Decay of accelerated particles

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We study how the decay properties of particles are changed by acceleration. It is shown that under the influence of acceleration (1) the lifetime of particles is modified and (2) new processes (such as the decay of the proton) become possible. This is illustrated by considering scalar models for the decay of muons, pions, and protons. We discuss the close conceptual relation between these processes and the Unruh effect. [S0556-2821(97)04414-7]

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I. INTRODUCTION

Usually, the lifetime of a particle is regarded as one of its inherent and characteristic properties. In spite of its statistical nature, the decay process possesses a certain regularity expressed by the decay rate or the linewidth. The lifetime of particles such as the pion or the muon can be calculated from the knowledge of the fundamental interaction which governs the decay (or a suitable model of it). Other particles, such as the electron or the proton, are regarded as stable; i.e., they do not decay at all (in the standard model).

In the present paper, we will show that the decay properties of particles are less fundamental than commonly thought. One can manipulate the lifetime of an unstable particle by exposing it to a large *acceleration*, e.g., in a storage ring or a collider. This effect is not to be confused with the ordinary special relativistic time dilation. Instead, acceleration causes a modification of particle lifetimes with respect to their own proper time, i.e., in their accelerated rest frame.

A different, even more exciting effect is that supposedly stable particles can decay under the influence of acceleration. Usually forbidden processes such as the decay of the proton become possible, leading to a finite lifetime for these particles. That does not mean that new fundamental interactions are involved as in grand unified theories. The calculation given below stays entirely within the framework of presently established interactions. The effect can therefore be regarded as prediction of the standard model.

To illustrate these general statements, we will consider three specific processes in a toy model approach: the decay of muons, of pions, and of protons,

$$\mu^- \to e^- \overline{\nu_e} \nu_\mu ,$$

$$\pi^- \to \mu^- \overline{\nu_\mu} ,$$

$$p^+ \to n e^+ \nu_e .$$

The first two processes occur already for nonaccelerated particles. We will investigate how the corresponding decay rates are influenced by acceleration. The third process, proton decay, is forbidden without acceleration. It can be regarded as the inverse of the neutron decay. Here, the decay products are heavier than the original decaying particle. Obviously, the missing energy must be supplied by the accelerating device.

The effects discussed in this paper should not come so much as a surprise if one realizes that also in other branches of quantum physics apparently inherent properties of quantum objects can be modified by external influences. Examples are provided by cavity quantum electrodynamics, where it is demonstrated that ''constants'' such as the spontaneous emission rate of an atom or the Lamb shift of energy levels are changed inside a cavity [1]. Elementary particles are subject to these environmental influences too, as has been shown for the magnetic moment of an electron inside a cavity (see, e.g., [2]) or near a topological defect [3].

It is also known that acceleration can influence quantum field theoretical effects in a nontrivial way. The most prominent example is the Unruh effect [4]: the spontaneous excitation of a uniformly accelerated two-level atom. This process is not possible for inertially moving atoms, in close analogy to the proton decay discussed above. Other examples of quantum properties that are modified by acceleration include the spontaneous emission rate of an atom [5,6] and the Lamb shift [7]. We will discuss the connection between the processes considered here and the Unruh effect in more detail below.

We also note that quantum field theory in accelerated frames is conceptually closely related to the quantum theory in curved spacetimes. There, effects like black hole radiation are theoretically derived which have attracted considerable interest. However, very few predictions of both theories have a chance of being tested in laboratory experiments in the foreseeable future (e.g., the Bell-Leinaas proposal [8] for detecting the Unruh effect). Some of the effects presented in this paper appear to be not totally out of the range of experimental possibilities. If detected, they may help in gaining a deeper insight into the more fundamental aspects of quantum field theory under noninertial conditions.

II. MUON DECAY

We first consider a model for the decay of an accelerated muon:

$$\mu^- \to e^- \overline{\nu_e} \nu_\mu ,$$

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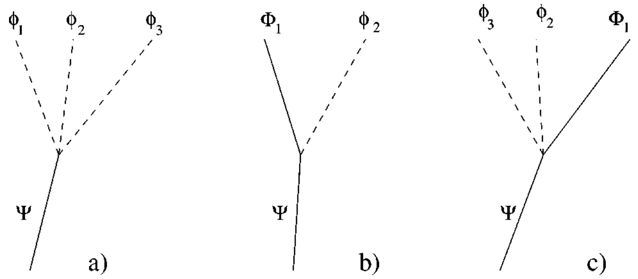

FIG. 1. The decay processes considered in this paper: Scalar model of (a) muon decay, (b) pion decay, and (c) proton decay. Dashed lines denote massless particles; solid lines denote massive particles.

$$\mu^+ \to e^+ \nu_e \overline{\nu_\mu}.$$

These are weak interaction processes which are well described by Fermi's four-fermion contact interaction. We are mainly interested in the structural features of the acceleration-induced modifications. We therefore construct a toy model of the actual physical interaction which simplifies the calculation considerably. We neglect the complicated spin dynamics of Dirac particles and consider scalar quantum fields instead. The Fermi interaction is replaced by a quadrilinear coupling of the four fields with a suitably chosen coupling constant G. As a result of this simplification, we will be able to evaluate the modification to the inertial decay rate analytically and to give an estimate of their relative magnitude.

