Renormalizability of a lattice chiral fermion in the overlap formulation

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Renormalizability of a lattice chiral fermion is studied at the one loop level in the overlap formulation in four dimensions. The fermion chirality is examined including the self-energy corrections due to gauge interactions. Divergent terms breaking the chiral symmetry do not appear and the chiral fermion is renormalized, preserving the correct chiral properties without adding new counterterms or tuning the parameters involved. The divergent part of the wave function renormalization factor agrees with that of the continuum theory. The lattice chiral fermion in the overlap formulation has passed the important test, renormalizability, at the one loop level. [S0556-2821(97)05523-9]

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The overlap formulation [1] is a formulation of a lattice chiral fermion, inspired by the idea of the domain wall fermion [2]. It has been shown to have promising analytic properties [3–8] and gives results in good agreement with continuum theories in numerical simulations of two-dimensional systems [9]. This formulation provides the possibility of investigating chiral gauge theories in the strong coupling region, as well as a clearer analysis of the phenomenon of chiral symmetry breaking in vector gauge theories such as QCD. Moreover, it has been recently argued that this formulation can be applied in the study of supersymmetric gauge theories [10] and strongly correlated fermion systems in three (two plus one) dimensions [11]. However, one of the most important tests of the validity of that formulation, the renormalizability of a chiral fermion, has not yet been examined. Since the existence of the triangle anomaly means that it is impossible to regularize a chiral fermion in a chiral-invariant way [12], even if a chiral fermion is obtained in the continuum limit in a (lattice) regularization at the tree level [13], it is not evident that the chirality of the regularized fermion is preserved after including quantum corrections. The fact that the chiral anomaly is correctly reproduced in the overlap formulation [4] shows that the chirality of the regularized fermion is preserved in the triangle anomaly diagram. However, the dynamics of gauge fields does not play any role in this analysis [14]. Therefore, that fact does not guarantee that the chirality of the regularized fermion is preserved after including the radiative corrections by the dynamics of the gauge bosons [15]. In this paper, we study the fermion propagator in the overlap formulation regularized on a lattice, including self-energy corrections due to the SU(N) gauge interactions at one loop level, and directly examine the chirality of the fermion and its renormalizability. (We will follow the notation of Ref. [5] in our analysis.)

In the overlap formulation, the effective action of a chiral fermion in the presence of gauge fields is expressed by the overlap of the two vacua $|A\pm\rangle$ of the Hamiltonians $\mathcal{H}_{\pm}(A)$, where

$$\mathcal{H}_{\pm}(A)=\int_p \psi^{\dagger}(p)H_{\pm}(p)\psi(p)+\mathcal{V}(A), \tag{1}$$

$$H_{\pm}(p)=\gamma_5\left[\sum_{\mu=1}^{4} i\tilde{p}_{\mu}\gamma_{\mu}+T_c X_{\pm}(p)\right],$$

$$X_{\pm}(p)=\pm\frac{\lambda}{a}+\frac{ar}{2}\hat{p}^2. \tag{2}$$

In Eq. (1), the momentum integral is over the Brillouin zone $[-\pi/a,\pi/a]$ and the term $\mathcal{V}(A)$ describes the gauge interactions, which we treat as perturbations. In Eq. (2), $\tilde{p}_{\mu}=(1/a)\sin(p_{\mu}a)$, $\hat{p}_{\mu}=(2/a)\sin(p_{\mu}a/2)$, a is the lattice spacing, and $T_c=\pm 1$ determines the fermion chirality, as will be seen later. The Hamiltonians $\mathcal{H}_{\pm}$ describe time evolution of a Dirac fermion in four plus one dimensions and this Dirac fermion is reduced to a Weyl fermion in four dimensions [1,7]. The operator $\psi(p)$ is expanded in terms of creation and annihilation operators as

$$\psi(p)=\sum_{\sigma}\left[u_{\pm}(p,\sigma)b_{\pm}(p,\sigma)+v_{\pm}(p,\sigma)d^{\dagger}_{\pm}(p,\sigma)\right], \tag{3}$$

