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Inelastic rescattering and CP asymmetries in $D\rightarrow\pi^+\pi^-$, $\pi^0\pi^0$

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We study direct CP violation induced by inelastic final state interaction rescattering in $D\rightarrow\pi\pi$ modes, and find that the resultant CP asymmetry is about 10^{-4} which is larger than ϵ' in the K system. Our estimation is based on well-established theories and experimentally measured data, so there are almost no free parameters except the weak phase δ_{13} in the CKM matrix.
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I. INTRODUCTION

The study of CP violation phenomena is so important for understanding the mechanisms in particle physics that the subject has attracted great attention from both experimentalists and theoreticians of high energy physics for several decades already. The first and so far only observation of CP violation is the measurement of ϵ in the neutral K system, as $\epsilon\sim2.3\times10^{-3}$ [1]. The nonzero ϵ is due to mixing of $K^0-\overline{K^0}$ through box diagrams, which is theoretically evaluated at the quark level, and the result predicts an approximate mass of the charmed quark [2]. However, the parameter ϵ' which is related to processes such as direct CP violation has not been reliably measured yet; it is believed that ϵ'/ϵ is nonzero and is expected to be between 10^{-4} and 3×10^{-3} [1].

In the well-known theory, the mechanism resulting in a nonzero ϵ' is due to the interference of two pions in neutral K decays [3], and all the details are given in Ref. [4]. Since 2π can be in both $I=0$ and 2 isospin eigenstates, if the two isospin channels have different weak and strong phases, their interference would result in a CP asymmetry proportional to $\sin(\delta_0-\delta_2)\sin(\theta_0-\theta_2)$ where δ_I is the weak phase and θ_I corresponds to the phase shifts emerging in the strong interaction rescattering processes of $I=0$ and 2 channels, respectively.

Generally, direct CP violation must be realized through interferences between at least two amplitudes having different weak and strong phases, no matter at quark or hadron levels. The weak phase is determined by an underlying theory, for example, the Cabibbo-Kobayashi-Maskawa (CKM) theory [5], two-Higgs-doublet model [6], etc., while the strong phase can be either produced in the strong scattering at the hadron level as in the K system [4] or may occur at the quark level. If an absorptive part of loops exists, the strong phase is nonzero. For instance, in high energy processes, B physics [7], top quark physics [8], and high energy collisions [9], the main part of the strong phases comes from the absorptive part of loops, even though in the hadronization process, the hadron rescattering can also cause a strong phase.

As the observation concerns hadron products, the strong final state interaction is not negligible [10] and CP asymmetry may occur due to phase shifts in the rescattering. Recently, by studying the single pion exchange inelastic final state interaction (FSI) for $D\rightarrow VP$ processes, we pointed out that the inelastic strong FSI due to t-channel particle exchange may play important roles for producing CP violation in D and B hadronic decays [11]. A very recent estimation by Blok, Gronau, and Rosner [12] shows that the inelastic FSI for $B\rightarrow\pi\pi$, $K\overline{K}$ may produce CP asymmetry as large as 10–20 %, disregarding time-dependent B-$\overline{B}$ mixing effects. Because of the Glashow-Iliopoulos-Maiani (GIM) mechanism, the D^0-$\overline{D^0}$ mixing is small compared to B^0-$\overline{B^0}$ [13,14], even though models beyond the three-generation standard model may result in a larger mixing effect [15]. Recently, Browder and Pakvasa [16] restudied experimental implications of large CP violation and final state interactions (FSI's) in a search for D^0-$\overline{D^0}$ mixing and they concluded that the FSI is important. Close and Lipkin [17] studied a possible strong FSI due to exotic resonances in D exclusive decays. They constrain their analysis at the most Cabibbo favored channels where the rescattering is elastic only. Obviously, it would be interesting to study CP asymmetry in the D system due to the inelastic strong FSI.