In our scalar model, we are dealing with the process shown in Fig. 1(a):

$$\Psi \to \phi_1 \phi_2 \phi_3. \tag{1}$$

Here Ψ denotes the decaying massive particle while the decay products ϕ_1, ϕ_2, ϕ_3 are assumed to belong to three different scalar particle species. As the textbook calculation of the muon decay shows (see, for example, [9,10]), the electron mass can be neglected to a good approximation, the corrections being only of the order $(m_e/m_\mu)^2$. We will adopt this approximation in the following; i.e., all three final particles will be regarded as massless. In this paper, capital Greek letters generally denote massive particles, while small Greek letters mean massless particles.

In the standard treatment one starts from Fermi's golden rule and determines the decay rate by evaluating the momentum integrals over the phase space of the decay products. We will choose a different approach because it turns out that the usual procedure is quite impractical for treating accelerated particles. We will instead use a Green's-function-based formalism.

To describe the decay (1) we assume an interaction of the form

$$\mathcal{L}_I = G\Psi(x)\phi_1(x)\phi_2(x)\phi_3(x)\sqrt{-g}, \tag{2}$$

where G is the coupling constant and $x=(t,\mathbf{x})$. All fields are real scalar quantum fields. The theory defined by this interaction is not renormalizable, in accordance with the original Fermi theory. This will not represent a problem for our calculations since we are only interested in tree-level processes.

The probability amplitude for the decay of the Ψ particle into a state with definite momenta $\mathbf{k}_j$ for the ϕ_j is given by

$$A = iG\int d^4x\sqrt{-g}\langle \mathbf{k}_1\mathbf{k}_2\mathbf{k}_3|\Psi(x)\phi_1(x)\phi_2(x)\phi_3(x)|i\rangle, \tag{3}$$

where $|i\rangle$ is the initial state containing only the accelerated Ψ. To obtain the total decay probability we have to sum the squared amplitude over all possible final momenta:

$$\begin{aligned} P &= \sum_{\mathbf{k}_1}\sum_{\mathbf{k}_2}\sum_{\mathbf{k}_3}|A|^2 \\ &= G^2\int d^4x\sqrt{-g}\int d^4x'\sqrt{-g'}\langle i|\Psi(x)|0\rangle \\ &\quad\times\langle 0|\Psi(x')|i\rangle\prod_{k=1}^{3}\langle 0|\phi_k(x)\phi_k(x')|0\rangle. \end{aligned}$$

To model the physical situation of a particle beam in an accelerator, we assume that the initial accelerated particle is prepared in a narrow beam sufficiently concentrated around the uniformly accelerated trajectory. It can be described by a wave packet whose center of mass follows this trajectory. Electric or magnetic fields prevent its spreading. Let $f_i(x)$ denote the mode function corresponding to the initial state of Ψ. Then Eq. (4) becomes

$$\begin{aligned} P &= G^2\int d^4x\sqrt{-g}\int d^4x'\sqrt{-g'}f_i^*(x)f_i(x') \\ &\quad\times\prod_{k=1}^{3}\langle 0|\phi_k(x)\phi_k(x')|0\rangle. \end{aligned} \tag{4}$$

As seen from the accelerated frame, the particle is essentially at rest. Its energy in this frame is therefore just the rest mass M, and the wave function can be written as

$$f_i(x) = h_i(\mathbf{x}(\tau))e^{-iM\tau}. \tag{5}$$

Here, τ denotes the *proper time* with respect to the accelerated trajectory $x(\tau)=(t(\tau),\mathbf{x}(\tau))$. The function $h_i(\mathbf{x})$ gives the spatial form of the wave packet (see, e.g., [11]).

Because we have assumed a very narrow wave packet, the correlation function of the ϕ fields in Eq. (4) can be evaluated essentially along the trajectory $\mathbf{x}(\tau)$ of the Ψ particle. We therefore write

$$\begin{aligned} P &= G^2\kappa\int d\tau\int d\tau' e^{iM(\tau-\tau')} \\ &\quad\times\prod_{k=1}^{3}\langle 0|\phi_k(t(\tau),\mathbf{x}(\tau))\phi_k(t(\tau'),\mathbf{x}(\tau'))|0\rangle, \end{aligned} \tag{6}$$

with

$$\kappa = \left|\int d^3x\sqrt{-g^{(3)}}h_i(\mathbf{x})\right|^2. \tag{7}$$

The quantity κ depends on the detailed shape of the wave packet. Its exact value will not be necessary for our purposes, but for physically realistic wave packets, it is of order unity.

