Nonrenormalization theorem for the $d=1$, $\mathcal{N}=8$ vector multiplet

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Sigma models describing low energy effective actions on $D0$-brane probes with $\mathcal{N}=8$ supercharges are studied in detail using a manifestly $d=1$, $\mathcal{N}=4$ superspace formalism. Two $(0+1)$-dimensional $\mathcal{N}=4$ multiplets together with their general actions are constructed. We derive the condition for these actions to be $\mathcal{N}=8$ supersymmetric and apply these techniques to various D-brane configurations. We find that if in addition to $\mathcal{N}=8$ supersymmetry the action must also have Spin(5) invariance, the form of the σ model metric is uniquely determined by the one-loop result and is not renormalized perturbatively or nonperturbatively. [S0556-2821(97)01424-0]

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I. INTRODUCTION AND SUMMARY

Recent developments [1–3] have emphasized the crucial role played by $D0$-branes in probing space-time structure at substringy scales as well as in a nonperturbative definition of 11-dimensional M theory. The basic feature that enables D-particles to test short distances in string theory is that their low energy dynamics is a quantum mechanics of the lightest open string degrees of freedom. The geometrical background in the substringy domain is reproduced by quantum open string effects, while the classical background at distances larger than the string scale is described by supergravity results which are essentially mediated by massless closed strings. As discussed in [1], in the cases with enough supersymmetry, the two regimes are continuously connected by factorization of the open string annulus diagram. The general behavior in such cases is that the long distance supergravity results coincide with the one-loop quantum corrections to the probe moduli space. By analogy with higher dimensional field theories, it is plausible that higher order perturbative corrections as well as nonperturbative ones vanish, leading to nonrenormalization theorems. A similar nonrenormalization result for a higher derivative interaction proves to be essential [2] in the matrix theory formulation of M theory.

The purpose of the present work is to study the $\mathcal{N}=8$ quantum mechanics of a $D0$-brane probe moving in different $D4$-brane and/or orientifold plane backgrounds. The low energy degrees of freedom in the probe theory are five bosons and eight fermions. A single $D0$-$D4$ configuration has $\mathcal{N}=8$ supersymmetry and a Spin(5) rotational symmetry in the transverse directions under which the bosons transform as a vector and the fermions as a spinor. We construct two $\mathcal{N}=4$ multiplets that together have these degrees of freedom, but are not manifestly Spin(5) symmetric. We call the pair of these multiplets the $d=1$, $\mathcal{N}=8$ vector multiplet. Our main result is that the condition for Spin(5) invariance of the vector multiplet action is compatible with the condition for it to have $\mathcal{N}=8$ supersymmetry and that when taken together these invariances uniquely determine the form of the target space metric. The form of the metric we find agrees with the one-loop results of [1], and we conclude that it cannot receive perturbative or nonperturbative corrections.

The plan of this paper is as follows. In Sec. II we develop a manifestly $\mathcal{N}=4$ superspace formalism in $0+1$ dimensions and describe $\mathcal{N}=4$ chiral and linear multiplets that together contain the right number of bosonic and fermionic degrees of freedom. We then find in Sec. III the condition for this action to admit four additional supersymmetries and argue that this condition is essentially unique. Requiring also Spin(5) invariance leads to the nonrenormalization theorem. This result is applied in Sec. IV to various D-brane configurations. Finally, we discuss the range of validity of this theorem in connection with three-dimensional analogues and string duality.

II. $\mathcal{N}=4$ MULTIPLETS IN ONE DIMENSION

The $d=1$, $\mathcal{N}=4$ superspace is parametrized by one commuting coordinate t and four noncommuting ones arranged as an SU(2) spinor θ_α and its complex conjugate $\overline{\theta^\alpha}$. The covariant derivatives and supercharges are given by (our conventions are summarized in Appendix A)

$$D_\alpha=\frac{\partial}{\partial\theta^\alpha}-i\overline{\theta_\alpha}\partial_0,\quad \overline{D_\alpha}=\frac{\partial}{\partial\overline{\theta^\alpha}}-i\theta_\alpha\partial_0,$$

