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Can the supersymmetric flavor problem be solved by decoupling?

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It has been argued that the squarks and sleptons of the first and second generations can be relatively heavy without destabilizing the weak scale, thereby improving the situation with too-large flavor-changing neutral current (FCNC) and CP -violating processes. In theories where the soft supersymmetry-breaking parameters are generated at a high scale (such as the Planck scale), we show that such a mass spectrum tends to drive the scalar top quark mass squared $m_{\tilde{Q}_3}^2$ negative from two-loop renormalization group evolution. Even ignoring CP violation and allowing $O(\lambda) \sim 0.22$ alignment, the first two generation scalars must be heavier than 22 TeV to suppress FCNCs. This in turn requires the boundary condition on $m_{\tilde{Q}_3} > 4$ TeV to avoid negative $m_{\tilde{Q}_3}^2$ at the weak scale. Some of the models in the literature employing the anomalous $U(1)$ in string theory are excluded by our analysis. [S0556-2821(97)50121-4]

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The biggest embarrassment of low-energy supersymmetry (SUSY) is the flavor problem: the superparticles may generate too-large flavor-changing (FC) effects such as K^0 - \bar{K}^0 mixing or $\mu \rightarrow e \gamma$, or too-large neutron and electron electric dipole moments. Traditionally, one assumes that the SUSY-breaking parameters are universal and real at a high scale to avoid these problems, as done in the “minimal supergravity” framework. Supergravity, however, does not have a fundamental principle to guarantee the universality of scalar masses nor their reality. Several ideas have been proposed to solve this supersymmetric flavor problem. If SUSY-breaking is mediated by gauge interactions [1], or if the dominant SUSY breaking effect is in the dilaton multiplet of string theory [2], the soft breaking parameters are flavor blind and the problem is eradicated. Alternately, flavor symmetries can guarantee sufficient degeneracy amongst the first- and second-generation sfermions [3], or alignment between quark and squark mass matrices [4].

It would be simplest, however, to push up the masses of the first- and second-generation scalars high enough to avoid the flavor problem [5–8]. Since the first two generations

have small Yukawa couplings to the Higgs doublets, it is conceivable that they can be heavy while maintaining natural electroweak symmetry breaking (EWSB), which is the very motivation for low-energy SUSY. We of course still need to keep the masses of third-generation scalars, gauginos and Higgsinos close to the weak scale for this purpose.

Such a spectrum was studied in [6] and it was argued that too-heavy first- and second-generation scalars lead to a fine-tuning in EWSB because they give a too large contribution to the Higgs boson mass squared via two-loop renormalization group equations (RGEs). It was concluded that the heavy scalars need to be (at least) lighter than 5 TeV to avoid a fine-tuning of more than 10% in EWSB. A subsequent analysis [7] required that the mass splitting between the different generations be preserved by the two-loop RGEs, obtaining a similar constraint. However, the constraints based on this type of discussion are somewhat subjective: the results depend on how large a fine-tuning one allows, or exactly what is meant by the preservation of the mass splitting. More recently, such a split mass spectrum was argued to be