where $u_{\pm}(p,\sigma)$ and $v_{\pm}(p,\sigma)$ are positive and negative energy eigenspinors of the one-particle Hamiltonian $H_{\pm}(p)$, respectively, i.e., $H_{\pm}(p)u_{\pm}(p,\sigma)=\omega_{\pm}(p)u_{\pm}(p,\sigma)$ and $H_{\pm}(p)v_{\pm}(p,\sigma)=-\omega_{\pm}(p)v_{\pm}(p,\sigma)$. The label σ denotes the spin states and $\omega_{\pm}(p)=\sqrt{\tilde{p}^2+X^2_{\pm}(p)}$. The spinors $u_{\pm}$ and $v_{\pm}$ are given by

$$u_{\pm}(p,\sigma)=\frac{\omega_{\pm}+X_{\pm}-i\sum_{\mu}\tilde{p}_{\mu}\gamma_{\mu}T_c}{\sqrt{2\omega_{\pm}(\omega_{\pm}+X_{\pm})}}\chi(\sigma),$$

$$v_{\pm}(p,\sigma)=\frac{\omega_{\pm}-X_{\pm}+i\sum_{\mu}\tilde{p}_{\mu}\gamma_{\mu}T_c}{\sqrt{2\omega_{\pm}(\omega_{\pm}-X_{\pm})}}\chi(\sigma), \tag{4}$$

and the spinor $\chi(\sigma)$ satisfies $\gamma_5 T_c\chi(\sigma)=\chi(\sigma)$. The two sets $(b_{+}(p,\sigma),d_{+}(p,\sigma))$ and $(b_{-}(p,\sigma),d_{-}(p,\sigma))$ are related by a Bogoliubov transformation as

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$$b_-(p,\sigma)=\cos\beta(p)b_+(p,\sigma)-\sin\beta(p)d_+^\dagger(p,\sigma),$$

$$d_-^\dagger(p,\sigma)=\sin\beta(p)b_+(p,\sigma)+\cos\beta(p)d_+^\dagger(p,\sigma), \quad (5)$$

with $\cos\beta(p)=u_+^\dagger(p,\sigma)u_-(p,\sigma)$. The Dirac vacua $|\pm\rangle$ of the Hamiltonian $\mathcal{H}_\pm(0)$ are defined as $b_\pm(p,\sigma),d_\pm(p,\sigma)|\pm\rangle=0$, and the Dirac vacua $|A\pm\rangle$ are given by the integral equations

$$|A\pm\rangle=\alpha_\pm(A)[1-G_\pm(\mathcal{V}-\Delta E_\pm)]^{-1}|\pm\rangle,$$

$$G_\pm=\frac{1-|\pm\rangle\langle\pm|}{E_\pm(0)-H_\pm(0)}, \quad (6)$$

$$|\alpha_\pm(A)|^2=1-\langle A\pm|[\mathcal{V}-\Delta E_\pm]G_\pm^2[\mathcal{V}-\Delta E_\pm]|A\pm\rangle, \quad (7)$$

where $\Delta E_\pm=E_\pm(A)-E_\pm(0)$. $E_\pm(A)$ and $E_\pm(0)$ are the ground state energies of the Hamiltonians $\mathcal{H}_\pm(A)$ and $\mathcal{H}_\pm(0)$, respectively. The fermion propagator is defined by the path integral

$$\frac{\int \mathcal{DA}\langle A+|\Omega(p,q)|A-\rangle e^{-S(A)}}{\int \mathcal{DA}\langle A+|A-\rangle e^{-S(A)}}, \quad (8)$$

where $\Omega(p,q)=\{\psi(p)\overline{\psi(q)}-\overline{\psi(q)}\psi(p)\}/2$, $\overline{\psi}=\psi^\dagger\gamma_5$ and $S(A)$ is the action of the gauge field. We find it convenient for later calculations to decompose $\Omega(p,q)$ in the form [using the Bogoliubov transformation (5)], $\Omega(p,q)=\langle+|\Omega(p,q)|-\rangle+:\Omega(p,q):$

$$\langle+|\Omega(p,q)|-\rangle=(2\pi)^4\delta_P^4(p-q)\frac{1}{2}[S_+(p)-S_-(p)], \quad (9)$$

$$\begin{aligned}:\Omega(p,q):=\frac{1}{\cos\beta(p)\cos\beta(q)}&\times\sum_{\sigma,\tau}[-u_+(p,\sigma)\overline{u_-}(q,\tau)b_+^\dagger(q,\tau)b_-(p,\sigma)\\&+u_+(p,\sigma)\overline{v_+}(q,\tau)b_-(p,\sigma)d_-(q,\tau)\\&+v_-(p,\sigma)\overline{u_-}(q,\tau)d_+^\dagger(p,\sigma)b_+^\dagger(q,\tau)\\&+v_-(p,\sigma)\overline{v_+}(q,\tau)d_+^\dagger(p,\sigma)d_-(q,\tau)]. \quad (10)\end{aligned}$$

