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Constraint on axial-vector meson mixing angle from the nonrelativistic constituent quark model

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In a nonrelativistic constituent quark model we find a constraint on the mixing angle of the strange axial-vector mesons, $35° \lesssim \theta_K \lesssim 55°$, determined solely by two parameters: the mass difference of the a_1 and b_1 mesons and the ratio of the constituent quark masses. [S0556-2821(97)50215-3]

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I. INTRODUCTION

It is known that the decay of the $I=\frac{1}{2}$ $1\,^3P_1$ and $1\,^1P_1$ mesons, $K_1(1270)$ and $K_1(1400)$, with masses 1273 ± 7 MeV and 1402 ± 7 MeV, respectively [1], satisfies a dynamical selection rule such that

$$\Gamma(K_1(1270)\to K\rho)\gg\Gamma(K_1(1270)\to K^*\pi),$$

$$\Gamma(K_1(1400)\to K^*\pi)\gg\Gamma(K_1(1400)\to K\rho),$$

which, following the classical example of neutral kaons, suggests a large mixing (with a mixing angle close to 45°) between the $I=\frac{1}{2}$ members of two axial-vector and nonets, K_{1A} and K_{1B}, respectively, leading to the physical K_1 and K_1' states [2]. Carnegie *et al.* [3] obtained the mixing angle $\theta_K=(41\pm4)°$ as the optimum fit to the data as of 1977. In a recent paper by Blundell *et al.* [4], who have calculated strong Okubo-Zweig-Iizuka- (OZI-) allowed decays in the pseudoscalar emission model and the flux-tube-breaking model, the K_{1A}-K_{1B} mixing angle obtained is $\simeq45°$. Theoretically, in the exact SU(3) limit the K_{1A} and K_{1B} states do not mix, similarly to their $I=1$ counterparts a_1 and b_1. As for the s-quark mass greater than the u- and d-quark masses, SU(3) is broken and these states do mix to give the physical K_1 and K_1'. If the K_{1A} and K_{1B} are degenerate before mixing, the mixing angle will always be $\theta_K=45°$ [5,6]. As pointed out by Suzuki [7], the data on $K\pi\pi$ production in τ decay may confirm or refute this simple picture: if $\theta_K=45°$, production of the $K_1(1270)$ and $K_1(1400)$ would be one-to-one up to the kinematic corrections, since in the SU(3) limit only the linear combination $[K_1(1270)+K_1(1400)]/\sqrt{2}$ would have the right quantum numbers to be produced there. After phase-space correction, the $K_1(1270)$ production would be favored over the $K_1(1400)$ one by nearly a factor of 2. However, current experimental data are very uncertain. The measurements made by the TPC/Two-Gamma Collaboration give [8]

$$B(\tau\to\nu K_1(1270))=(0.41^{+0.41}_{-0.35})\times10^{-2}, \tag{1}$$

$$B(\tau\to\nu K_1(1400))=(0.76^{+0.40}_{-0.33})\times10^{-2}, \tag{2}$$

$$B(\tau\to\nu K_1)=(1.17^{+0.41}_{-0.37})\times10^{-2}. \tag{3}$$

Alemany [9] combines the CLEO and ALEPH data [10] to obtain

$$B(\tau\to\nu K_1)=(0.77\pm0.12)\times10^{-2}, \tag{4}$$

which is smaller, but consistent with, the TPC/Two-Gamma results. Conversely, the claim from the CLEO Collaboration is that the τ decays preferentially into the $K_1(1270)$. If one assumes, however, that the production of the $K_1(1400)$ is favored over that of $K_1(1270)$ by nearly a factor of 2 [as follows from Eqs. (1) and (2) if the experimental errors are ignored], one would arrive at $\theta_K\approx33°$ [7]. A very recent analysis by Suzuki of the experimental data on the two-body decays of the J/ψ and ψ' into an axial vector and a pseudoscalar meson from the BES Collaboration [11] shows that any value of θ_K between 30° and 60° can be consistent with the 1^+0^- modes of both the J/ψ and ψ' that have been so far measured [12].