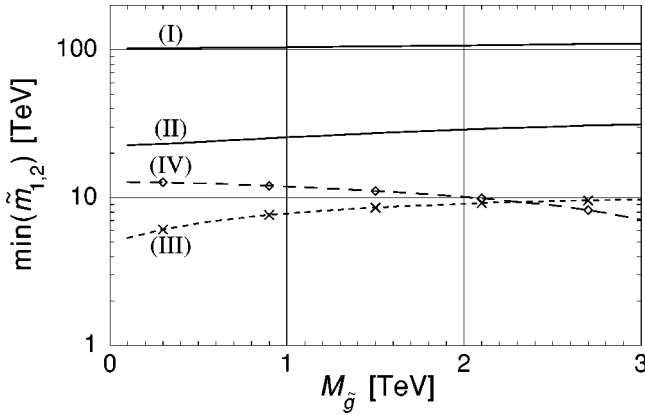


FIG. 1. The minimum mass of the first- and second-generation scalars $\min(\tilde{m}_{1,2}^2)$ to keep $(\Delta m_K)_{\tilde{q},\tilde{g}} < (\Delta m_K)_{\text{obs}}$ as a function of the gluino mass for four different cases: (I) $(\delta_{12}^d)_{LL} = (\delta_{12}^d)_{RR} = 1$, (II) $(\delta_{12}^d)_{LL} = (\delta_{12}^d)_{RR} = 0.22$, (III) $(\delta_{12}^d)_{LL} = (\delta_{12}^d)_{RR} = 0.05$, and (IV) $(\delta_{12}^d)_{LL} = 0.22$ and $(\delta_{12}^d)_{RR} = 0$. Including CP -violating phase of $\pi/4$ makes the lower bound stronger by a factor of 9.2.

best from the phenomenological point of view [8]. Furthermore, it was pointed out that the D -term contributions from the anomalous $U(1)$ gauge group in string theory may naturally lead to such a split mass spectrum [9,10]. Therefore, it is useful to study the phenomenological viability of this type of spectrum.

The purpose of this letter is to point out that such a split scalar mass spectrum tends to drive the mass squared of third-generation squarks or sleptons negative, breaking color and charge. This constraint is purely phenomenological and does not depend on any naturalness criteria. Indeed, the mass patterns proposed in some stringy anomalous $U(1)$ models do not satisfy our constraint and are hence not phenomenologically viable. Throughout the letter we assume that SUSY-breaking parameters are generated at a high scale such as the Planck scale. Our results are then not an immediate concern for models where all effects of SUSY breaking shut off at scales of a few orders of magnitude above the weak scale, as in the ‘‘more minimal’’ scenario [8], since the negative contributions to the scalar masses are not enhanced by a large logarithm. Nevertheless, our results are strong enough to suggest that in any concrete realization of a ‘‘more minimal’’ scenario, a detailed check of the radiative corrections to third-generation scalar masses must be done to ensure that they are not driven negative.

Our analysis has three steps. We first determine the minimum mass of the first- and second-generation scalars which make the SUSY contribution to K^0 - \bar{K}^0 mixing smaller than the observed value. Next, we determine the smallest allowed ratio of the scalar mass of the third generation to that of other generations consistent with the requirement that $m_{\tilde{Q}_3}^2$ is not driven negative by the two-loop RGE; this constraint is independent of the discussion of FCNC. Finally, we combine the two analyses to obtain the minimum boundary value for $m_{\tilde{Q}_3}^2$ consistent both with K^0 - \bar{K}^0 mixing and with positivity of $m_{\tilde{Q}_3}^2$ at the weak scale. We find it is difficult to keep the third-generation scalars below the TeV

scale, even ignoring CP violation and allowing $O(\lambda)$ degeneracy or alignment in scalar mass matrices of the first two generations. This observation strengthens the case for flavor symmetries or dynamical mechanisms for degeneracy.