Finally we note that the vacuum expectation value of the product of field operators is just the Wightman function

$$G^+(x,x')=\langle 0|\phi(x)\phi(x')|0\rangle = \frac{1}{2(2\pi)^3}\int \frac{d^3k}{\omega_{\mathbf{k}}}\exp[-i\omega_{\mathbf{k}}(\Delta t-i\epsilon)+i\mathbf{k}\Delta x], \tag{8}$$

where $\Delta t=t-t'$ and $\Delta\mathbf{x}=\mathbf{x}-\mathbf{x}'$. Its explicit form for real massless scalar fields is given by (see, e.g., [12])

$$G^+(x,x')=-\frac{1}{(2\pi)^2}\frac{1}{(\Delta t-i\epsilon)^2-|\Delta\mathbf{x}|^2}. \tag{9}$$

For all stationary trajectories (which follow the orbit of a timelike Killing vector field), the Wightman function $G^+(x(\tau),x(\tau'))$ depends only on the proper time interval $u=\tau-\tau'$. In this case, it is useful to divide out an infinite proper time integral and to consider the decay rate Γ, i.e., the transition probability per unit proper time:

$$\Gamma=-\frac{G^2\kappa}{(2\pi)^6}\int_{-\infty}^{\infty}du\,e^{iMu}\frac{1}{[(\Delta t-i\epsilon)^2-|\Delta\mathbf{x}|^2]^3}. \tag{10}$$

This equation is the starting point for our calculation of the lifetime of an accelerated Ψ particle. The fact that the decay rate considered here refers to the proper time in the accelerated frame shows that the special relativistic time dilation is automatically included when a transformation to the laboratory time is performed.

In the following, we will concentrate on a uniformly accelerated particle which follows the trajectory

$$t(\tau)=\frac{1}{a}\sinh(a\tau),\quad z(\tau)=\frac{1}{a}\cosh(a\tau),\quad x(\tau)=y(\tau)=0. \tag{11}$$

Using

$$(\Delta t-i\epsilon)^2-|\Delta\mathbf{x}|^2=\frac{4}{a^2}\sinh^2\left(\frac{a}{2}(\tau-\tau')-i\epsilon\right), \tag{12}$$

we have to evaluate

$$\Gamma=-\frac{G^2\kappa}{(2\pi)^6}\frac{a^6}{64}\int_{-\infty}^{\infty}du\,e^{iMu}\frac{1}{\{\sinh[(a/2)u-i\epsilon]\}^6}. \tag{13}$$

The integral can be easily calculated by closing the integration contour at $\mathrm{Im}(u)=2\pi i/a$. We obtain

$$\Gamma=\Gamma_0\frac{1}{1-e^{-2\pi M/a}}\left[1+5\left(\frac{a}{M}\right)^2+4\left(\frac{a}{M}\right)^4\right], \tag{14}$$

where

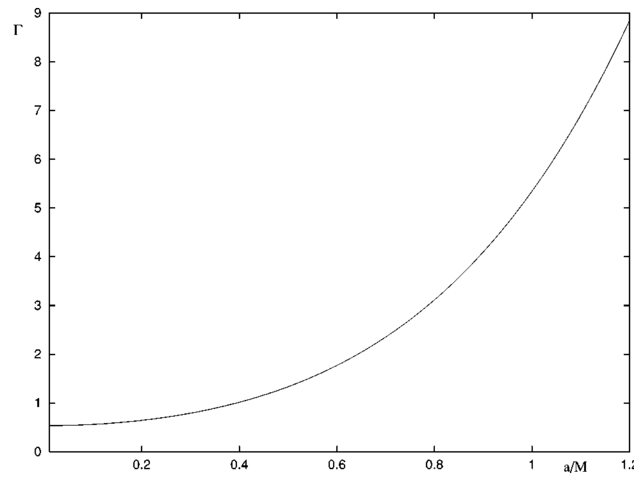

FIG. 2. Decay rate in the scalar model of the muon decay. The rate is shown in units of $G^2\kappa/(64(2\pi)^5)$ as a function of a/M.

$$\Gamma_0=\frac{G^2\kappa}{3840\pi^5} \tag{15}$$

is the decay rate of the unaccelerated particle in our scalar model. It depends on the coupling constant and on the quantity κ. Since it appears as a common factor for all terms in Eq. (14), its numerical value is unimportant for our purposes because we are only interested in the *relative* magnitude of the corrections to the inertial decay rate.

A plot of Γ as a function of a/M is shown in Fig. 2. We see that the decay rate increases monotonically with the acceleration. For small accelerations, however, which are experimentally relevant, the acceleration-induced modification grows only quadratically with a [cf. Eq. (14)], making the effect difficult to detect in this regime.

Let us estimate the magnitude of the effect for accelerations that can be achieved in present experiments. From Eq. (14) we see that the modification of the decay rate from its inertial value is governed by the dimensionless parameter $a/M=a/[(5\times10^{29}\ \mathrm{m/s^2})\times(\text{mass in MeV})]$. This indicates that very large accelerations are needed for an appreciable effect. The situation is not hopeless, however. Muons can be accelerated very efficiently in storage rings or ring colliders where they are subject to a large circular acceleration. This kind of acceleration has a similar effect as the linear acceleration considered here.