The purpose of this work is to consider the K_{1A}-K_{1B} mixing within the framework of a constituent quark model. In our previous papers [13,14] this model was successfully applied to P- and D-wave meson spectroscopy in order to explain the common mass near-degeneracy of two pairs of nonets $(1\,^3P_0,\ 1\,^3P_2)$, $(1\,^3D_1,\ 1\,^3D_3)$, in the isovector and isodoublet channels, as observed in experiment, and to make predictions regarding the masses of missing and problematic $q\bar{q}$ states. As we shall see, the nonrelativistic constituent quark model provides a very simple constraint on the K_{1A}-K_{1B} mixing angle determined solely by the mass difference of the isovector counterparts of the corresponding nonets, the a_1 and b_1 mesons, and the ratio of the constituent quark masses.

II. NONRELATIVISTIC CONSTITUENT QUARK MODEL

In the constituent quark model, conventional mesons are bound states of a spin 1/2 quark and spin 1/2 antiquark bound by a phenomenological potential which has some basis in QCD [15]. The quark and antiquark spins combine to give a total spin 0 or 1 which is coupled to the orbital angular momentum L. This leads to meson parity and charge conjugation given by $P=(-1)^{L+1}$ and $C=(-1)^{L+S}$, respectively. One typically assumes that the $q\bar{q}$ wave function is a solution of a nonrelativistic Schrödinger equation with the

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generalized Breit-Fermi Hamiltonian[1] H_{BF},

$$H_{\mathrm{BF}}\psi_n(\mathbf{r})\equiv[H_{\mathrm{kin}}+V(\mathbf{p},\mathbf{r})]\psi\_n(\mathbf{r})=E_n\psi_n(\mathbf{r}), \tag{5}$$

where $H_{\mathrm{kin}}=m_1+m_2+\mathbf{p}^2/2\mu-(1/m_1^3+1/m_2^3)\mathbf{p}^4/8$, $\mu=m_1m_2/(m_1+m_2)$, m_1 and m_2 are the constituent quark masses, and to first order in $(v/c)^2=\mathbf{p}^2c^2/E^2\simeq\mathbf{p}^2/m^2c^2$, $V(\mathbf{p},\mathbf{r})$ reduces to the standard nonrelativistic result

$$V(\mathbf{p},\mathbf{r})\simeq V(r)+V_{SS}+V_{LS}+V_T, \tag{6}$$

with $V(r)=V_V(r)+V_S(r)$ being the confining potential which consists of a vector and a scalar contribution, and V_{SS}, V_{LS}, and V_T the spin-spin, spin-orbit, and tensor terms, respectively, given by [15]

$$V_{SS}=\frac{2}{3m_1m_2}\mathbf{s}_1\cdot\mathbf{s}_2\Delta V_V(r), \tag{7}$$

$$\begin{aligned}V_{LS}=\frac{1}{4m_1^2m_2^2}\frac{1}{r}\bigg(&\{[(m_1+m_2)^2+2m_1m_2]\mathbf{L}\cdot\mathbf{S}_+\\&+(m_2^2-m_1^2)\mathbf{L}\cdot\mathbf{S}_-\}\frac{dV_V(r)}{dr}-[(m_1^2+m_2^2)\mathbf{L}\cdot\mathbf{S}_+\\&+(m_2^2-m_1^2)\mathbf{L}\cdot\mathbf{S}_-]\frac{dV_S(r)}{dr}\bigg),\end{aligned} \tag{8}$$

$$V_T=\frac{1}{12m_1m_2}\left(\frac{1}{r}\frac{dV_V(r)}{dr}-\frac{d^2V_V(r)}{dr^2}\right)S_{12}. \tag{9}$$

