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Dissipative field theory with the Caldeira-Leggett method and its application to disoriented chiral condensation

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The effective field theory including the dissipative effect is developed based on the Caldeira-Leggett theory at the classical level. After the integration of the small field fluctuations considered as the field radiation, the integrodifferential field equation is given and shown to include dissipative effects. In that derivation, special care should be taken for the boundary condition of the integration. Application to the linear σ model is given, and the decay process of the chiral condensate is calculated with it, both analytically in the linear approximation and numerically. With these results, we discuss the stability of chiral condensates within the quenched approximation. [S0556-2821(98)03803-X]

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I. INTRODUCTION

Recently, there has been great interest in the collective phenomena that might be observed after heavy ion collisions, especially in the state called the disoriented chiral condensate (DCC) [1,2].

The quantum chromodynamics (QCD) that describes the strong-interaction physics has the chiral symmetry $SU_L(2) \otimes SU_R(2)$ that is explicitly broken with the order of the pion mass m_π. The behavior of the vacuum state can be represented by the order parameters $\langle\sigma\rangle = \langle\overline{q}q\rangle$ and $\langle\pi\rangle = \langle\overline{q}\gamma_5 q\rangle$, where q is a quark field. At zero temperature, the effective potential $V_{\rm eff}(\langle\sigma\rangle,\langle\pi\rangle)$ has a bottom circle with a radius f_π, so that this symmetry is spontaneously broken and the vacuum state takes $\langle\sigma\rangle \neq 0$ and $\langle\pi\rangle = 0$.

The DCC state is defined as that on the bottom circle of $V_{\rm eff}$ but with $\langle\pi\rangle \neq 0$, and is expected to be produced after high-energy proton or heavy-ion collisions. About the formation of the DCC, there have been a lot of discussions, especially concerning the quenching or annealing scenarios [2–7], but the present situation is still controversial.

As has been pointed out in [8], another important problem is the decay process of the DCC state. Because of the explicit breaking of the chiral symmetry, the DCC state is only metastable and considered to decay into a true vacuum radiating many coherent pions [9]. If the lifetime of the DCC is very short compared to the formation time, it will be very difficult to observe the DCC state through the pion signal. The entire process of the DCC can be formulated as the dissipative system of the collective coordinates (the order parameters $\langle\sigma\rangle$ and $\langle\pi\rangle$) under background pion radiation, so that we can analyze the DCC, especially the decay of it, with the method of the dissipative theory in semiclassical dynamics.

There has been much work done to derive the equation of motion with radiative damping in classical electrodynamics and thermostatistical theory. The motion of the charged particle with electromagnetic radiation has been much discussed since the prequantum mechanical era [10]. Here we should mention that the advanced Green function for radiation was introduced by Dirac [11] to cancel the divergent electron's radiation mass and was justified by Wheeler and Feynman [12].

In thermostatistical theory, dissipative dynamics has been considered with the Brownian motion and the Langevin equation has been one of the most important tools to attack the problem [13]:

$$M\ddot{q} + \gamma\dot{q} + V'(q) = R(t), \tag{1}$$

where q and R are the particle coordinate and the fluctuating force and γ is the dissipative coefficient. However, in Langevin theory, the dissipative coefficient is the given parameter and cannot be fixed by itself.

In 1983, Caldeira and Leggett proposed a method to derive the dissipation from the microscopic theory [14]. They start with the system-plus-reservoir model (the interacting system of the collective coordinates and the background degrees of freedom) and, by integrating out the background degrees of freedom, the dissipative equation for the collective coordinates can be obtained. This method was formulated in particle dynamics, and has been applied to the polaron motion in a crystal (acoustic polaron), diffusion of charged interstitials in normally conducting metals, and the dynamics of Josephson junctions [13]. In the latter case, the reduction of the tunneling probability was predicted for the macroscopic quantum tunneling effect between superconductor and insulator with Caldeira-Leggett theory and was confirmed experimentally [15,16].

The application of the Caldeira-Leggett theory to the nuclear-hadron physics is very interesting; especially when considering phenomena with multiparticle production such as the emission of a meson, photon, and so on. The energy of the system (the excited nucleus or the fireballs, for example) is considered to decrease "dissipatively" with the particle emission and Caldeira-Leggett theory can be applied to them, where the reservoir corresponds to the emitted particle. Another interesting application is for the DCC phenomenon. The DCC process is described by the nonlinear σ model, and the collective coordinates and the reservoir are just the condensates $\langle\sigma\rangle$, $\langle\pi\rangle$, and the radiative pion through the formation and decay processes.