Here $\delta_P^4(p-q)$ is the periodic δ function on the lattice, $S_+(p)=\Sigma_\sigma u_+(p,\sigma)\overline{u_-}(p,\sigma)/\cos\beta(p)$, and $S_-(p)=\Sigma_\sigma v_-(p,\sigma)\overline{v_+}(p,\sigma)/\cos\beta(p)$. The first term (9) is the propagator for $A=0$ and the second term (10) satisfies the relation $\langle+|:\Omega(p,q):|-\rangle=0$. Near the origin $p\simeq0$,

$$S_\pm(p)\simeq\frac{\lambda}{a}\frac{1}{p^2}\left[\frac{1}{2}(1+\gamma_5T_c)(\mp i\not{p})+\frac{a}{2\lambda}p^2\gamma_5\right], \quad (11)$$

and thus the propagator $[S_+(p)-S_-(p)]/2$ describes a chiral fermion in this region. At each corner of the Brillouin zone, $p_\mu\simeq\pm\pi/a+q_\mu$, the propagator takes the following form:

$$\frac{1}{2}[S_+(p)-S_-(p)]\simeq\frac{1}{\sqrt{(4n^2r^2-\lambda^2)}+O(a^2q^2)}(c_1+c_2\gamma_5), \quad (12)$$

where $n=1,\ldots,4$ is the number of momentum components which lie near the corner of the Brillouin zone and $c_{1,2}$ are constants. The gauge symmetry of the Hamiltonians (1) becomes a chiral gauge symmetry near the origin of the Brillouin zone, while chiral noninvariant contributions coming from each corner are suppressed due to the $\sqrt{(4n^2r^2-\lambda^2)}$ mass. (We restrict ourselves to the range of parameters $0<\lambda<2r$.)

Now we consider the one loop correction to the propagator $\langle+|\Omega(p,q)|-\rangle$. Inserting the decomposition of $\Omega(p,q)$ into Eq. (8), Eq. (9) yields the propagator at tree level, while the quantum corrections arise from $\langle A+|:\Omega(p,q):|A-\rangle$. To obtain the one loop correction, the interaction $\mathcal{V}(A)$ in Eq. (1) should be expanded up to the second order in the gauge coupling constant g; $\mathcal{V}(A)=\mathcal{V}_1(A)+\mathcal{V}_2(A)$, where

$$\mathcal{V}_1=ig\int_{p,q}\overline{\psi(p)}\sum_\mu V_{1\mu}(p+q)A_\mu(p-q)\psi(q), \quad (13)$$

$$\begin{aligned}\mathcal{V}_2=\frac{1}{4}ag^2\int_{p,t,q}\overline{\psi(p)}\sum_{\mu,\nu}V_{2\mu}(p+q)\\\times\delta_{\mu\nu}\{A_\mu(t),A_\nu(p-t-q)\}\psi(q), \quad (14)\end{aligned}$$

$V_{1\mu}(p)=\gamma_\mu\cos(p_\mu a/2)-irT_c\sin(p_\mu a/2)$, and $V_{2\mu}(p)=rT_c\cos(p_\mu a/2)-i\gamma_\mu\sin(p_\mu a/2)$. Then evaluating $|A\pm\rangle$ perturbatively in Eq. (6) the quantum correction $\langle A+|:\Omega:|A-\rangle$, up to the order g^2 [16], is

$$\begin{aligned}&\langle+|:\Omega:G_-\mathcal{V}_2|-\rangle+\langle+|\mathcal{V}_2G_+:\Omega:|-\rangle\\&+\langle+|\mathcal{V}_1G_+:\Omega:G_-\mathcal{V}_1|-\rangle+\langle+|\mathcal{V}_1G_+\mathcal{V}_1G_+:\Omega:|-\rangle\\&+\langle+|:\Omega:G_-\mathcal{V}_1G_-\mathcal{V}_1|-\rangle. \quad (15)\end{aligned}$$

These terms are evaluated by rewriting the fermion operators in $\mathcal{V}_1$ and $\mathcal{V}_2$ in terms of the creation and annihilation operators defined in Eq. (3). Then performing the path integral over the gauge fields, the first two terms and the last three terms lead to the quantum corrections described by the Feynman diagrams Figs. 1(a) and 1(b), respectively.

