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CDM models with a smooth component

Michael S. Turner
Department of Physics, The University of Chicago, Chicago, Illinois 60637-1433;
NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, Illinois 60510-0500;
and Department of Astronomy and Astrophysics, Enrico Fermi Institute, The University of Chicago, Chicago, Illinois 60637-1433

Martin White
NASA/Fermilab Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, Illinois 60510-0500;
and Department of Astronomy and Astrophysics, Enrico Fermi Institute, The University of Chicago, Chicago, Illinois 60637-1433
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The inflationary prediction of a flat Universe is at odds with current determinations of the matter density ($\Omega_M \simeq 0.2-0.4$). This dilemma can be resolved if a smooth component contributes the remaining energy density ($\Omega_X = 1-\Omega_M$). We parametrize the smooth component by its equation of state, $p_X = w\rho_X$, and show that XCDM with $w \simeq -0.6$, $\Omega_M \simeq 0.3$, and $h \simeq 0.7$ is the best fit to all present cosmological data. Together, the position of the peak in the CMB angular power spectrum and the type Ia supernova magnitude-redshift diagram provide a crucial test of XCDM. [S0556-2821(97)50220-7]

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I. INTRODUCTION

Inflation is a bold and expansive cosmological paradigm which makes three firm and testable predictions: a flat universe; nearly scale-invariant spectrum of density perturbations; and nearly scale-invariant spectrum of gravitational waves [1]. (The first prediction can be relaxed at the expense of more complicated models and tuning the amount of inflation [2].) Flatness implies that the total energy density is equal to the critical density ($\Omega_{\rm tot}=1$). However, it makes no prediction about the form(s) that the critical energy takes.

Together, the first and second predictions lead to the cold dark matter (CDM) scenario of structure formation which holds that most of the matter consists of slowly moving elementary particles such as axions or neutralinos and that structure in the Universe developed hierarchically, from galaxies to clusters of galaxies to superclusters. Both the density perturbations and the gravitational waves lead to characteristic signatures in the anisotropy of the cosmic microwave background radiation (CMB) [3].

The CDM picture is generally consistent with a wide array of cosmological observations: CMB anisotropy, determinations of the power spectrum of inhomogeneity from redshift surveys and peculiar-velocity measurements, the evolution of galaxies as recently revealed by the Hubble Space Telescope and the Keck telescope, x-ray studies of clusters of galaxies, and more. Actually, there are several CDM models, distinguished by their ''invisible'' matter content (e.g., Ref. [4] and references therein): baryons + CDM only (sCDM, *s* for simple); baryons + CDM + neutrinos with $\Omega_\nu \sim 0.15$ (νCDM); baryons + CDM + cosmological constant (ΛCDM); baryons + CDM + larger energy density in relativistic particles (τCDM). Cosmological parameters also affect the predictions of each model: Hubble parameter $H_0 = 100h$ km s^{-1} Mpc^{-1}, baryon density $\Omega_B h^2$, power-law index characterizing the spectrum of density perturbations n, and gravitational radiation described by its contribution to the quadrupole CMB anisotropy relative to that of density perturbations (T/S) and the power-law index characterizing its spectrum (n_T). For each CDM variant there are values of the cosmological parameters for which the model is consistent with most—but possibly not all—of the data.

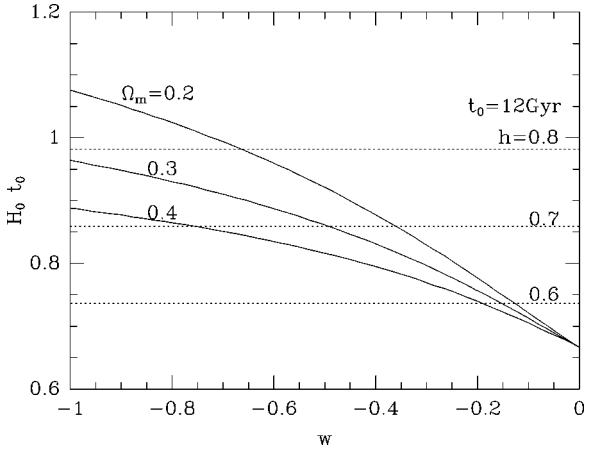

FIG. 1. The age in Hubble units as a function of w for $\Omega_M = 0.2$, 0.3, and 0.4. Horizontal lines indicate the value of $H_0 t_0$ required for $t_0 = 12$ Gyr with $h = 0.6$, 0.7, and 0.8. Note that the ''age constraint'' is strongly w dependent.