$$Q_\alpha=\frac{\partial}{\partial\theta^\alpha}+i\overline{\theta_\alpha}\partial_0,\quad \overline{Q_\alpha}=\frac{\partial}{\partial\overline{\theta^\alpha}}+i\theta_\alpha\partial_0,$$

and satisfy the algebra

$$\{D_\alpha,\overline{D_\beta}\}=2i\epsilon_{\alpha\beta}\partial_0,\quad \{Q_\alpha,\overline{Q_\beta}\}=-2i\epsilon_{\alpha\beta}\partial_0,\quad (2.1)$$

with all the other anticommutators vanishing. The manifest supersymmetry transformations are generated by $\epsilon^\alpha Q_\alpha+\overline{\epsilon^\alpha}\overline{Q_\alpha}$ acting on the various multiplets.

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A. Chiral multiplet

As in the $d=4$, $\mathcal{N}=1$ case, the chiral and antichiral multiplets are defined by the constraints $\bar{D}\Phi = D\bar{\Phi}=0$, which are solved by functions of $y=t-i\theta^\alpha\bar{\theta}_\alpha$ and $\bar{y}=t+i\theta^\alpha\bar{\theta}_\alpha$. In component form they are given by

$$\Phi(y)=\Phi(y)+2\theta^\alpha\psi_\alpha(y)+\theta\theta F(y)$$
$$=\Phi-i\theta^\alpha\bar{\theta}_\alpha\dot{\Phi}+\frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\ddot{\phi}+2\theta^\alpha\psi_\alpha$$
$$-i\theta\theta\bar{\theta}_\alpha\dot{\psi}^\alpha+\theta\theta F,$$

and

$$\bar{\Phi}(\bar{y})=\bar{\Phi}(\bar{y})-2\bar{\theta}_\alpha\bar{\psi}^\alpha(\bar{y})-\bar{\theta}\bar{\theta}F^*(\bar{y})$$
$$=\bar{\Phi}+i\theta^\alpha\bar{\theta}_\alpha\dot{\bar{\Phi}}+\frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\ddot{\bar{\Phi}}-2\bar{\theta}_\alpha\bar{\psi}^\alpha$$
$$+i\bar{\theta}\bar{\theta}\theta_\alpha\dot{\bar{\psi}}^\alpha-\bar{\theta}\bar{\theta}F^*,$$

which are the $d=4$, $\mathcal{N}=1$ chiral and antichiral multiplets reduced to one dimension. The physical on-shell degrees of freedom arising from these multiplets are two bosons and four fermions.

B. Linear multiplet

The $d=4$, $\mathcal{N}=1$ vector multiplet dimensionally reduced to $D=3$ becomes equivalent [4] to the real linear multiplet G defined by the constraints

$$D^2G=\bar{D}^2G=0,$$

where D and $\bar{D}$ denote the spinor derivatives of the $d=3$, $\mathcal{N}=2$ superspace. They are solved by

$$G=D\bar{D}V,$$

with V an arbitrary real superfield. The physical degrees of freedom consist of a real scalar boson, a three-dimensional vector field, and their fermionic superpartners. The real scalar can be thought of as the fourth component of the four-dimensional vector field. Further reduction to two dimensions yields [4,5] the twisted chiral multiplet Σ_{+-} defined [4] by the constraints

$$\bar{D}_+\Sigma_{+-}=D_-\Sigma_{+-}=0.$$

The solution of these constraints can be expressed similarly in terms of a real superfield

$$\Sigma_{+-}=\frac{1}{\sqrt{2}}\bar{D}_+D_-V,$$

describing the dynamics of the two real scalars obtained by dimensional reduction of the four-dimensional vector field plus their superpartners.

