

Ω and $\Sigma^0\Lambda$ transition magnetic moment in QCD sum rules

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The method of QCD sum rules in the presence of the external electromagnetic $F_{\mu\nu}$ field is used to calculate the Ω magnetic moment μ_Ω and the $\Sigma^0\Lambda$ transition magnetic moment $\mu_{\Sigma^0\Lambda}$, with the susceptibilities obtained previously from the study of octet baryon magnetic moments. The results $\mu_\Omega = -1.92\mu_N$ and $\mu_{\Sigma^0\Lambda} = 1.5\mu_N$ are in good agreement with the recent experimental data. [S0556-2821(98)03003-3]

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According to the quark model the Ω^- is composed of three strange quarks with parallel spins. This state is particularly interesting because of the large symmetry breaking that cannot be accommodated consistently in the naive quark model. In Ref. [1], it has been shown that nonstatic baryon-dependent magnetic effects are large. In this paper we investigate these symmetry breaking effects and evaluate the Ω moment in the QCD sum rule [3]. The Ω moment has been studied in literatures [2] and various theoretical results range from $-1.3\mu_N$ to $-2.7\mu_N$. We find our result is in good agreement with the recent data [4,5].

In a typical hadronic scale the quantum chromodynamics (QCD) is highly nonperturbative which makes a direct analytical first-principles calculation impossible. In this work we adopt the method in the presence of an external electromagnetic field [6,7] to calculate the Ω and the $\Sigma^0\Lambda$ transition magnetic moment $\mu_{\Sigma^0\Lambda}$. The important aspect of this investigation is that the various susceptibilities in this calculation have already been determined from previous studies of octet baryon magnetic moments [6-9], and as a result our calculation is parameter free.

In the method of QCD sum rules [6,7], the two-point correlation function $\Pi(p)$ in the presence of an external electromagnetic field $F_{\alpha\beta}$ is written as

$$\begin{aligned} \Pi_\Omega(p) &= i \int d^4x \langle 0 | T \{ \eta_\mu(x), \bar{\eta}^\mu(0) \} | 0 \rangle_{F_{\alpha\beta}} e^{ip \cdot x} \\ &= \Pi_0(p) + \Pi_1(p) (\sigma \cdot F \hat{p} + \hat{p} \sigma \cdot F) + \dots, \end{aligned} \quad (1)$$

$$\begin{aligned} \Pi_{\Sigma\Lambda}(p) &= i \int d^4x \langle 0 | T \{ \eta_{\Sigma^0}(x), \bar{\eta}_\Lambda(0) \} | 0 \rangle_{F_{\alpha\beta}} e^{ip \cdot x} \\ &= \Pi_2(p) (\sigma \cdot F \hat{p} + \hat{p} \sigma \cdot F) + \dots, \end{aligned} \quad (2)$$

where $\Pi_0(p)$ is the polarization operator without the external field $F_{\alpha\beta}$. The η_μ , η_{Σ^0} , and η_Λ are the currents with Ω , Σ^0 , and Λ quantum numbers

$$\eta_\mu(x) = \epsilon^{abc} [s^{aT}(x) C \gamma_\mu s^b(x)] s^c(x), \quad (3)$$

$$\begin{aligned} \eta_{\Sigma^0}(x) &= \epsilon^{abc} \frac{1}{\sqrt{2}} \{ [u^{aT}(x) C \gamma_\mu d^b(x)] \gamma_5 \gamma^\mu s^c(x) \\ &\quad + [d^{aT}(x) C \gamma_\mu u^b(x)] \gamma_5 \gamma^\mu s^c(x) \}, \end{aligned} \quad (4)$$

$$\begin{aligned} \eta_\Lambda(x) &= \epsilon^{abc} \sqrt{\frac{2}{3}} \{ [u^{aT}(x) C \gamma_\mu s^b(x)] \gamma_5 \gamma^\mu d^c(x) \\ &\quad - [d^{aT}(x) C \gamma_\mu s^b(x)] \gamma_5 \gamma^\mu u^c(x) \}, \end{aligned} \quad (5)$$

