

## Higher order graviton scattering in $M$ (atrix) theory

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In matrix theory the effective action for graviton-graviton scattering is a double expansion in the relative velocity and inverse separation. We discuss the systematics of this expansion and subject matrix theory to a new test. Low-energy supergravity predicts the coefficient of the  $v^6/r^{14}$  term, a two-loop effect, in agreement with explicit matrix model calculations. [S0556-2821(97)50118-4]

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Matrix theory is a remarkable proposal for the fundamental degrees of freedom and their Hamiltonian. In the original paper [1], one of the principal tests was a successful comparison of graviton-graviton scattering in the matrix theory and in eleven-dimensional supergravity. In subsequent work this has been extended to scattering of extended objects [2,3] and scattering with nonzero momentum transfer  $q_{11}$  [4]. This work has dealt with the leading term both at low velocity and long distance, for example  $v^4/r^7$  in graviton-graviton scattering at  $q_{11}=0$ . Matrix theory predicts a series of corrections both in  $v$  and  $r$ , and if it is correct these must all be understood in eleven-dimensional terms.

Two of the present authors [5] have recently reported on the  $v^4/r^{10}$  term at  $q_{11}=0$ , a two-loop matrix theory effect. It vanishes as required by the matrix theory conjecture. In this note we would like to develop some of the systematics of the double expansion in  $v$  and  $r$  for graviton-graviton scattering at  $q_{11}=0$  and to report on a new test of matrix theory. We observe that eleven-dimensional supergravity predicts the coefficient of the  $v^6/r^{14}$  term. In matrix theory this is a two-loop effect, and by an extension of the calculation [5] we find agreement.<sup>1</sup>

Higher velocity corrections have recently been considered in Ref. [7]. In that work there appeared to be a mismatch between the supergravity and matrix theory amplitudes. However, as noted by those authors, the mismatch is sub-leading in the large- $N$  expansion. To make the comparison one must therefore have a precise understanding of the meaning of the finite- $N$  matrix theory. Happily, this has recently been supplied in an important paper by Susskind [8] (see also [9]). Finite  $N$  is to be identified with compactification of a null direction (henceforth the  $-$  direction), not a spacelike direction. We will see that the velocity expansion at fixed  $p_-$  is simpler than in Ref. [7].

Let us consider first the matrix theory perturbation expansion. The bosonic part of the matrix theory action is [1]

<sup>1</sup>A recent paper by Ganor, Gopakumar, and Ramgoolam [6] considers scattering of a graviton from an  $R^8/Z_2$  fixed point, finding a discrepancy at two loops. The extension of matrix theory to such less symmetric backgrounds is an open issue. These authors also discuss the general form of higher-order matrix theory amplitudes.

$$S = \int d\tau \text{Tr} \left( \frac{1}{2R} D_\tau X^i D_\tau X^i + \frac{M^6 R}{4} [X^i, X^j]^2 \right), \quad (1)$$

where  $R$  is the radius of eleventh dimension and  $M$  the eleven-dimensional Planck mass up to a convention-dependent numerical coefficient; the signs are appropriate for Hermitian  $X$ . By rescaling  $\tau = u/R$  and  $X^i = y^i/M^3$ , the action becomes

$$S = \frac{1}{M^6} \int du \text{Tr} \left( \frac{1}{2} D_u y^i D_u y^i + \frac{1}{4} [y^i, y^j]^2 \right). \quad (2)$$

It follows that  $M^6$  is the loop-counting parameter, and that the effective action at  $L$  loops is of the form

$$\begin{aligned} S_L &= M^{6L-6} \int du f_L(y^i, D_u) \\ &= R M^{6L-6} \int d\tau f_L(M^3 X^i, R^{-1} D_\tau). \end{aligned} \quad (3)$$

Finally we have dimensional analysis:  $f_L$  must have units of (length)<sup>6L-8</sup>. For the leading low-energy effective action, depending on the velocity  $v^i = D_\tau X^i$ , but not the acceleration, this becomes

$$S_L = R M^{6-3L} \int d\tau r^{4-3L} g_L \left( \frac{X^i}{r}, \frac{v^i}{R M^3 r^2} \right), \quad (4)$$

where  $r^2 = X^i X^i$ . Let us write out the first few terms in the expansion of the effective Lagrangian, indicating the dependence on  $v$  and  $r$ , but suppressing the dependence on  $M$ ,  $R$ , and  $X^i/r$ :<sup>2</sup>