In those phenomena, the dynamical variables of the system are the fields corresponding to the emitted particles. For the application to such a process, we have to extend Caldeira-Leggett theory for field dynamics. That extension is very interesting by itself because it produces the dissipative field equation.

In this paper, we formulate Caldeira-Leggett theory in field dynamics, and apply it to the DCC process. The extension of the theory for field theory will be given in the first half of this paper as generally as possible. The resultant formulation is applied for the nonlinear σ model, and the existence of dissipative effects is shown analytically and numerically. In the final part of the paper, the decay process of the DCC is discussed qualitatively within the present formulation.

II. DISSIPATIVE FIELD EQUATION WITH CALDEIRA-LEGGETT THEORY

To illustrate the application of Caldeira-Leggett theory to the field equation, we take a simple system of the order parameter $\Phi(x)$ and the background field $f(x)$ that describes the particle with the mass m:

$$L=\frac{1}{2}(\partial\Phi)^2-V(\Phi)-F(\Phi)f+\frac{1}{2}[(\partial f)^2-m^2f^2], \tag{2}$$

where $V(\Phi)$ is the effective potential for Φ and $F[\Phi]f$ gives the interaction between Φ and f. Taking variations with Φ and f, we obtain the following field equations from Eq. (2):

$$\partial^2\Phi+\frac{\delta V}{\delta\Phi}+\frac{\delta F}{\delta\Phi}f=0, \tag{3a}$$

$$(\partial^2+m^2)f+F[\Phi]=0. \tag{3b}$$

They are easily checked to satisfy the energy-momentum conservation.

Equation (3b) is formally solved with the Green functions $G(x-y;m)$ that satisfy

$$(\partial^2+m^2)G(x-y)=-\delta^4(x-y). \tag{4}$$

The fluctuation f is then written as[1]

$$f(x)=\int d^4yG(x-y;m)F[\Phi(y)], \tag{5}$$

and, substituting it into Eq. (3a), we obtain the integrodifferential equation for Φ

$$\partial^2\Phi(x)+\frac{\delta V[\Phi(x)]}{\delta\Phi}+\frac{\delta F[\Phi(x)]}{\delta\Phi}\times\int d^4yG(x-y;m)F(\Phi[y])=0. \tag{6}$$

This equation is essentially equivalent to Eqs. (3a) and (3b).

[1]The homogeneous part can be dropped safely because it can be absorbed with proper adjustment of the boundary condition.

The Green function in Eq. (6) can be written as

$$G(x-y;m)=\int\frac{d^4k}{(2\pi)^4}G(k;m)e^{-ik(x-y)}, \tag{7}$$

where $G(k;m)$ is the Fourier transform of $G(x-y;m)$ and generally takes a complex value. In Caldeira-Leggett theory, the imaginary component of $G(k;m)$ can be considered as the dissipative term caused by the emission of particles for the background field f. It is suggested by the analysis of the simple Newtonian equation including the dissipative terms $m\ddot{x}+\eta\dot{x}=F$ that after Fourier transformation, the dissipative term $\eta\dot{x}(t)$ is similar to that of pure imaginary $i\eta\omega x(\omega)$ with the spectral function $x(\omega)$ for $x(t)$. Further, we can interpret this dissipation as the particle emission because $\mathrm{Im}G(k;m)$ includes $\delta^4(k^2-m^2)$ (on-mass shell). The real part $\mathrm{Re}G(k;m)$ is considered to give the modification of the effective potential that comes from the background absorption and emission, and can be absorbed into $V(\Phi)$ by redefinition. Thus we can drop it because the parameters in $V(\Phi)$ are adjusted from the experimental data. Finally, we obtain the dissipative field equation corresponding to Eq. (6):

$$\partial^2\Phi(x)+\frac{\delta V[\Phi(x)]}{\delta\Phi}+\frac{\delta F[\Phi(x)]}{\delta\Phi}\tilde{f}=0, \tag{8}$$

where

$$\tilde{f}(x)=\int d^4y\int\frac{d^4k}{(2\pi)^4}i\mathrm{Im}G(k;m)e^{-ik(x-y)}F(\Phi[y]). \tag{9}$$

Equation (9) is still ambiguous because a variety of Green functions (advanced, retarded, causal, or their combinations) can be used in it and give solutions with different boundary conditions. We should select the proper Green functions that show the dissipative effect when $t\to\infty$ with real $\tilde{f}$. In the application to the linear σ model, it will be shown that the advanced Green function satisfies both conditions and a proper selection.