Before examining these terms separately, we briefly discuss their general features and the strategy of our analysis. Each term in Eq. (15) yields the contribution to the tree level propagator of the form $S_\varepsilon(p)a\Sigma(p)S_{\varepsilon'}(p)$ where ε and ε' denote $\pm$, and $\Sigma(p)$ is a self-energy given by integral over the loop momentum. Here, we explicitly factor out the lattice spacing a so that $\Sigma(p)$ has the correct dimension (one) of the self-energy for fermions. To compute the divergent part of the renormalization factors, $\Sigma(p)$ should be evaluated up to the logarithmically divergent part in the continuum limit. We find that in general, in this limit $\Sigma(p)$ gives rise to the following expression:

$$\Sigma(p)=\frac{1}{a}[\sigma_1(\lambda,r,\xi)+\sigma_2(\lambda,r,\xi)\gamma_5]$$
$$+\frac{1}{\lambda}C\frac{\overline{g^2}}{16\pi^2}\{1-(1-\xi)\}\ln(p^2a^2)\frac{1}{2}(1-\gamma_5T_c)i\not{p}$$
$$+(\text{finite terms}), \tag{16}$$

where $\sigma_{1,2}(\lambda,r,\xi)$ are the constants obtained by the integration over the loop momentum and C is a numerical coefficient. Here ξ is the gauge fixing parameter, defined later, and $\overline{g^2}=(N^2-1)/(2N)g^2$. This expression is obtained, for example, by expanding $\Sigma(p)$ with respect to the external momentum p as in Refs. [12,17,18], or by splitting the integration region of the loop momentum into two pieces as in Ref. [12]. In both methods, the ultraviolet divergent parts are evaluated by explicitly factoring out the a dependence from the momentum integral with a rescaling of the loop momentum $k\to\overline{k/a}$. Logarithmic divergences appear as the infrared divergences of the integration with respect to the rescaled variable $\overline{k}$. Following the strategy of Refs. [12,17,18], we expand $\Sigma(p)$ up to the first order in p, where zero and first order terms give rise to the ultraviolet divergence in the continuum limit as

$$\Sigma(0)\to\frac{1}{a}[\sigma_1(\lambda,r,\xi)+\sigma_2(\lambda,r,\xi)\gamma_5]+\text{finite term}, \tag{17}$$

$$\sum_\mu p_\mu\frac{\partial\Sigma(p)}{\partial p_\mu}\bigg|_{p=0}\to\frac{1}{\lambda}C\frac{\overline{g^2}}{16\pi^2}\{1-(1-\xi)\}\ln(\kappa^2a^2)$$
$$\times\frac{1}{2}(1-\gamma_5T_c)i\not{p}+\text{finite term}, \tag{18}$$

where κ is the infrared regulator. The remaining finite part is given by

$$\Sigma(p)-\Sigma(0)-\sum_\mu p_\mu\frac{\partial\Sigma(p)}{\partial p_\mu}\bigg|_{p=0}$$
$$\to\frac{1}{\lambda}C\frac{\overline{g^2}}{16\pi^2}\{1-(1-\xi)\}\ln(p^2/\kappa^2)\frac{1}{2}(1-\gamma_5T_c)i\not{p}. \tag{19}$$

Expression (16) is the sum of Eqs. (17), (18), and (19). The infrared divergence at $\kappa\to0$ is cancelled between Eqs. (18) and (19). In Eq. (18), the logarithmic divergence is obtained if the integral over the rescaled loop momentum $\overline{k}=ak$ exhibits infrared divergences. When the integral over $\overline{k}$ is infrared finite, there is no logarithmic term in Eq. (19). The renormalization factors are evaluated by inserting Eq. (16) into the expression $S_\varepsilon(p)a\Sigma(p)S_{\varepsilon'}(p)$. In Eq. (16), the Lorentz structure of the linearly divergent terms is completely determined by the discrete symmetry on the lattice, e.g., $(k\leftrightarrow-k)$. These terms are reduced to a finite wave function renormalization factor