In one dimension the closest analogue of the above conditions would be

$$D^2\Sigma=\bar{D}^2\Sigma=0. \tag{2.2}$$

Proceeding naively we take

$$\Sigma=\bar{D}_\alpha\Theta^\alpha,$$

where Θ^α is a superfield, as the general solution of the second constraint in Eq. (2.2). By making use of $[\bar{D}_\alpha,D^2]=4iD_\alpha\partial_0$, the first condition in Eq. (2.2) is satisfied if

$$\bar{D}_\alpha D^2\Theta^\alpha+4iD_\alpha\partial_0\Theta^\alpha=0,$$

which is unacceptable since the time dependence of Θ^α is restricted. A natural modification would be to consider a triplet of superfields which we denote by $\Sigma_{\alpha\beta}$. More precisely, the linear multiplet $\Sigma_{\alpha\beta}$ is defined by

$$D^\gamma D^\alpha\Sigma_{\alpha\beta}=\bar{D}^\gamma\bar{D}^\alpha\Sigma_{\alpha\beta}=0 \tag{2.3}$$

and the reality condition

$$\bar{\Sigma}_{\alpha\beta}\equiv\Sigma^{\alpha\beta}=\epsilon^{\alpha\gamma}\Sigma_{\gamma\delta}\epsilon^{\delta\beta}. \tag{2.4}$$

The unique solution of these constraints with no restriction on the time dependence is given by

$$\Sigma_{\alpha\beta}=\bar{D}_{(\alpha}D_{\beta)}V, \tag{2.5}$$

where V is the real superfield of $d=4$, $\mathcal{N}=1$ reduced to one dimension and parentheses denote symmetrization. The second constraint in Eq. (2.3) is identically satisfied, while the first one follows from the algebra (2.1):

$$D^2\Sigma_{\alpha\beta}=\tfrac{1}{2}(D^2\bar{D}_\alpha D_\beta+D^2\bar{D}_\beta D_\alpha)V=i(\epsilon_{\beta\alpha}+\epsilon_{\alpha\beta})\partial_0V. \tag{2.6}$$

The component form of $\Sigma_{\alpha\beta}$ is given by

$$\Sigma_{\alpha\beta}=-\sigma^i_{\alpha\beta}x_i+i(\theta_\alpha\bar{\lambda}_\beta+\theta_\beta\bar{\lambda}_\alpha)+i(\bar{\theta}_\beta\lambda_\alpha+\bar{\theta}_\alpha\lambda_\beta)-(\bar{\theta}_\beta\theta_\alpha$$
$$+\bar{\theta}_\alpha\theta_\beta)D+\frac{i}{2}(\theta_\beta\bar{\theta}^\gamma\sigma^i_{\gamma\alpha}+\theta_\alpha\bar{\theta}^\gamma\sigma^i_{\gamma\beta}+\bar{\theta}_\beta\theta^\gamma\sigma^i_{\gamma\alpha}$$
$$+\bar{\theta}_\alpha\theta^\gamma\sigma^i_{\gamma\beta})\dot{x}_i+\frac{1}{2}\bar{\theta}\bar{\theta}(\theta_\beta\dot{\lambda}_\beta+\theta_\alpha\dot{\lambda}_\beta)-\frac{1}{2}\theta\theta(\bar{\theta}_\beta\dot{\bar{\lambda}}_\alpha$$
$$+\bar{\theta}_\alpha\dot{\bar{\lambda}}_\beta)+\frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\sigma^i_{\alpha\beta}\ddot{x}_i \tag{2.7}$$

or, alternatively, by

$$\Sigma^i\equiv\frac{1}{2}\sigma^{i\alpha\beta}\Sigma_{\alpha\beta}$$
$$=-x^i+i\theta_\gamma\sigma^{i\gamma\delta}\bar{\lambda}_\delta+i\bar{\theta}_\gamma\sigma^{i\gamma\delta}\lambda_\delta-\bar{\theta}_\gamma\sigma^{i\gamma\delta}\theta_\delta D$$
$$+\epsilon^{ijk}\bar{\theta}_\gamma\sigma^{j\gamma\delta}\theta_\delta\dot{x}^k-\frac{1}{2}\bar{\theta}\bar{\theta}\theta_\gamma\sigma^{i\gamma\delta}\dot{\lambda}_\delta$$
$$+\frac{1}{2}\theta\theta\bar{\theta}_\gamma\sigma^{i\gamma\delta}\dot{\bar{\lambda}}_\delta+\frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\ddot{x}^i, \tag{2.8}$$

which is more convenient for our purposes. We stress that supersymmetry forces us to consider $(\Sigma^1,\Sigma^2,\Sigma^3)$ as a *vector* of superfields and not treat each component separately. This can be seen by observing that the three scalars x^i enter the supersymmetry transformation of λ and $\bar{\lambda}$ symmetrically. When Σ^i is taken separately, x^i enters through the lowest order in θ component, while the other two scalars appear in the $\theta\theta$ component. Thus only an invariant and real combination, such as $\Sigma_{\alpha\beta}\Sigma^{\alpha\beta}$ or Σ^2, may appear in a supersymmetric action.