The dissipative effects given in that manner are essentially the energy dissipation through the radiation of the on-mass shell physical mode (particle) represented by the field f. We should comment about the dissipation by the stochastic fluctuations in the finite temperature condensates. This is dictated by the dissipation-fluctuation theorem and, in the form of the Langevin equation (1), the dissipative coefficient γ is given by the correlation integral of the fluctuating force $R(t)$.

In the present application to DCC, the low-temperature limit is assumed, and the effects of the stochastic fluctuation can be neglected. However, at finite temperature, the latter effects are important because they are essential for the thermal equilibration of the system.

III. APPLICATION TO THE LINEAR σ MODEL

We apply the formulation given in the last section to the linear σ model by Gell-Mann and Lévy [17] and discuss the dynamical behavior of the order parameter of the SU(2) chiral symmetry.

The Lagrangian of the σ model is

$$\mathcal{L}=\frac{1}{2}[(\partial\sigma)^2+(\partial\pi)^2]-V(\sigma,\pi), \tag{10}$$

where σ and $\pi=(\pi^1,\pi^2,\pi^3)$ are the σ and pion fields. In this paper, we discuss only the phenomena related to the neutral pion so that π in Eq. (10) is a single field from now on. The application to the charged pion goes in the same way but we have to modify the Lagrangian (10) to include the photon degrees of freedom. The effective potential $V(\sigma,\pi)$ is taken to the forth order of the fields:

$$V(\sigma,\pi)=\frac{\lambda}{4}(\sigma^2+\pi^2-\nu^2)^2-H\sigma, \tag{11}$$

where the parameters λ, ν, and H are written as

$$\lambda=\frac{m_\sigma^2-m_\pi^2}{2f_\pi^2},\quad \nu^2=f_\pi^2\frac{m_\sigma^2-3m_\pi^2}{m_\sigma^2-m_\pi^2},\quad H=f_\pi m_\pi^2, \tag{12}$$

with the σ and pion mass $m_{\sigma,\pi}$ and the pion decay constant f_π.

The last term in Eq. (11) breaks the chiral symmetry explicitly and produces the finite pion mass. It is the simplest potential that describes the explicit symmetry breaking phenomena, and is consistent with the low-energy theorem.

The fields σ and π can be divided into two parts:

$$\sigma=\langle\sigma\rangle+\delta\sigma,\quad \pi=\langle\pi\rangle+\delta\pi, \tag{13}$$

where $\langle\sigma\rangle$ and $\langle\pi\rangle$ correspond to the order parameter of the chiral symmetry for the (disoriented) condensed state Ψ: $\langle\sigma\rangle=\langle\Psi|\overline{q}q|\Psi\rangle$ and $\langle\pi\rangle=\langle\Psi|\overline{q}\tau\gamma_5 q|\Psi\rangle$ with the quark field $q=(u,d)$. (The condensed states may be described with the coherent or the squeezed states as quantum states [18].) $\delta\sigma$ and $\delta\pi$ are the fluctuations around them, and represent the σ and π meson degrees of freedom. The decomposition (13) is essentially the same one found in Bogoliubov theory for the weak-interacting Bose liquid. In that theory, the fluctuation represents elementary excitations (phonon/roton) that cause the dissipative effects in superfluids [19].

Minimizing the effective potential V, we obtain the vacuum state $|0\rangle$ with which the order parameters become

$$\langle\sigma\rangle=\langle 0|\sigma|0\rangle\equiv f_\pi,\quad \langle\pi\rangle=\langle 0|\pi|0\rangle=0, \tag{14}$$

and the fluctuations $\delta\sigma$ and $\delta\pi$ describe the mesons with mass m_σ and m_π.