In $D\rightarrow\pi\pi$ decays, there are both elastic and inelastic rescatterings, in general, and amplitudes of $D\rightarrow\pi\pi$ should be

$$T(D\rightarrow f)=T^{tree}(D\rightarrow f)+T^{FSI}(D\rightarrow f) \tag{1}$$

and

$$T^{FSI}=i\sum_n \langle f|T|n\rangle\rho_n\langle n|H_{eff}|D\rangle, \tag{2}$$

where $|n\rangle$ is a complete set of the strong interaction states satisfying four-momentum conservation, and ρ_n is the density of states $|n\rangle$. At the $\sqrt{s}=M_D$ energy region, the most important intermediate states are $\pi\pi,K\overline{K},\rho\rho,K^*\overline{K^*}$, $a_1\pi,a_2\pi,K_1K$. These intermediate mesons can be on their mass shell, thus real particles. The off-shell contribution can be attributed to the quark level because the intermediate particles are virtual.

In our present paper we only consider the elastic $\pi\pi\to\pi\pi$ and the inelastic $K\bar{K}\to\pi\pi$ rescattering processes which have experimental measurements [18] at the energy of the D meson mass and therefore are well constrained. This is different from the B meson case, where the inelastic scattering $\pi\pi\rightleftharpoons K\bar{K}$ amplitudes can be only estimated by theoretical models [12]. Contributions from other intermediate states may modify the results by adding a factor of around unity, but are unlikely to change the whole scenario and order of magnitude of the CP asymmetry.

Considering the direct tree level transition amplitude, $\pi\pi\to\pi\pi$ and $K\bar{K}\to\pi\pi$ FSI, we have

$$T(D\to\pi\pi)\equiv T^{tree}(D\to\pi\pi)+T^{FSI}(D\to\pi\pi\to\pi\pi)+T^{FSI}(D\to K\bar{K}\to\pi\pi). \tag{3}$$

Here only the inelastic rescattering from $K\bar{K}$ intermediate states can induce a direct CP violation, but not the elastic ones. Since the tree amplitude of $D^0\to\pi\pi$ and $K\bar{K}$ have different weak phases $Arg(V_{cd}^*V_{ud})$ and $Arg(V_{cs}^*V_{us})$, respectively, and the phase shifts in the inelastic rescattering $K\bar{K}\rightleftharpoons\pi\pi$ are nonzero, the interference between the two parts $T^{tree}(D\to\pi\pi)$ and $T^{FSI}(D\to K\bar{K}\to\pi\pi)$ would result in a nonzero CP violation. The contribution of elastic scattering does not change the weak phase of $T^{tree}(D\to\pi\pi)$, but they can cause a strong phase shift to T^{tree}. Namely,

$$\tilde{T}^{tree}(D\to\pi\pi)=T^{tree}(D\to\pi\pi)+T^{FSI}(D\to\pi\pi\to\pi\pi)=T^{tree}\times fe^{i\theta},$$

where f is a scattering probability amplitude.

Moreover, both $D\to K\bar{K}$ and $D\to\pi\pi$ are Cabibbo suppressed modes, and so their tree amplitudes have the same order of magnitude. Even though one expects that the FSIs may change their relative ratios somehow [19], the order remains the same. Their interference may be large, since two parts suffer the same Cabibbo suppression.

Our numerical results show that the CP asymmetry can be about 10^{-4}.

In the next section, we present the formulation for evaluating the CP asymmetries, in Sec. III we give the numerical results, and Sec. IV is devoted to our conclusion and discussion.

II. FORMULATION

A. Tree level amplitudes

The effective Hamiltonians for $c\to d+u+\bar{d}$ and $c\to s+u+\bar{s}$ are [20]

$$H_{eff}^{(1)}=\frac{G_F}{\sqrt{2}}V_{cd}^*V_{ud}[a_1\bar{d}\gamma_\mu(1-\gamma_5)c\,\bar{u}\gamma^\mu(1-\gamma_5)d+a_2\bar{d}\gamma_\mu(1-\gamma_5)d\,\bar{u}\gamma^\mu(1-\gamma_5)c]+\text{H.c.} \tag{4}$$

