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Hybrid inflation in supergravity

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We propose a simple realization of hybrid inflation in supergravity. Inflation in this scenario is relatively short, and inflationary density perturbations fall down on large scales, which corresponds to a blue spectrum with $n>1$. Superheavy strings which are formed after inflation in this scenario produce density perturbations of comparable magnitude. In this mixed perturbation (MP) model one may have microwave anisotropy produced by strings and galaxy formation due to inflationary perturbations. [S0556-2821(97)50316-X]

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For the last 15 years cosmologists have believed that something like inflation is necessary in order to construct an internally consistent cosmological theory. For more than 20 years the best hopes for the theory of all fundamental interactions have been related to supersymmetry. Unfortunately it is not that simple to combine these two ideas. It is possible to construct inflationary models based on globally supersymmetric theories, but things become increasingly complicated when one investigates supergravity and superstring theory [1].

The main problem is that the effective potential for the inflaton field σ in supergravity typically is too curved, growing as $\exp(C\sigma^2/M^2)$ at large values of the inflaton field σ. Here $M=2.4\times10^{18}$ GeV is the stringy Planck mass. The typical value of the parameter C in the theories with the minimal Kähler potential is $O(1)$, which makes inflation impossible because the inflaton mass in this theory becomes of the order of the Hubble constant. One can avoid this problem by introducing theories with specific nonminimal Kähler potentials, see, e.g., [2], but it is hard to find any independent motivation for potentials of that type in particle physics. The situation does not become better in superstring theory. A typical shape of the scalar field potential in superstring theory is $\exp(C\sigma/M)$ with $C=O(1)$, which also prevents inflation for $\sigma>M$.

As a result, after many years of development of supersymmetric theories we still do not have a consistent cosmological theory based on supergravity and superstrings. Of course, interpretation of string theory changes every year, so one may simply postpone investigation of this problem and study only globally supersymmetric models. A more constructive approach is to find those versions of superstring theory and supergravity where a consistent cosmological theory can be developed.

This is a difficult problem, but not entirely hopeless. Perhaps the easiest way to find a consistent inflationary model in supersymmetric theories is based on the hybrid inflation scenario [3]. In particular, recently a very interesting class of hybrid inflation models have been proposed based on the use of the D term in the effective potential, which does not contain the exponentially growing factors [4]. However, in string-inspired versions of these models one typically obtains too large density perturbations. In our paper we will concentrate on the model with the minimal Kähler potential and the simplest choice of the superpotential which leads to hybrid inflation [5]

$$W=S(\kappa\bar{\phi}\phi-\mu^2), \tag{1}$$

with $\kappa\ll1$. Here ϕ and $\bar{\phi}$ denote conjugate pairs of superfields transforming as nontrivial representations of some gauge group G under which the superfield S is neutrally charged. As noted in [6], the superpotential (1) is of the most general form consistent with an R symmetry under which $S\rightarrow e^{i\alpha}S$, $W\rightarrow e^{i\alpha}W$ and $\bar{\phi}\phi$ is invariant. The hybrid inflation scenario with this superpotential in a context of a globally supersymmetric theory was studied by many authors; see, e.g., [5–8]. However, the possibility to have inflation in this model with an account taken of supergravity corrections was not fully investigated. This will be the purpose of our paper.

The effective potential in a globally supersymmetric theory with the superpotential (1) is given by

$$V=\frac{\kappa^2|\sigma|^2}{2}(|\phi|^2+|\bar{\phi}|^2)+|\kappa\bar{\phi}\phi-\mu^2|^2+D\ \text{term}. \tag{2}$$

Here $\sigma=\sqrt{2}S$ is a canonically normalized scalar field. The absolute minimum appears at $\sigma=0$, $\phi=\bar{\phi}=\mu/\sqrt{\kappa}$. However, for $\sigma>\sigma_c=\sqrt{2}\mu/\sqrt{\kappa}$, the fields ϕ and $\bar{\phi}$ possess a positive mass squared and stay at the origin. This potential for $\phi=\bar{\phi}=0$ is exactly flat in the σ direction. If one simply adds a mass term $m^2\sigma^2/2$ which softly breaks supersymmetry, one obtains a simple realization of the hybrid inflation scenario [5]: Initially the scalar field σ is large. It slowly rolls down to $\sigma=\sigma_c$. Then the curvature of the effective potential in the ϕ direction becomes negative, the fields rapidly roll to the absolute minimum of the effective potential leading to the symmetry breaking of the group G, and inflation ends, as in the original version of the hybrid inflation scenario [3].