Now we consider the dynamical behaviors of the disoriented-condensed vacuum state $|\Psi\rangle\neq|0\rangle$, taking $\langle\sigma\rangle\neq f_\pi$ and $\langle\pi\rangle\neq 0$ as the field Φ given in the last section and the fluctuations $\delta\sigma$ and $\delta\pi$ as the background field f. Substituting Eq. (13) into Eq. (10), we obtain

$$L=\frac{1}{2}[(\partial\langle\sigma\rangle)^2+(\partial\langle\pi\rangle)^2]-V(\langle\sigma\rangle,\langle\pi\rangle)+H\langle\sigma\rangle +\partial\langle\sigma\rangle\partial\delta\sigma+\partial\langle\pi\rangle\partial\delta\pi+\frac{1}{2}[(\partial\delta\sigma)^2-m_\sigma^2\delta\sigma^2] +\frac{1}{2}[(\partial\delta\pi)^2-m_\pi^2\delta\pi^2], \tag{15}$$

where we assume that the background fields are small and describe the real σ and π meson degrees of freedom. Here we dropped $\mathcal{O}(\delta\sigma^3,\delta\pi^3)$ terms and the $\delta\sigma^2$ and $\delta\pi^2$ terms are adjusted to be the mass term with the real pion and σ masses m_σ and m_π as in Eq. (2). In this paper, we discuss the case where the condensates are on the chiral circle, $\langle\sigma\rangle^2+\langle\pi\rangle^2=\nu^2$, so that $\langle\sigma\rangle$ and $\langle\pi\rangle$ can be parametrized with the chiral angle ϕ:

$$\langle\sigma\rangle=\nu\cos\phi,\quad \langle\pi\rangle=\nu\sin\phi. \tag{16}$$

Substituting Eq. (16) into Eq. (15), it is obtained that

$$L=\frac{\nu^2}{2}(\partial\phi)^2+H\nu\cos\phi+\nu\partial\cos\phi\partial\delta\sigma+\nu\partial\sin\phi\partial\delta\pi +\frac{1}{2}[(\partial\delta\sigma)^2-m_\sigma^2\delta\sigma^2]+\frac{1}{2}[(\partial\delta\pi)^2-m_\pi^2\delta\pi^2]. \tag{17}$$

Taking the variation with ϕ, $\delta\sigma$, and $\delta\pi$, we obtain a set of Euler-Lagrange equations:

$$\nu^2\partial^2\phi+H\nu\sin\phi-\nu\sin\phi\partial^2\delta\sigma+\nu\cos\phi\partial^2\delta\pi=0, \tag{18a}$$

$$(\partial^2+m_\sigma^2)\delta\sigma+\nu\delta^2\cos\phi=0, \tag{18b}$$

$$(\partial^2+m_\pi^2)\delta\pi+\nu\delta^2\sin\phi=0, \tag{18c}$$

which correspond to Eqs. (3a) and (3b).

Now we can apply the formulation developed in the last section to Eqs. (18a), (18b), and (18c), and we obtain the dissipative field equation to Eq. (10)

$$\partial^2\phi(x)+\frac{H}{\nu}\sin\phi-\nu\sin\phi\partial^2\widetilde{\delta\sigma}+\nu\cos\phi\partial^2\widetilde{\delta\pi}=0, \tag{19}$$

where $\widetilde{\delta\sigma}$ and $\widetilde{\delta\pi}$ are functions similar to those found in Eq. (9).

For $G(x-y;m)$, we take the advanced Green function

$$G_{\text{adv}}(x-y;m)=\int\frac{d^4k}{(2\pi)^4}\frac{e^{-ik(x-y)}}{k^2-m^2-i\epsilon\,\text{sgn}(k^0)}. \tag{20}$$

The reality of $\widetilde{\delta\sigma}$ and $\widetilde{\delta\pi}$ can be checked easily using the direct calculation. In the next chapter, we will show that the advanced Green function really gives the dissipative solutions. From Eq. (20), we can read off the imaginary part of $G(k;m)$:

$$i\,\mathrm{Im}G(k;m)=i\pi\,\mathrm{sgn}(k^0)\,\delta^4(k^2-m^2) \tag{21}$$

so that $\delta\tilde{\sigma}$ and $\delta\tilde{\pi}$ become

$$\partial^2\delta\tilde{\sigma}(x)=\frac{1}{2}\int_{x_0}^{\infty}dy^0\int d^3y\,\Delta(x-y;m_\sigma)\partial_y^2\cos\,\phi(y), \tag{22a}$$

$$\partial^2\delta\tilde{\pi}(x)=\frac{1}{2}\int_{x_0}^{\infty}dy^0\int d^3y\,\Delta(x-y;m_\sigma)\partial_y^2\sin\,\phi(y), \tag{22b}$$

where $\Delta(x-y;m)$ is the invariant Δ function

$$\Delta(x-y;m)=\frac{1}{i(2\pi)^3}\int d^4k\,\delta^4(k^2-m^2)\mathrm{sgn}(k^0)e^{-ik(x-y)}. \tag{23}$$

As discussed in the previous section, we take out the on-mass-shell contributions from the fluctuations $\delta\sigma$ and $\delta\pi$ because they are essential for the energy dissipation by the radiative effects, and assume that the off-mass-shell part can be renormalized in the chiral potential. One of the important effects in the off-mass-shell part is the quantum fluctuations that should be included in the correlation function $\langle\pi(x)\pi(y)\rangle$, and a rigorous treatment of them can be seen in [7].

