Gravitational violation of *R* parity and its cosmological signatures

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Discrete R parity (R_P) is usually imposed in the minimal supersymmetric standard model (MSSM) as an unbroken symmetry. In this paper we study very weak gravitationally induced R-parity breaking, described by nonrenormalizable terms inversely proportional to the Planck mass. The lightest supersymmetric particle, a neutralino, is unstable but its lifetime exceeds the age of the Universe and thus it can serve as a dark matter (DM) particle. The neutralino lifetime is severely constrained from below due to the production of positrons and antiprotons, diffuse gamma radiation, etc. The violation of R_P generated gravitationally by dimension-five operators in the MSSM is shown to violate these constraints if they are suppressed only by the Planck scale. A general theoretical analysis of gravitationally induced R_P violation is performed and two plausible and astrophysically consistent scenarios for achieving the required suppression are identified and discussed. [S0556-2821(97)02223-6]

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I. INTRODUCTION

A discrete symmetry, called R parity (R_P) is usually imposed [1] on the minimal supersymmetric standard model (MSSM) [2]. This assumption makes the lightest supersymmetric particle (LSP) stable. The most natural candidate for the LSP in the MSSM is a neutralino. Indeed, in the MSSM with soft supersymmetry- (SUSY-)breaking terms and radiatively induced electroweak symmetry-breaking calculations one can show that one of the neutralinos is the LSP in a wide range of allowed parameters. Moreover, the relic density of neutralinos in this scenario satisfies the requirements for cold dark matter density in large areas of SUSY parameter space [3].

The hypothesis of the neutralino as a dark matter (DM) particle is amenable to experimental verification [3]. The neutralinos can be detected directly through their elastic scattering off nuclei [4]. The annihilation of neutralinos can produce remarkable indirect signals in the form of high-energy neutrino radiation from the Sun and Earth [5], in the form of galactic antiprotons and positrons [6], and some others.

If R_P is very mildly broken instead of being exactly conserved then the LSP can be a DM particle, but an unstable one. Naively one would expect that a neutralino with a lifetime of the order of the age of the Universe could provide the DM. But specific neutralino decay channels, e.g., containing positrons, antiprotons, pions, are severely constrained from observations. They typically require the neutralino lifetime to be much larger than the age of the Universe [7].

While it is possible for R_P to remain unbroken on technical grounds, there is no deep theoretical reason for R_P to be a symmetry of nature. In fact many models of R_P violation have been proposed [8] but in the absence of fine-tuning they lead to large R_P violation inconsistent with the LSP as a dark matter particle. Only very weak violation of R_P can make the decaying neutralino a realistic dark matter particle. Such a possibility was studied in [9] in the context of a specific mechanism [10] of spontaneous R_P violation through a right handed sneutrino vacuum expectation value (VEV) close to the weak scale and a very tiny $\nu^c \ell H_2$ Yukawa coupling.

A more natural possibility is given by R_P violation due to gravitational effects. In fact, it is well known that quantum gravity effects, associated with worm holes or ''virtual'' black holes violate all nongauge symmetries including the discrete ones and R_P in particular [11–17]. In this paper we shall describe a gravitational breaking of R_P by nonrenormalizable terms inversely proportional to the Planck mass. We shall discuss also the relevance of these terms to the wormhole effects.

Although the gravitational violation of R_P seems to be a realistic possibility, one cannot ignore the alternative case of exactly conserved R_P in the presence of all gravitational effects. Such theories were indeed constructed [18–22]. A discrete symmetry is respected by all interactions including quantum gravity if it is a remnant of a spontaneously broken gauge symmetry. In the case of R_P, the matter parity $[\equiv(-1)^{3(B-L)}]$ forms a discrete subgroup of the gauged B-L symmetry. Hence in the presence of a gauged B-L symmetry, R_P could arise as a discrete gauge symmetry [21,22] in the low-energy theory if the breaking of B-L is accomplished by a Higgs fields with appropriate values of B-L [19,20]. In this case R_P is exactly conserved.