<sup>2</sup>The systematics of this expansion have also been considered by Fischler and Susskind.

$$\begin{aligned}
\mathcal{L}_0 &= c_{00} v^2, \\
\mathcal{L}_1 &= c_{11} \frac{v^4}{r^7} + c_{12} \frac{v^6}{r^{11}} + c_{13} \frac{v^8}{r^{15}} + \dots, \\
\mathcal{L}_2 &= c_{21} \frac{v^4}{r^{10}} + c_{22} \frac{v^6}{r^{14}} + c_{23} \frac{v^8}{r^{18}} + \dots, \\
\mathcal{L}_3 &= c_{31} \frac{v^4}{r^{13}} + c_{32} \frac{v^6}{r^{17}} + c_{33} \frac{v^8}{r^{21}} + \dots.
\end{aligned} \tag{5}$$

In writing this expansion we have used the fact that the supersymmetry algebra with 16 supercharges prevents renormalization of the coefficient of  $v^2$ , and that the expansion must be even in  $v$  by time-reversal invariance.

Now let us consider the supergravity prediction. We will study the scattering of gravitons of momenta  $p_- = N_1/R$ ,  $N_2/R$ , with  $N_1$  large enough that the first graviton can be considered as a classical source for the gravitational field. Ultimately, to understand the full form of the supergravity amplitude, we will need to develop the Feynman rules for supergravity with lightlike compactification, but we leave that for future work.

Our conventions are  $x^\pm = x^{11} \pm t$ , and the time parameter of the light-cone quantization is  $\tau = \frac{1}{2}x^+$ . These are chosen so that the large- $N$  limit of the lightlike quantization is consistent with the large- $N$  limit of spacelike compactification. Note that at large  $N$  all particles are moving approximately along lines of  $\delta x^{11} = \delta t$ , so that with  $x^- = x^{11} - t$  the periodicity is  $2\pi R$  both in  $x^-$  and  $x^{11}$ . Also,  $\delta\tau = \frac{1}{2}(\delta x^{11} + \delta t) \sim \delta t$  along any world line. Finally,  $p_-$  is positive.

The source graviton is taken to have vanishing transverse velocity. Its world line is  $x^- = x^i = 0$ , and it produces the Aichelburg-Sexl metric [11]

$$G_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{6}$$

where the only nonvanishing component of  $h_{\mu\nu}$  is

$$h_{--} = \frac{2\kappa_{11}^2 p_-}{7\omega_8 r^7} \delta(x^-) = \frac{15\pi N_1}{RM^9 r^7} \delta(x^-). \tag{7}$$

Here  $\kappa_{11}^2 = 16\pi^5/M^9$  (see Ref. [3], for example) and  $\omega_8$  is the volume of  $S_8$ . This metric can be thought of as obtained from the Schwarzschild metric by taking the limit of infinite boost in the  $+$  direction, while the mass is taken to zero; the latter accounts for the absence of higher-order terms in  $1/r$  or  $N_1$ . A more detailed derivation of this metric can be found in the Appendix. The source graviton is in a state of definite  $p_-$  and so we average over the  $x^- \in (0, 2\pi R)$  direction to give

$$h_{--} = \frac{15N_1}{2R^2 M^9 r^7}. \tag{8}$$

For the action of the ‘‘probe’’ graviton in this field, we use the following trick. Begin with the action for a massive scalar (spin effects fall more rapidly with  $r$ ) in eleven dimensions:

$$\begin{aligned}
S &= -m \int d\tau (-G_{\mu\nu} \dot{x}^\mu \dot{x}^\nu)^{1/2} \\
&= -m \int d\tau (-2\dot{x}^- - v^2 - h_{--} \dot{x}^- \dot{x}^-)^{1/2},
\end{aligned} \tag{9}$$

where we have used the form of the Aichelburg-Sexl metric. A dot denotes  $\partial_\tau$  and  $v^2 = \dot{x}^i \dot{x}^i$ . This action vanishes if we take  $m \rightarrow 0$  with fixed velocities, but for the process being considered here, it is  $p_-$  that is to be fixed. We therefore carry out a Legendre transformation on  $x^-$ :

$$p_- = m \frac{1 + h_{--} \dot{x}^-}{(-2\dot{x}^- - v^2 - h_{--} \dot{x}^- \dot{x}^-)^{1/2}}. \tag{10}$$

