} = \frac{N_C}{2\pi^2 g^2 f_\eta} \varepsilon^{\mu\nu\alpha\beta} \eta \times \left(-\sqrt{\frac{2}{3}} \sin\theta + \frac{1}{\sqrt{3}} \cos\theta \right) \partial_\mu \rho^i_\nu \partial_\alpha \rho^i_\beta ,$$

$$\mathcal{L}_{\eta'\rho\rho} = \frac{N_C}{2\pi^2 g^2 f_{\eta'}} \varepsilon^{\mu\nu\alpha\beta} \eta' \times \left(\sqrt{\frac{2}{3}} \cos\theta + \frac{1}{\sqrt{3}} \sin\theta \right) \partial_\mu \rho^i_\nu \partial_\alpha \rho^i_\beta , \tag{2}$$

where $\theta = -20°$ and $f_\eta = f_{\eta'} = f_\pi$ are taken. g is a universal coupling constant and is determined to be 0.39 in Ref. [6]. $\mathcal{L}_{\eta\rho\rho}$ was used in Ref. [5]. Besides the vertices (2), there are other vertices which contribute to these two decay modes. Using the same method we used to derive Eqs. (2) [4], we obtain

$$\mathcal{L}_{\eta\rho\pi\pi}=\frac{2}{\pi^2 g f_\pi^3}\left(-\sqrt{\frac{2}{3}}\sin\theta+\frac{1}{\sqrt{3}}\cos\theta\right)\times\left(1-\frac{4c}{g}-\frac{2c^2}{g^2}\right)\varepsilon^{\mu\nu\alpha\beta}\epsilon_{ijk}\eta\partial_\mu\rho^i_\nu\partial_\alpha\pi^j\partial_\beta\pi^k,$$

$$\mathcal{L}_{\eta'\rho\pi\pi}=\frac{2}{\pi^2 g f_\pi^3}\left(\sqrt{\frac{2}{3}}\cos\theta+\frac{1}{\sqrt{3}}\sin\theta\right)\times\left(1-\frac{4c}{g}-\frac{2c^2}{g^2}\right)\varepsilon^{\mu\nu\alpha\beta}\epsilon_{ijk}\eta'\partial_\mu\rho^i_\nu\partial_\alpha\pi^j\partial_\beta\pi^k, \tag{3}$$

where $c=f_\pi^2/2gm_\rho^2$. Using vector meson dominance (VMD), the direct couplings of $\eta(\eta')\pi\pi\gamma$ are obtained from the Lagrangians (3). As studied in Ref. [9] long ago, the abnormal Lagrangians (3) are from the box anomaly. Therefore, the decays $\tau\to\eta\pi\pi\nu$ provide a test ground for the box anomaly [9]. All the parameters in Eqs. (3) have been fixed. In the final states of the decay $\tau\to\eta(\eta')\pi\pi\nu$ caused by the vertices (2) the two pions have a ρ-resonance structure, while for the decay amplitudes determined by Eqs. (3) there are no ρ resonances. Because of the cancellation in the factor $1-4c/g-2c^2/g^2$ the contributions of Eqs. (3) are smaller. This result has been obtained by Braaten, Oakes, and Tse [5]. This is similar to the decay $\omega\to\pi\pi\pi$ which is dominated by the vertex $\omega\rho\pi$ and the contribution of the contact term $\omega\pi\pi\pi$ is small [3,4]. Using the VMD and Eqs. (2),(3), the decay branching ratios are calculated:

$$B(\tau\to\eta\pi\pi\nu)=1.9\times10^{-3},$$
$$B(\tau\to\eta'\pi\pi\nu)=0.44\times10^{-5}. \tag{4}$$

The calculation shows that 28% of $B(\tau\to\eta\pi\pi\nu)$ is from the anomalous contact terms (3). The data are

$$B(\tau\to\eta\pi\pi\nu)=(1.71\pm0.28)\times10^{-3}\ \ [10],$$
$$B(\tau\to\eta'\pi\pi\nu)<0.8\times10^{-4}\ \ [8].$$

Theoretical results agree with the data.