Another example is given [23] in a $SU(5)\times SU(5)$ model where R_P is conserved in the presence of nonrenormalizable Planck scale terms.

In all these examples matter parity is conserved and one has a standard neutralino as a stable dark matter particle.

II. ASTROPHYSICAL CONSTRAINTS ON R_P VIOLATION

Let us first quantitatively discuss constraints on the amount of R_P violation. We can parametrize the effective R_P violating interactions responsible for neutralino decay in terms of the MSSM fields as follows:

$$W_{\rm eff}=\lambda_1(U^cD^cD^c)_F+\lambda_2(LLE^c)_F+\lambda_3(QD^cL)_F+\epsilon(LH_2)_F. \tag{1}$$

The notation for the fields is standard. Each of the λ_i has three generation indices, which for simplicity we have suppressed. Equation (1) includes renormalizable terms relevant for neutralino decay. However, as will be understood from the discussion below, this expression has wider generality.

The lightest superposition of w-ino $\widetilde{W}$, b-ino $\widetilde{B}$, and two Higgsinos, $\widetilde{H}_1$ and $\widetilde{H}_2$ is associated with the DM particle χ in the usual way:

$$\chi=Z_{\chi\widetilde{W}_3}\widetilde{W}_3+Z_{\chi\widetilde{B}}\widetilde{B}+Z_{\chi\widetilde{H}_1}\widetilde{H}_1+Z_{\chi\widetilde{H}_2}\widetilde{H}_2. \tag{2}$$

The above interactions result in neutralino decay to three fermions. The width for this decay depends upon whether it proceeds through the Higgsino or gaugino component. In the former case,

$$\Gamma_\chi=\lambda_i^2Z_{\chi\widetilde{H}}^2\frac{G_Fm_f^2}{192(2\pi)^3}\frac{m_\chi^5}{\widetilde{m}_f^4}, \tag{3}$$

where m_χ, $\widetilde{m}_f$, and m_f are masses of the neutralino, sfermion, and fermion, respectively. In the case of quarks the width should be multiplied by the number of colors. When the decay proceeds through the b-ino component $\widetilde{B}$ of the neutralino the decay width is

$$\Gamma_\chi=\lambda_i^2Z_{\chi\widetilde{B}}^2\frac{\alpha_{em}Y_{f_R}^2}{192(2\pi)^2\cos^2\theta_W}\frac{m_\chi^5}{\widetilde{m}_f^4}, \tag{4}$$

where Y_{f_R} is hypercharge of the right fermion and θ_W is the Weinberg angle.

Finally, the width of $\chi\rightarrow\nu+e^++e^-$ due to the last term of Eq. (1) is

$$\Gamma_\chi=\epsilon^2Z_{\chi\widetilde{H}}^2\left(\frac{1}{4}+\sin^2\theta_W+\frac{4}{3}\sin^4\theta_W\right)\frac{G_F^2m_\chi^3}{192\pi^3}. \tag{5}$$

Constraints on λ_i and ϵ come from the condition $\tau_\chi>t_0$, where t_0 is the age of the Universe. However, much more stringent limits follow from production of positrons in our Galaxy [7,9], from diffuse gamma-radiation [7,9], and from neutrino-induced muons [24]. From the analysis presented in [9] it follows that when the decay to positrons is unsuppressed as in the present case, the strongest constraints on both λ_i and ϵ follow from the observed flux of positrons in our Galaxy. The lower limit on neutralino lifetime from this flux is [9]

$$\tau_\chi(\chi\rightarrow e^++\text{anything})>7\times10^{10}\sqrt{m_{100}}t_0h, \tag{6}$$

where $m_{100}=m_\chi/100$ GeV and h is dimensionless Hubble constant.