However, if one takes into account radiative corrections and supergravity effects, the behavior of the fields becomes somewhat different. The one-loop effective potential in this model is easily calculated from the spectrum of the theory composed by two pairs of real and pseudoscalar fields with mass squared $\kappa^2\sigma^2/2\pm\kappa\mu^2$ and a Dirac fermion with mass $\kappa\sigma/\sqrt{2}$. The one-loop effective potential is given by [6]

$$V_1=\frac{\kappa^2}{128\pi^2}\left[(\kappa\sigma^2-2\mu^2)^2\ \ln\frac{\kappa\sigma^2-2\mu^2}{\Lambda^2}\right.\\ \left.+(\kappa\sigma^2+2\mu^2)^2\ln\frac{\kappa\sigma^2+2\mu^2}{\Lambda^2}-2\kappa^2\sigma^4\ln\frac{\kappa\sigma^2}{\Lambda^2}\right], \tag{3}$$

where Λ indicates the renormalization scale.

At the stage of inflation, when $\sigma\gg\sigma_c$, the total effective potential is $V=\mu^4[1+(\kappa^2/8\pi^2)\ln(\sigma/\sigma_c)+\dots]$. The Hubble constant practically does not change during inflation. In units $M=1$, which we will use throughout the paper, one has $H=\mu^2/\sqrt{3}$.

It is convenient to use time t measured in units of H^{-1}. In these units time t is directly related to the number of e folds N. The usual equation $3H\dot\sigma=-V'$ for the field σ in this system of units looks particularly simple: $\dot\sigma=-V'/V=-\kappa^2/8\pi^2\sigma$. This gives $\sigma_0^2-\sigma^2(t)=(\kappa^2/4\pi^2)t$. This means that the universe expands during the inflationary stage as follows:

$$a(t)=a(0)e^t=a(0)\exp\left(\frac{4\pi^2}{\kappa^2}[\sigma_0^2-\sigma^2(t)]\right). \tag{4}$$

The total number of e folds of inflation is obtained by taking $\sigma(t)=0$: $N=\ln[a(t)/a(0)]\approx 4\pi^2\sigma_0^2/\kappa^2$. The universe expands e^N times when the field σ rolls down from $\sigma_N=\kappa\sqrt{N}/2\pi$. In particular, the structure of the observable part of the universe, corresponding to $N\sim 60$, is formed at $\sigma_{60}\sim 1.2\kappa\ll 1$.

Now let us calculate density perturbations [1]:

$$\frac{\delta\rho}{\rho}\sim\frac{\sqrt{3}}{5\pi}\frac{V^{3/2}}{V'}=\frac{8\pi\sqrt{3}}{5\kappa^2}\ \mu^2\sigma. \tag{5}$$

Using Eq. (4) one can express this result in terms of the ratio of the wavelength l of a perturbation to the wavelength l_0 of a perturbation which had the wavelength H^{-1} at the end of inflation:

$$\frac{\delta\rho}{\rho}\sim\frac{4\sqrt{3}\mu^2}{5\kappa}\ln^{1/2}\frac{l}{l_0}. \tag{6}$$

The COBE normalization is [9]

$$\frac{V^{3/2}}{V'}=\frac{8\pi^2}{\kappa^2}\mu^2\sigma\sim 5.3\times10^{-4}. \tag{7}$$

For $\sigma\sim 1.2\kappa$ it gives $\mu\sim 2.5\times10^{-3}\sqrt{\kappa}$ (in units $M=1$). This can be achieved, e.g., for $\kappa\sim 0.1$, and $\mu\sim 2\times10^{15}$ GeV; we will make a more accurate estimate shortly. Note that after the symmetry breaking in this scenario one may encounter cosmic strings production on a very interesting mass scale $\phi\sim\mu/\sqrt{\kappa}\sim 10^{16}$ GeV. This may happen, for instance, during the second stage of the symmetry breaking of the supersymmetric SO(10) grand unified theory (GUT) group, $SO(10)\to SU(3)_c\otimes SU(2)_L\otimes U(1)_R\otimes U(1)_{B-L}\to SU(3)_c\otimes SU(2)_L\otimes U(1)_Y\otimes Z_2$. Even though during the first stage of this symmetry breaking topologically stable monopoles can be formed, they are subsequently diluted at the inflationary stage preceding the phase transition when ϕ and $\bar\phi$ acquire a vacuum expectation value and $SU(3)_c\otimes SU(2)_L\otimes U(1)_R\otimes U(1)_{B-L}$ breaks down to $SU(2)_L\otimes U(1)_Y\otimes Z_2$. In such a case the $\phi+\bar\phi$ have to be identified with a $\mathbf{16}+\overline{\mathbf{16}}$ or a $\mathbf{126}+\overline{\mathbf{126}}$ pair of Higgs fields and S is an SO(10) singlet [7]. As we will see, inflation ends well before this second phase transition takes place and therefore the cosmic strings formed when ϕ and $\bar\phi$ roll down to their true values are not diluted.