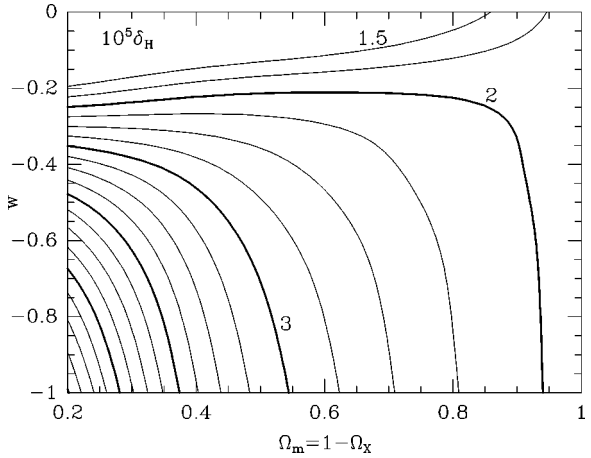

FIG. 2. Power-spectrum normalization $10^5 \delta_H$ as determined by the four-year COBE Differential Microwave Radiometer (DMR) results. The COBE 1σ error is approximately 10%.

II. FLATNESS PROBLEM

From the very beginning, the prediction of a flat universe has been troublesome: Put simply there has never been strong evidence for $\Omega_M = 1$. Today, almost all determinations of the matter density are consistent with $\Omega_M = 0.2 - 0.4$ [5]. [This does provide general support for the existence of CDM since big-bang nucleosynthesis (BBN) constrains $0.007h^{-2} \leqslant \Omega_B \leqslant 0.024h^{-2} < 0.1$ for $h > 0.5$ [6].] Strong support for $\Omega_M \sim 0.3$ comes from measurements of peculiar velocities and the cluster baryon fraction. Relating galactic peculiar velocities to the distribution of galaxies allows the mean density to be sampled in a very large volume, about $(30h^{-1}\ \mathrm{Mpc})^3$, and several studies indicate that Ω_M is at least 0.25, but probably significantly less than 1 [7]. x-ray observations of clusters of galaxies determine the baryon-to-total mass ratio in a system of sufficient size to be representative of the universal value (Ω_B/Ω_M). This, together with the BBN value for Ω_B, implies $\Omega_M (h/0.7)^{1/2} = (0.3 \pm 0.2)$ [8]. Indirect support for $\Omega_M < 1$ comes from the fact that a flat, matter-dominated universe (age $t_0 = \frac{2}{3} H_0^{-1}$) may be too young to be consistent with determinations of the age of the oldest stars ($t_0 = 15 \pm 2$ Gyr) [9] and the Hubble parameter ($h = 0.7 \pm 0.1$) [10].

In defense of a flat, matter-dominated universe it should be said that there has yet to be a convincing measurement of the matter density in a sufficiently large volume to provide a definitive determination of Ω_M—important systematic and interpretational uncertainties remain even in the peculiar-velocity and cluster-baryon-fraction methods. While the age of the Universe coupled with large values of the Hubble parameter argue for $\Omega_M < 1$, the errors in t_0 and h are still significant. Finally, some methods continue to favor higher values of Ω_M: velocity power-spectrum measurements, redshift-space distortions, void outflow, linear vs nonlinear power-spectrum measurements, galaxy counts, and the problem of galaxy antibiasing (see, e.g., Ref. [11]).

A cosmological constant can resolve the flatness dilemma [12,13]. Since it corresponds to a uniform energy density (vacuum energy) that does not clump, its presence is not detected in determinations of the matter density. Because of the accelerated expansion associated with a cosmological constant, the expansion age is larger for a given Hubble parameter (see Fig. 1). Until very recently, ΛCDM was the model preferred by the observations [14].