Here $\mathbf{S}_+\equiv\mathbf{s}_1+\mathbf{s}_2$, $\mathbf{S}_-\equiv\mathbf{s}_1-\mathbf{s}_2$, and

$$S_{12}\equiv3\left(\frac{(\mathbf{s}_1\cdot\mathbf{r})(\mathbf{s}_2\cdot\mathbf{r})}{r^2}-\frac{1}{3}\mathbf{s}_1\cdot\mathbf{s}_2\right). \tag{10}$$

For constituents with spin $s_1=s_2=1/2$, S_{12} may be rewritten in the form

$$S_{12}=2\left(3\frac{(\mathbf{S}\cdot\mathbf{r})^2}{r^2}-\mathbf{S}^2\right),\qquad \mathbf{S}=\mathbf{S}_+\equiv\mathbf{s}_1+\mathbf{s}_2. \tag{11}$$

Since $(m_1+m_2)^2+2m_1m_2=6m_1m_2+(m_2-m_1)^2$, $m_1^2+m_2^2=2m_1m_2+(m_2-m_1)^2$, the expression for V_{LS}, Eq. (8), may be rewritten as

$$\begin{aligned}V_{LS}=\frac{1}{2m_1m_2}\frac{1}{r}\bigg[&\left(3\frac{dV_V(r)}{dr}-\frac{dV_S(r)}{dr}\right)\\&+\frac{(m_2-m_1)^2}{2m_1m_2}\left(\frac{dV_V(r)}{dr}-\frac{dV_S(r)}{dr}\right)\bigg]\mathbf{L}\cdot\mathbf{S}_+\\&+\frac{m_2^2-m_1^2}{4m_1^2m_2^2}\frac{1}{r}\left(\frac{dV_V(r)}{dr}-\frac{dV_S(r)}{dr}\right)\mathbf{L}\cdot\mathbf{S}_-\equiv V_{LS}^++V_{LS}^-.\end{aligned} \tag{12}$$

Since two terms corresponding to the derivatives of the potentials with respect to r are of the same order of magnitude, the above expression for V_{LS}^+ may be rewritten as

$$\begin{aligned}V_{LS}^+=\frac{1}{2m_1m_2}\frac{1}{r}&\left(3\frac{dV_V(r)}{dr}-\frac{dV\_S(r)}{dr}\right)\mathbf{L}\cdot\mathbf{S}\\&\times\left[1+\frac{(m_2-m_1)^2}{2m_1m_2}\times O(1)\right].\end{aligned} \tag{13}$$

III. *P*-WAVE MESON SPECTROSCOPY

We now wish to apply the Breit-Fermi Hamiltonian to the P-wave mesons. By calculating the expectation values of different terms of the Hamiltonian defined in Eqs. (7), (11), and (12), taking into account the corresponding matrix elements $\langle\mathbf{L}\cdot\mathbf{S}\rangle$ and S_{12} [15], one obtains the relations [4,13]

$$M(^3P_0)=M_0+\tfrac{1}{4}\langle V_{SS}\rangle-2\langle V_{LS}^+\rangle-\langle V_T\rangle,$$

$$M(^3P_2)=M_0+\tfrac{1}{4}\langle V_{SS}\rangle+\langle V_{LS}^+\rangle-\tfrac{1}{10}\langle V_T\rangle,$$

$$M(a_1)=M_0+\tfrac{1}{4}\langle V_{SS}\rangle-\langle V_{LS}^+\rangle+\tfrac{1}{2}\langle V_T\rangle,$$

$$M(b_1)=M_0-\tfrac{3}{4}\langle V_{SS}\rangle,$$

$$\begin{pmatrix}M(K_1)\\M(K_1')\end{pmatrix}=\begin{pmatrix}M_0+\tfrac{1}{4}\langle V_{SS}\rangle-\langle V_{LS}^+\rangle+\tfrac{1}{2}\langle V_T\rangle & \sqrt{2}\langle V_{LS}^-\rangle\\ \sqrt{2}\langle V_{LS}^-\rangle & M_0-\tfrac{3}{4}\langle V_{SS}\rangle\end{pmatrix}\times\begin{pmatrix}K_{1A}\\K_{1B}\end{pmatrix},$$

where M_0 stands for the sum of the constituent quark masses and binding energies in either case. The V_{LS}^- term acts only on the $I=1/2$ singlet and triplet states giving rise to the spin-orbit mixing between these states,[2] and is responsible for the physical masses of the K_1 and K_1'. The masses of the K_{1A} and K_{1B} are determined by relations which are common for all eight $I=1,1/2$ P-wave mesons, b_1, a_0, a_1, a_2, K_{1B}, K_0^*, K_{1A}, and K_2^*:

[1] The most widely used potential models are the relativized model of Godfrey and Isgur [16] for the $q\overline{q}$ mesons, and Capstick and Isgur [17] for the qqq baryons. These models differ from the nonrelativistic quark potential model only in relatively minor ways, such as the use of $H_{\mathrm{kin}}=\sqrt{m_1^2+\mathbf{p}_1^2}+\sqrt{m_2^2+\mathbf{p}_2^2}$ in place of that given in Eq. (5), the retention of the m/E factors in the matrix elements, and the introduction of coordinate smearing in the singular terms such as $\delta(\mathbf{r})$.

[2] The spin-orbit $^3P_1$-$^1P_1$ mixing is a property of the model we are considering; the possibility that another mechanism is responsible for this mixing, such as mixing via common decay channels [6] should not be ruled out, but is not included here.

$$M(^1P_1)=m_1+m_2+E_0-\frac{3}{4}\frac{a}{m_1m_2}, \tag{14}$$

$$M(^3P_0)=m_1+m_2+E_0+\frac{1}{4}\frac{a}{m_1m_2}-\frac{2b}{m_1m_2}-\frac{c}{m_1m_2}, \tag{15}$$

$$M(^3P_1)=m_1+m_2+E_0+\frac{1}{4}\frac{a}{m_1m_2}-\frac{b}{m_1m_2}+\frac{c}{2m_1m_2}, \tag{16}$$

$$M(^3P_2)=m_1+m_2+E_0+\frac{1}{4}\frac{a}{m_1m_2}+\frac{b}{m_1m_2}-\frac{c}{10m_1m_2}, \tag{17}$$

where a, b, and c are related to the matrix elements of V_{SS}, V_{LS}, and V_T [see Eqs. (7), (9), and (13)], and assumed to be the same for all of the P-wave states. E_0 is a nonrelativistic binding energy which may in general be absorbed in the definition of a constituent quark mass [13,14]. We assume also SU(2) flavor symmetry: $m(u)=m(d)\equiv n$, $m(s)\equiv s$.

The correction to V_{LS}^+ in the formula (13), because of the difference in the masses of the n and s quarks, is ignored. Indeed, these effective masses, as calculated from Eqs. (14)–(17) in the case where E_0 is absorbed into their definition, are[3]

$$n=\frac{3b_1+a_0+3a_1+5a_2}{24}, \tag{18}$$

$$s=\frac{6K_{1B}+2K_0^*+6K_{1A}+10K_2^*-3b_1-a_0-3a_1-5a_2}{24}. \tag{19}$$

With the physical values of the meson masses (in GeV), $a_1\simeq b_1\cong 1.23$, $a_0\simeq a_2\cong 1.32$, $K_{1A}\simeq K_{1B}\cong 1.34$, $K_0^*\simeq K_2^*\cong 1.43$, the above relations give

$$n\simeq 640 \text{ MeV}, \quad s\simeq 740 \text{ MeV}, \tag{20}$$

so that the abovementioned correction, according to Eq. (13), is $\sim 100^2/(2\times 640\times 740)\simeq 1\%$, i.e., comparable to isospin breaking on the scale considered here, and so completely negligible.

In the expressions (20), the nonrelativistic binding energies are absorbed in the constituent quark masses. The same constituent quark masses appear also in the denominators of the hyperfine interaction terms in Eqs. (14)–(17). Since this is usually done only for the lowest S-wave states, we briefly review the precedent and argument for the generality of these forms.