where $u^a(x)$, T , and C are the quark field, the transpose operators, and the charge conjugate operators. a, b, c are the color indices. The interpolating currents couples to the baryon states with the overlap amplitude λ :

$$\langle 0 | \eta_\mu(0) | \Omega \rangle = \lambda_\Omega \nu_\mu(p), \quad (6)$$

$$\langle 0 | \eta_{\Sigma^0}(0) | \Sigma \rangle = \lambda_\Sigma \nu_\Sigma(p), \quad (7)$$

$$\langle 0 | \eta_\Lambda(0) | \Lambda \rangle = \lambda_\Lambda \nu_\Lambda(p), \quad (8)$$

where ν_μ is a vectorial spinor and satisfies $(\hat{p} - m_\Omega) \nu_\mu = 0$, $\bar{\nu}_\mu \nu_\mu = -2m_\Omega$, and $\gamma_\mu \nu^\mu = p_\mu \nu^\mu = 0$ in the Rarita-Schwinger formalism. $\nu(p)$ is a Dirac spinor.

On the hadronic level the correlators $\Pi_1(p)$ and $\Pi_2(p)$ are expressed in terms of the chirality-odd tensor structure $(\sigma \cdot F \hat{p} + \hat{p} \sigma \cdot F)$:

$$\begin{aligned} \Pi_1(p) &= -\frac{1}{4} \mu_\Omega \frac{\lambda_\Omega^2}{(p^2 - m_\Omega^2)^2} \left\{ \frac{10}{9} + \frac{4}{9m_\Omega^2} (p^2 - m_\Omega^2)^2 \right\} \\ &\quad \times (\sigma \cdot F \hat{p} + \hat{p} \sigma \cdot F) + \dots, \end{aligned} \quad (9)$$

$$\Pi_2(p) = -\frac{1}{4} \mu_{\Sigma^0\Lambda} \frac{\lambda_\Sigma \lambda_\Lambda}{(p^2 - m^2)^2} (\sigma \cdot F \hat{p} + \hat{p} \sigma \cdot F) + \dots, \quad (10)$$

where $\{\dots\}$ is 1 for the nucleon magnetic moment. The deviation from unity in Eq. (9) is due to the Rarita-

Schwinger formalism for the spin $\frac{3}{2}$ field, of which the details may be found in Ref. [10]. $\bar{m} = (m_{\Sigma^0} + m_{\Lambda})/2$. We treat the Σ and Λ mass as degenerate due to their small mass difference since we never come across the poles and always work in the virtuality $p^2 < 0$. We denote the continuum and non-diagonal transition contributions simply by ellipses.

The external field $F_{\mu\nu}$ may induce changes in the physical vacuum and modify the propagation of quarks. Up to dimension six ($d \leq 6$), we introduce three induced condensates,

$$\begin{aligned} \langle 0 | \bar{q} \sigma_{\mu\nu} q | 0 \rangle_{F_{\mu\nu}} &= e_q \chi F_{\mu\nu} \langle 0 | \bar{q} q | 0 \rangle, \\ g_s \langle 0 | \bar{q} \frac{\lambda^n}{2} G_{\mu\nu}^n q | 0 \rangle_{F_{\mu\nu}} &= e_q \kappa F_{\mu\nu} \langle 0 | \bar{q} q | 0 \rangle, \\ g_s \epsilon^{\mu\nu\lambda\sigma} \langle 0 | \bar{q} \gamma_5 \frac{\lambda^n}{2} G_{\lambda\sigma}^n q | 0 \rangle_{F_{\mu\nu}} &= i e_q \xi F^{\mu\nu} \langle 0 | \bar{q} q | 0 \rangle, \end{aligned} \quad (11)$$