Two problems now loom for ΛCDM: The limits to Ω_Λ from (1) the frequency of gravitational lensing of distant quasistellar objects (QSOs), $\Omega_\Lambda < 0.66$(95% C.L.) [15], and (2) the magnitude-redshift (Hubble) diagram of type Ia supernovae (SNe-Ia) $\Omega_\Lambda < 0.51$ (95%) [16]. Neither deals a death blow to ΛCDM— Ω_Λ as low as 0.5 still retains many of the beneficial features and several systematic uncertainties associated with the SNe-Ia determination remain—but a dark shadow has been cast.

III. *X*CDM

Though ΛCDM is the ''best fit'' CDM model, the theoretical motivation is weak. The best argument for considering the tiny vacuum energy required, $\rho_{\mathrm{vac}} \sim 10^{-8}$ eV^4, is the absence of a reliable calculation of the quantum vacuum energy [17]. (Naive estimates of the vacuum energy range from 50 to 125 orders of magnitude larger than this.) Given the weak motivation for a cosmological constant and the apparent observational evidence against one, as well as the strong motivation for inflation and the evidence against $\Omega_M = 1$, we think it worthwhile to take a broader view.

Other possibilities have been suggested for a smooth component [18]: relativistic particles [12]; a tangled network of light strings [19]; texture [20]; and a decaying cosmological constant (i.e., scalar-field energy) [21–23]. For definiteness, as well as to facilitate a comprehensive analysis, we parametrize the effective equation of state of the unknown, smooth component by $w \equiv p_X/\rho_X$ with $w < 0$ [24]. The energy density of the smooth component ρ_X decreases as $R^{-3(1+w)}$ where $R(t)$ is the cosmic scale factor; vacuum energy corresponds to $w = -1$ and texture or tangled strings correspond to $w = -\frac{1}{3}$.

For the reasons described above, we insist that X matter remain approximately smooth on all scales. Naively a component with $w < 0$ should be highly unstable to the growth of small-scale perturbations. However, vacuum energy, by definition, is constant in space and time. Tangled strings, rela-

tivistic particles, and scalar-field energy are all relativistic by nature and hence very ''stiff''; thus, in spite of the clumping of matter around them, they should remain (nearly) smooth. (In fact, it has been shown [22,23] that scalar-field energy remains approximately smooth.) Relativistic particles, by virtue of their high speeds, do not clump [12]. Likewise, it is easy to show that the effect of clumpy matter on an otherwise straight string segment is to bend it (similar to the bending of light) by an angle of order $\delta\Phi/c^2$, where $\delta\Phi\sim10^{-5}$ is the typical magnitude of the large-scale perturbed gravitational potential in the universe. Thus, a tangled string network should remain approximately smooth. We consider our smoothness (or stiff X component) approximation to be a reasonable starting point [25].

We note that there are reasons for only considering $w<0$. The first is the age problem, which is even more severe for $w\geqslant0$ (see Fig. 1). The second is that for $w>0$ the energy density in the smooth component decreases faster than R^{-3}, implying that the ratio of the energy density in the smooth component to the matter component was even larger at earlier times. This suppresses the growth of density perturbations, and when the spectrum of density perturbations is fixed on large scales by the Cosmic Background Explorer (COBE), this leads to too little inhomogeneity on small scales (see Fig. 2) [26]. The case $w=0$ corresponds to the smooth component behaving like pressureless matter; if the smooth component clumped, but only on large enough scales to evade detection ($>50h^{-1}$ Mpc), the flatness problem could be solved and the COBE normalization would be the same as sCDM because the growth of density perturbations on large scales would be unaffected. However, the growth of perturbations on small scales would be affected and the problem of producing sufficient small scale structure would be similar to that of hot dark matter. Thus, we dismiss this possibility.

The formation of cosmic structure in a CDM model is dictated by the power spectrum of density perturbations. There are two important changes brought about by the presence of a smooth component: the normalization of the power spectrum based upon the accurate COBE determination of CMB anisotropy on angular scales of around 10° and the transfer function that describes the growth of density perturbations from the inflationary epoch to the present. For fixed inflationary perturbations, CMB anisotropy on COBE scales is larger (due to the integrated Sachs-Wolfe effect [27]); because of the smooth component there is less growth of density perturbations from the inflationary period until the present.