and

$$H_{eff}^{(2)}=\frac{G_F}{\sqrt{2}}V_{cs}^*V_{us}[a_1\bar{s}\gamma_\mu(1-\gamma_5)c\,\bar{u}\gamma^\mu(1-\gamma_5)s+a_2\bar{s}\gamma_\mu(1-\gamma_5)s\,\bar{u}\gamma^\mu(1-\gamma_5)c]+\text{H.c.}, \tag{5}$$

where the color indices are omitted and a_1,a_2 are parameters,

$$a_1=c_1+\xi_1c_2,\quad a_2=c_2+\xi_2c_1 \tag{6}$$

and

$$\xi_1\equiv\frac{1}{N_c}+\frac{r_1}{2},\quad \xi_2\equiv\frac{1}{N_c}+\frac{r_2}{2}, \tag{7}$$

where r_1 and r_2 correspond to a nonfactorizable contribution of the hadronic matrix elements $\langle\lambda^a\lambda^a\rangle$ [21],

$$c_1=\frac{c_++c_-}{2},\quad c_2=\frac{c_+-c_-}{2}, \tag{8}$$

where $c_\pm$ can be derived with the renormalization group equation, numerically at the energy scale of the charm quark,

$$c_1\simeq1.26,\quad c_2\simeq-0.51.$$

By fitting data, Cheng obtained $r_1\simeq r_2\sim-0.67$ for $D\to PP$ decay [21]. Then, in our later calculations, we use $a_1=1.26$ and $a_2=-0.51$.

As many authors suggested, we can ignore the contributions from the W exchange and annihilation quark diagrams [22], and so the amplitude at the tree level can be obtained with the vacuum saturation approximation and nonfactorization effects are absorbed into the parameters r_1 and r_2. We have

$$\begin{aligned}\langle\pi^+\pi^-|H_{eff}^{(1)}|D^0\rangle&=\frac{G_F}{\sqrt{2}}V_{cd}^*V_{ud}a_1f_\pi p_\pi^\mu\times\langle\pi^-|\bar{d}\gamma_\mu(1-\gamma_5)c|D^0\rangle\\&=\frac{G_F}{\sqrt{2}}V_{cd}^*V_{ud}a_1f_\pi[f_+^{D\pi}(M_D^2-m_\pi^2)+f_-^{D\pi}m_\pi^2],\end{aligned} \tag{9}$$

where $f_\pm^{D\pi}$ are the form factors in the D to π transition. With the multipole approximation [23]

$$f_+(q^2)=F_1(q^2)=F_1(q_m^2)\left(\frac{(M_{D*}^2-q_m^2)}{(M_{D*}^2-q^2)}\right)^n, \tag{10}$$

$$f_-(q^2)=-\left(\frac{M_D^2-q^2}{M_D^2}\right)F_1(q^2), \tag{11}$$

where $M_{D*}=2.010$ GeV is the nearest pole, $q_m^2\equiv=(M_D-m_\pi)^2$. In the following, we take the single pole approximation as $n=1$ in Eq. (10).

For $D^0\to\pi^0\pi^0$ and $D^0\to K^+K^-$ we have similar expressions. Ignoring the W exchange and annihilation, the tree amplitude for $D^0\to K^0\overline{K^0}$ is zero and the transition can only be realized through elastic and inelastic FSI rescattering.

The calculated $f_\pm^{DK}$ values have been checked by using the method given in Ref. [24] and found they coincide with each other very well. But Roberts' parameters [24] are obtained by fitting the data of the $D\to K$ transition only, and so the results of $f_\pm^{D\pi}$ obtained in the multipole approximation deviate from that calculated in terms of the parameters of Ref. [24] assuming an SU(3) symmetry. Therefore we later take only the values $f_\pm^{D\pi}$ obtained in the multipole approximation.

B. Elastic and inelastic FSI's

The S matrix for the strong interaction is

$$s_{mn}=\delta_{mn}+2i\sqrt{\rho_m}T_{mn}\sqrt{\rho_n}, \tag{12}$$

where the T matrix is the nontrivial part determined by the strong interaction Lagrangian.