Finally we note that by analogy with the $d=4$, $\mathcal{N}=1$ case one can define chiral and antichiral field-strength multiplets by

$$W_\alpha \equiv \bar{D}^\beta \Sigma_{\alpha\beta}, \quad \bar{W}_\alpha \equiv D^\beta \Sigma_{\alpha\beta}. \tag{2.9}$$

Then,

$$-\tfrac{1}{6}(W^\alpha W_\alpha|_{\theta\theta} + \bar{W}^\alpha \bar{W}_\alpha|_{\bar{\theta}\bar{\theta}})$$

yields the same kinetic terms as $\Sigma^i\Sigma^i|_{\theta\theta\bar{\theta}\bar{\theta}}$.

III. $\mathcal{N}=8$ SUPERSYMMETRY AND NONRENORMALIZATION

We will ultimately be concerned with applications to a $D0$-brane in $D4$-branes and orientifold backgrounds. As will be explained in Sec. IV, the low energy degrees of freedom on the $D0$-brane world line are precisely described by the pair (Φ,Σ), which is the $d=1$, $\mathcal{N}=8$ multiplet. Such systems have only eight supersymmetries, and so quadratic terms in the velocities are generally not protected from renormalization. In the regime where the velocity of the $D0$-brane is small, we may restrict our attention to an action which is quadratic in velocities and neglect higher order terms. A general such action with four manifest supersymmetries is given by

$$\int d^2\theta\, d^2\bar{\theta}\, K(\Phi,\bar{\Phi},\vec{\Sigma}^2), \tag{3.1}$$

where K is an arbitrary real prepotential. It is possible to add a superpotential integrated over half of superspace, but it will not contribute to the metric. This remark actually applies to a wider class of actions. We may think of Eq. (3.1) as the first term in an expansion of the form

$$\mathcal{L} = \int d^2\theta\, d^2\bar{\theta}(K_2 + K_4 \partial_0\Sigma^i\partial_0\Sigma^i + \tilde{K}_4 \partial_0\Phi\,\partial_0\bar{\Phi} + \cdots),$$

where each successive K_i produces an ith power in velocity term in the Lagrangian (we do not have to consider an expansion in covariant derivatives of the multiplets since these lead to cubic terms in the velocities). Again, $K_4, \tilde{K}_4, \ldots$ cannot give metric terms, and so the nonrenormalization result we will prove below applies also to the metric terms in these actions as well.

The metric can be read from the kinetic terms arising from the superspace integration:

$$\frac{1}{4} K_{\Sigma^i\Sigma^i}[\dot{x}^j\dot{x}^j + i(\bar{\lambda}\dot{\lambda} + \lambda\dot{\bar{\lambda}})] - K_{\Phi\bar{\Phi}}[\dot{\Phi}\dot{\bar{\Phi}} + i(\bar{\psi}\dot{\psi} + \psi\dot{\bar{\psi}})], \tag{3.2}$$

and consists of only two undetermined functions $\frac{1}{4}K_{\Sigma^i\Sigma^i}$ and $-K_{\Phi\bar{\Phi}}$. Note the absence of mixed derivative terms—this will prove crucial for the applications to the 0-4 system.