IV. SOLUTION OF THE DISSIPATIVE FIELD EQUATION

A. Asymptotic behavior of solutions

We consider the asymptotic behavior of the solutions that satisfy Eq. (19) and show the dissipative nature of it. The order parameter $\phi(x)$ is assumed to decrease when $t\to\infty$, so that it can be regarded as small. Then we can expand Eq. (19) about ϕ and approximate to first order (the linear approximation). Instead of $\phi(t,\mathbf{x})$, we use the Fourier component

$$\phi_{\mathbf{k}}(t)=\frac{1}{(2\pi)^3}\int d^3\mathbf{x}\,\phi(t,\mathbf{x})e^{-i\mathbf{k}\cdot\mathbf{x}}, \tag{24}$$

for which Eq. (19) is approximated to be

$$\ddot{\phi}_{\mathbf{k}}(t)+\mathbf{k}^2\phi_{\mathbf{k}}(t)+\alpha m_\pi^2\phi_{\mathbf{k}}(t)+\frac{1}{2}\frac{m_\pi^2}{\omega_{\mathbf{k}}}\int_t^{\infty}ds\sin\,\omega_{\mathbf{k}}(t-s)\times\{\ddot{\phi}_{\mathbf{k}}(s)+\mathbf{k}^2\phi_{\mathbf{k}}(s)\}=0, \tag{25}$$

with $\omega_{\mathbf{k}}=\sqrt{m_\pi^2+\mathbf{k}^2}$ and $\alpha=f_\pi/\nu\sim1.05$.[2] It should be noted that no correlations exist among different-momentum modes in Eq. (25) in the linear approximation.

[2]This slight shift from 1 results from the deformation of the chiral circle because of the explicit symmetry breaking.

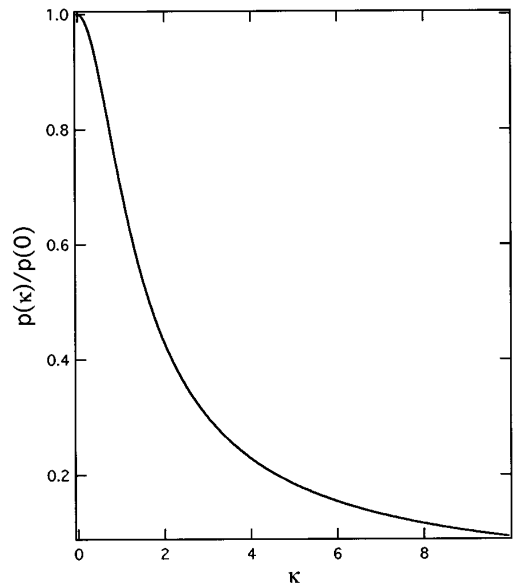

FIG. 1. The momentum dependence of the dissipative coefficient $p(\kappa)$ given by Eq. (27). κ is the pion-mass scaled momentum $\kappa=|\mathbf{k}|/m_\pi$. $p(\kappa)$ is normalized at $\kappa=0$ with the value $p(0)=0.35m_\pi$.

Substituting the ansatz $\phi_{\mathbf{k}}(t)=e^{-a(\mathbf{k})t}$ into Eq. (25), we obtain the characteristic equation for the index $a(\mathbf{k})$

$$a(\mathbf{k})^4+(2\kappa^2+1/2+\alpha)m_\pi^2a(\mathbf{k})^2+\{\kappa^4+(1/2+\alpha)\kappa^2+\alpha\}m_\pi^4=0, \tag{26}$$

with $\kappa=|\mathbf{k}|/m_\pi$. One of the solutions can be written as $a(\mathbf{k})=p(\kappa)+iq(\kappa)$,