Using this limit and keeping in mind indirect production of positrons through decay of other particles, we obtain the following constraints on λ_i and ϵ:

$$\lambda<4\times10^{-21}Z_{\chi\widetilde{H}}^{-1}\left(\frac{\widetilde{m}_f}{1\ \text{TeV}}\right)^2\left(\frac{100\ \text{GeV}}{m_\chi}\right)^{9/8}\left(\frac{1\ \text{GeV}}{m_f}\right)^{1/2},$$

$$\epsilon<6\times10^{-23}Z_{\chi\widetilde{H}}^{-1}m_{100}^{-7/4}\ \text{GeV}. \tag{7}$$

III. DIFFICULTIES WITH GRAVITATIONAL VIOLATION OF R PARITY

The above estimates demonstrate that if the neutralino is a DM particle the R-parity-violating parameters are very strongly limited from above. With the superpotential terms given explicitly by Eq. (2) this implies a fine-tuning. However, if R parity is broken by gravitational effects these parameters may result from the effective nonrenormalizable interactions and therefore can be very small.

Let us start with a systematic analysis of nonperturbative gravitationally induced R-parity-violating operators in the MSSM. It is convenient to perform this analysis in terms of effective R_P-violating operators of different dimensions.

The first terms in the expansion of R-parity-breaking operators in powers of the Planck mass $M_{\rm Pl}$ have dimension $d\leqslant4$. They are given by Eq. (2) and we already discussed them. Very small coupling constants needed for the LSP as DM particle are extremely artificial.

The next terms in the expansion are proportional to $M_{\rm Pl}^{-1}$ and they are given by the following $d=5$ operators:[1]

$$\frac{\beta_1}{M_{\rm Pl}}(H_1H_2^*E^c)_D,\quad \frac{\beta_2}{M_{\rm Pl}}(QL^*U^c)_D,$$

$$\frac{\beta_3}{M_{\rm Pl}}(U^cD^{c*}E^c)_D,\quad \frac{\beta_4}{M_{\rm Pl}}(LH_2H_1H_2)_F. \tag{8}$$

The first three terms in the above equation contain the auxiliary F terms of the antichiral fields H_2^*, L^*, and D^{c*}, respectively. These are determined in the supersymmetric limit by the standard R_P-conserving superpotential of the MSSM. For example, the dimension-five term associated with the first operator leads to an effective interaction:

$$(H_1H_2^*E^c)_D\sim(\widetilde{H}_1{}^0E^c)(\mu H_1^-+\lambda_u\widetilde{D}\widetilde{U}^c). \tag{9}$$

As long as the charged Higgs H_1^- is heavier than the LSP, the above interaction leads to three-body decays such as $\widetilde{\chi}\rightarrow\ell s\overline{c}$, where $\ell=e,\mu,\tau$. The resulting rate involves

[1]Operators such as $(QH_1QQ)_F$ would lead to lightest neutralino decays only at loop level. Moreover, since this operator violates baryon number rather than lepton numbers, its coexistence with lepton-number-violating operators may be strongly constrained by proton decay.

small Yukawa couplings and can be shown to satisfy the astrophysical constraint Eq. (7) for reasonable values of β_i. Similar considerations also apply to the next two operators in Eq. (8). In contrast, the decay induced by the last term cannot be suppressed kinematically and leads to the effective R_P-breaking operator displayed in Eq. (1) with

$$\epsilon \sim \beta_4 M_W^2 / M_{\rm Pl} \sim \beta_4 10^{-15} \ \text{GeV}.$$

This value of ϵ is extremely small and leads to a LSP lifetime longer than the age of the Universe. But surprisingly enough it is in conflict with the astrophysical constraints (7) by several orders of magnitude, unless the parameter β_4 is suppressed, $\beta_4 \lesssim 10^{-5}$. This situation is similar to the gravitationally induced axion mass [16,17] where the quantum gravitational corrections are not small enough to suppress it adequately.

If $1/M_{\rm Pl}$ terms are forbidden (for example, by some unbroken symmetry), then $1/M_{\rm Pl}^2$ terms ($d=6$ operators) become important. An example of such an operator is

$$\frac{\beta}{M_{\rm Pl}^2}[(LH_2)(H_1H_2)^*]_D. \tag{10}$$

This term gives rise to a neutrino-Higgsino mixing ($\nu \widetilde{H}_2$) with the mixing parameter

$$\epsilon \sim \beta \frac{\langle H_1^* \rangle \langle F_{H_2^*} \rangle + \langle H_2^* \rangle \langle F_{H_1^*} \rangle}{M_{\rm Pl}^2} \sim \beta \frac{\mu M_W^2}{M_{\rm Pl}^2} \sim 10^{-32} \ \text{GeV},$$

which is around 10 orders of magnitude less than needed to produce observable effects.