It was shown in [18] that a pure scalar potential contributes to the effective constituent quark mass. Bag models suggest that the kinetic energy also contributes to the effective constituent quark mass in the case of no potential [19]. These results were generalized further by Cohen and Lipkin [20] who have shown that both the kinetic and potential energy are included in the effective mass parameter which appears also in the denominators of the hyperfine interaction terms in the case of a scalar confining potential. The analyses of experimental data suggest that the nonstrange and strange quarks are mainly subject to the scalar part of the confining potential (whereas charmed and bottom quarks are more dominantly affected by the Coulomblike vector part) [15]. Moreover, the generality of the arguments by Cohen and Lipkin [20] allows one to apply them to any partial wave. Therefore, the constituent quark masses can be defined for any partial wave, through relations of the form in Eqs. (14)–(17); in this case they vary with the energies of the corresponding mass levels. Such an energy dependence of the constituent quark masses was considered in Refs. [21,22]. Also, a QCD-based mechanism which generates dynamical quark mass growing with L in a Regge-like manner was considered by Simonov [23].

We note that one could, in principle, fit the measured masses of the P-wave mesons with the conventional values of the quark masses, $n=306$ MeV and $s=487$ MeV which serve S-wave meson spectroscopy [13], being used in Eqs. (14)–(17) along with nonzero E_0. In this case, as shown below, the final constraint will be tightened at most.

It follows from Eqs. (14)–(17) that

$$\frac{9a}{m_1m_2}=M(^3P_0)+3M(^3P_1)+5M(^3P_2)-9M(^1P_1), \tag{21}$$

$$\frac{12b}{m_1m_2}=5M(^3P_2)-3M(^3P_1)-2M(^3P_0), \tag{22}$$

$$\frac{18c}{5m_1m_2}=3M(^3P_1)-2M(^3P_0)-M(^3P_2). \tag{23}$$

By expressing the ratio n/s in three different ways, viz., dividing the expressions (21) and (23) for the $I=1/2$ and $I=1$ mesons by each other, one obtains the relations

$$\frac{n}{s}=\frac{K_0^*+3K_{1A}+5K_2^*-9K_{1B}}{a_0+3a_1+5a_2-9b_1}=\frac{5K_2^*-3K_{1A}-2K_0^*}{5a_2-3a_1-2a_0}=\frac{2K_0^*+K_2^*-3K_{1A}}{2a_0+a_2-3a_1}. \tag{24}$$

It follows from the last relation of Eq. (24) that

$$(K_2^*-K_0^*)(a_2-a_1)=(K_2^*-K_{1A})(a_2-a_0). \tag{25}$$

This formula explains the common mass degeneracy of the scalar and tensor meson nonets in the isovector and isodoublet channels. Using now Eqs. (24) and (25), one arrives, by straightforward algebra, at

$$\frac{n}{s}=\frac{K_{1A}-K_{1B}}{a_1-b_1}. \tag{26}$$

This relation is an intrinsic property of the model we are considering; it depends neither on the values of the input parameters, n,s,a,b,c, nor the presence of E_0 in the rela-

[3]In the following, a_0 stands for the mass of the a_0, etc.

tions (14)–(17). We shall now use this relation in order to obtain a constraint on the K_{1A}-K_{1B} mixing angle.

IV. CONSTRAINT ON THE K_{1A}-K_{1B} MIXING ANGLE

Since, on general grounds, $n \leqslant s$, it follows from Eq. (26) that

$$|K_{1A}-K_{1B}| \leqslant |a_1-b_1| \equiv \Delta, \tag{27}$$

which may be rewritten as

$$K_{1A}^2+K_{1B}^2-2K_{1A}K_{1B} \leqslant \Delta^2. \tag{28}$$

Moreover, independent of the mixing angle,

$$K_{1A}^2+K_{1B}^2=K_1^2+K_1'^2. \tag{29}$$

It then follows from Eqs. (28) and (29) that

$$2K_{1A}K_{1B} \geqslant K_1^2+K_1'^2-\Delta^2. \tag{30}$$

To obtain a constraint on the K_{1A}-K_{1B} mixing angle, we now use the formula [7]

$$\tan^2(2\theta_K)=\left(\frac{K_1^2-K_1'^2}{K_{1B}^2-K_{1A}^2}\right)^2-1,$$

which may be rewritten as

$$\cos^2(2\theta_K)=\left(\frac{K_{1B}^2-K_{1A}^2}{K_1^2-K_1'^2}\right)^2. \tag{31}$$