where q refers to u, d , and s quark, and e_q is the charge. The χ, κ , and ξ in Eq. (5) are the quark condensate susceptibilities and their values have been the subject of various studies. Ioffe and Smilga [6] found $\chi \approx -8 \text{ GeV}^{-2}$ with $\kappa = 0, \xi = 0$ in order to have $\mu_p = 3.0 \mu_N$ and $\mu_n = -2.0 \mu_N$ ($\pm 10\%$). Balitsky and Yung [7] estimated

$$\chi = -3.3 \text{ GeV}^{-2}, \quad \kappa = 0.22, \quad \xi = -0.44 \quad (12)$$

using the one-pole approximation. Belyaev and Kogan [8] extended the calculation and obtained an improved estimate

$$\begin{aligned} iS_q^{ab}(p) &= \delta^{ab} \frac{i}{\hat{p} - m_q} + \frac{i}{4} \frac{\lambda^n}{2} g_s G_{\mu\nu}^n \frac{1}{(p^2 - m_q^2)^2} [\sigma^{\mu\nu}(\hat{p} + m_q) + (\hat{p} + m_q)\sigma^{\mu\nu}] + \frac{i}{4} e_q \delta^{ab} F_{\mu\nu} \frac{1}{(p^2 - m_q^2)^2} \\ &\times [\sigma^{\mu\nu}(\hat{p} + m_q) + (\hat{p} + m_q)\sigma^{\mu\nu}] - \delta^{ab} \frac{\langle \bar{q} q \rangle}{12} (2\pi)^4 \delta^4(p) - \delta^{ab} \frac{\langle g_s \bar{q} \sigma \cdot G q \rangle}{192} (2\pi)^4 g^{\mu\nu} \partial_\mu \partial_\nu \delta^4(p) - \delta^{ab} e_q \frac{\langle \bar{q} q \rangle}{192} \\ &\times \left[\sigma \cdot F g^{\mu\nu} - \frac{1}{3} \gamma^\mu \sigma \cdot F \gamma^\nu \right] (2\pi)^4 \partial_\mu \partial_\nu \delta^4(p) - \delta^{ab} e_q \chi \frac{\langle \bar{q} q \rangle}{24} \sigma \cdot F (2\pi)^4 \delta^4(p) - \delta^{ab} e_q \kappa \frac{\langle \bar{q} q \rangle}{192} \\ &\times \left[\sigma \cdot F g^{\mu\nu} - \frac{1}{3} \gamma^\mu \sigma \cdot F \gamma^\nu \right] (2\pi)^4 \partial_\mu \partial_\nu \delta^4(p) + i \delta^{ab} e_q \xi \frac{\langle \bar{q} q \rangle}{768} \\ &\times \left[\sigma^{\delta\rho} g^{\mu\nu} - \frac{1}{3} \gamma^\mu \sigma^{\delta\rho} \gamma^\nu \right] \gamma_5 \epsilon_{\alpha\beta\delta\rho} F^{\alpha\beta} (2\pi)^4 \partial_\mu \partial_\nu \delta^4(p) + \dots \end{aligned} \quad (16)$$

in momentum space with $\hat{p} \equiv p_\mu \gamma^\mu$. Here we follow [6,7] and do not introduce induced condensates of higher dimensions, while we find that the strange quark mass correction is important so that we have explicitly kept it in our calculation.