A. Nonmanifest supersymmetries

If the action (3.1) admits more supersymmetries, their form is severely constrained by the following considerations. First, they must be realized as spinorial derivatives acting on superfields so that the supersymmetry algebra is satisfied. The four manifest supersymmetries of each multiplet are already generated by the supercharges acting on it. Therefore, additional supersymmetries, if they exist, must be generated by spinorial derivatives acting on the other multiplets. This means that $\Sigma_{\alpha\beta}$ will enter the nonmanifest transformations of Φ, $\bar{\Phi}$ and that Φ, $\bar{\Phi}$ enter symmetrically the nonmanifest variation of $\Sigma_{\alpha\beta}$. The form of these variations is further constrained and, in fact, determined up to a constant by requiring the variations to respect the defining constraints of the multiplets. Thus we conclude that if there are four additional supersymmetries their form is

$$\delta\Phi \propto i\bar{\epsilon}^\beta \bar{D}^\alpha \Sigma_{\alpha\beta},$$

$$\delta\bar{\Phi} \propto i\epsilon^\beta D^\alpha \Sigma_{\alpha\beta},$$

$$\delta\Sigma_{\alpha\beta} \propto i(\epsilon_{(\alpha}D_{\beta)}\Phi - \bar{\epsilon}_{(\alpha}\bar{D}_{\beta)}\bar{\Phi}). \tag{3.3}$$

[The chiral and antichiral constraints of $\delta\Phi$ and $\delta\bar{\Phi}$ follow directly from Eq. (2.3). The variation of $\Sigma_{\alpha\beta}$ can be seen to satisfy the conditions (2.4) and (2.3) by using the algebra (2.1). It is also easy to verify that the commutator of two nonmanifest variations closes on translations.]

A straightforward (and a little laborious) calculation in components shows that the action (3.1) admits the four nonmanifest supersymmetries

$$\delta\Phi = \frac{-2i}{3}\,\bar{\epsilon}^\beta \bar{D}^\alpha \Sigma_{\alpha\beta},$$

$$\delta\bar{\Phi} = \frac{-2i}{3}\,\epsilon^\beta D^\alpha \Sigma_{\alpha\beta},$$

$$\delta\Sigma_{\alpha\beta} = i(\epsilon_{(\alpha}D_{\beta)}\Phi - \bar{\epsilon}_{(\alpha}\bar{D}_{\beta)}\bar{\Phi}), \tag{3.4}$$

provided that the following condition holds:

$$K_{\Sigma^i\Sigma^i} + 4K_{\Phi\bar{\Phi}} = 0. \tag{3.5}$$

This is actually also a *necessary* condition. The explicit form of the action shows that it cannot be invariant under the supersymmetry transformations of the form (3.3) unless Eq. (3.5) holds, that is, unless the action depends only one arbitrary function. As argued above, the form of the nonmanifest variations is unique, and so we conclude that any $\mathcal{N}=4$ supersymmetric action is automatically $\mathcal{N}=8$ supersymmetric if and only if Eq. (3.5) holds. The metric, the action, and all the supersymmetry transformations are now determined by

$$f = -\tfrac{1}{4}K_{\Sigma^i\Sigma^i} = K_{\Phi\bar{\Phi}} \tag{3.6}$$

and are given in Appendix B (f enters the variation laws once the auxiliary fields are solved for). Differentiating f twice with respect to Σ^i and with respect to Φ and $\overline{\Phi}$, and using Eq. (3.5), shows that the metric satisfies

$$f_{\Sigma^i\Sigma^i}+4f_{\Phi\overline{\Phi}}=0 \tag{3.7}$$

as well.

B. Spin (5) invariance and nonrenormalization

The five scalars in the vector multiplet can be thought of as local coordinates, $y_1,\ldots,y_5$, on a five-dimensional target space manifold by making the change of variables

$$y_i=x^i, \quad i=1,2,3,$$

$$y_4=\frac{1}{2}(\Phi+\overline{\Phi}),$$

$$y_5=\frac{1}{2i}(\Phi-\overline{\Phi}).$$

In these coordinates the condition (3.7) satisfied by the metric f is precisely the Spin(5)-invariant Laplace equation. Any function of r^2, r being the five-dimensional radius, must also satisfy this equation, and therefore the condition for a Spin(5)-invariant metric is compatible with the condition for $\mathcal{N}=8$ supersymmetry. This conclusion depends crucially on the relative sign and factor in Eq. (3.2) and would not have been valid otherwise. Furthermore, f is now determined up to two constants. The condition (3.7) on a Spin(5)-invariant function reduces to

$$r^2f''+\tfrac{5}{2}f'=0 \tag{3.8}$$

and is solved by

$$f=C'+\frac{C}{r^3}, \tag{3.9}$$

where C and C' are arbitrary constants. We conclude that the metric of a general action compatible with the above symmetries is not renormalized either perturbatively or nonperturbatively.