We use the COBE four-year results [28] to normalize the power spectrum (assuming negligible gravity waves and $n=1$) using the method of Ref. [29]. Writing the (linear) power spectrum today as

$$P(k)\equiv\langle|\delta_k|^2\rangle=\frac{2\pi^2}{H_0^3}\,\delta_H^2(k/H_0)^nT^2(k)\,, \tag{1}$$

$$T(k)=\frac{\ln(1+2.34q)}{2.34q}\times[1+3.89q+(16.1q)^2+(5.46q)^3+(6.71q)^4]^{-1/4}, \tag{2}$$

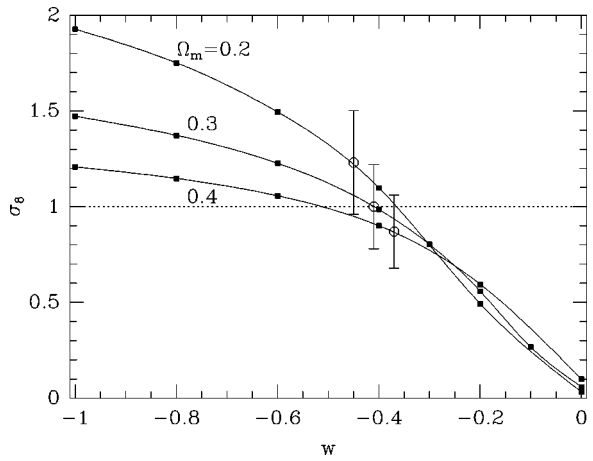

FIG. 3. σ_8 as a function of w with $\Gamma=0.25$ and $n=1$. Points with 2σ error bars on the curves show σ_8 from cluster abundance for that value of Ω_M. Note, σ_8 scales as $(\Gamma/0.25)^{1.3}\exp[-3.1(n-1)]$.

where δ_k is the Fourier transform of the density field, $q=k/\Gamma$ (h/Mpc) and $\Gamma\simeq\Omega_Mh$ is the ''shape'' parameter [30,31]. The quantity δ_H, which corresponds to the amplitude of density perturbations on the Hubble scale today, is a convenient normalization whose value is shown as a function of Ω_X and w in Fig. 2. The transfer function, $T(k)$, is well fit by the form quoted for $w<0$, with more small-scale power than this form predicts when $w\to0$.

There are many constraints on CDM models. The two most stringent for the power spectrum in low-Ω_M models are the shape parameter $\Gamma=0.25\pm0.05$ (for $n=1$) [32] and the abundance of rich clusters. The latter can be reduced to a constraint on σ_8 (the rms mass fluctuation in spheres of radius $8h^{-1}$ Mpc):

$$\sigma_r^2\equiv\int_0^\infty\frac{dk}{k}\,\frac{k^3P(k)}{2\pi^2}\left(\frac{3j_1(kr)}{kr}\right)^2, \tag{3}$$

with $r=8h^{-1}$ Mpc. There is no consensus on the precise value of σ_8 or its scaling with Ω_M; differences arise due to different input data and calculational schemes [33]. Further, the scaling with Ω_M depends slightly upon w, through the relation between virial mass and cluster temperature. Nevertheless, there is a general consensus about this important constraint and as a middle-of-the-road estimate we use $\sigma_8=(0.55\pm0.06)\Omega_M^{-0.5}$ which is consistent with most published estimates [33] and slightly conservative (low σ_8) near $\Omega_M\sim0.3$.

There are two nice features of xCDM: The shape constraint can be satisfied with $h\sim0.7$ and $\Omega_X\sim0.6$ for which the σ_8 constraint can be readily satisfied with $w\sim-1/2$ (see Fig. 3). For ΛCDM ($w=-1$) tilt (i.e., $n<1$) and/or gravity waves are needed to reduce σ_8 and for an open universe (closely approximated by $w=-\frac{1}{3}$) σ_8 is too small unless Ω_M is large or $n>1$ [34].