Therefore, while in MSSM $1/M_{\rm Pl}^2$ terms are too small, the $1/M_{\rm Pl}$ terms are too large and need additional suppression, i.e., small β.

There exist at least two possibilities to obtain the required strong suppression in the coefficient of the dimension-5 operator β.

(1) If wormhole effects are responsible for the terms we are discussing, they can contain a topological suppression leading to very small β [16]. Generically, this suppression is described by factor e^{-S}, where S is an action of a wormhole that absorbs the R_P charge. In the semiclassical approach $S \sim 10$. In particular, for Peccei-Quinn symmetry such estimates give $S \sim \ln(M_{\rm Pl}/f_{\rm PQ}) \approx 16$. It results in the suppression factor $\beta \sim 10^{-7}$, which is needed in our case. A detailed discussion of a suppression factor for wormhole effects is given in [16]. It is shown that the action S is connected with the size of the throat of the wormhole $R(0)$ and can vary from $S \approx 6.7$ for naive estimate $R(0) \approx M_{\rm Pl}^{-1}$, to a very large value $8\pi^2/g_{\rm str}^2 \approx 190$ in string inspired models. Thus the wormhole effects have the suppression needed to ensure a long-lived neutralino $\beta \sim 10^{-7} - 10^{-5}$ in case the action is close to the semiclassical value.

(2) Suppression of dimension-5 operators can occur due to some additional symmetry. Let us assume that there exists a singlet sector that communicates with the MSSM sector only gravitationally through nonrenormalizable terms in the Lagrangian. R parity can be broken spontaneously in this sector, for example, due to some R_P-odd field η developing a nonzero VEV. R_P violation can penetrate the MSSM sector through nonrenormalizable interactions between η and the MSSM fields. In contrast to the first case, gravity is not directly responsible for breaking of R_P but it leads to effective R_P violation in the observable sector through the presence of nonrenormalizable interactions. We shall discuss this possibility first in a model-independent way and then provide an example.

IV. SPONTANEOUS R PARITY VIOLATION IN HIDDEN SECTOR

(1) Let us assume the existence of a singlet field η beyond the MSSM fields and assume that η couples to the MSSM fields only through nonrenormalizable terms. This can be achieved by a proper symmetry as we shall discuss. There are four dimension-5 operators involving η that lead to R_P violation:

$$O_1 = \frac{\alpha_1}{M_{\rm Pl}}(U^c D^c D^c \eta)_F, \quad O_2 = \frac{\alpha_2}{M_{\rm Pl}}(LLE^c \eta)_F,$$

$$O_3 = \frac{\alpha_3}{M_{\rm Pl}}(QD^c H_1 \eta)_F, \quad O_4 = \frac{\alpha_4}{M_{\rm Pl}}(LH_2 \eta^*)_D, \tag{11}$$

where $\alpha_{1,2,3,4}$ are parameters of order one. The operators displayed above conserve R_P if the field η is chosen odd. The vacuum expectation value of η then breaks R_P and leads to effective interactions displayed in Eq. (2) with coupling constants given by ($i=1,2,3$)

$$\lambda_i = \alpha_i \langle \eta \rangle / M_P, \quad \epsilon = \alpha_4 \langle F_{\eta^*} \rangle / M_P. \tag{12}$$