The decay process $\tau\to\eta\pi\pi\pi\nu$ is more complicated than the ones studied above. Only the axial-vector current contributes to these processes. The WZW anomalous interactions of mesons cause these decays. More vertices are involved. The Feynman diagrams of these decays are shown in Fig. 1. The derivation of these vertices is lengthy. As done in Ref. [4] by calculating $-(2i/f_\pi)m\eta\langle\bar{\psi}\gamma_5\psi\rangle$ and $-(2i/f_\pi)m\eta\langle\bar{\psi}\lambda_8\gamma_5\psi\rangle$, we derive all anomalous vertices contributing to these decays:

$$\mathcal{L}_{\eta aa}=\frac{f_a^2}{2\pi^2 f_\eta}\varepsilon^{\mu\nu\alpha\beta}\eta\left(-\sqrt{\frac{2}{3}}\sin\theta+\frac{1}{\sqrt{3}}\cos\theta\right)\partial_\mu a^i_\nu\partial_\alpha a^i_\beta, \tag{5}$$

$$\mathcal{L}_{\eta\rho\pi\pi}=\frac{1}{4\pi^2 g}\left(\frac{2}{f_\pi}\right)^3\left(1-\frac{4c}{g}-\frac{2c^2}{g^2}\right)\left(-\sqrt{\frac{2}{3}}\sin\theta+\frac{1}{\sqrt{3}}\cos\theta\right)\epsilon_{ijk}\varepsilon^{\mu\nu\alpha\beta}\eta\partial_\mu\rho^i_\nu\partial_\alpha\pi_j\partial_\beta\pi_k, \tag{6}$$

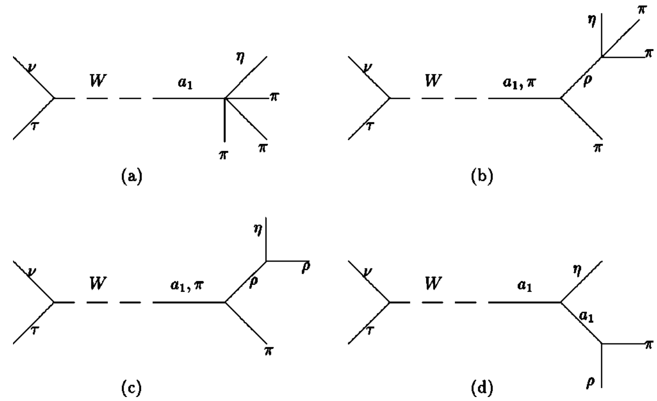

FIG. 1. Diagrams of the decay $\tau\to\eta\pi\pi\pi\nu$.

$$\mathcal{L}_{\eta a\pi\pi\pi}=\frac{N_C f_a}{(4\pi)^2}\left(\frac{2}{f_\pi}\right)^4\left(-\sqrt{\frac{2}{3}}\sin\theta+\frac{1}{\sqrt{3}}\cos\theta\right)\times\left(1-\frac{10}{3}\frac{c}{g}+\frac{8}{3}\frac{c^2}{g^2}\right)\varepsilon^{\mu\nu\alpha\beta}\eta\partial_\mu a^i_\nu\partial_\alpha\pi^2\partial_\beta\pi^i, \tag{7}$$

where a^i_μ is the a_1 meson field and $f_a^{-1}=g(1-1/2\pi^2g^2)^{1/2}$. The vertices related to η' meson are obtained by using the substitution

$$\left(-\sqrt{\frac{2}{3}}\sin\theta+\frac{1}{\sqrt{3}}\cos\theta\right)\eta\to\left(\sqrt{\frac{2}{3}}\cos\theta+\frac{1}{\sqrt{3}}\sin\theta\right)\eta'$$

in Eqs. (5)–(7).

The vertices $\mathcal{L}_{a\rho\pi}$ and $\mathcal{L}_{\rho\pi\pi}$ are involved in $\tau\to\eta\pi\pi\pi\nu$. They are presented in Refs. [4,6]:

$$\mathcal{L}_{a_1\rho\pi}=\epsilon_{ijk}\{Aa^i_\mu\rho^{j\mu}\pi^k-Ba^i_\mu\rho^j_\nu\partial^{\mu\nu}\pi^k\}, \tag{8}$$