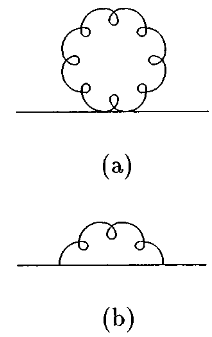

FIG. 1. The Feynman diagrams describing the contributions of (a) the first two terms and (b) the last three terms in Eq. (15).

$$\frac{\lambda}{a}\frac{1}{p^2}\left[\frac{1}{2}(1+\gamma_5T_c)(-\varepsilon i\not{p})+\frac{a}{2\lambda}p^2\gamma_5\right]$$
$$\times a\frac{1}{a}[\sigma_1+\sigma_2\gamma_5]\cdot\frac{\lambda}{a}\frac{1}{p^2}\left[\frac{1}{2}(1+\gamma_5T_c)(-\varepsilon' i\not{p})\right.$$
$$\left.+\frac{a}{2\lambda}p^2\gamma_5\right]\to\frac{1}{2}[\sigma_1T_c(-\varepsilon+\varepsilon')+\sigma_2(\varepsilon+\varepsilon')$$
$$+O(a)]\frac{\lambda}{a}\frac{1}{2}(1+\gamma_5T_c)\frac{-i\not{p}}{p^2}. \tag{20}$$

From this expression, we can see that only an ultraviolet divergence of $\Sigma(p)$ more severe than quadratic can invalidate the chiral property of the regularized fermion. The logarithmically divergent term in Eq. (18) directly appears in the wave function renormalization factor as a logarithmic divergence.

Next we discuss each contribution in Eq. (15) separately. The contributions of the first two terms to the tree level propagator are given by $-S_\mp(p)a\Pi^a_\pm(p)S_\pm(p)$ with

$$\Pi^a_\pm(p)=-\frac{\overline{g^2}}{2}\frac{1}{2\omega_\pm(p)}\sum_\mu V_{2\mu}(2p)\int_k D_{\mu\mu}(k)\to\frac{\overline{g^2}}{4\lambda}\sigma^a(\xi)$$
$$\times\left(\frac{4rT_c}{a}-i\not{p}\right), \tag{21}$$

where the upper (lower) sign should be taken for the contribution containing G_+ (G_-), and $D_{\mu\nu}(k)$ is the gauge boson propagator

$$D_{\mu\nu}(k)=\frac{1}{\hat{k}^2}\left(\delta_{\mu\nu}-(1-\xi)\frac{\hat{k}_\mu\hat{k}_\nu}{\hat{k}^2}\right). \tag{22}$$

In Eq. (21), $\sigma^a(\xi)$ is the constant obtained by the integration over the rescaled loop momentum $\overline{k}=ak$. No logarithmic divergence appears, since the rescaled integral is infrared finite. As was discussed previously, the contributions (21) are reduced to a finite wave function renormalization factor.

Next consider the last three terms in Eq. (15). The third term leads to the following contributions to the tree level propagator: $S_+(p)a\Sigma_+^b(p)S_+(p)+S_-(p)a\Sigma_-^b(p)S_-(p)$ with

$$\Sigma_\pm^b(p)=\pm\frac{1}{a}\overline{g}^2\sum_{\mu,\nu}\int_k\left[\frac{1}{\omega_+(p)+\omega_+(k)}\right]\times\left[\frac{1}{\omega_-(p)+\omega_-(k)}\right]V_{1\mu}(p+k)S_\mp(k)\times V_{1\nu}(p+k)D_{\mu\nu}(p-k). \tag{23}$$