It is noted that the δ_{mn} term corresponds to a no-interaction scattering transition (or the trivial part of the S matrix), and so is exactly the ''tree'' part of Eq. (1). For the elastic and the inelastic rescattering contributions, the amplitudes read

$$T^{FSI}(D^0\to\pi^+\pi^-\to\pi^+\pi^-)=i\langle\pi^+\pi^-|T|\pi^+\pi^-\rangle \times\rho_\pi\langle\pi^+\pi^-|H_{eff}^{(2)}|D^0\rangle, \tag{13}$$

$$T^{FSI}(D^0\to\pi^0\pi^0\to\pi^+\pi^-)=i\langle\pi^+\pi^-|T|\pi^0\pi^0\rangle \times\rho_\pi\langle\pi^0\pi^0|H_{eff}^{(2)}|D^0\rangle, \tag{14}$$

$$T^{FSI}(D^0\to K^+K^-\to\pi^+\pi^-)=i\langle\pi^+\pi^-|T|K^+K^-\rangle \times\rho_K\langle K^+K^-|H_{eff}^{(2)}|D^0\rangle. \tag{15}$$

With help of the isospin analysis, for the elastic scattering,

$$\rho_\pi\langle\pi^+\pi^-|T|\pi^+\pi^-\rangle=\frac{2}{3}T_0e^{i\theta_0}+\frac{1}{3}T_2e^{i\theta_2}, \tag{16}$$

$$\rho_\pi\langle\pi^+\pi^-|T|\pi^0\pi^0\rangle=\sqrt{\frac{2}{3}}(-T_0e^{i\theta_0}+T_2e^{i\theta_2}), \tag{17}$$

where T_0,θ_0 and T_2,θ_2 are the measured scattering amplitudes and phase shifts of $I=0$ and $I=2$ channels [18], respectively. The transitions to the $\pi^0\pi^0$ final state have similar expressions.

The contributions from the elastic FSI of $\pi\pi\to\pi\pi$ can be absorbed into the tree amplitudes; in our case they do not provide a different weak phase from the tree amplitudes, but result in a strong phase shift. Including the elastic scattering, the amplitudes for $D^0\to\pi^+\pi^-$ and $\pi^0\pi^0$ can be written as

$$\begin{aligned}\widetilde{T}^{tree}(D^0\to\pi^+\pi^-)&\equiv T^{tree}(D^0\to\pi^+\pi^-)+T^{FSI}(D^0\to\pi^+\pi^-\to\pi^+\pi^-)+T^{FSI}(D^0\to\pi^0\pi^0\to\pi^+\pi^-)\\&=\frac{G_F}{\sqrt{2}}V_{cd}^*V_{ud}f_\pi[f_+^{D\pi}(M_D^2-m_\pi^2)+f_-^{D\pi}m_\pi^2]\left[a_1+\left(\frac{2}{3}a_1-\sqrt{\frac{2}{3}}a_2\right)iT_0e^{i\theta_0}+\left(\frac{1}{3}a_1+\sqrt{\frac{2}{3}}a_2\right)iT_2e^{i\theta_2}\right],\end{aligned} \tag{18}$$

$$\begin{aligned}\widetilde{T}^{tree}(D^0\to\pi^0\pi^0)&\equiv T^{tree}(D^0\to\pi^0\pi^0)+T^{FSI}(D^0\to\pi^+\pi^-\to\pi^0\pi^0)+T^{FSI}(D^0\to\pi^0\pi^0\to\pi^0\pi^0)\\&=\frac{G_F}{\sqrt{2}}V_{cd}^*V_{ud}f_\pi[f_+^{D\pi}(M_D^2-m_\pi^2)+f_-^{D\pi}m_\pi^2]\left[a_2+\left(\frac{1}{3}a_2-\sqrt{\frac{2}{3}}a_1\right)iT_0e^{i\theta_0}+\left(\frac{2}{3}a_2+\sqrt{\frac{2}{3}}a_1\right)iT_2e^{i\theta_2}\right],\end{aligned} \tag{19}$$

where the notation $\widetilde{T}^{tree}$ refers to the tree amplitude modified by the elastic scattering and T_i, θ_i are measured values.