It follows from Eqs. (29) and (30) that

$$\begin{aligned}(K_{1B}^2-K_{1A}^2)^2&=(K_{1A}^2+K_{1B}^2)^2-4K_{1A}^2K_{1B}^2\\ &\leqslant (K_1^2+K_1'^2)^2-(K_1^2+K_1'^2-\Delta^2)^2\\ &\cong 2\Delta^2(K_1^2+K_1'^2),\end{aligned} \tag{32}$$

since $\Delta \sim 50$ MeV (see below), and therefore $\Delta^2 \ll K_1^2+K_1'^2$. Thus, Eq. (31) finally reduces to

$$\cos^2(2\theta_K) \leqslant \frac{2\Delta^2(K_1^2+K_1'^2)}{(K_1^2-K_1'^2)^2}, \tag{33}$$

and therefore

$$|\cos(2\theta_K)| \leqslant \frac{\Delta\sqrt{2(K_1^2+K_1'^2)}}{|K_1^2-K_1'^2|}. \tag{34}$$

The value of Δ is determined by current experimental data on the a_1 and b_1 meson masses [1]: $a_1=1230\pm 40$ MeV, $b_1=1231\pm 10$ MeV. Therefore, $\Delta \leqslant 50$ MeV, and one obtains, from Eq. (34),

$$33.6° \leqslant \theta_K \leqslant 56.4°, \tag{35}$$

consistent with the recent result of Suzuki [12], $30° \leqslant \theta_K \leqslant 60°$. The above constraint may be tightened further by using the ratio of the constituent quark masses given in Eq. (20). Then from Eq. (26) we obtain

$$|K_{1A}-K_{1B}|=\frac{n}{s}|a_1-b_1| \leqslant \frac{0.64}{0.74}50 \text{ MeV} \simeq 43 \text{ MeV} \equiv \Delta'. \tag{36}$$

With this Δ' being used in Eq. (34) in place of Δ, one obtains

$$35.3° \leqslant \theta_K \leqslant 54.7°. \tag{37}$$

The constraint (35) is tightened at most if one uses the conventional values of the quark masses, $n=306$ MeV and $s=487$ MeV, in Eq. (36). In this case,

$$|K_{1A}-K_{1B}| \leqslant \frac{0.306}{0.487}50 \text{ MeV} \simeq 31 \text{ MeV} \equiv \Delta'',$$

and with this Δ'' Eq. (34) yields

$$38.0° \leqslant \theta_K \leqslant 52.0°. \tag{38}$$

The three ranges, (35), (37), and (38), are consistent with the value $\theta_K=(37.3\pm 3.2)°$ obtained in our previous work [13].

V. CONCLUDING REMARKS

As we have shown, a nonrelativistic constituent quark model provides a simple constraint on the K_{1A}-K_{1B} mixing angle, in terms of the mass difference of the a_1 and b_1 mesons and the squared masses of the physical states K_1 and K_1'. The numerical value of the allowed interval for the mixing angle, $33.6° \leqslant \theta_K \leqslant 56.4°$, is consistent with that provided by the very recent analysis by Suzuki [12]. This interval may be constrained further by using the ratio of the constituent quark masses. In the mass degenerate case $a_1=b_1$, the model considered shows a similar mass degeneracy for the corresponding strange mesons, $K_{1A}=K_{1B}$, independent of the input parameters, and so requiring a precise 45° mixing. We conclude, therefore, that more precise experimental data on the mass of the a_1 meson are required to obtain a better estimate of the K_{1A}-K_{1B} mixing angle.

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