Radiative corrections change the way inflation ends in the hybrid inflation model. In the original version of this scenario the structure of the universe appears due to the perturbations generated at $\sigma\sim\sigma_c=\sqrt{2}\mu/\sqrt{\kappa}$ [3]. For $\kappa\sim 0.1$, and $\mu\sim 2\times10^{15}$ GeV one has $\sigma_c\sim 10^{16}$ GeV. Meanwhile in our scenario this happens for $\sigma_{60}\sim 1.2\kappa\ll 1$. For $\kappa\sim 0.1$ one has $\sigma_{60}\sim 3\times10^{17}$ GeV$\sim 30\sigma_c$. Therefore when one calculates density perturbations and evaluates a possible significance of supergravity (SUGRA) corrections to the effective potential in our scenario, one should do it not near σ_c, but at $\sigma\sim kM\gg\sigma_c$.

The supergravity potential for $\phi=\bar\phi=0$ (ignoring the one-loop corrections calculated above) is given by [5] $V_{\rm SUGRA}=\mu^4 e^{\sigma^2/2}(1-\sigma^2/2+\sigma^4/4)=\mu^4(1+\sigma^4/8+\cdots)$. Inflation with this potential is possible because of the cancellation of the quadratic term $\sim\mu^4\sigma^2=3H^2\sigma^2$, but still it may occur only at $\sigma\lesssim 1$. Notice that the cancellation of this quadratic term derives from the general form of the typical superpotential during hybrid inflation, $W\sim\mu^2 S$. However, this cancellation is only operative with the choice of minimal Kähler potential $K=S^\dagger S$.

Since the Kähler potential is not protected by any nonrenormalization theorem, it is reasonable to expect the presence of terms like $\delta\mathcal{L}=\lambda\int d^4\theta(S^\dagger S)^2=2\lambda\mu^4\sigma^2$ in the effective Lagrangian. This mass term, however, is not important for our consideration if $\lambda\mu^4\sigma<{\rm Max}\{V_1',V_{\rm SUGRA}'\}$ in the range $\sigma_0<\sigma<\sigma_N$. This translates into the bound bound $\lambda\lesssim 0.03\kappa$, which is restrictive but not dangerous if $\delta\mathcal{L}$ is generated at the one-loop level.

One must admit that the Kähler potential may be nonminimal from the very beginning. Then the constraint $\lambda\lesssim 0.03\kappa$ requires that λ should be fine-tuned with accuracy about 10^{-2} for our scenario to work. However, it is not such a big price to pay if one compares it with the value of the constant $\lambda\sim 10^{-13}$ in a nonsupersymmetric inflationary model $\lambda\phi^4/4$ [1]. In any case, our main purpose here was to remove the standard obstacle preventing inflation in supergravity in the models with a minimal Kähler potential, and we are going to show that this problem is indeed resolved in our model.

First of all, as we will see now, at small σ the one-loop SUSY potential is more important, so that at the last stages of inflation the effective potential is always dominated by the one-loop effects.

Indeed, for $\sigma<1$ the effective potential is approximately given by μ^4, so it remains to compare its derivative due to one-loop and SUGRA terms. The condition $V_1'\gtrsim V_{\rm SUGRA}'$ reads $\sigma\lesssim\sqrt{\kappa}/3$. Comparing this expression with the expression for σ_N, one concludes that the last $N_{\rm SUSY} \sim 4\kappa^{-1}$ e folds of inflation are controlled by the loop effects. In particular, in order to neglect SUGRA effects during the last 60 e folds of inflation one would need to have $\kappa \lesssim 6\times 10^{-2}$. This condition is reasonable, but it is not automatically guaranteed, as expected in [6].