The appropriate Lagrangian for  $x^i$  at fixed  $p_-$  is (minus) the Routhian,

$$\mathcal{L}'(p_-) = -\mathcal{R}(p_-) = \mathcal{L} - p_- \dot{x}^-(p_-). \tag{11}$$

Equation (10) determines  $\dot{x}^-(p_-)$ ; it is convenient before solving to take the limit  $m \rightarrow 0$ , where it reduces to  $G_{\mu\nu} \dot{x}^\mu \dot{x}^\nu = 0$ . Then

$$\dot{x}^- = \frac{\sqrt{1 - h_{--} v^2} - 1}{h_{--}}. \tag{12}$$

In the  $m \rightarrow 0$  limit at fixed  $p_-$  the effective Lagrangian becomes

$$\begin{aligned}
\mathcal{L}' \rightarrow -p_- \dot{x}^- &= p_- \left\{ \frac{v^2}{2} + \frac{h_{--} v^4}{8} + \frac{h_{--}^2 v^6}{16} + O(h_{--}^3 v^8) \right\} \\
&= \frac{N_2}{2R} v^2 + \frac{15}{16} \frac{N_1 N_2}{R^3 M^9} \frac{v^4}{r^7} + \frac{225}{64} \frac{N_1^2 N_2}{R^5 M^{18}} \frac{v^6}{r^{14}} + O\left(\frac{v^8}{r^{21}}\right).
\end{aligned} \tag{13}$$

The  $r$  and  $v$  dependences match the diagonal terms in the series (5), and the  $N$  dependences are consistent with the leading large- $N$  behavior  $N^{L+1}$ . The  $v^4/r^7$  agrees with the one-loop matrix model amplitude, as asserted in Ref. [1] and worked out in detail in Ref. [3]. The two-loop calculation in Ref. [5] extended to  $v^6$  gives for SU(2) the value

$$\frac{225}{32} \frac{1}{R^5 M^{18}} \frac{v^6}{r^{14}}. \tag{14}$$

In [5],  $RM^3$  was implicitly set to one, but we have restored it by dimensional analysis and used the relation  $g = 2R$  for  $g$  defined in Ref. [5], as follows from the tree level term in Eq. (13). The separate contributions of the various two-loop graphs are given in Table I. The  $N$  dependence can be reconstructed as follows. In double-line notation every graph involves three index loops, and so is of order  $N^3$ . Terms proportional to  $N_1^3$  or  $N_2^3$  would only involve one block (graviton) and so could not depend on  $r$ . Symmetry under interchange of 1 and 2 thus determines that the SU(2) result (14) is multiplied by

$$\frac{N_1 N_2^2 + N_1^2 N_2}{2}, \tag{15}$$

TABLE I. Coefficients of  $gv^6/r^{14}$ . Graphs are labeled as in Ref. [5]. We have included factors 1/2 for diagrams involving two cubic vertices of the same type directly in this table.

$(a)_1$	$+\frac{287481}{28682}$	$(b)_3$	$-\frac{4717523}{229376}$	$(c)_1$	$+\frac{107251}{114688}$	$(d)_2$	0
$(a)_2$	$-\frac{27519}{7168}$	$(b)_4$	$+\frac{16965}{4096}$	$(c)_2$	$+\frac{892261}{688128}$	$(d)_3$	$+\frac{4615}{672}$
$(b)_1$	$-\frac{2366913}{114688}$	$(b)_5$	$+\frac{13311}{4096}$	$(c)_3$	$+\frac{231}{1024}$	$(d)_4$	$+\frac{7995}{224}$
$(b)_2$	$+\frac{31595}{14336}$	$(b)_6$	$+\frac{315}{2048}$	$(d)_1$	$-\frac{698165}{43008}$	sum	$+\frac{225}{64}$

in agreement with the supergravity result (13) for the term of interest.