We consider four patterns for the first two generation squark mass matrices. Working in the basis of superfields where the down-quark mass matrix is diagonal, it is convenient to characterize the patterns by the ratio (δ_{12}^d) of the off-diagonal (1,2) elements $(m_{\tilde{d}}^2)_{12}$ of the \tilde{d} mass squared matrices to the arithmetic mean of the squared mass eigenvalues $\tilde{m}_{1,2}^2$, for both left- and right-handed scalars. In case (I), the first- and second-generation fields are heavy with masses of the same order of magnitude, but with order 1 off-diagonal elements, i.e., $(\delta_{12}^d)_{LL} = (\delta_{12}^d)_{RR} = 1$. Case (II) assumes an $O(\lambda)$ alignment, $(\delta_{12}^d)_{LL} = (\delta_{12}^d)_{RR} = 0.22$. In case (III), we assume that there is some small amount of degeneracy $\sim 1/5$ between the first two generation scalars, on top of an $O(\lambda)$ alignment, so that the off-diagonal elements are $(\delta_{12}^d)_{LL} = (\delta_{12}^d)_{RR} = 0.05$. Finally in case (IV), we assume that the only mixing is between left-handed squarks and is $O(\lambda)$: $(\delta_{12}^d)_{LL} = 0.22$ and $(\delta_{12}^d)_{RR} = 0$. This case is motivated by our lack of knowledge of the mixing between right-handed quarks, although it is somewhat artificial. Our analysis is then very simple. We require the squark-gluino contribution $(\Delta m_K)_{\tilde{q},\tilde{g}}$ to K^0 - \bar{K}^0 mixing (using the formulas in [11]) to be less than the observed size $(\Delta m_K)_{\text{obs}}$. We give the lower bounds on $\tilde{m}_{1,2}^2$ for each pattern of squark mixing, as a function of the gluino mass $M_{\tilde{g}}$. The results are plotted in Fig. 1. In all cases, this lower bound ranges from 100 TeV to the multi-TeV range. Allowing a phase $e^{i\theta}$ in the off-diagonal elements strengthens the lower bounds on $\tilde{m}_{1,2}^2$ by a factor of $13\sin\theta$. Therefore the scalar masses of the first two generations make important *negative* contributions to the RGE of third-generation scalar masses due to gauge interactions at the two-loop level. We thus turn our attention to the RGE analysis.

First note that the heavy first- and second-generation scalars of the same generation must have certain degeneracies among themselves to avoid inducing a too-large Fayet-Illiopoulos D -term D_Y for the hypercharge gauge group at one-loop. Since their mass scale is high, such a contribution would induce negative mass squared to either $\tilde{\tau}$ or \tilde{L}_3 depending on its sign [6,8]. Therefore we require the scalars within each of the $\mathbf{5}^*$, $\mathbf{10}$ $SU(5)$ -multiplets to be degenerate [12], and consider cases where N_5 of the $\mathbf{5}^*$'s and N_{10} of the $\mathbf{10}$'s are heavy. $N_5 = N_{10} = 2$ is relevant for all patterns of squark masses (I–IV), while $N_5 = 0, N_{10} = 2$ is possible for case (IV). Second, we take the gaugino masses universal ($=M_0$) at the GUT-scale for simplicity. Third, we run all scalar masses starting from the GUT-scale $M_{\text{GUT}} = 2 \times 10^{16}$ GeV. If the scale where the SUSY-breaking effects are transmitted is lower, the effects of running will be smaller and the constraints will be weaker. On the other hand, the string-derived case starts at the Planck scale and the constraints are stronger. We chose the (GUT) grand unified theory scale as a compromise for the presentation. Finally, we omit all the Yukawa couplings in the

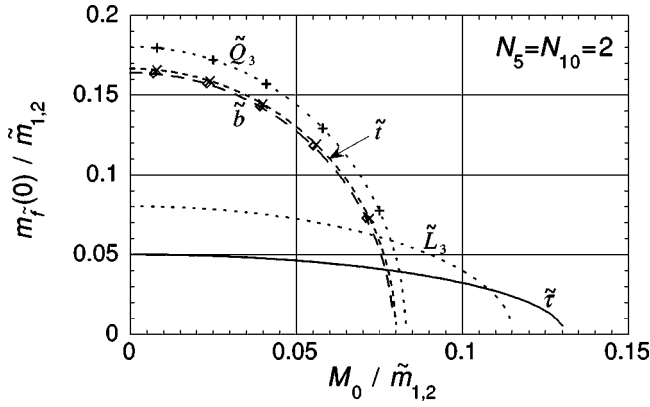


FIG. 2. Constraint on the mass ratio between the first- and second-generation scalars $\tilde{m}_{1,2}$ and the third-generation scalars $m_{\tilde{f}}(0)$ from the requirement that none of the third-generation scalars acquire negative mass squared at the weak scale. The regions below the curves are excluded. Constraints are shown for the case $N_5=N_{10}=2$. See the text for details of our conservative assumptions.