At the moment, the lifetime of the muon can be measured with an accuracy of 10^{-5}. For a detection of the acceleration-induced modification of the decay rate, the effect had to be larger. This is achieved for $a/M=0.0014$ or, if we insert the muon mass, $a=7\times10^{27}g$. This must be compared with the acceleration $a\approx10^{19}g$ which has been achieved already 20 years ago at CERN's muon storage ring (Ref. [13] contains a precision measurement of the muon lifetime obtained at this facility) or with $a\approx10^{22}g$ at the projected Brookhaven muon-muon collider. Since none of these rings were designed for a large acceleration (which even leads to undesirable synchrotron radiation), there seems to be some chance of observing the present effect in future experiments.

We want to stress again that the present calculation cannot yield exact numerical predictions since it approximates fermionic particles by scalar quantum fields. Nevertheless, we can expect to get at least an estimate of the order of magnitude of the effect.

III. PION DECAY

Let us now consider the decay of an accelerated pion:

$$\pi^{\pm} \to \mu^{\pm} \overline{\nu_{\mu}}.$$

As in the previous section we will simplify the analysis by treating a scalar model of the process [Fig. 1(b)]

$$\Psi \to \Phi_1 \phi_2. \tag{16}$$

An accelerated Ψ particle decays into a massive Φ_1 and a massless ϕ_2 scalar field. The decay is described by the interaction Lagrangian

$$\mathcal{L}_I = G\Psi(x)\Phi_1(x)\phi_2(x)\sqrt{-g}. \tag{17}$$

We use the same model assumptions as before and arrive at the analogue of Eq. (6):

$$P = G^2\kappa \int d\tau \int d\tau' e^{iM(\tau-\tau')} \langle 0|\Phi_1(x(\tau))\Phi_1(x(\tau'))|0\rangle \times \langle 0|\phi_2(x(\tau))\phi_2(x(\tau'))|0\rangle. \tag{18}$$

The Wightman function for the massive scalar field is more complicated now than in the massless case:

$$\langle 0|\Phi_1(x)\Phi_1(x')|0\rangle = \frac{m}{2(2\pi)^2} \frac{K_1(m\sqrt{|\Delta x|^2-(\Delta t - i\epsilon)^2})}{\sqrt{|\Delta x|^2-(\Delta t - i\epsilon)^2}}, \tag{19}$$

where K_1 is a modified Bessel function. If we insert the uniformly accelerated trajectory (13) and consider the rate with respect to the proper time τ, we arrive at the expression

$$\Gamma = -i\frac{G^2\kappa}{(2\pi)^4}\frac{\pi m a^2}{8}\int_{-\infty}^{\infty} dx e^{i2Mx/a} \times \frac{H_1^{(2)}[(2m/a)\sinh(x-i\epsilon)]}{[\sinh(x-i\epsilon)]^3}, \tag{20}$$

where $x = a(\tau-\tau')/2$ and $H_1^{(2)}$ is a Hankel function of the second kind.

The integral in Eq. (20) cannot be evaluated analytically because the hyperbolic sine in the argument of the Hankel function leads to a very complicated branch cut structure. Instead, we have used numerical methods. However, even the numerical integration of Eq. (20) turns out to be exceedingly difficult because of the highly singular nature of the integrand at $x=0$. To circumvent this difficulty we adopt the following strategy: Using the ascending series expansion around $x=0$ of the Hankel function [see Eqs. (9.1.10) and (9.1.11) of Ref. [14]], we isolate all contributions of the integrand in Eq. (20) which are singular at $x=0$. They can either be treated analytically, or a solvable auxiliary integral which has the same singularity structure as the original one can be found. These contributions are subtracted from the original integral in Eq. (20), leaving a well-behaved integrand which is finite at $x=0$ and can be treated with the usual numerical methods. To obtain the decay rate, the analytically calculated integrals of the singular parts must be added to the numerical result so that Γ is determined according to

$$\Gamma = \int_{-\infty}^{\infty} dx[(\text{original integrand}) - (\text{singular parts})] + (\text{integral of singular parts}). \tag{21}$$

For our particular integral (20), the singular parts of the integrand are $G^2\kappa m\mathcal{A}^{(\pi)}/(2\pi)^4$ with

$$\mathcal{A}^{(\pi)} = \frac{a^3}{8m}\int_{-\infty}^{\infty} dx \frac{e^{i2Mx/a}}{[\sinh(x-i\epsilon)]^4} + \frac{am}{8}(1-2\gamma-i\pi) \times \int_{-\infty}^{\infty} dx \frac{e^{i2Mx/a}}{[\sinh(x-i\epsilon)]^2} \tag{22}$$

$$- \frac{a^2}{4}\sum_{k=0}^{1}\frac{(-1)^k}{k!(k+1)!}\left(\frac{m}{a}\right)^{2k+1} \times \int_{-\infty}^{\infty} dx e^{i2Mx/a} \ln\left(\frac{m}{a}\sinh(x-i\epsilon)\right) \times [\sinh(x-i\epsilon)]^{2k-2}, \tag{23}$$

where γ is Euler's constant. The integrals used for the implementation of the above calculation scheme are

$$\mathcal{I}_1^{(\pi)} = \frac{a^3}{8m}\int_{-\infty}^{\infty} dx \frac{e^{i2Mx/a}}{[\sinh(x-i\epsilon)]^4} = \frac{\pi M}{3m}(a^2+M^2)[1-e^{-2\pi M/a}]^{-1},$$