The effective R_P violation among the MSSM fields is governed by two physically distinct scales. The $\langle \eta \rangle$ signifying the R_P violation determines the trilinear interactions of Eq. (1), while the scale of SUSY breaking in the hidden sector determines the bilinear term ϵ. In general, these two scales could be quite different. The constraints derived in Eq. (7) imply

$$\langle \eta \rangle \lesssim 10^{-1} \ \text{GeV}, \quad F_{\langle \eta \rangle *} \lesssim 10^{-2} \ \text{GeV}^2 \tag{13}$$

if $\lambda \lesssim 10^{-20}$ and $\epsilon \lesssim 10^{-21}$ GeV, respectively. If SUSY remains unbroken in the singlet sector then the constraint on ϵ is trivially satisfied. Even when SUSY is broken through the usual soft terms, it is possible to satisfy the constraint on F_η without significant fine-tuning as we shall demonstrate through a specific example. In contrast, the constraint on the trilinear coupling implies a very small VEV for the singlet field, which may be unnatural and one should forbid the corresponding dimension-5 operators in this case. If the dimension-5 terms are absent then the dominant R_P violation would arise from dimension-6 interactions. For example, the operator

$$\frac{1}{M_{\rm Pl}^2}[(lH_2)(H_1H_2)\eta]_F \tag{14}$$

results in $\epsilon \sim 10^{-34} \langle \eta \rangle$ GeV. The effect of this term could be observable provided R_P violation in the singlet sector occurs at a large scale close to the grand unification scale. Let us consider now a specific realization of this scenario.

(2) Two basic ingredients are needed to realize the above scenario. Firstly one needs a symmetry that forbids dangerous R_P-violating terms of dimensionality four and five. Moreover this symmetry needs to be a gauged discrete symmetry in order to prevent the gravitational breaking by dimension-5 terms. The existence of such local symmetry depends clearly on the structure of the theory at high scale, which is unknown. But one must ensure that the symmetry imposed here could arise as a remnant of some gauge symmetry. This is done by imposing the discrete gauge anomaly constraints [21,22].

It is clear that R_P by itself cannot satisfy the above criteria. It can be a gauge symmetry as already mentioned but in the presence of an R_P-odd field η required for spontaneous breaking, R_P cannot prevent a renormalizable coupling like $LH_2\eta$ as required for suppression of R_P breaking. We thus consider an alternative class of symmetry. This corresponds to a Z_N symmetry assumed to act nontrivially on the Grassman variable θ. Such symmetries are already considered in [21] with the idea of forbidding proton decay in the MSSM. Here we consider them with a different motivation and in the context of an extension of the MSSM containing η. The θ is assumed to carry Z_N charge -1. The Z_N charge of one of the observable superfields can be chosen to be zero by appropriate redefinition of the Z_N generators. The charges of the remaining fields are then determined in terms of two parameters (called x and y below) by requiring that the standard R_P-conserving couplings of the MSSM fields are allowed by the Z_N symmetry. The charge assignments of the various fields are given below:

Q	U^c,H_1	D^c,H_2	L	E^c	Y	η
0	x	$2-x$	y	$2-(x+y)$	2	$N/2$

where we have introduced a singlet field Y in addition to η in order to obtain spontaneous R_P violation. Due to the above charge assignments dimension-4 terms respect R_P and the η,Y do not couple to the MSSM fields in the renormalizable Lagrangian as long as $x\neq 2$ and $x-y\neq 0,-2,N/2$. The most general Z_N-invariant renormalizable superpotential in this case can be written as [25]

$$W=W_{\rm MSSM}+\delta Y(\eta^2-f^2). \tag{15}$$

The above superpotential leads to a VEV for η at the supersymmetric minimum. This VEV would lead to effective R_P breaking for the MSSM fields through the operator of dimensionality $\geqslant 5$. The choice $2+y-x=N/2$ allows the dimension-6 operator of Eq. (14). The allowed higher-dimensional terms are given in this case by

$$\mathcal{L}_{\rm NR}=\frac{\beta_5}{M_P}(LH_2\eta^*)_D+\frac{1}{M_P^2}[\delta_1(LLe^c\eta^*)_D + \delta_2(QD^cL\eta^*)_D+\delta_3(LH_2\eta^*Y)_D + \delta_4(LH_2H_1H_2\eta)_F]. \tag{16}$$