$$p(\kappa)=\frac{m_\pi}{2}\sqrt{2\rho(\kappa)-(2\kappa^2+1/2+\alpha)},$$

$$q(\kappa)=\frac{m_\pi}{2}\sqrt{2\rho(\kappa)+(2\kappa^2+1/2+\alpha)}, \tag{27}$$

where $\rho(\kappa)=\sqrt{\kappa^4+(1/2+\alpha)\kappa^2+\alpha}$. It can be shown easily that $p(\kappa)$ takes a real and positive value when $\kappa\geqslant0$, so that the asymptotic behavior of $\phi_{\mathbf{k}}$ is found to be

$$\phi_{\mathbf{k}}(t)\sim e^{-p(\kappa)t}\sin[q(\kappa)t+\delta],\quad(t\to\infty) \tag{28}$$

with constant phase. Summarizing all the results, we obtain the asymptotic differential equation to Eq. (19),

$$\ddot{\phi}_{\mathbf{k}}+\mathbf{k}^2\phi_{\mathbf{k}}+\gamma\dot{\phi}_{\mathbf{k}}+\sqrt{\alpha}\sin\phi_{\mathbf{k}}\sim0, \tag{29}$$

where the dissipative coefficients are $\gamma=2p(\kappa)$. The momentum dependence of $p(\kappa)$ is shown in Fig. 1.

B. Numerical results

The fields $\delta\tilde{\sigma}$ and $\delta\tilde{\pi}$ defined by Eqs. (22a) and (22b) satisfy the differential equations

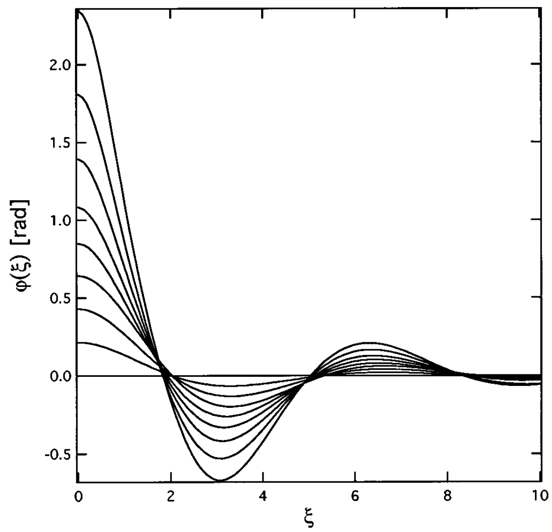

FIG. 2. A series of solutions of the integrodifferential equation (19) for the space-independent uniform case $\phi(x)\equiv\phi(t)$. The ξ is the pion-mass scaled time $\xi=m_\pi t$ and $\phi(t)$ is a chiral angle of the order parameter. The initial conditions are given at $\xi=0$ with $d\phi/d\xi=0$.

$$(\partial^2+m_\sigma^2)\delta\widetilde{\sigma}+\partial^2\cos\phi=0,\quad (\partial^2+m_\pi^2)\delta\widetilde{\pi}+\partial^2\sin\phi=0. \tag{30}$$

Hence the solutions of the original integrodifferential equations are given as those of the three differential equations (19) and (30).

In the following, we show the numerical solutions for two cases: the uniform and the expanding solutions. For physical quantities, we took

$$f_\pi=92.5\ \text{MeV},\quad M=940\ \text{MeV},\quad m_\pi=135\ \text{MeV}, \tag{31}$$

and $m_\sigma=600$ MeV was used for the mass of the σ meson. With these values, the parameters in Eq. (3b) are fixed as $\lambda=20.0$ and $\nu=87.4$ MeV.

1. Uniform solution

The solution that is uniform for the space dependence is characterized by $\phi=\phi(t)$ [and correspondingly, $\delta\widetilde{\sigma}=\delta\widetilde{\sigma}(t)$ and $\delta\widetilde{\pi}=\delta\widetilde{\pi}(t)$]. In this case, Eqs. (19) and (30) are reduced to the ordinary differential equations, which can be easily solved. The numerical results are shown in Fig. 2, where the scaled time $\xi=m_\pi t$ has been used. In this figure, the damped oscillation behavior proved analytically in the last section is easily confirmed.

The dissipating behaviors can be read off in the phase diagram (Fig. 3) too, where each line is a phase trajectory to these solutions; the spiral pattern around the origin shows that they behave as the damped oscillator asymptotically.