$$\mathcal{I}_2^{(\pi)} = -\frac{ma}{4}\int_{-\infty}^{\infty} dx \frac{e^{i2Mx/a}}{(x-i\epsilon)^2}\ln\left(\frac{m}{a}(x-i\epsilon)\right) = \pi m M\left[1-\gamma - i\frac{\pi}{2} + \ln\left(\frac{m}{2M}\right)\right],$$

$$\mathcal{I}_3^{(\pi)} = \frac{am}{8}(1-2\gamma-i\pi)\int_{-\infty}^{\infty} dx \frac{e^{i2Mx/a}}{[\sinh(x-i\epsilon)]^2} = -\frac{\pi}{2}mM(1-2\gamma-i\pi)[1-e^{-2\pi M/a}]^{-1},$$

$$\mathcal{I}_4^{(\pi)} = \left(\frac{ma}{12} + \frac{m^3}{8a}\right)\int_{-\infty}^{\infty} \ln\left(\frac{m}{a}x\right) e^{-Mx/a} = -\left(\frac{ma^2}{6M} + \frac{m^3}{4M}\right)\left[\ln\left(\frac{M}{m}\right) + \gamma\right]. \tag{24}$$

Some further remarks concerning the numerical treatment are in order: First, for $x \to \pm\infty$, the integrand is oscillatory with monotonically decreasing magnitude. This is difficult to handle for ordinary integration routines. We have therefore

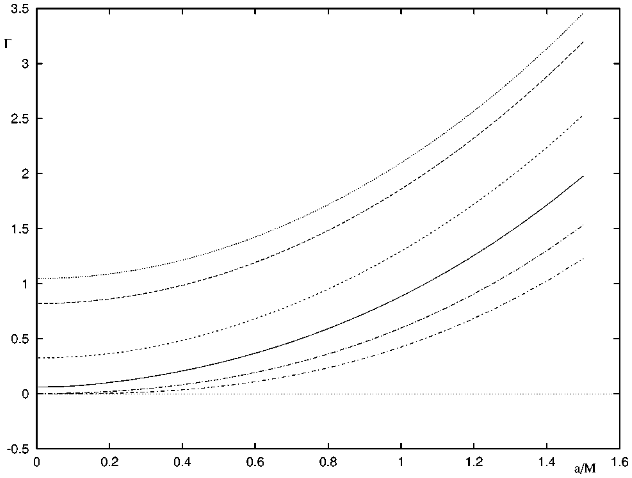

FIG. 3. Decay rate for the process $\Psi \rightarrow \Phi_1 \phi_2$ (''pion'' decay) in units of $G^2\kappa/(2\pi)^4$. The curves from top to bottom show the decay rate as a function of the acceleration a/M for different values of $m/M=$ 0, 0.2, 0.5, 0.757, 1.0, 1.2. The solid line corresponds to the ratio of muon to pion mass.

used fast Fourier methods to compute the integral at large $|x|$. Second, very close to the origin numerical extinction occurs and the integrand cannot be reliably evaluated. This is circumvented by fitting the integrand to a cubic polynomial in this region. In spite of the smoothness of the integrand this is the main source of error in the integration scheme. Finally we note that the decay rate given by Eq. (20) is formally a complex quantity. The condition $\mathrm{Im}(\Gamma)=0$ can serve as a consistency check and provides an estimate of the numerical error in the actual calculation.

The results of the numerical integration are shown in Figs. 3 and 4. Figure 3 displays the decay rate of the Ψ particle [in units of $G^2\kappa/(2\pi)^4$] as a function of the acceleration a/M for different values of the mass ratio m/M. The solid line corresponds to the ratio of muon to pion mass (i.e., $m/M=105.7$ MeV/139.6 MeV). Generally, the decay rate decreases as the mass of the decay product rises (from top to bottom). A larger acceleration leads to a reduction of the lifetime, although the slope of the curves is very small for low accelerations.

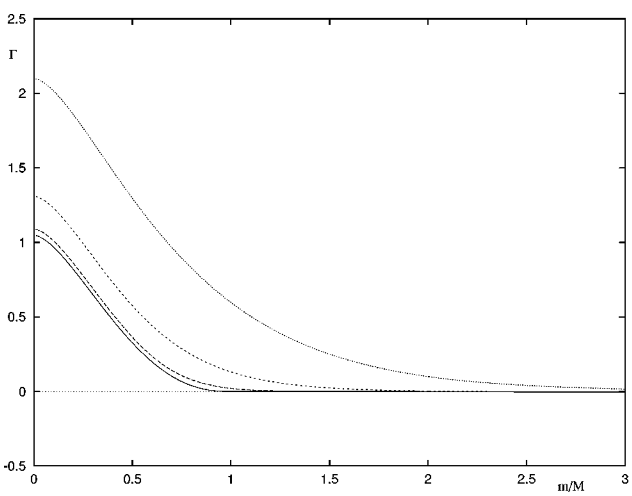

FIG. 4. Decay rate of the ''pion'' decay as a function of the mass ratio m/M. From bottom to top, the curves are for $a=0$, 0.2, 0.5, 1.0.