$$A=\frac{2}{f_\pi}gf_a\left\{\frac{m_a^2}{g^2f_a^2}-m_\rho^2+p^2\left[\frac{2c}{g}+\frac{3}{4\pi^2g^2}\left(1-\frac{2c}{g}\right)\right]+q^2\left[\frac{1}{2\pi^2g^2}-\frac{2c}{g}-\frac{3}{4\pi^2g^2}\left(1-\frac{2c}{g}\right)\right]\right\}, \tag{9}$$

$$B=-\frac{2}{f_\pi}gf_a\frac{1}{2\pi^2g^2}\left(1-\frac{2c}{g}\right), \tag{10}$$

$$\mathcal{L}^{\rho\pi\pi}=\frac{2}{g}\epsilon_{ijk}\rho^i_\mu\pi^j\partial^\mu\pi^k-\frac{2}{\pi^2f_\pi^2g}\times\left\{\left(1-\frac{2c}{g}\right)^2-4\pi^2c^2\right\}\epsilon_{ijk}\rho^i_\mu\partial_\nu\pi^j\partial^{\mu\nu}\pi^k, \tag{11}$$

where p is the momentum of the ρ meson and q is the momentum of a_1. These vertices have been used to calculate the decay widths of $a_1 \to \rho\pi$, 3π, the ratio of d/s, the width of $\rho \to \pi\pi$, and the width of $\tau \to 3\pi\nu$ [4,6]. The theoretical results agree with the data.

The axial-vector current is needed to calculate the decay rates of $\tau \to \eta\pi\pi\pi\nu$. In terms of chiral symmetry and dynamical chiral symmetry breaking, the expression of the axial-vector current of mesons is obtained in Ref. [6] [Eqs. (10),(23),(24),(25)]. We used this axial-vector current to calculate some τ mesonic decay rates in Ref. [6]. Theoretical results agree with data. Using the axial-vector current and the vertices [Eqs. (5)–(11)], the decay rate of $\tau \to \eta\pi\pi\pi\nu$ is calculated. In the subprocesses shown in Fig. 1 there are anomalous vertices and normal vertices. The anomalous vertices are at the fourth order in low energy expansion, hence the strengths of anomalous vertices are weaker. This argument is supported by the narrow decay widths of the ω and $f_1(1280)$ mesons whose decays are a result of anomalous vertices [4]. Because the vertices (8),(11) are normal and at the second order in the low energy expansion, the strengths of the vertices (8),(11) are stronger. Therefore, the ρ and the a_1 mesons have broader widths. Only the anomalous vertex $\mathcal{L}_{\eta a\pi\pi\pi}$ contributes to the subprocess shown in Fig. 1(a), therefore, the contribution of this process to the decay is small. For the subprocess [Fig. 1(b)] there are normal vertices and anomalous ones. For the anomalous vertex $\mathcal{L}_{\eta\rho\pi\pi}$ (6), because of the cancellation in the factor $(1-4c/g-2c^2/g^2)$, the anomalous vertex $\mathcal{L}_{\eta\rho\pi\pi}$ (6) is very weak. The contribution of this subprocess is small. In the third subprocess [Fig. 1(c)] there is a ρ resonance. The minimum of q^2 of this ρ is $m_\rho + m_\eta = 1.32$ GeV which is much greater than m_ρ. This effect suppresses the contribution of this subprocess. Numerical calculation supports these arguments. The contribution of these subprocesses is less than 10% which is ignored in this paper. Finally, only the fourth subprocess [Fig. 1(d)] survives. There is no cancellation in the anomalous vertex $\mathcal{L}_{\eta aa}$ and the strength of the normal vertex $\mathcal{L}_{a\rho\pi}$ is strong. Therefore, we expect that the diagram Fig. 1(d) is the major contributor of the decay $\tau \to \eta\pi\pi\pi\nu$.

The discussion above leads to that the process $\tau \to \eta\rho\pi\nu$ is dominant the decay $\tau \to \eta\pi\pi\pi\nu$. The Feynman diagram, Fig. 1(d), shows that there are two subprocesses in this decay: $\tau \to \eta a_1 \nu$ and $a_1 \to \rho\pi$. As mentioned above, the former is caused by the anomalous vertex $\mathcal{L}_{\eta aa}$ (5) which has never been tested before and the latter is a result of the vertex (8) which has been tested. It is necessary to point out that the anomalous vertices $\mathcal{L}_{\eta vv}$ $(v=\rho,\omega,\phi)$ have been tested by $\eta \to \gamma\gamma$, $\rho \to \eta\gamma$, $\omega \to \eta\gamma$, $\phi \to \eta\gamma$, and $\tau \to \eta\pi\pi\nu$. However, it is the first time that the anomalous WZW vertex $\mathcal{L}_{\eta aa}$ has been tested.