RGE: since the Yukawa couplings always drive the scalar masses smaller, this is a conservative choice. Given a model with specific predictions for the scalar mass spectrum, the analysis must be repeated with Yukawa couplings included in the RGEs. Without a concrete model in mind, our choice suffices for this letter.

We take the two-loop RGEs in the dimensional reduction with modified minimal subtraction ($\overline{\text{DR}}'$) scheme [13]. We neglect all two-loop terms subdominant to the ones involving the heavy scalar masses. Neglecting Yukawa couplings as discussed above, the RGEs have only two important contributions: the one-loop gaugino contributions and the two-loop contributions from the heavy scalars. Furthermore, the running of the heavy scalar masses is negligible. The RGE for the third-generation scalar species \tilde{f} is then given by

$$\begin{aligned} \frac{d}{dt} m_{\tilde{f}}^2 = & -8 \sum_i \tilde{\alpha}_i C_i^f M_i^2 + 8 \left[\left(\frac{1}{2} N_5 + \frac{3}{2} N_{10} \right) \sum_i \tilde{\alpha}_i^2 C_i^f \right. \\ & \left. + (N_5 - N_{10}) \frac{3}{5} Y_f \tilde{\alpha}_1 \left(\frac{4}{3} \tilde{\alpha}_3 - \frac{3}{4} \tilde{\alpha}_2 - \frac{1}{12} \tilde{\alpha}_1 \right) \right] \tilde{m}_{1,2}^2. \end{aligned} \quad (1)$$

Here, $\tilde{\alpha}_i = g_i^2/16\pi^2$ and C_i^f is the Casimir for f , in SU(5) normalization, and Y_f is its hypercharge. The two-loop contribution is decoupled at the scale $\mu' \sim \tilde{m}_{1,2}$ of the heavy scalars, which we approximate as ~ 10 TeV [14]. The positive gaugino mass contribution, however, survives down to the scale where the gauginos decouple, which we approximate as $\mu \sim 1$ TeV. The RGEs can be solved analytically and the solutions are given by

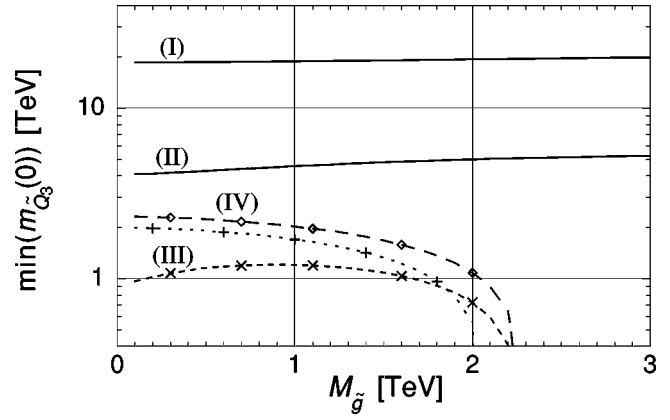


FIG. 3. The minimum boundary mass of the left-handed scalar top $\min(m_{\tilde{Q}_3}(0))$ required to avoid negative $m_{\tilde{Q}_3}^2$ at the weak scale, while keeping the $\min(\tilde{m}_{1,2})$ within the constraints from Fig. 1. As in Fig. 1, four cases are considered: (I) $(\delta_{12}^d)_{LL} = (\delta_{12}^d)_{RR} = 1$, (II) $(\delta_{12}^d)_{LL} = (\delta_{12}^d)_{RR} = 0.22$, (III) $(\delta_{12}^d)_{LL} = (\delta_{12}^d)_{RR} = 0.05$, and (IV) $(\delta_{12}^d)_{LL} = 0.22$ and $(\delta_{12}^d)_{RR} = 0$. Two curves are shown for the last case. The upper curve is for $N_5 = N_{10} = 2$ as in other cases, and the constraint is slightly weaker if $N_5 = 0$.