Note that we have not distinguished radial and transverse velocities. Any term proportional to the radial velocity is equivalent by parts to a term involving the acceleration. All matrix theory calculations to date have considered straight-line motion and so are insensitive to such terms. Thus, we write  $v^2$  with the understanding that only the transverse part is relevant.

It is difficult to be certain which of the many tests of matrix theory actually test that conjecture and not just the weaker and less controversial assumption that the IIA string has an eleven-dimensional limit. In the present case the numerical agreement is impressive. Moreover, it is difficult to see how supersymmetry alone would determine the normalization of the  $v^6/r^{14}$  term in the supersymmetric quantum mechanics effective action, suggesting that an additional structure (eleven-dimensional Lorentz invariance) is present. A die-hard skeptic might still argue as follows. A ‘‘normal’’ supersymmetric invariant, obtained from a multiple commutator with all sixteen supercharges (the analog of an integral over all of superspace), would be at least of order  $v^8$ . The  $v^6$  term is therefore ‘‘chiral’’ and so might be constrained by nonrenormalization theorems. Then one could continue from the eleven-dimensional supergravity limit where one calculation is valid, to the IIA string limit where the other calculation is valid, and the answers must agree independent of the matrix theory conjecture. But the skeptic is not willing to bet that the  $v^8/r^{21}$  term, a three-loop effect, will show a discrepancy.

It is interesting to consider higher corrections in the supergravity theory. The  $v^6/r^{14}$  term can be thought of as arising from the graph of Fig. 1(a), with a second-order coupling to the probe.<sup>3</sup> Figure 1(b) would represent a nonlinear correction to the metric (8), which as we have noted is absent. The ladder graph is second order in the effective Lagrangian and the crossed ladder is absent in the light-cone frame. It appears that each graviton coupling to the source brings at least an  $r^7$  from the field (8), so that the leading large- $r$  behavior at order  $N_1^k$  would be the diagonal  $k$ -loop term that we have considered.

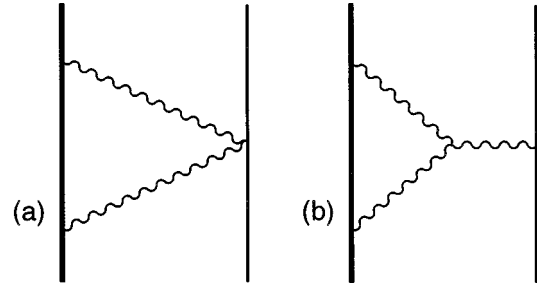


FIG. 1. (a) Graphical representation of probe graviton (thin straight line) interacting with the metric of source graviton (heavy straight line) at second order. (b) Vanishing nonlinear correction to the metric of the source.

Dimensionally, higher derivative operators in the low-energy supergravity theory bring in additional powers of  $v$  and  $1/r$  and so correspond to matrix model amplitudes that lie, for a given power of  $N_1$ , to the right of and/or below  $N_1^k v^{2k+2}/r^{7k}$  in the series (5). However, higher-order local curvature invariants  $R^4 + R^6 + \dots$  in the  $D=11$  supergravity action are not expected to change the low-energy scattering of two gravitons when one of them has large  $p_-$ , i.e., when it can be treated as a source for the gravitational field. The reason is that (in contrast to the Schwarzschild solution, for example) the corresponding plane-fronted wave background (8) is not modified by  $R^n$  corrections to the action: according to the standard argument (see, e.g., [12]), the existence of a covariantly constant null Killing vector implies the vanishing of all second rank tensors constructed out of curvature and the metric except the Ricci one (equivalently, corrections to Schwarzschild disappear in the infinite boost, zero mass limit). Supergravity loop effects, being weak at low energy, should also lie to the right of and below the  $N_1^k v^{2k+2}/r^{7k}$  term. Terms below the diagonal that arise in this way are subleading in  $N_1$  for the given number of loops.<sup>4</sup> It would be interesting to relate these supergravity effects to the matrix model, even at one matrix model loop where the whole series is known [13]. In passing we would like to mention the observation that the coefficient  $c_{12}$  of the next higher one-loop term  $v^6/r^{11}$  actually vanishes.