Note that the dimension-5 operator displayed above cannot be forbidden if the dimension-6 term in Eq. (14) is to be allowed. But as discussed above it does not lead to large R_P violation as long as SUSY remains unbroken in the singlet sector. This indeed happens with the choice of superpotential as in Eq. (15). In a realistic situation, soft breaking of SUSY can introduce terms that will make $F_{\eta,Y}$ nonzero. If $\langle\eta\rangle$ is of $O(M_{EW})$ or smaller than the conventional soft breaking of SUSY can be shown to lead to

$$F_{\langle\eta\rangle}\sim\frac{2\langle\eta\rangle^3\delta^2A}{m_Y^2},$$

where $m_Y\sim A$ signify soft SUSY breaking. Mild fine-tuning in δ allows one to satisfy constraint (13), e.g., $\delta\sim10^{-2}$, $m_Y\sim10^3$ GeV, $A\sim10^2$ GeV, and $\langle\eta\rangle\leqslant100$ GeV lead to $F_\eta\leqslant10^{-2}$ GeV^2. If $\langle\eta\rangle$ is much larger than the weak scale then F_η would also be large and would induce large ϵ. This can be prevented by means of a symmetry. Specifically, if the kinetic energy terms for the singlet fields are chosen to be no-scale type [26] then $F_{\eta,Y}$ vanish at the minimum of the potential and effective R_P breaking in this case would arise only from the dimension-6 operator. This operator could lead to observable signatures if $\langle\eta\rangle$ is very large, near the GUT scale.

The Z_N introduced above can be a gauge symmetry if it satisfies discrete gauge anomaly constraints. These are discussed in [22] and are given in our case as follows [27]:

$$-2N_g+6=k_1N,\quad N_g(y-4)+4=k_2N, \tag{17}$$

$$N_g(-7+y-x)+N/2-9=k_3N+\kappa k_4N/2, \tag{18}$$

where κ is 1 (0) for even (odd) N and $k_{1,2,3,4}$ are integers. The first constraint is automatically satisfied for the case of three ($N_g=3$) generations. The remaining constraints can also be satisfied for appropriate choices of x, y, and N. An example of a specific choice that satisfies all the anomaly constraints above and that leads to the required interactions displayed in Eqs. (15) and (16) is given by $N=3$, $x=1/6$ and $y=-1/3$. Clearly many more choices would be possible.

V. CONCLUSIONS

There is no deep theoretical motivation for R parity to be absolutely conserved. In case of R-parity violation the lightest supersymmetric particle, the neutralino, is unstable. In order to provide the DM the neutralino must be long lived, i.e., R-parity violation should be extremely small. It is severely constrained by astrophysical observations, with the strongest limit coming from neutralino decay to positron in our Galaxy. Barring fine-tuning Yukawa couplings being very tiny, only the gravitational interaction can be responsible for the required weakness of R-parity violation.

We demonstrated that dimension-5 operators suppressed by the Planck mass result in R-parity violation, which is too strong to satisfy the astrophysical restrictions. We discussed the additional suppression that can arise in these operators. One possibility is that these terms are induced by wormhole effects. In this case the additional suppression is given by [16] exp($-S$), where S is a wormhole action. The action in the semiclassical limit $S\sim 10$ reconciles the dimension-5 operators with astrophysical restrictions.

Another possibility for very weak R-parity breaking can be provided by the existence of an additional symmetry. We constructed a model with R-parity breaking in the hidden sector, which communicates to the MSSM fields only through gravity. Additional suppression of R-parity-breaking dimension-5 operators is provided by a Z_N symmetry.

The decaying neutralino can have interesting astrophysical signatures. In some models [9] the neutralino decay to Majoron J, $\chi\rightarrow\nu+J$ may be quite important, resulting in a detectable isotropic flux of monoenergetic neutrinos. In the more general case of R_P breaking by dimension-5 operators discussed above, the neutralino decay signature is weaker and is given by the ratio of the signals from the Sun and Earth to that from the Galactic halo. The signal from annihilation of neutralinos in the Earth and the Sun is the same as for a stable neutralino, while the positron and antiproton fluxes from the Galactic halo could be strongly enhanced due to neutralino decay.

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