The integration over the rescaled momentum $\overline{k}$ exhibits infrared divergences near $\overline{k}\simeq 0$, which in turn, give rise to logarithmic divergences in the form of Eq. (16) with $C=1/4$. Infrared divergences appear only in the region $\overline{k}\simeq 0$, as is seen from Eqs. (11) and (12). The contributions of the fourth and fifth terms in Eq. (15) are expressed as $-S_\mp(p)a[\Sigma_\pm^{b'}(p)+\Pi_\pm^b(p)]S_\pm$, where

$$\Sigma_\pm^{b'}(p)=\frac{1}{a}\overline{g}^2\sum_{\mu,\nu}\int_k\left[\frac{1}{\omega_\pm(p)+\omega_\pm(k)}\right]^2V_{1\mu}(p+k)\frac{\sin\beta(k)}{\cos\beta(k)}\times\sum_s\begin{bmatrix}u_+(k,s)\overline{v_+}(k,s) & (\text{for}+)\\ -v_-(k,s)\overline{u_-}(k,s) & (\text{for}-)\end{bmatrix}\times V_{1\nu}(p+k)D_{\mu\nu}(p-k), \tag{24}$$

$$\Pi_\pm^b(p)=-\frac{1}{a}\overline{g}^2\frac{1}{2\omega_\pm(p)}\sum_{\mu,\nu}\int_k\frac{1}{\omega_\pm(p)+\omega_\pm(k)}V_{1\mu}(p+k)\times\sum_s[u_\pm(k,s)\overline{u_\pm}(k,s)-v_\pm(k,s)\overline{v_\pm}(k,s)]\times V_{1\nu}(p+k)D_{\mu\nu}(p-k). \tag{25}$$

The upper (lower) sign should be taken for the contribution containing G_+ (G_-). The contributions (24) lead to linearly divergent terms as well as logarithmically divergent terms with $C=1/4$ in the form of Eq. (16), while the contributions (25) lead only to the linearly divergent terms. This is because the integrations over the rescaled loop momentum $\overline{k}$ give rise to infrared divergences in Eqs. (24), whereas in Eqs. (25) they are finite.

Summing up all the contributions $\Pi_\pm^{a,b}$, $\Sigma_\pm^{b,b'}$ together with the tree level propagator, the propagator is given at one loop level as

$$\frac{\lambda}{a}\frac{1}{2}(1+\gamma_5T_c)\frac{-i\not{p}}{p^2}\left[1+\frac{\overline{g}^2}{16\pi^2}\{1-(1-\xi)\}\times(\ln a^2p^2+\text{finite terms})\right]. \tag{26}$$

From this expression, we see that the chirality of the regularized fermion is properly preserved and the wave function renormalization factor is $Z=1+\overline{g}^2/16\pi^2[1-(1-\xi)](\ln a^2\mu^2+\text{const})$, where μ is the renormalization scale. The divergent part of the wave function renormalization factor agrees with that of the continuum theory.

We have studied the renormalization of a lattice chiral fermion due to the non-Abelian gauge interactions in the overlap formulation in four dimensions. Divergent terms breaking the chiral symmetry do not appear at one loop level, and accordingly there is no need to add new counterterms or to tune parameters in the theory to specific values to realize a chiral fermion. The divergent part of the wave function renormalization factor is correctly reproduced. Our study proves the renormalizability of a lattice chiral fermion in the overlap formulation at one loop level, and indicates, together with the analyses of the gauge boson n-point functions [5–7] the renormalizability of this formulation.

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[14] It has been also shown that the vacuum polarization [5,6] and

the gauge boson n-point functions [7] are correctly reproduced in this formulation. However, these analyses do not involve the loop of the gauge fileds, and therefore do not clarify the effects of the gauge interactions on the fermion chirality.

[15] Essentially the same problem emerges in the renormalization of a massless Wilson fermion. In that case, the chiral symmetry is broken by the Wilson term, which vanishes in the continuum limit at tree level. Thus at tree level, the chiral symmetry is preserved in the continuum limit. However, the Wilson term leads to a linearly divergent mass term at one loop level due to the gauge interactions [12] (see also Refs. [17,18] below) and the chiral symmetry is broken. To restore it in the continuum limit, one of the parameters, the hopping parameter, must be a critical value so as to cancel this divergent term.

[16] The functionals $\alpha_{\pm}(A)$ of A in Eq. (7) are expanded as $\alpha_{\pm}(A) \simeq 1 + O(g^2 A^2)$ including complex phases and do not contribute to $\langle A+|{:}\Omega(p,q){:}|A-\rangle$ in this order.

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