For a quantitative check, we consider the damped rigid oscillator

$$\ddot{\phi}+\gamma\dot{\phi}+\sin\phi=0, \tag{32}$$

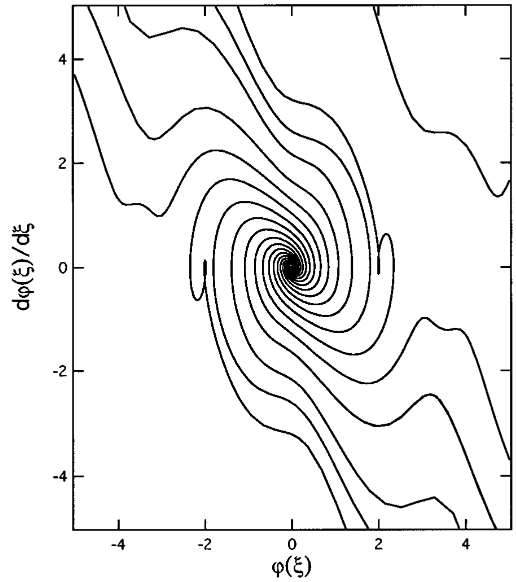

FIG. 3. The trajectories of the integrodifferential equation (19) for the space-independent uniform case $\phi(x)\equiv\phi(t)$. ϕ and $d\phi/d\xi$ are the chiral angle and the corresponding velocity with the scaled time $\xi=m_\pi t$. The initial conditions of each trajectory are chosen so they behave asymptotically [Eq. (28)] with $\delta=\pi n/5$ ($n=0,\pm1,\ldots,\pm5$).

with the damping coefficient γ consistent with the dissipative coefficient $p(k)$ in Eq. (27):

$$\gamma=2p(0)=0.7m_\pi. \tag{33}$$

The phase diagrams for Eq. (32) are given in Fig. 4. The trajectories in Fig. 3 are found to behave in a manner similar to those in Fig. 4, especially in the asymptotic region (close to the origin). In the nonasymptotic region (far from the origin), the trajectories in Fig. 3 have modulations that are close to those of the damped rigid oscillator. They come from the nonlinear dissipating behavior in Eqs. (19) and (30), which is more effective in the nonasymptotic region.

Through a comparison with the damped rigid oscillator, we can also realize the complicated behaviors at ($\phi=\pm\pi,\dot{\phi}=0$) in Fig. 3. They are the turning points of the rigid oscillator, and the trajectories around them are changed in a chaotic manner under small perturbation.

2. Expanding solution

Putting $\phi=\phi(\tau=\sqrt{t^2-x^2})$ in Eqs. (19) and (30), we get the expanding solution in the x direction (uniform in other directions). Originally, this type of solution was given by Blaizot and Krzywicki [20] with no dissipative effect, and more rigorous calculations have been done, including quantum fluctuation effects, by Cooper *et al.* [7].

The numerical solution is given as the rigid line in Fig. 5, where the scaled local time $\xi=m_\pi\tau$ was used. In this figure, we find that the expanding solution is damped faster than the uniform solution (shown as the dotted line in Fig. 4). To realize the expansion effect, we study the differential equation that Blaizot and Krzywicki solved. In our notation, it becomes

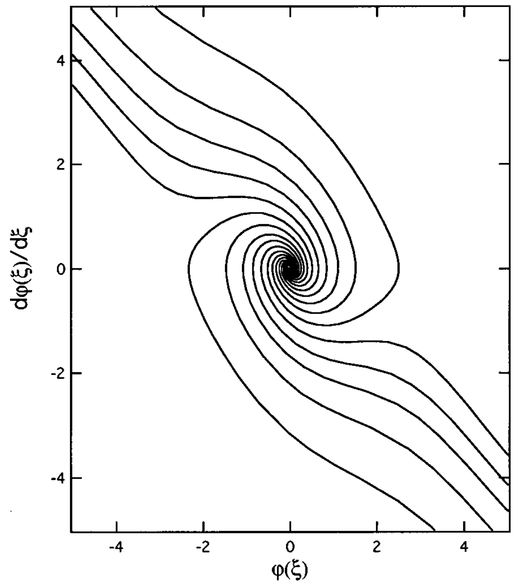

FIG. 4. The trajectories of the damped rigid oscillator (32). The dissipative coefficient is chosen to be consistent with the asymptotic value in the integrodifferential equation (19): $\gamma = 2p(0) = 0.7m_\pi$. ϕ and $d\phi/d\xi$ are the chiral angle and the corresponding velocity with the scaled time $\xi = m_\pi t$. The initial conditions of each trajectories are chosen as in Fig. 3.