Now let us study inflation at the stage when SUGRA effects are dominant. In this case equation of motion is particularly simple, $\dot\sigma = -\sigma^3/2$. It has a solution $\sigma^{-2}(t) - \sigma_0^{-2} = t$. Suppose that inflation begins at $\sigma_0 = 1$, $t=0$. Then $\sigma^{-2}(t) = 1+t$. Expressing everything in terms of the number of e folds from the beginning of inflation $N_{\rm SUGRA} = t$, one has $\sigma(t) = 1/\sqrt{1+N_{\rm SUGRA}} \approx 1/\sqrt{N_{\rm SUGRA}}$. As we already noted, the SUGRA term dominates only at $\sigma > \sqrt{\kappa}/3$, which gives the total number of e folds of expansion at the SUGRA stage: $N_{\rm SUGRA} \sim 9\kappa^{-1}$. Thus inflation consists of two long stages, one of which is determined by the one-loop effects; another, which is about two times longer, is determined by SUGRA corrections. In fact, the total duration of each of these stages is slightly shorter than what is indicated by our estimates because at an intermediate stage of inflation the contributions of radiative corrections and SUGRA terms are comparable. This increases the speed of rolling of the field σ and makes inflation slightly shorter. The total duration of inflation can be estimated by $N_{\rm SUGRA} + N_{\rm SUSY} \sim 10\kappa^{-1}$. Therefore we need $\kappa \lesssim 0.15$ in order to obtain 60 e folds of inflation in our scenario.

During the SUGRA stage the density perturbations have spectrum $\delta\rho/\rho \sim 2\mu^2/5\pi\sqrt{3}\sigma^3$. One can express it in terms of the ratio of the wavelength l to the wavelength $l_{\rm max}$ corresponding to the beginning of the first stage of inflation:

$$\frac{\delta\rho}{\rho} \sim \frac{6\mu^2}{5\pi} \ln^{-3/2}\frac{l_{\rm max}}{l}. \qquad (8)$$

Equations (6) and (8) match each other at the scale l^* at which SUGRA terms become smaller than the radiative corrections. This scale therefore should correspond to a maximum of the spectrum $(\delta\rho/\rho)(l)$.

At $l > l^*$ the spectrum decreases towards large scales, which corresponds to a blue spectrum. In the beginning, when the logarithm is rather large, the damping of the amplitude of density fluctuations is relatively insignificant. However, when l approaches $l_{\rm max}$ the spectrum falls down very sharply.

To evaluate the change of $\delta\rho/\rho$ during the SUGRA inflation (for $\kappa = 0.1$), one should note that in the beginning one has $\sigma \sim 1$, and in the end of SUGRA inflation one has $\sigma \sim \sqrt{\kappa}/3$. This implies that at the interval from l^* to $l_{\rm max}$ density perturbations fall down by $27\kappa^{-3/2} \sim 10^3$. This means that in order to make this model consistent with COBE data one should keep $l_{\rm max}$ somewhat greater than the size of the observable part of the universe.

To study this question in a more detailed way we solved equations for the field σ numerically, taking into account simultaneously the contribution of SUGRA terms and radiative corrections, assuming in the first approximations that these two types of terms are additive. The results of our investigation are shown in Figs. 1 and 2 for $\kappa = 0.1$. The total duration of inflation in this case is given by $N \sim 90$. The

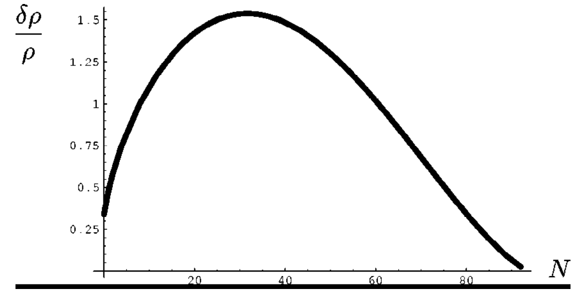

FIG. 1. The shape of density perturbations as a function of the number of e folds for $\kappa = 0.1$. The horizon scale corresponds to $N \sim 60$. COBE normalization in this figure is shown as $\delta\rho/\rho = 1$. Density perturbations sharply fall down on the scale corresponding to the beginning and the end of inflation.

spectrum of density perturbation depends on scale $l \sim e^N$. However, near $N \sim 60$ where the COBE normalization should be imposed this dependence is rather mild. It can be expressed in terms of the effective spectral index n, which in our case is given by $1 + 2V''$. At $N \sim 60$ one has $n - 1 \sim 0.1$, which is a noticeable deviation from the flat Harrison-Zeldovich spectrum. From COBE normalization in that case one finds $\mu \approx 1.2\times 10^{-3} M \sim 3\times 10^{15}$ GeV, and the symmetry breaking scale $\phi \sim \mu/\sqrt{\kappa} \sim 10^{16}$ GeV.