Using the vertices (5),(8),(11) and the axial-vector current of Ref. [6] in the chiral limit the amplitude of the decay $\tau \to \eta\rho\pi\nu$ is derived as

$$\begin{aligned}
&\langle \eta(p)\rho^0(k')\pi^-(k)|\overline{\psi}\tau_+\gamma_\mu\gamma_5\psi|0\rangle \\
&= \frac{i}{\sqrt{8\omega\omega' E}} \frac{f_a^2}{\pi^2 f_\pi}\left(-\sqrt{\frac{2}{3}}\sin\theta + \frac{1}{\sqrt{3}}\cos\theta\right) \\
&\times \frac{g^2 f_a m_\rho^2 - i f_a^{-1}\sqrt{q^2}\Gamma_a(q^2)}{q^2 - m_a^2 + i\sqrt{q^2}\Gamma_a(q^2)} \\
&\times \frac{1}{q_1^2 - m_a^2 + i\sqrt{q_1^2}\Gamma_a(q_1^2)} \\
&\times \{A(q_1^2) g_{\beta\lambda} + B k_\beta k_\lambda\}\varepsilon^{\mu\nu\alpha\beta} q_\nu q_{1\alpha}\epsilon^{*\lambda}, \qquad (12)
\end{aligned}$$

where $q=p+k+k'$, $q_1 = q-p$, and k, k', and p are momentum of the pion ρ and η, respectively. The width of the a_1 meson is derived from the vertex (8)

$$\begin{aligned}
\Gamma_a(q^2) = \frac{k_a}{12\pi}\frac{1}{\sqrt{q^2} m_a}\Bigg\{&\left(3+\frac{k_a^2}{m_\rho^2}\right)A^2(q^2) \\
&-2A(q^2)B(q^2+m_\rho^2)\frac{k_a^2}{m_\rho^2} + k_a^4\frac{q^2}{m_\rho^2}B^2\Bigg\}, \qquad (13)
\end{aligned}$$

where $k_a^2 = (1/4q^2)(q^2+m_\rho^2-m_\pi^2)^2 - m_\rho^2$. Making the substitution $q^2 \to q_1^2$ in Eq. (13), the $\Gamma_a(q_1^2)$ is obtained.

This matrix element has the strength of the vertex $\mathcal{L}_{a_1\rho\pi}$. In the matrix element (13) there is a second a_1 resonance,

$$\frac{1}{q_1^2 - m_a^2 + i\sqrt{q_1^2}\Gamma_a(q_1^2)}.$$

The minimum of q_1^2 is $(m_\rho + m_\pi)$ which is less than m_a. Therefore, the matrix element is enhanced by the second a_1 resonance. These two factors make the subprocess [Fig. 1(d)] the main contributor of the decay $\tau \to \eta\pi\pi\pi\nu$. The term $g^2 f_a m_\rho^2$ comes from dynamical chiral symmetry breaking which is the origin of the mass difference of the a_1 and the ρ mesons [6]. Using the matrix element, in the chiral limit the decay rate of $\tau \to \eta\rho\pi\nu$ is found to be

$$\begin{aligned}
d\Gamma(\tau^- \to \eta\rho^0\pi^-\nu) = &\frac{G^2}{(2\pi)^5}\cos^2\theta_c \frac{1}{1152}\frac{1}{m_\tau^3 q^2}(m_\tau^2-q^2)^2(m_\tau^2+2q^2)\left(\frac{f_a^2 a_\eta}{\pi^2 f_\pi}\right)^2 \frac{m_\rho^4 g^2 f_a^2 + f_a^{-2} q^2\Gamma_a^2(q^2)}{(q^2-m_a^2)^2 + q^2\Gamma_a^2(q^2)} \\
&\times \frac{1}{(q_1^2-m_a^2)^2 + q_1^2\Gamma_a^2(q_1^2)}\Bigg\{3A^2(q_1^2)\frac{(q\cdot p)^2}{q^2}(q_{\max}^2 - q_{\min}^2) - \frac{1}{6}A(q_1^2)B(q\cdot p)^2\frac{1}{q^4}[(q^2-q_{\min}^2)^3 \\
&-(q^2-q_{\max}^2)^3] + [A^2(q_1^2) - 2A(q_1^2)Bk\cdot k' + B^2(k\cdot k')^2] \\
&\times \frac{(q\cdot p)^2}{12 m_\rho^2 q^4}[(q^2-q_{\min}^2)^3 - (q^2-q_{\max}^2)^3]\Bigg\} dq^2 dq_1^2, \qquad (14)
\end{aligned}$$