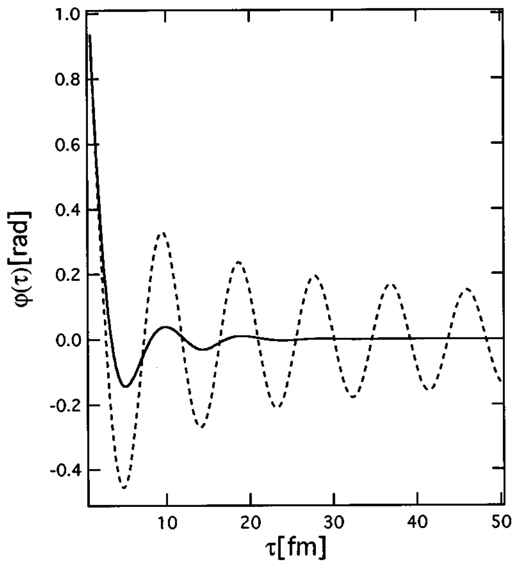

FIG. 5. One-dimensional expanding (scaling) solutions for the chiral angle $\phi(\xi)$ where ξ is a pion-mass scaled local time $\xi = m_\pi \tau = m_\pi \sqrt{t^2 - x^2}$. The rigid line is for the dissipative case: the solution of the integrodifferential equation (19), and the dashed line is for the nondissipative case: the original Blaizot-Kryzwicki solution of Eq. (34).

$$\phi'' + \frac{1}{\xi}\phi' + \sin\,\phi = 0, \tag{34}$$

where the differentiations are for $\xi = m_\pi \tau$. The second term in Eq. (34) is proportional to the time derivative of ϕ, and has the effect of dissipation with the time-dependent dissipative coefficient $1/\xi$. (This effect is not a real dissipation, but a smearing of ϕ brought on by volume expansion.) In the present case, this smearing effect has an additional effect to the real dissipative effect [represented by the second term in Eq. (32) asymptotically], and it causes faster damping in the expanding solution. The smearing effect is found to be more effective in the nonasymptotic region, because the effective dissipating coefficient is inversely proportional with the local time τ.

V. SUMMARY AND DISCUSSIONS

We formulated a dissipative field theory by applying the Caldeira-Leggett method. Explicit calculations were done for the linear σ model and the resultant field equations were shown to have dissipative properties both analytically and numerically.

As a phenomenological application, we discussed the disoriented chiral condensate that is expected to appear after the high-energy hadron collision. In the standard picture, the chiral symmetry is considered to be broken spontaneously at zero temperature and its order parameters take the expectation values $\langle\sigma\rangle \neq 0$ and $\langle\pi\rangle = 0$. The DCC is also in the broken phase, but is defined to be the state where the order parameters take different values: $\langle\sigma\rangle \neq 0$ and $\langle\pi\rangle \neq 0$. Because of the explicit chiral-symmetry breaking, the DCC state has higher energy, and it should be observed as a metastable state.

As written in the Introduction, there exist many controversies about the formation process of the DCC state after rebreaking of the chiral symmetry, but we concentrate on the decay process of it in the remainder of this paper. We consider the neutral DCC state with $\langle\pi^0\rangle \neq 0$ and $\langle\pi^\pm\rangle = 0$.[3] The main decay process of this state should be π^0 radiation, so we can apply the above-developed formula regarding π in Eq. (29) as π^0. The lifetime τ_L can be estimated with the dissipative constant $p(0)$ in Eq. (27):

$$\tau_L = 1/p(0) \sim 3m_\pi^{-1}. \tag{35}$$

If we take the quenching scenario for the DCC formation, the formation time τ_R is estimated to be [2]

$$\tau_R \sim \sqrt{2}m_\sigma^{-1} \sim 0.3m_\pi^{-1}. \tag{36}$$

It tells us that the lifetime τ_L in (35) is ten times longer than the formation time, so that the neutral DCC state will be metastable enough.

In this paper, we considered the case where the order parameters move only on the chiral circle $\langle\sigma\rangle^2 + \langle\pi\rangle^2 = f_\pi^2$, therefore, we could not consider the DCC formation process. The extension beyond the chiral circle will be given elsewhere.

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[3]For the charged DCC state, photon degrees of freedom are important and we have to extend our equations to include the dissipative effect by photon radiation.

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