On the matrix model side there is the important complication of bound state effects.<sup>5</sup> Matrix theory scattering calculations to date have treated the 0-branes in a bound state as being coincident with zero relative velocity. Note, however, that a term which is dimensionally of order  $v^8$  can have the structure  $v_1^2 v_2^6$  and even with the center of mass  $v_1$  vanishing can generate a  $v^8$  term proportional to the expectation value of the relative  $v^2$  in the bound state. This would not affect the present calculation because all  $v^8$  terms fall off more rapidly in  $r$ , but to determine some higher terms one needs an understanding of the bound state. One must also consider recoil, interactions causing the gravitons to deviate from a straight line. To the order we are working we believe that this corresponds to omitting the one-particle-reducible two-loop graphs, but at higher order it may be necessary to sepa-

<sup>3</sup>Note that this is second order in a first quantized description of the probe. This does not correspond directly to second order in a field theory action.

<sup>4</sup>This has also been noted by Susskind [10].

<sup>5</sup>We would like to thank David Gross for raising this issue. See also Ref. [6].

rate the light and heavy matrix model degrees of freedom in a more systematic way.

It is interesting to repeat the derivation of the Routhian for scattering at fixed spacelike momentum  $p_{11}$ . Here we have ( $\tau=t$ )

$$S = -m \int dt \{1 - (\dot{x}^{11})^2 - v^2 - h_{--}(\dot{x}^{11} - 1)^2\}^{1/2}. \quad (16)$$

Then one finds

$$\mathcal{L}' = -p_{11}\dot{x}^{11} = -p_{11} \left( 1 + \frac{\sqrt{1 - (1 + h_{--})v^2 - 1}}{1 + h_{--}} \right). \quad (17)$$

Where the earlier Routhian (13) had only terms of order  $v^2(v^2/r^7)^k$ , this now has higher velocity corrections, a double series  $v^{2+2l}(v^2/r^7)^k$ . Spacelike compactification of  $M$  theory gives the IIA string theory, and Eq. (17) is precisely the action for interaction of two  $D0$ -branes via classical supergravity. This is the more complicated expansion considered in Ref. [7], but we see that it has no direct relevance to finite  $N$  matrix theory. We emphasize that the result (13) is fully relativistic.

It is curious that the null and timelike Lagrangians (13) and (17) are related by the simple substitution  $h_{--} \rightarrow 1 + h_{--}$  (the transverse velocities are in direct correspondence because of our conventions, as noted earlier). To better understand the formal relation between the two cases, note that just as Eq. (17) is essentially the Lagrangian for a  $D0$ -brane probe moving in a  $D0$ -brane source background, Eq. (13) can be interpreted as a  $D0$ -brane probe Lagrangian in a  $D=10$  background resulting from reducing the  $D=11$  plane wave  $ds_{11}^2 = dx^+ dx^- + h_{--} dx^- dx^- + dx^i dx^i$ ,  $h_{--} = Q/r^7$ , along the null  $x^-$  direction<sup>6</sup> instead of the spatial  $x^{11}$  direction. While the reduction along  $x^{11}$  gives the standard 0-brane background, the reduction along  $x^-$  produces the following  $D=10$  (string-frame) metric, dilaton, and 1-form field

$$ds_{10}^2 = -h_{--}^{-1/2} d\tau^2 + h_{--}^{1/2} dx^i dx^i, \quad e^\phi = h_{--}^{3/4},$$

$$A = -h_{--}^{-1} d\tau, \quad (18)$$

where  $\tau = \frac{1}{2}x^+$ . This becomes the usual 0-brane solution if  $\tau \rightarrow t$  and  $h_{--} \rightarrow H = 1 + h_{--}$  (and  $A \rightarrow A + dt$ ). This relation is implied by the structure of the  $D=11$  plane wave metric (in particular, it remains invariant under  $x^+ \rightarrow x^+ - x^- = 2t$  and  $h_{--} \rightarrow h_{--} + 1$ ).

Thus Eq. (18) is formally the same as the *short-distance* (or “near-horizon”) limit of the 0-brane background: then  $h_{--} \gg 1$  so that  $H = 1 + h_{--} \approx h_{--}$ . Equivalently, it may be viewed as a large charge  $Q \sim Ng$  or *large  $N$*  (but *fixed distance  $r$* ) limit of the 0-brane solution. The fact that the two actions are formally related by  $h_{--} \rightarrow 1 + h_{--}$  implies that large  $r$ , small  $v$  expansion of the first action is simply the leading part of the expansion of the second action.