Next, we turn to the two worries of ΛCDM—the frequency of QSO lensing and the SNe-Ia constraint. Both in-

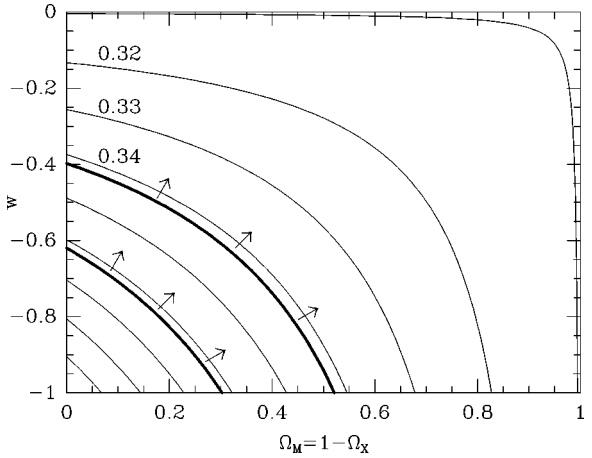

FIG. 4. Constraints from the SNe-Ia magnitude-redshift diagram. Contours are of $H_0 r(z=0.4)$ from 0.31 to 0.39 in steps of 0.01. The thick contours are the current 95% C.L. and 68% C.L. limits, and arrows indicate that values to the upper right of these curves are favored. (For $w=-1$ the constraint shown here is less stringent than that in Ref. [16] because we have two free parameters rather than one.)

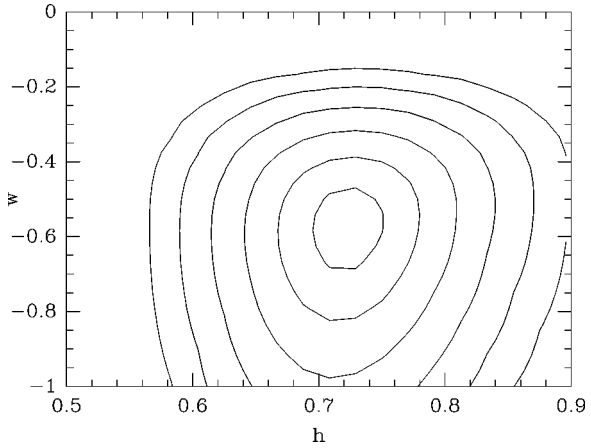

FIG. 5. The likelihood, marginalized over Ω_M. Contours are in units of $\frac{1}{2}\sigma$. (See text for the cosmological data and conservative errorbars used.)

volve the increased distance to a given redshift that comes with Λ. The proper distance today is given by the Robertson-Walker radial coordinate [35]

$$r(z) = \int_0^z \frac{dz}{H(z)} = H_0 \left[z - \frac{1}{2}(1+q_0)z^2 + \cdots \right], \qquad (4)$$

$$H^2(z) = H_0^2 [(1+z)^3 \Omega_M + (1+z)^{3(1+w)} \Omega_X], \qquad (5)$$

and the deceleration parameter $q_0 = -\ddot{R}_0 / R_0 H_0^2 = \frac{1}{2} + \frac{3}{2} w \Omega_X$. Note, $r(z)$ increases with decreasing w; this leads to more volume and more lenses between us and a QSO at redshift z and a higher frequency of lensing.

While the SNe-Ia limits on the distance redshift relation [16] are quoted for a flat universe with cosmological constant, they are readily translated into a constraint on r. Since the seven distant SNe-Ia have redshifts $z \sim 0.4$, that constraint can be expressed as $0.287(0.271) < H_0 r(z=0.4) < 0.342(0.362)$ at 68% C.L. (95% C.L.) [36]. Their results constrain Ω_M and w (see Fig. 4). Soon, Perlmutter's group should have results based on nearly four times as many SNe-Ia's and another group (the High-z Supernova Team) should have results based on a comparable number of SNe-Ia's. This will sharpen this important constraint to w significantly.

IV. CONCLUDING REMARKS

Inflation is a bold and compelling idea. It predicts a flat universe, but not the form which the critical energy density takes. Because of increasing evidence that the matter density is significantly less than the critical density, as well as the attractiveness of inflation and the successes of CDM, we have explored the possibility that most of the critical energy density resides in a smooth component of unknown nature, with equation of state $p_X = w\rho_X$ ($w<0$). Increasing w to around -0.6 retains the attractive features of ΛCDM and resolves the conflict with the SNe-Ia constraint; further tilt and/or gravity waves are not required to obtain the correct number of rich clusters observed at present.