It is also possible to restore manifest Spin(5) invariance in the full Lagrangian. After solving algebraically for the auxiliary fields, the superspace Lagrangian is given by[1]

$$-f(\dot{x}^i\dot{x}^i+i(\overline{\dot{\eta}}\eta+\eta\dot{\overline{\eta}}))+\dot{x}^if_{,j}(\eta\gamma^{ij}\overline{\eta})+\tfrac{1}{2}(f_{,ij}-\tfrac{1}{2}f_{,i}f_{,j})$$
$$\times(\eta\gamma^i\overline{\eta}\eta\gamma^j\overline{\eta}+\eta\gamma^i\eta\overline{\eta}\overline{\gamma^j}\overline{\eta}), \tag{3.10}$$

and has a natural geometric interpretation. Specifically, the bilinear fermion term, taking into account the fünfbein factors, is the pull back of the minimal spin connection made of the metric f. With a little algebra, the quadratic fermion term can also be seen to be

$$R_{\alpha\beta\gamma\delta}\eta^{\alpha}\overline{\eta^{\beta}}\eta^{\gamma}\overline{\eta^{\delta}},$$

with the curvature computed from the minimal connection. The manifest and nonmanifest supersymmetry (SUSY) transformations also match up in a nice way. The manifest ones can be recovered if in the five-dimensional SUSY transformations (B6) the parameter ϵ^5_α is taken as

$$\epsilon^5_\alpha=\begin{pmatrix}\epsilon_\alpha\\0\end{pmatrix}$$

and the nonmanifest ones if we take

$$\epsilon^5_\alpha=\begin{pmatrix}0\\\epsilon_\alpha\end{pmatrix}.$$

Since the target space is odd dimensional, these restrictions cannot be made in an invariant way, but together they combine into an $\mathcal{N}=8$ SUSY parameter. This is again due to the consistency of the Spin(5) invariance and $\mathcal{N}=8$ conditions.

IV. *D*0-*D*4 SYSTEM

The formalism developed in the previous sections can be applied to the study of low energy effective actions of $D0$-brane probes in different type-IIA backgrounds. Extending the analysis of [1], we consider $D0$-brane probes in type-I$'$ theory realized as an orientifold of the type-IIA theory compactified on a 5-torus T^5 [6,7]. More precisely, one starts with type-I theory on T^5 and performs T duality on all the five circles of the torus. The resulting theory is type IIA on $T^5/Z_2\Omega$ with 16 pairs of $D4$-branes in the background to cancel the charge of the 32 orientifold fixed planes. In the normalization of [6,7], the Ramond-Ramond (RR) charge of a fixed plane is -1, while the charge of a 4-brane is 1 such that cancellation holds globally. Local cancellation occurs in a configuration with a 4-brane at each orientifold plane. The supersymmetric probes for this background are $D0$-branes whose world line effective action is expected to reproduce the string background [1]. We will consider two distinct configurations:

(i) n $D4$-branes coalesce away from an orientifold fixed plane. In $d=1$, $\mathcal{N}=8$ language, the degrees of freedom on the $D0$-brane world line consist of an Abelian vector multiplet and a neutral hypermultiplet arising from 0-0 strings and n hypermultiplets in the fundamental of the U(1) gauge group arising from 0-4 strings. The space-time positions of the 4-branes correspond to bare masses $\vec{m}_i$ of the charged multiplets in the gauge theory on the probe. When the branes come together, one obtains SU(n) gauge symmetry enhancement in space-time corresponding to SU(n) global symmetry enhancement in the probe theory.

When the 0-brane is away from the 4-brane, the massive 0-4 string states can be integrated out. The surviving low energy degrees of freedom in the world line theory are the

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[1] To avoid clutter, the same tangent space indices are used to denote the flat space carried by the γ matrices.