As was already mentioned above, it is the “null reduction” action that is in direct correspondence with the matrix theory results for finite  $N$ . Remarkably, this conclusion extends also to more complicated cases of graviton scattering off  $M$ -branes discussed in [2]. Again, the supergravity potentials corresponding to the “fixed  $p_-$ ” case can be obtained from the relevant  $D$ -brane probe actions in the  $D=10$  backgrounds following upon reduction along the “null” direction  $x^-$ . These actions are found from the “fixed  $p_{11}$ ” actions by replacing the 0-brane harmonic function  $H$  by its “short-distance” (or large  $N$ ) limit  $H - 1 = h_{--}$ . The resulting long-distance interaction potentials [containing in general both static and velocity-dependent terms like  $V = (1/r^n)(a + bv^2 + cv^4) + O(1/r^{2n})$ ] are then in precise agreement with the one-loop matrix model potentials with no need to assume that the number of 0-branes  $N$  is large as was done in [2] to be able to ignore additional subleading terms present in the fixed  $p_{11}$  picture. This provides another test of the proposal of Ref. [8].

Finally, let us note that from another point of view, discrete light-cone quantization can be regarded as a limit of spacelike compactification as follows [14]. The null direction has zero invariant length, so by a boost should be related to the  $R_{11} \rightarrow 0$  limit. The naive  $R_{11} \rightarrow 0$  limit is simply dimensional reduction to the  $p_{11}R_{11} = 0$  sector. Here one takes instead the  $p_{11}R_{11} = N$  sector, subtracts the overall  $N/R_{11}$ , and rescales to

$$H_{\text{eff}} = \frac{H - N/R_{11}}{R_{11}} \quad (19)$$

at fixed momentum  $p^i$ . Noting that  $v^i = O(R_{11}p^i)$  and  $h_{--} = O(R_{11}^{-2})$ , this yields Eq. (13) from Eq. (17).

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## APPENDIX

In this appendix we would like to derive the form of the Aichelburg-Sexl metric (7). This follows closely the original derivation of [11]. Start with the Einstein field equations

$$R_{\mu\nu} - \frac{1}{2}G_{\mu\nu}R = \kappa_{11}^2 T_{\mu\nu}, \quad (A1)$$

and approximate

$$G_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad (A2)$$

where  $(h_{\mu\nu})^2 \approx 0$ . This gives the linearized field equations

$$(\partial_t^2 - \Delta)\psi^{\mu\nu} = 2\kappa_{11}^2 T^{\mu\nu}, \quad (A3)$$

where

$$\psi^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}h_\lambda{}^\lambda. \quad (A4)$$

For a massless particle moving in  $x_{11}$  direction with the velocity of light, the energy momentum tensor is

<sup>6</sup>More precisely, the direction  $x^-$  is null in flat space, but is spacelike in the curved plane wave background.

$$T^{\mu\nu} = p_- \delta(x^-) \delta(x_\perp) s^\mu s^\nu, \quad (\text{A5})$$

where  $s^\mu = \delta_0^\mu + \delta_{11}^\mu$  and  $\delta(x_\perp) = \prod_{i=1}^9 \delta(x_i)$ . Inserting Eq. (A5) into Eq. (A3) gives a determining equation for  $\psi^{\mu\nu}$ . To solve it make the ansatz

$$\psi^{\mu\nu} = 2\kappa_{11}^2 p_- \delta(x^-) G_9(x_\perp) s^\mu s^\nu. \quad (\text{A6})$$

Then  $G_9$  satisfies nine-dimensional Poisson equation

$$\Delta G_9(x_\perp) + \delta(x_\perp) = 0. \quad (\text{A7})$$

The solution is given by

$$G_9(x_\perp) = \frac{15}{2(2\pi)^4} \frac{1}{r^7}. \quad (\text{A8})$$

Therefore

$$\psi^{\mu\nu} = \frac{15\pi N_1}{RM^9 r^7} \delta(x^-) s^\mu s^\nu. \quad (\text{A9})$$

This determines the form of the fluctuation of the metric. The only nonvanishing component of  $h_{\mu\nu}$  is  $h_{--}$  with the result (7).

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