For $\kappa < 0.05$ the total duration of inflation is $N \sim 200$, and the last 60 e folds are determined by the one-loop effects. In that case the effective spectral index at the horizon scale is very close to 1, μ is two times smaller than for $\kappa < 0.1$, and the symmetry breaking scale is $\phi \sim 7\times 10^{15}$ GeV. For κ approaching 0.15 the total duration of inflation approaches $N \sim 60$. In this case the spectrum of density perturbation sharply falls down on the horizon scale, which corresponds to $n \gg 1$, in contradiction with observations. Finally, for $\kappa > 0.15$ inflation is too short to incorporate the observable part of the universe.

This behavior is rather interesting and unusual. Most of the inflationary models describe the universe with a long stage of inflation, and with a spectrum slowly growing on large scale. Hybrid inflation provided a natural way to obtain a blue spectrum of perturbations, decreasing on large scale. However, in the simplest versions of this scenario the spectrum was nearly flat. Recently a new class of models was introduced, which was called tilted hybrid inflation [10]. In such models one may have a rather short stage of inflation and blue spectrum of perturbations. Both of these features

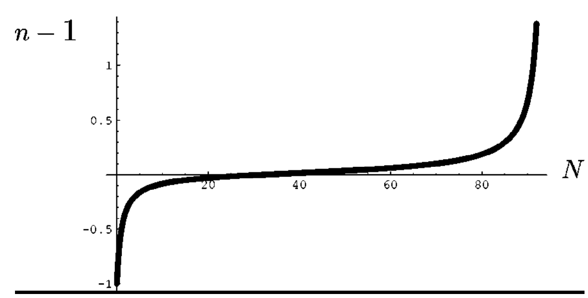

FIG. 2. Deviation of the effective spectral index from $n = 1$ at different scales for $\kappa = 0.1$. For $\kappa > 0.15$ the duration of inflation becomes smaller than 60 e folds, and the spectral index rapidly grows near the horizon.

may be useful for constructing realistic models of open inflationary universe. It was not quite clear, however, whether the models with such properties could be constructed in the context of a realistic theory of elementary particles. It is very interesting (and even somewhat unexpected) that when one makes an attempt to implement the hybrid inflation scenario in the context of a supersymmetric model with the simplest superpotential (1), one obtains the hybrid inflation scenario of this new type.

Until now we ignored density perturbations produced by strings formed after inflation. These perturbations are proportional to $\phi^2 \sim \mu^2/\kappa$. According to [7], stringy perturbations in this model are expected to be of the same order as inflationary ones, though a little smaller. Our results taking into account supergravity corrections confirm this conclusion: Strings which are generated in this model appear as a result of spontaneous symmetry breaking with $\phi \sim 10^{16}$ GeV, which is the right scale for stringy density perturbations. The relative importance of the two types of perturbations depends on the choice of parameters in our model. For example, by decreasing κ from 0.1 to 0.05 and decreasing μ two times one suppresses stringy perturbations by a factor of 2 without changing the inflationary ones. Perturbations of both types are proportional to μ^2. Therefore keeping all other parameters fixed, one can easily adjust the combined amplitude of fluctuations to the COBE normalization without affecting much of our results. Finally, one can modify the theory in such a way as to avoid topological defects altogether; see, e.g., [11].

We will call the model where inflationary and stringy perturbations have comparable magnitude *a mixed perturbation* (MP) model. An interesting possibility which appears in the MP model is related to the different scale dependence of these perturbations. Stringy perturbations are approximately scale independent. If we consider a scenario with a blue spectrum of inflationary perturbations, one may encounter a situation where the perturbations of metric on the galaxy scale are dominated by adiabatic inflationary perturbations, whereas the perturbations on the horizon scale, which show up in the large angle anisotropy of the microwave background radiation, are dominated by strings. It seems that the hybrid inflation scenario may indeed live up to its name.

After this work was finished, we learned about a related work by C. Panagiotakopoulos, Phys. Rev. D **55**, R7335 (1997). The author also studied SUGRA effects in the model (1), but he did not consider a combined scenario containing both SUGRA terms and radiative corrections. Moreover, he calculated density perturbations near σ_c and concluded that one needs to have extremely small values of the parameter κ. We disagree with his conclusion. As we mentioned in our paper, density perturbations should be evaluated at $\sigma_{60} \gg \sigma_c$, which allows to have small density perturbations with a reasonably large κ.

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