$\mathcal{N}=8$ vector multiplet and neutral hypermultiplet. The later decouples, and so the low energy effective action is the theory of an interacting $\mathcal{N}=8$ vector multiplet. If the positions of the 4-branes coincide, the system is rotationally invariant in the five transverse directions, and so the theory has Spin(5)$\simeq$Sp(2) symmetry under which the bosons transform in the **5** and fermions in the **4**.

The result of the previous section applies to this configuration, and it remains to determine the constants in Eq. (3.9). In the present case,

$$C' = \frac{1}{g_s}$$

is the asymptotic value of the dilaton far from the 4-branes and at the same time the classical coupling constant of the gauge theory on the probe. The second constant C is determined by the one-loop effects of the n charged hypermultiplets [1] to be

$$C = n.$$

Therefore, we conclude that the one-loop results of [1] are exact already in this order and do not receive further corrections. This statement is true as long as the theory is described in terms of the multiplets introduced above, but it may break down in a description in terms of different variables. A similar phenomenon is encountered in three-dimensional gauge theories [8–10] where the monopole corrections become visible only after dualizing the photon. As we will see later, string duality suggests that this happens in the present case as well.

If the 4-branes are localized at different points of coordinates $\vec{m}_i$ in transverse space, the SU(n) global symmetry on the probe is broken since the hypermultiplets have different masses. In this case one-loop metric is given by

$$f(\vec{x}) = \frac{1}{g_s} + \frac{1}{|\vec{x}-\vec{m}_1|^3} + \cdots + \frac{1}{|\vec{x}-\vec{m}_n|^3}. \tag{4.1}$$

This configuration is no longer Spin(5) symmetric in the transverse directions, but the nonrenormalization result still holds. The system is still $\mathcal{N}=8$ supersymmetric, and so the exact f must still satisfy the five-dimensional Laplace equation. The boundary conditions on the exact metric close to $\vec{m}_i$ are given by

$$f = \frac{1}{|\vec{x}-\vec{m}_i|^3},$$

since near any of the n $D4$-branes the remaining $n-1$ hypermultiplets (corresponding to the the rest of the $D4$-branes) are very massive and can be neglected. As f is uniquely determined by the boundary conditions, the one-loop result (4.1) is exact.

(ii) n $D4$-branes coalesce at an orientifold fixed plane. The degrees of freedom on the $D0$-brane consist now of a non-Abelian SU(2) vector multiplet plus an adjoint hypermultiplet arising from 0-0 strings and n hypermultiplets in the fundamental of the gauge group. The n coalescing 4-branes can be viewed as a collection of $2n$ branes pairwise identified by the Z_2 projection. Therefore, they are localized at points $\vec{m}_i$ and $-\vec{m}_i$ in transverse space. The space-time gauge symmetry is enhanced to SO($2n$) when the branes coincide with the orientifold plane. As before, this corresponds to an SO($2n$) global symmetry enhancement on the probe. The SU(2) gauge group on the world line is spontaneously broken to U(1) by expectation values of the five scalars in the vector multiplet which parametrize the Coulomb branch of the theory. Strictly speaking, this terminology is inappropriate as there is no real moduli space in quantum mechanics. Nevertheless, one can still refer to a quantum-mechanical moduli space in the Born-Oppenheimer approximation [8]. In this sense the low energy effective action on the probe is a U(1) gauge theory with $\mathcal{N}=8$ supersymmetry. When the 4-branes coalesce at the fixed plane, there is a global Spin(5) symmetry rotating the five scalars in the Abelian vector multiplet. Therefore, the general action is exactly of the form (3.10). The only difference with respect to the previous case is reflected in the value of the constant C,

$$C = 2n-1,$$

where the negative term represents the one-loop contribution of the non-Abelian vector multiplet to the effective action. As above, there are no quantum corrections beyond one loop. When the 4-branes are in general positions, the one-loop metric is

$$f(\vec{x}) = \frac{1}{g_s} + \frac{1}{|\vec{x}-\vec{m}_1|^3} + \frac{1}{|\vec{x}+\vec{m}_1|^3} + \cdots + \frac{1}{|\vec{x}-\vec{m}_n|^3} + \frac{1}{|\vec{x}+\vec{m}_n|^3} - \frac{1}{|\vec{x}|^3},$$

and by the same argument used for Eq. (4.1) does not get renormalized beyond this order.