For the inelastic scattering, $K\overline{K}\to\pi\pi$, there are experimental measurements [18] We can decompose K^+K^- and $\pi^+\pi^-$ in terms of the basis of isospin as

$$|K^+K^-\rangle=\frac{1}{\sqrt{2}}[|1,0\rangle+|0,0\rangle]_K, \tag{20}$$

$$|\pi^+\pi^-\rangle=\left[\sqrt{\frac{2}{3}}|0,0\rangle+\sqrt{\frac{1}{3}}|2,0\rangle\right]_\pi. \tag{21}$$

Thus

$$\begin{aligned}T^{inelastic}(D^0\to\pi^+\pi^-)&\equiv T^{FSI}(D^0\to K^+K^-\to\pi^+\pi^-)\\&=\frac{G_F}{\sqrt{2}}V_{cs}^*V_{us}f_Ka_1[f_+^{DK}(M_D^2-m_K^2)\\&\quad+f_-^{DK}m_K^2]i\sqrt{\frac{1}{3}}T_Ke^{i\theta_K}\sqrt{\rho_K/\rho_\pi},\end{aligned} \tag{22}$$

where $\rho_K/\rho_\pi=\sqrt{1-4m_K^2/M_D^2}/\sqrt{1-4m_\pi^2/M_D^2}$.

Similar expressions can be easily derived for $T^{FSI}(D^0 \to K^+K^- \to \pi^0\pi^0)$.

C. CP violation

As long as there are more than two channels with different weak and strong phases, their interferences can result in direct CP violation. If the total transition amplitude T is a superposition of two independent amplitudes A_1 and A_2,

$$T = A_1 e^{i\delta_1} e^{i\phi_1} + A_2 e^{i\delta_2} e^{i\phi_2}, \tag{23}$$

while its CP conjugate amplitude is

$$\overline{T} = A_1 e^{-i\delta_1} e^{i\phi_1} + A_2 e^{-i\delta_2} e^{i\phi_2}. \tag{24}$$

Thus a direct CP asymmetry is defined as

$$\begin{aligned} R &= \frac{|T|^2 - |\overline{T}|^2}{|T|^2 + |\overline{T}|^2} \\ &= \frac{2A_1A_2 \sin(\delta_1 - \delta_2)\sin(\phi_1 - \phi_2)}{A_1^2 + A_2^2 + 2A_1A_2\cos(\delta_1-\delta_2)\cos(\phi_1-\phi_2)}. \end{aligned} \tag{25}$$

In our case of $D \to \pi^+\pi^-(\pi^0\pi^0)$, the two interfering amplitudes are the modified "tree" part $\tilde{T}^{tree}(D \to \pi\pi)$, Eq. (18), and the pure "inelastic FSI" part $T^{inelastic}$ given in Eq. (22), which have different weak phases δ_i and strong phases ϕ_i. For $D^0 \to \pi^0\pi^0$ the expressions are similar.

Let us work within the framework of the CKM matrix [1],

$$V_{ud}V_{cd}^* = (c_{12}c_{13})(-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}})^*, \tag{26}$$

$$V_{us}V_{cs}^* = (s_{12}c_{13})(c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}})^*. \tag{27}$$

Thus

$$\delta^{D\to\pi} = \arctan\frac{-c_{12}s_{23}s_{13}\sin\delta_{13}}{s_{12}c_{23} + c_{12}s_{23}s_{13}\cos\delta_{13}}, \tag{28}$$

$$\delta^{D\to K} = \arctan\frac{s_{12}s_{23}s_{13}\sin\delta_{13}}{c_{12}c_{23} - s_{12}s_{23}s_{13}\cos\delta_{13}}. \tag{29}$$

In $\tilde{T}^{tree}(D^0 \to \pi^+\pi^-) \equiv T^{tree}(D^0 \to \pi^+\pi^-) + T^{FSI}(D^0 \to \pi\pi \to \pi^+\pi^-)$, the weak phase is that of the tree amplitude as $\delta^{D\to\pi}$, while that of $T^{FSI}(D^0 \to K^+K^- \to \pi^+\pi^-)$ is $\delta^{D\to K}$; therefore, they are different. One can also notice that $|\delta^{D\to K}| \ll |\delta^{D\to\pi}|$ in this convention, even though the final result is independent of the convention adopted in the calculations.