Only the condensates with even dimensions contribute to the structure $(\sigma \cdot F \hat{p} + \hat{p} \sigma \cdot F)$. They are $1, \chi \langle 0 | \bar{q} q | 0 \rangle m_s, m_s \langle 0 | \bar{q} q | 0 \rangle, \langle 0 | g_s^2 G_{\alpha\beta}^n G_{\alpha\beta}^n | 0 \rangle, \chi \langle 0 | \bar{q} q | 0 \rangle^2, \chi \langle 0 | \bar{q} q | 0 \rangle \langle 0 | g_s \bar{q} \sigma \cdot G q | 0 \rangle, \kappa \langle 0 | \bar{q} q | 0 \rangle^2$, and $\xi \langle 0 | \bar{q} q | 0 \rangle^2$

$\chi = -5.7 \text{ GeV}^{-2}$ using the two-pole approximation. Chiu *et al.* [9] also estimated the susceptibilities with the two-pole model and obtained

$$\chi = -4.4 \text{ GeV}^{-2}, \quad \kappa = 0.4, \quad \xi = -0.8. \quad (13)$$

The values of these susceptibilities are consistent with one another except that the earliest result $\chi = -8 \text{ GeV}^{-2}$ in [6], is considerably larger (in magnitude) due to their neglect of κ, ξ in the fitting procedure. In what follows, we shall adopt the condensate parameters $\chi = -4.5 \text{ GeV}^{-2}, \kappa = 0.4, \xi = -0.8$ which represent the average in the last three analyses.

The correlation functions $\Pi_1(p)$ and $\Pi_2(p)$ at the quark level are

$$\begin{aligned} \langle 0 | T \eta_\mu(x) \bar{\eta}^\mu(0) | 0 \rangle_F &= -2i \epsilon^{abc} \epsilon^{a'b'c'} (\text{Tr} \{ S^{bb'}(x) \gamma_\mu C [S^{aa'}(x)]^T C \gamma_\mu \} \\ &\times S^{cc'}(x) + 2S^{bb'}(x) \gamma_\mu C [S^{aa'}(x)]^T C \gamma_\mu S^{cc'}(x)), \end{aligned} \quad (14)$$

$$\begin{aligned} \langle 0 | T \eta_{\Sigma^0}(x) \bar{\eta}^\Lambda(0) | 0 \rangle_F &= -\frac{2}{\sqrt{3}} i \epsilon^{abc} \epsilon^{a'b'c'} \{ \gamma_5 \gamma^\mu S_s^{aa'}(x) \gamma_\nu C [S_u^{bb'}(x)]^T \\ &\times C \gamma_\mu S_d^{cc'}(x) \gamma^\nu \gamma_5 \\ &- \gamma_5 \gamma^\mu S_s^{aa'}(x) \gamma_\nu C [S_d^{bb'}(x)]^T C \gamma_\mu S_u^{cc'}(x) \gamma^\nu \gamma_5 \}, \end{aligned} \quad (15)$$

where $iS^{ab}(x)$ is the quark propagator in the presence of the external electromagnetic field $F_{\mu\nu}$ [6]. We find

up to dimension eight. The up and down quark is treated as massless. The gluon condensate $\langle 0 | g_s^2 G_{\alpha\beta}^n G_{\alpha\beta}^n | 0 \rangle$ always appears with a small numerical factor $1/(2\pi)^4$ through the two-loop integration [6]. Its contribution was found to be negligible through the direct calculation [9]. Following Refs. [3,6,7] the four quark condensate $\langle 0 | \bar{q} \Gamma_1 q \bar{q} \Gamma_2 q | 0 \rangle$ is treated by the factorization approximation as the susceptibilities are estimated under the vacuum dominance hypothesis. The calculation is straightforward by substituting the

quark propagator into Eqs. (9) and (14). Here we present the final result after Borel transformation:

$$\begin{aligned} & \frac{9}{4} e_s \left\{ M_B^6 E_2(y_1) L^{4/27} - \frac{6}{5} \chi m_s a_s M_B^4 E_1(y_1) L^{-4/9} \right. \\ & \quad + \frac{6}{5} m_s a_s M_B^2 E_0(y_1) L^{4/27} - \frac{4}{5} \chi a_s^2 M_B^2 E_0(y_1) L^{4/9} \\ & \quad \left. + a_s^2 L^{28/27} \left[\frac{14}{15} + \frac{\chi m_0^2}{30} L^{-10/9} - \frac{22}{15} \kappa - \frac{4}{15} \xi \right] \right\} \\ & = (2\pi)^4 \lambda_\Omega^2 e^{-\bar{m}^2/M_B^2} \mu_\Omega (1 + A_1 M_B^2), \end{aligned} \quad (17)$$