$$\begin{aligned} m_{\tilde{f}}^2(t) = & m_{\tilde{f}}^2(0) + \sum_i \frac{2}{b_i} [M_0^2 - M_i^2(t)] C_i^f \\ & - 8 \tilde{m}_{1,2}^2 \left[\left(\frac{1}{2} N_5 + \frac{3}{2} N_{10} \right) \sum_i \frac{1}{2 b_i} [\tilde{\alpha}_{\text{GUT}} - \tilde{\alpha}_i(t')] C_i^f \right. \\ & - (N_5 - N_{10}) \frac{3}{5} Y_f \left(\frac{4}{3} \frac{\tilde{\alpha}_{\text{GUT}}}{b_1 - b_3} \frac{1}{2} \ln \frac{\tilde{\alpha}_1(t')}{\tilde{\alpha}_3(t')} \right. \\ & \left. - \frac{3}{4} \frac{\tilde{\alpha}_{\text{GUT}}}{b_1 - b_2} \frac{1}{2} \ln \frac{\tilde{\alpha}_1(t')}{\tilde{\alpha}_2(t')} \right. \\ & \left. \left. + \frac{1}{12} \frac{1}{2 b_1} [\tilde{\alpha}_{\text{GUT}} - \tilde{\alpha}_1(t')] \right] \right]. \end{aligned} \quad (2)$$

In the above, the b_i stand for the gauge coupling beta-function coefficients, $t=0$ corresponds to the GUT-scale, $\tilde{\alpha}_{\text{GUT}} = \tilde{\alpha}_i(0)$, and the final scales are at $t' = \ln \mu'/M_{\text{GUT}}$. The final results can be written down explicitly in terms of the universal gaugino mass $M_0 = M_i(0)$, the heavy scalar mass $\tilde{m}_{1,2}$ and the boundary value of the third-generation scalar mass $m_{\tilde{f}}(0)$ as

$$\begin{aligned} m_{\tilde{f}}^2(t) = & m_{\tilde{f}}^2(0) + (0.245 C_1^f + 0.599 C_2^f + 3.20 C_3^f) M_0^2 \\ & - 10^{-2} (0.157 C_1^f + 0.292 C_2^f + 0.750 C_3^f) \\ & - 0.097 Y_f N_{10} \tilde{m}_{1,2}^2 - 10^{-2} (0.052 C_1^f + 0.097 C_2^f \\ & + 0.250 C_3^f + 0.097 Y_f) N_5 \tilde{m}_{1,2}^2 \end{aligned} \quad (3)$$

with $\tilde{\alpha}_{\text{GUT}}^{-1} = 25 \times 4\pi$. The combinations of $m_{\tilde{f}}(0)/\tilde{m}_{1,2}$ and $M_0/\tilde{m}_{1,2}$ giving vanishing $m_{\tilde{f}}(t)$ for each f are plotted in Fig. 2 for the case with $N_5 = N_{10} = 2$. The regions below the curves are all excluded.

Combining this plot with the Δm_K constraints, we obtain lower bounds on $m_{\tilde{Q}_3}(0)$ so that $(\Delta m_K)_{\tilde{q},\tilde{g}}$ is on the experimental bound while retaining positivity of $m_{\tilde{Q}_3}(t)$. The results are shown in Fig. 3 for each pattern of squark masses (I–IV). For the cases (I) and (II), $m_{\tilde{Q}_3}(0)$ must be at least larger than 4 TeV, which is clearly beyond whatever can be regarded as natural. For instance, the fine-tuning in EWSB quantified in [15] scales as $10\% \times [m_{\tilde{Q}_3}(0)/300 \text{ GeV}]^{-2}$, $m_{\tilde{Q}_3}(0) > 4 \text{ TeV}$ requires a severe fine-tuning in EWSB worse than the 10^{-3} level. It is clear that one needs further alignment or degeneracy to keep $m_{\tilde{Q}_3}(0)$ within a natural range. Case (III), where $(\delta_{12}^d)_{LL} = (\delta_{12}^d)_{RR} = 0.05$, marginally allows $m_{\tilde{Q}_3}(0) \sim M_g \sim 1 \text{ TeV}$. However this mass range still incurs a fine-tuning in EWSB at the 1% level. Case (IV) is no better than this. In this case $(\delta_{12}^d)_{RR} = 0$, and there is an option to keep the $\mathbf{5}^*$ fields of first and second generations at the weak scale. Figure 3 shows two curves for this case depending on $N_5 = N_{10} = 2$ as in the other cases or $N_5 = 0$, $N_{10} = 2$ which gives the most conservative constraint. None of the patterns for scalar mass matrices we considered allow $m_{\tilde{Q}_3}(0)$ in the most natural range $\sim 100 \text{ GeV}$. Recall that the actual constraint is stronger than what we presented; we ignored CP violation in the K^0 - \bar{K}^0 mixing and the top Yukawa coupling in the RGE. We conclude that pushing up the first- and second-generation scalar masses does not solve the flavor problem, and hence either a relatively strong flavor symmetry or a dynamical mechanism to generate degenerate scalar masses is necessary.