where

$$q^2_{\rm max}=m^2_\rho+\frac{1}{\sqrt{q^2_1}}(q^2-q^2_1)(\sqrt{m^2_\rho+l^2}+l),$$

$$q^2_{\rm min}=m^2_\rho+\frac{1}{\sqrt{q^2_1}}(q^2-q^2_1)(\sqrt{m^2_\rho+l^2}-l),$$

where $l=(1/2\sqrt{q^2_1})(q^2_1-m^2_\rho)$ and $a_\eta=-\sqrt{2/3}\sin\theta+(1/\sqrt{3})\cos\theta$. The branching ratio is computed to be

$$B(\tau^-\to\eta\rho^0\pi^-\nu)=2.93\times10^{-4}. \tag{15}$$

Theoretical result agrees with the data reasonably well. This theory predicts a ρ resonance structure in the two pion state and a a_1 resonance in the three pion state of the decay $\tau\to\eta\pi\pi\pi\nu$.

Because of the value of $m_{\eta'}$ the decay $\tau\to\eta'\rho\pi\nu$ is forbidden. Only the diagrams, Figs. 1(a) and 1(b), contribute to the decay. Because of kinematic reason the ρ resonance amplitude in the amplitude derived from the vertex $\mathcal{L}_{\eta'\rho\pi\pi}$ provides a very strong suppression. The contribution of this diagram is completely negligible. The decay rate derived from the anomalous vertex $\mathcal{L}_{\eta' a\pi\pi\pi}$ is

$$d\Gamma(\tau^-\to\eta'\pi^+\pi^-\pi^-\nu)=\frac{G^2\cos^2\theta}{9216(2\pi)^7}\frac{p^2}{48m^3_\tau q^4}F^2(m^2_\tau-q^2)^2(m^2_\tau+2q^2)[(q^2+p^2-m^2_{\eta'})^2-4q^2p^2]^{1/2}$$
$$\times\left[\frac{1}{4q^2}(q^2+m^2_{\eta'}-p^2)^2-m^2_{\eta'}\right][(q^2+p^2-m^2_{\eta'})^2-q^2p^2]\frac{m^4_\rho g^4 f^4_a+q^2\Gamma^2_a(q^2)}{(q^2-m^2_a)^2+q^2\Gamma^2_a(q^2)}dq^2dp^2, \tag{16}$$

$$F=\frac{2N_C}{(4\pi)^2}\left(\frac{2}{f_\pi}\right)^4\left(\sqrt{\frac{2}{3}}\cos\theta+\frac{1}{\sqrt{3}}\sin\theta\right)\left(1-\frac{10}{3}\frac{c}{g}+\frac{8}{3}\frac{c^2}{g^2}\right), \tag{17}$$

where $q^2=(p_\tau-p_\nu)^2$ and $p^2=(q-p_{\eta'})^2$,

$$B(\tau\to\eta'\pi\pi\pi\nu)=0.64\times10^{-8}.$$

To conclude, it is predicted that the decay rate of $\tau\to\eta'\pi\pi\nu$ is two orders of magnitude smaller than $\tau\to\eta\pi\pi\nu$. The decay mode $\tau\to\eta\pi\pi\pi\nu$ provides the first test on the anomalous vertex $\mathcal{L}_{\eta aa}$. Theoretical results agree with data. A a_1 resonance structure in the final state of $\tau\to\eta\pi\pi\pi\nu$ is predicted. A small decay rate for $\tau\to\eta'\pi\pi\pi\nu$ is predicted too. The formulas of the anomalous vertices presented in this paper are model independent. The coupling constants in these vertices are determined by fitting to other experiments. The vertex $a\rho\pi$ is derived from the effective theory [4] and has been tested in many physical processes [4,6].

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