$$\begin{aligned} & \frac{1}{\sqrt{3}} (e_u - e_d) \left\{ M_B^6 E_2(y_2) L^{-4/9} - \chi m_s a M_B^4 E_1(y_2) L^{-28/27} \right. \\ & \quad + \frac{4}{3} m_s a M_B^2 E_0(y_2) L^{-4/9} - m_s a_s M_B^2 E_0(y_2) L^{-4/9} \\ & \quad - \frac{2}{3} \chi a a_s M_B^2 E_0(y_2) L^{-4/27} + \frac{1}{9} a a_s L^{4/9} \\ & \quad \left. \times \left[4 + \kappa - 2\xi + \frac{3}{4} \chi m_0^2 L^{-10/9} \right] \right\} \\ & = (2\pi)^4 \lambda_\Sigma \lambda_\Lambda e^{-\bar{m}^2/M_B^2} \mu_{\Sigma^0\Lambda} (1 + A_2 M_B^2), \end{aligned} \quad (18)$$

where $m_\Omega = 1.672$ GeV, $\bar{m} = 1.15$ GeV, $m_s = 150$ MeV, $y_1 = W_1^2/M_B^2$ and $y_2 = W_2^2/M_B^2$. $E_n(y) = 1 - e^{-y} \sum_{k=0}^n 1/k! y^k$ are the factors used to subtract the continuum contribution [6]. $W_1^2 = 5.0$ GeV² and $W_2^2 = 3.4$ GeV² are the continuum thresholds which are determined together with the overlap amplitudes $(2\pi)^4 \lambda_\Omega^2 = 5.56$ GeV⁶, $(2\pi)^4 \lambda_\Sigma^2 = 1.88$ GeV⁶, and $(2\pi)^4 \lambda_\Lambda^2 = 1.64$ GeV⁶ from the Ω , Σ , and Λ mass sum rules [11,12]. We adopt the ‘‘standard’’ values for the various condensates $a = -(2\pi)^2 \langle 0 | \bar{u}u | 0 \rangle = 0.55$ GeV³, $a_s = -(2\pi)^2 \langle 0 | \bar{s}s | 0 \rangle = 0.55 \times 0.8$ GeV³, $am_0^2 = (2\pi)^2 g_s \langle 0 | \bar{u} \cdot G u | 0 \rangle$, $m_0^2 = 0.8$ GeV². $L = \ln(M_B/\Lambda_{\text{QCD}})/\ln(\mu/\Lambda_{\text{QCD}})$, Λ_{QCD} is the QCD parameter, $\Lambda_{\text{QCD}} = 100$ MeV, and $\mu = 0.5$ GeV is the normalization point to which the used values of condensates are referred.

We further improve the numerical analysis by taking into account of the renormalization group evolutions of the sum rules (17) and (18) through the anomalous dimensions of the various condensates and currents. A_1 and A_2 are constants to be determined from the sum rule, which arise from the non-diagonal transitions $\Omega \gamma \rightarrow \Omega^*$, $\Sigma^* \rightarrow \Lambda \gamma$, or $\Sigma \rightarrow \Lambda^* \gamma$ [6]. The working intervals of the Borel mass M_B^2 for the sum rules (17) and (18) are $2.0 \text{ GeV}^2 \leq M_B^2 \leq 4.0 \text{ GeV}^2$ and $1.3 \text{ GeV}^2 \leq M_B^2 \leq 3.0 \text{ GeV}^2$, respectively, where both the continuum contribution and power corrections are controllable. Moving the factor $(2\pi)^4 \lambda_\Omega^2 e^{-\bar{m}^2/M_B^2}$ and $(2\pi)^4 \lambda_\Sigma \lambda_\Lambda e^{-\bar{m}^2/M_B^2}$ on the right-hand side to the left and fitting the new sum rules with a straight line approximation we may extract the μ_Ω and $\mu_{\Sigma\Lambda}$. We show the new sum rules as a function of the Borel mass in Fig. 1. The new sum rules are almost stable and independent of M_B^2 , which implies that the nondiagonal transition contributions are small.