Figure 4 shows the decay rate as a function of the mass ratio m/M for fixed acceleration a/M. The solid line corresponds to the inertial decay rate for $a=0$. For nonaccelerated particles no decays are possible for which $m>M$; i.e., the decay product is *heavier* than the decaying particle. The figure shows that this is possible for accelerated particles. This is one of the main results of the present paper.

To get an estimate for the absolute magnitude of the effect, we note that the modification of the pion decay rate amounts to 1% for $a/M\approx 0.03$, i.e., $a=2\cdot 10^{30}$ m/s^2. This is far above the presently achievable accelerations.

IV. PROTON DECAY

A most intriguing effect is that particles which are usually considered to be stable can decay if they are accelerated. As a prototype of such a process, we consider the decay of the proton via the reaction

$$p^+ \rightarrow n e^+ \nu_e . \tag{25}$$

This ''inverse neutron decay'' does not occur for inertial protons because the sum of the rest energies of the decay products is greater than the proton mass itself. In this section, we will show that this process is possible if the initial proton is accelerated and give an estimate of its magnitude.

As in the previous sections, we consider a scalar model [Fig. 1(c)]

$$\Psi \rightarrow \Phi_1 \phi_2 \phi_3 \tag{26}$$

with the interaction Lagrangian

$$\mathcal{L}_I = G\Psi(x)\Phi_1(x)\phi_2(x)\phi_3(x)\sqrt{-g}. \tag{27}$$

The model differs from the one of Sec. II only in that Φ_1 is now a massive scalar field (m denotes the mass of Φ_1, while the mass of the initial Ψ is M). Again, this fact leads to considerable computational difficulties.

The decay probability is given by

$$P = G^2\kappa \int d\tau \int d\tau' e^{iM(\tau-\tau')} \langle 0|\Phi_1(x(\tau))\Phi_1(x(\tau'))|0\rangle \times \prod_{k=1}^{2} \langle 0|\phi_k(x(\tau))\phi_k(x(\tau'))|0\rangle . \tag{28}$$

If we insert the explicit expressions for the Wightman functions evaluated along the accelerated trajectory, we obtain for the decay rate

$$\Gamma = i\frac{G^2\kappa a^4 m}{64(2\pi)^5}\int_{-\infty}^{\infty} dx e^{i2Mx/a} \frac{H_1^{(2)}[(2m/a)\sinh(x-i\epsilon)]}{[\sinh(x-i\epsilon)]^5}. \tag{29}$$

As in the previous section, the integration must be performed numerically, and we can use the scheme developed there. We use again the ascending series expansion of the Hankel function to isolate all contributions which diverge at $x=0$. These contributions are in the present case given by $G^2\kappa m \mathcal{A}^{(p)}/64(2\pi)^5$, where

$$\mathcal{A}^{(p)}=-\frac{a}{\pi m}\int_{-\infty}^{\infty}dx\,\frac{e^{i2Mx/a}}{[\sinh(x-i\epsilon)]^6}+\frac{2}{\pi}\sum_{k=0}^{2}\frac{(-1)^k}{k!(k+1)!}\left(\frac{m}{a}\right)^{2k+1}\int_{-\infty}^{\infty}dx e^{i2Mx/a}\ln\left(\frac{m}{a}\sinh(x-i\epsilon)\right)[\sinh(x-i\epsilon)]^{2k-4}$$
$$-\frac{1}{\pi}\sum_{k=0}^{1}\frac{(-1)^k}{k!(k+1)!}\left(\frac{m}{a}\right)^{2k+1}\left(2\psi(k+1)+\frac{1}{k+1}-i\pi\right)\int_{-\infty}^{\infty}dx e^{i2Mx/a}[\sinh(x-i\epsilon)]^{2k-4}, \tag{30}$$

where $\psi(k)$ is the logarithmic derivative of the Γ function. Put together, the following integrals possess the same singularity structure at $x=0$ as the original integrand and can therefore be subtracted from the latter to obtain a finite integrand for the numerical integration routines:

$$\mathcal{I}_1^{(p)}=\frac{2m}{\pi a}\int_{-\infty}^{\infty}dx e^{i2Mx/a}\left[\frac{\ln[(m/a)|x-i\epsilon|]}{(x-i\epsilon)^4}-\frac{2}{3}\frac{\ln[(m/a)|x-i\epsilon|]}{(x-i\epsilon)^2}+\frac{1}{6}\frac{1}{(x-i\epsilon)^2}-i\pi\frac{\theta(-x)}{(x-i\epsilon)^4}+\frac{2}{3}i\pi\frac{\theta(-x)}{(x-i\epsilon)^2}\right]$$
$$=\frac{8m}{9a}\left(\frac{M}{a}\right)^3\left[11-6\gamma+6\ln\left(\frac{m}{2M}\right)-3\pi i\right]+\frac{16m}{3a}\left(\frac{M}{a}\right)\left[\frac{3}{4}-\gamma+\ln\left(\frac{m}{2M}\right)-i\frac{\pi}{2}\right],$$