Finally, we would like to comment on the anomalous U(1) models [9,10,16–18] which naturally generate a split mass spectrum between scalars of different generations [19]. These models do not fall into any of the patterns (I–IV) we discussed, and hence require a separate discussion. The model in [10] suppresses $(\Delta m_K)_{\tilde{q},\tilde{g}}$ by assigning the same anomalous U(1) charges to the first and second generations [20], thereby making them highly degenerate. However, it predicts a mass spectrum with $m_{\tilde{f}}(0)/\tilde{m}_{1,2} = 0.1$ and $M_0/\tilde{m}_{1,2} = 0.01$, which is clearly excluded by Fig. 2, because $m_{\tilde{Q}_3}^2$ is driven negative. In [16], a similar choice of anomalous U(1) charges is made, with $(\delta_{12}^d)_{LL,RR} \sim m_c/m_t \leq 0.01$. It was claimed that the flavor problem (including the constraint from ϵ_K allowing order 1 CP -violating phases) is solved

with the first two generations in the few-TeV range, while keeping the third generation and Higgs fields beneath a TeV to achieve natural EWSB. However, the constraint from ϵ_K used in [16] was too weak. For $(\delta_{12}^d)_{LL,RR} \sim 0.01$ and CP -violating phase of $\pi/4$, we find that $\tilde{m}_{1,2}$ must be heavier than 9.2 TeV, and $m_{\tilde{Q}_3}(0)$ must be heavier than 1.7 TeV in order to avoid being driven negative. Reference [17] tries to correlate the fermion mass hierarchy to the charges under the anomalous U(1), and is hence more realistic. For instance the scenario D in [17] needs one $\mathbf{5}^*$ at 5.0 TeV, another one at 6.1 TeV, and $\mathbf{10}^*$ multiplets at 6.1 TeV and 7.0 TeV, respectively, even ignoring CP violation. We obtain $m_{\tilde{Q}_3}(0) > 1.0 \text{ TeV}$, and hence our analysis does not allow $m_{\tilde{Q}_3}(0)$ in the indicated range of 500 GeV–1 TeV. The model is not excluded, but is not better than any of the patterns (I–IV) we considered. If one further implements quark-squark-alignment [18], the situation may be better. However, it is then not clear that it is the heavy $\tilde{m}_{1,2}$ which is helping rather than the flavor symmetries.

In summary, we examined the question of whether making first- and second-generation scalars heavy can solve the flavor problem without relying on flavor symmetries or particular dynamical mechanisms to obtain degenerate squark masses. In the case where SUSY-breaking parameters are generated at a high scale, our conclusion is negative. Even with an $O(\lambda)$ alignment, one needs $\tilde{m}_{1,2} > 22 \text{ TeV}$, and the contributions to the two-loop RGE of $m_{\tilde{Q}_3}^2$ drives it negative unless $m_{\tilde{Q}_3}(0) > 4 \text{ TeV}$. Our constraints are conservative because we do not include the top Yukawa coupling in the RGE and ignored possible CP violation. A significant degeneracy or much stronger alignment is necessary to keep third-generation scalars within their natural range $\lesssim (\text{a few} \times 100) \text{ GeV}$.

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