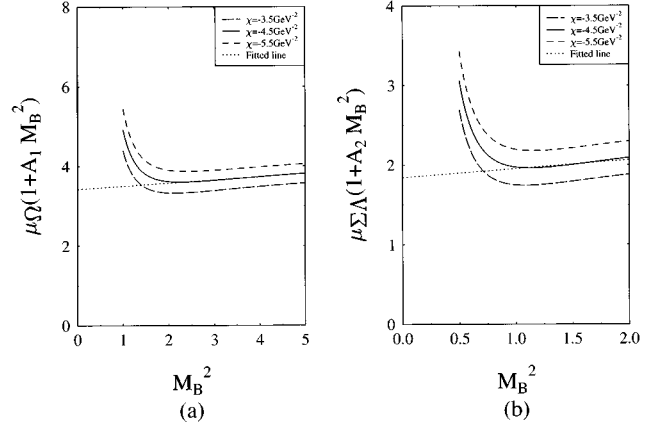


FIG. 1. (a) The Borel mass dependence of the Ω magnetic moment. The long-dashed, solid, and short-dashed curve are the QCD sum rule predictions for $\chi = -3.5$, -4.5 , and -5.5 GeV⁻², respectively, from Eq. (17) after the numerical factor $(2\pi)^4 \lambda_\Omega^2 e^{-\bar{m}^2/M_B^2}$ is moved to the left-hand side. The dotted line is a straight-line approximation. The intersect with the Y axis is the Ω magnetic moment in units of $e/2m_\Omega$. The Borel mass M_B^2 is in units of GeV². (b) The Borel mass dependence of the $\Sigma\Lambda$ transition magnetic moment. The notations are the same as those in (a).

The sum rules are insensitive to the susceptibilities κ and ξ because of their small values. Their contributions are less than 5%. The dependence on χ is shown in Fig. 1. When χ varies from -4.5 to -3.5 GeV⁻² or to -5.5 GeV⁻², the sum rules change within 10%. The correction from the strange quark mass is important and contributes about 20% to both of the sum rules. The SU(3)_f flavor symmetry breaking parameter $\gamma = \langle \bar{s}s \rangle / \langle \bar{u}u \rangle$ needs to take the standard value of 0.8 in order to yield a good agreement with the experimental data in Eq. (17). The Ω magnetic moment would increase 30% if $\gamma = 1$, in contradiction with the experimental data. Our final results are $\mu_\Omega = -3.41$ in units of $e/2m_\Omega$ and $\mu_{\Sigma^0\Lambda} = 1.85$ in units of $e/2\bar{m}$, where $e/2m_B$ is a natural unit in QCD sum rule analyses. In unit of nuclear magneton $\mu_\Omega = -1.92\mu_N$ and $\mu_{\Sigma^0\Lambda} = 1.5\mu_N$, in good agreement with the recent experimental data.

Baryon magnetic moments are important physical observables as masses. The method of QCD sum rules in the presence of an external electromagnetic field was successfully employed to calculate the octet baryon magnetic moments. The results are in reasonable agreement with the experimental data. In this work we have extended the same method to calculate the magnetic moment of the long-lived decuplet member, the Ω and $\Sigma^0\Lambda$ transition magnetic moments simultaneously, which may serve both as a consistency check of the various susceptibilities and a check of the method of the external field itself. Our results are in good agreement with the recent experimental data.

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