Later we will evaluate

$$R_1 \equiv \frac{\Gamma(D^0 \to \pi^+\pi^-) - \Gamma(\overline{D^0} \to \pi^+\pi^-)}{\Gamma(D^0 \to \pi^+\pi^-) + \Gamma(\overline{D^0} \to \pi^+\pi^-)}, \tag{30}$$

$$R_2 \equiv \frac{\Gamma(D^0 \to \pi^0\pi^0) - \Gamma(\overline{D^0} \to \pi^0\pi^0)}{\Gamma(D^0 \to \pi^0\pi^0) + \Gamma(\overline{D^0} \to \pi^0\pi^0)}. \tag{31}$$

In the next section, we will present our numerical results of R_1 and R_2.

III. NUMERICAL RESULTS

(i) All the CKM entries involved in our calculations have been measured, even though there are some uncertainties [1]. Thus with the optimistic $\sin\delta_{13} = 1$, we have

$$\delta^{D\to\pi} \simeq 3.2\times10^{-4} - 1.1\times10^{-3}, \quad \delta^{D\to K} \simeq 5.3\times10^{-5}.$$

Later we will use the most favorable values for the CP violation calculations.

(ii) We obtain, in terms of the dipole approximation,

$$f_+^{D\pi}(q^2 = m_\pi^2) \approx 0.8, \quad f_-^{D\pi}(q^2 = m_\pi^2) \approx -0.8,$$

$$f_+^{D}(q^2 = m_K^2) \approx 0.7, \quad f_-^{DK}(q^2 = m_K^2) \approx -0.65.$$

(iii) For the elastic and inelastic scattering $\pi\pi \rightleftharpoons \pi\pi$, $K\overline{K} \rightleftharpoons \pi\pi$, the transition probability amplitude T and the phase shift θ are experimentally measured [18] as

$$T_0(\pi\pi \rightleftharpoons \pi\pi) \approx -0.48, \quad \theta_0 \approx 308°,$$

$$T_2(\pi\pi \rightleftharpoons \pi\pi) \approx -0.45, \quad \theta_2 \approx -50°,$$

$$T^{inelastic} = T_K(K\overline{K} \rightleftharpoons \pi\pi) \approx 0.1, \quad \theta_K(K\overline{K} \to \pi\pi) \approx 310°,$$

for the energy range of M_D. So by the notation of Eq. (25) $\sin(\delta_1 - \delta_2) \sim -1.1\times10^{-3}$, while $\phi_2 \approx 310°$, but ϕ_1 of Eq. (25) must be evaluated by Eqs. (18) and (19), in our case.

(iv) The CP asymmetries. With the information given above, we obtain

$$R_1 = -1.1\times10^{-4}, \tag{32}$$

$$R_2 = 2.2\times10^{-4}. \tag{33}$$

In these calculations we almost do not have any free parameters, except the CKM phase δ_{13}. We have taken $\sin\delta_{13} = 1$ and neglected the contribution from other intermediate states. They may modify the results, but are unlikely to change the whole scenario and order of magnitude of the CP asymmetry. It is noted that R_1 and R_2 have opposite signs; as a matter of fact, due to the uncertainty of δ_{13} in the CKM matrix, the absolute sign of R_i is not important, but only the relative sign is meaningful.

IV. CONCLUSION AND DISCUSSION

In this work, we discuss the CP violation effects caused by the inelastic FSI rescattering in $D \to \pi^+\pi^-$, $\pi^0\pi^0$ modes and obtain the CP asymmetry ratios of order 10^{-4}.