$$\mathcal{I}_2^{(p)}=-\frac{a}{\pi m}\int_{-\infty}^{\infty}dx\frac{e^{i2Mx/a}}{[\sinh(x-i\epsilon)]^6}=\frac{8M}{15m}\left[4+5\left(\frac{M}{a}\right)^2+\left(\frac{M}{a}\right)^4\right][1-e^{-2\pi M/a}]^{-1},$$

$$\mathcal{I}_3^{(p)}=-\frac{m^3}{\pi a^3}\int_{-\infty}^{\infty}dx\frac{e^{i2Mx/a}}{(x-i\epsilon)^2}\ln\left(\frac{m}{a}(x-i\epsilon)\right)=\frac{4m^3M}{a^4}\left[1-\gamma+\ln\left(\frac{m}{2M}\right)-i\frac{\pi}{2}\right],$$

$$\mathcal{I}_4^{(p)}=\left(\frac{44m}{45\pi a}+\frac{2m^3}{3\pi a^3}+\frac{m^5}{3\pi a^5}\right)\int_{-\infty}^{\infty}\ln\left(\frac{m}{a}x\right)e^{-Mx/a}=\left(-\frac{44m}{45\pi a}-\frac{2m^3}{3\pi a^3}-\frac{m^5}{3\pi a^5}\right)\left[\gamma+\ln\left(\frac{M}{m}\right)\right],$$

$$\mathcal{I}_5^{(p)}=-\frac{m}{a\pi}(1-2\gamma-i\pi)\int_{-\infty}^{\infty}dx\frac{e^{i2Mx/a}}{[\sinh(x-i\epsilon)]^4}=-\frac{8mM}{3a^4}(a^2+M^2)(1-2\gamma-i\pi)[1-e^{-2\pi M/a}]^{-1},$$

$$\mathcal{I}_6^{(p)}=\frac{m^3}{2\pi a^3}\left(\frac{5}{2}-2\gamma-i\pi\right)\int_{-\infty}^{\infty}dx\frac{e^{i2Mx/a}}{[\sinh(x-i\epsilon)]^2}=-\frac{2m^3M}{a^4}\left(\frac{5}{2}-2\gamma-i\pi\right)[1-e^{-2\pi M/a}]^{-1},$$

where $\theta(x)$ is Heaviside's function.

The numerical integration leads to the results shown in Figs. 5 and 6. Figure 5 shows the decay rate of the Ψ particle as a function of the acceleration a/M. The rate is plotted in units of $G^2\kappa/(64(2\pi)^5)$ for various choices of the mass m of the decay product. The solid line corresponds to the neutron/proton mass ratio $m/M=1.0014$. As expected, the rate is zero for $a=0$ and rises very slowly with increasing a. The top curve is for $m=0$; it is identical to the one displayed in Fig. 2.

In Fig. 6, the decay rate is shown as a function of the mass ratio m/M. The solid curve corresponds to $a=0$ and vanishes for $m>M$. The curves for $a>0$ show that under the influence of acceleration, the decay is possible even if the decay products are heavier than the initial particle.

Getting a numerical estimate of the size of the effect is somewhat more difficult than in the previous sections. There, the acceleration-modified decay rates could be directly compared with their inertial values. However, for the ''proton'' decay considered here, there is no effect at $a=0$. Therefore, we calculate in our model the decay rate of a neutron at rest, which is analytically possible, and adjust our coupling constants to the experimentally known value of the neutron lifetime. The estimation of the proton lifetime is further complicated by the fact that even in the largest proton accelerators, the ratio a/M is exceedingly small. For example, the circular acceleration achieved at the CERN Large Hadron Collider (LHC) will only lead to $a/M\approx 10^{-11}$. Since in the limit $a/M\to 0$ the expressions used in the numerical treatment do not behave well, we cannot directly calculate the decay rate for such a small value of a/M. Instead we have to resort to a numerical extrapolation of our data. We find that the slow rising of the solid curve in Fig. 5 is well described for small a/M by $(a/M)^4$. If we assume the validity of this behavior down to $a=0$ we find 10^{26} years for the proton lifetime at LHC accelerations. Although this estimate gives a smaller lifetime than the one predicted by grand unified theories, it is evident that the effect is much too small to be detectable. We only mention that we obtain for the proton lifetime at earth's acceleration approximately 10^{70} years.

V. CONNECTION WITH THE UNRUH EFFECT

Let us finally discuss in more detail the relation between the effects presented in this paper and the Unruh effect. In the latter, one considers a two-level system accelerating through the quantum vacuum. If it is prepared in the ground state initially, there is a nonvanishing probability to find it in the excited state at some later time. An acceleration-induced spontaneous excitation has taken place which is forbidden at $a=0$.