For the sake of illustration we have used the following cosmological data to find the best fit XCDM model: $t_0 = 15 \pm 2$ Gyr, $h = 0.7 \pm 0.07$, $\Omega_B h^2 = 0.02$, $\Gamma = 0.25 \pm 0.1$, $\sigma_8 \Omega_M^{0.5} = 0.55 \pm 0.1$, $[H_0 r(z=0.4)]^2 = 0.10 \pm 0.015$, and the COBE four-year data set. (For several constraints we have inflated the error bars to be conservative.) We have marginalized over Ω_M with prior $\Omega_M = 0.3 \pm 0.05$. For $n=1$, an XCDM model with $w=-0.6$ and $h=0.7$ has maximum likelihood (see Fig. 5). (The unmarginalized likelihood prefers $\Omega_M \sim 0.4$, $h=0.7$, and $w=-0.4$ but is quite broad.) In passing, we note that a ''tangled'' network of walls or a wall wrapped around the universe (supposing space is $S^2 \times S^1$) would lead to a smooth component with $w = -\frac{2}{3}$.

Introducing w to the list of CDM parameters brings the total to at least ten (w, n, h, $\Omega_B h^2$, n_T, T/S, $\Omega_{\rm tot}$, Ω_ν, Ω_X, and $\Omega_{\rm rad}$). While this is a daunting number, the flood of cosmological data coming—larger redshift surveys, accurate measurements of the expansion rate and deceleration rate of the universe, high resolution observations of clusters with x rays, the Sunyaev-Zel'dovich effect and weak lensing, studies of galactic evolution by HST and Keck, and especially measurements of CMB anisotropy on angular scales from arcminutes to tens of degrees—should eventually overdetermine the parameters of CDM + inflation. Then the data will not only sharply test inflation, but also discriminate between different CDM models and even provide information about the underlying inflationary potential [37].

In the near term, the SNe-Ia magnitude-redshift diagram and CMB angular power spectrum will provide an important test of xCDM: the position of features (e.g., the first peak) in the angular power spectrum tests flatness, but is less sensitive to w, and given $\Omega_{\rm tot}$ (and Ω_M), SNe-Ia can determine w (see Fig. 4).

ACKNOWLEDGMENTS

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[25] In addition to the clumping that is caused by the lumpy matter distribution in the universe, the X component could also have been slightly perturbed (density variations of order 10^{-5}) to begin with; this would naturally occur if the underlying density perturbations are curvature perturbations (as predicted by inflation). The small amount of clumping of scalar-field energy, tangled strings, or any approximately smooth component, while certainly unimportant for considerations of the distribution of matter today or the evolution of the cosmic-scale factor, will have a small effect on large-angle CMB anisotropy and hence COBE normalization. When the X component is becoming dominant (at relatively recent times), it affects large-angle CMB anisotropy due to the spatially and time-varying gravitational potential it creates (''late-time, integrated Sachs-Wolfe effect''). While the effect of the time-varying potential is included in our work, and is significant, the effect of spatial variations is not. However, in Ref. [22] the effect of spatial variations on the COBE normalization is shown to be small for scalar-field energy.

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[35] The Taylor series expansion is not very accurate, even for $z \sim 0.4$. Padé approximants provide a more accurate approximation, e.g., the second-order Padé approximant is $H_0 r(z) = z/1 + (1+q_0)z/2]$. For all of our results we have numerically integrated the expression for $r(z)$.

[36] The 95% C.L. limit to Ω_Λ quoted in Ref. [16] is more stringent than that which follows from the 95% C.L. limit to $r(z)$ because the authors of Ref. [16] reduce the number of free parameters from two (Ω_M, Ω_Λ) to one.

[37] See, e.g., E. J. Copeland, E. W. Kolb, A. R. Liddle, and J. E. Lidsey, Phys. Rev. Lett. **71**, 219 (1993); Phys. Rev. D **48**, 2529 (1993); M. S. Turner, *ibid.* **48**, 3502 (1993); **48**, 5539 (1993).