(i) The observed indirect CP violation in the K system which is characterized by ϵ is of order 10^{-3}. Even though the direct CP violation ϵ'/ϵ has not been reliably measured yet, present data incline to confirm it to be $(1.5\pm0.8)\times10^{-3}$. Anyhow, one has many reasons to believe it is nonzero and of order 10^{-4}–3×10^{-3} [1]; namely, the superweak mechanism is almost ruled out by experiments. Our estimation of $D \to \pi\pi, K\overline{K}$ shows that the direct CP asymmetries here are about 2 orders of magnitude larger than ϵ'.

(ii) In our expression, one can see that direct CP violation is caused by the interference between the tree amplitude modified by the elastic rescattering (denoted as $\widetilde{T}^{tree}$ in this paper) and that induced by the inelastic FSI rescattering while the two parts have different weak and strong phases. The interference is proportional to a product of the two amplitudes $|\widetilde{T}^{tree}||T^{FSI}|$ and the differences of weak and strong angles $|\sin(\delta^{D\pi} - \delta^{DK})\sin(\phi_1 - \phi_2)|$. There are two factors which suppress the CP asymmetry values.

The first one is that in the framework of the CKM matrix; the weak phase is about order 10^{-3} which is independent of convention.

The second suppression factor comes from the measured strong phase difference $|\sin(\phi_1 - \phi_2)| = 0.3$ and amplitude $T_K(K\overline{K} \to \pi\pi) \sim 0.1$.

As discussed in Refs. [11,19], the final state interaction effects can be described in a hadronic triangle diagram and the absorptive part of the loop gives rise to a strong phase. In our previous work, we only estimated the absorptive part and stressed that the inelastic FSI is important in many processes, so that it cannot be ignored. The real part is hard to evaluate properly because of the ultraviolet divergence and the obtained results would depend on the renormalization scheme. In this work, as suggested in the literature [25], we only deal with the FSI; namely, the intermediate hadrons are real particles, i.e., on their mass shell. The dispersive part of the loop is small compared to the tree amplitude, and in fact, its contribution is effectively absorbed into the phenomenological parameters a_1, a_2 which may slightly deviate from the values derived in terms of the renormalization group equation, in our case (note that it is not always true). Thus, we use only the data directly obtained from corresponding experiments, and so can avoid any ambiguity caused by theoretical uncertainties.

(iii) Since the proposed channels to be observed are Cabibbo suppressed, the decay rates could be smaller than the Cabibbo favored channels by $\sin^2\theta_C$ roughly.

(iv) The branching ratio of $D \to K\overline{K}, \pi\pi$ is about 2×10^{-3}. The production cross section σ of $D^0\overline{D^0}$ is measured at BEPC [26],

$$\sigma(D^0\overline{D^0}) = 11.63 \pm 1.1 \ \text{nb} \quad \text{at BEPC energy.}$$

Taking the most optimistic values evaluated in the framework of CKM theory, the number of events for observing the Cabibbo suppressed decay channels $D \to K\overline{K}, \pi\pi$ would be

$$N = L \times 2 \times 10^{-3} \times \sigma \times \tau \times f, \tag{34}$$

where L is the luminosity, τ is the measuring time period, and f is the observation efficiency.

For the proposed charm-tau factory, L can reach 10^{34} $\text{cm}^{-2}\,\text{sec}^{-1}$, and so

$$N \approx 7.3 \times 10^6 \times f \times n,$$

where n is the number of necessary years. Since the CP asymmetry is in the range of about 10^{-4}, and so to the reasonable statistical level for observation of CP violation, N at least must be 10^7–10^8; it would need a charm-tau factory with a luminosity of 10^{34} $\text{cm}^{-2}\,\text{sec}^{-1}$ to run for 15 years. Even though this number is not very encouraging, as suggested by Browder and Pakvasa [16], if there is new physics which can provide us with a larger weak phase, the observation becomes very possible. Even with this small CP asymmetry, there is still the possibility to make the measurement.

Therefore, for measuring direct CP violation which is one of the main interests in the field of high energy physics, a high luminosity τ-charm factory would be extremely helpful.

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