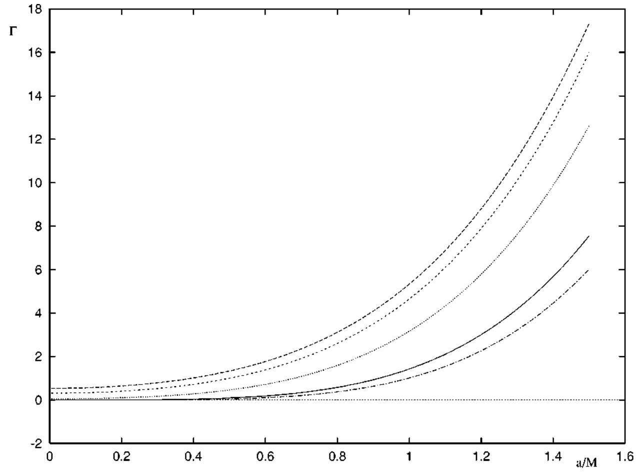

FIG. 5. Decay rate for the process $\Psi \to \Phi_1 \phi_2 \phi_3$ (model for the proton decay $p \to n e^+ \nu_e$). The rate is plotted in units of $G^2\kappa/(64(2\pi)^5)$ as a function of the acceleration a/M. The curves correspond to different mass ratios m/M (m/M=0, 0.2, 0.5, 1.0014, 1.2 from top to bottom). The solid line corresponds to the neutron/proton mass ratio.

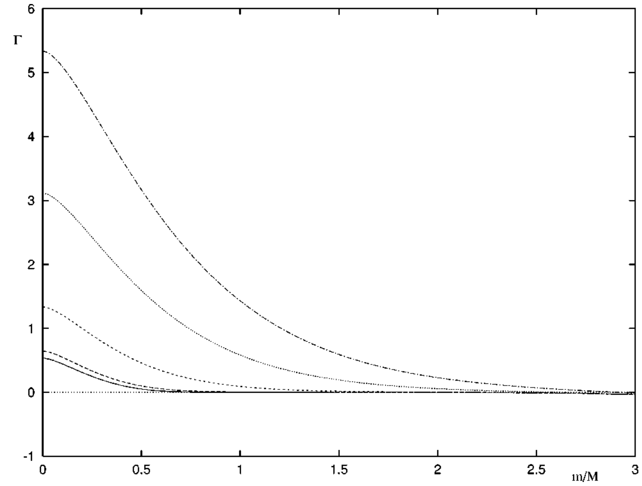

FIG. 6. Decay rate for the process $\Psi \to \Phi_1 \phi_2 \phi_3$ as a function of the mass ratio m/M. From bottom to top: $a/M=$ 0 (solid line), 0.2, 0.5, 0.8, 1.0.

The direct analogue of this effect is the proton decay discussed in the previous section. The two levels are replaced by the rest energies of proton and neutron. The spontaneous excitation occurring in the Unruh effect corresponds to the transition where the initial state (proton) has a lower rest energy than the final state (neutron). This similarity shows up already at the formal level. In the evaluation of the Unruh effect, for example, the Fourier transformation of the Wightman function has to be evaluated along the accelerated trajectory (see [12,15]). Equation (28) shows that in the calculation of the proton decay, the product of three Wightman functions appears, one of them corresponding to a massive field. The difference in the calculation schemes comes from the fact that a simple two-level approximation would be too crude for the quantitative evaluation of the proton decay. A correct treatment must take into account all possible final momenta of the decay products (which are integrated out for finding the total decay rate).

The processes discussed in Secs. II and III are not forbidden inertially. Instead the rates of existing decay channels are modified under the influence of acceleration. Strictly speaking, these reactions are not analogous to the Unruh effect but to the modification of spontaneous emission which was found in Ref. [5]. There, an inertially existing process (spontaneous emission) is modified in the presence of acceleration in a similar manner as the particle decays discussed in the present paper.

VI. CONCLUSION

We have studied how the decay properties of particles are modified by acceleration. We have shown that (1) the lifetime of an unstable particle is modified by acceleration and (2) particles that are stable when unaccelerated acquire a finite lifetime under the influence of acceleration.

Using a scalar model of the Fermi weak interaction theory, we have investigated the decay of the muon and of the pion. With regard to a possible experimental realization, the modification of the muon decay rate appeared to be the most promising candidate. Furthermore, we have shown that under the influence of a sufficiently large acceleration, the proton becomes unstable and can decay via an inverse neutron decay process. The estimate of the decay rate showed, however, that enormous accelerations are necessary to detect this effect.

The effects derived here have nothing in common with the classical special-relativistic time dilation which can be understood from purely kinematical reasoning. Instead they are of quantum field theoretical origin. Moreover, in their derivation no nonstandard particle physics (like grand unified theories) enter. Therefore, the effects are a direct prediction of quantum field theory in accelerated frames. Apart from being interesting in its own right, this theory may also help to understand the quantum theory in curved spacetime more thoroughly.

The results presented in this paper are derived by approximating Dirac particles with scalar quantum fields. Although we can gain a qualitative understanding of the physical processes in this way, the numerical predictions derived with such a model can only be regarded as rough order-of-magnitude estimates. An important next step is therefore to obtain more quantitative statements by basing the calculation on Dirac fields and the Fermi interaction. A further interesting question is how decay processes are influenced by spacetime curvature (instead of acceleration). This may have important consequences in the early universe where such curvature-induced corrections (or new processes) may influence particle reactions in a non-negligible way.

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