V. DISCUSSION

We have shown above that the one-dimensional action describing five bosons and eight fermions in the **5** and **4** of Sp(2) is, up to two constants, uniquely determined by requiring $\mathcal{N}=8$ supersymmetry and Spin(5) invariance. Since the form of the action is fixed by a solution of a differential equation, we cannot determine in our formalism the constants that appear in the metric. Indeed, if the probe is near an orientifold fixed plane with all 4-branes far away, the metric becomes negative definite at a finite distance in moduli space. The description of the physics in terms of the $\mathcal{N}=8$ vector multiplet degrees of freedom breaks down, and one has to look for another set of variables. Similar phenomena occur in three-dimensional gauge theories where the equivalent description involves dualizing the photon [8–10]. In the new variables the three-dimensional nonrenormalization theorem is violated by an infinite series of monopole corrections [9].

This is also likely to be the case here since a dual set of variables will not necessarily have a Spin(5) symmetry. Further evidence for this conclusion can be inferred from string duality arguments analogous to those presented in [8,9] for the three-dimensional case. The type-I' orientifold studied above is T dual to type-I theory on a 5-torus T^5, which is in turn S dual to heterotic string theory on the same T^5. This is further dual to type-IIA theory on $K3\times S^1$ and after T duality on the S^1 factor to type-IIB theory on $K3\times\widetilde{S}^1$. The $D0$-brane probe is mapped by the first duality in the chain to a type-I $D5$-brane wrapped on T^5, while the $D4$-branes in the background are mapped to the 32 $D9$-branes of type-I theory. According to the analysis of [11], the zero modes of the type-I $D5$-brane wrapped on a 4-torus correspond to the world sheet degrees of freedom of the type-IIB string in static gauge. In our case the 5-brane is wrapped on an extra circle, and thus it maps to a fundamental type-IIB string wrapped on the extra circle $\widetilde{S}^1$, which is a particle in the five noncompact dimensions. This is the image of the initial $D0$-brane probe through the above chain of dualities. The resulting σ model is very different from the one we started with. It represents the motion of the particle on $K3\times\widetilde{S}^1$, and thus the target space metric is the product of a hyper-Kähler metric on $K3$ and a trivial metric on S^1. The fermions are target space vectors, and the symmetry is reduced to a product $U(1)\times G$, where G is the isometry group of the hyper-Kähler metric.[2] The orientifold background is mapped to a noncompact hyper-Kähler manifold asymptotic to an S^1 bundle over the projective space RP^2. In particular, the metric is smooth and positive definite due to nonperturbative corrections [9]. The singularities at infinite distance corresponding to 4-branes are mapped to orbifold singularities in the complex structure of the hyper-Kähler surface.

While in the three-dimensional analysis of [8,9] the string duality picture is entirely reproduced by electric-magnetic duality on a $D2$-brane probe, the present situation is less clear. One could try to define an analogue of higher dimensional duality transformations for the linear multiplet, but we leave this for further study.

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APPENDIX A: SPINOR CONVENTIONS

1. Sp(1) spinors

The anticommuting coordinates θ_α and $\overline{\theta^\alpha}\equiv(\theta_\alpha)^*$ are spinors of $SU(2)\simeq Sp(1)$. Raising and lowering indices is done with the Sp(1)-invariant metric as

$$\theta_\alpha=\epsilon_{\alpha\beta}\theta^\beta,\quad \theta^\alpha=\epsilon^{\alpha\beta}\theta_\beta,$$

$$\overline{\theta_\alpha}=\epsilon_{\beta\alpha}\overline{\theta^\beta},\quad \overline{\theta^\alpha}=\epsilon^{\beta\alpha}\overline{\theta_\beta}. \tag{A1}$$

Complex conjugation of anticommuting numbers is defined by

$$(\eta_\alpha\xi_\beta)^*=\overline{\xi^\beta}\,\overline{\eta^\alpha}, \tag{A2}$$

which implies that $\partial_\alpha^*=-\overline{\partial^\alpha}$ and $\partial^{\alpha *}=-\overline{\partial_\alpha}$. [This is necessary for the solution $\Sigma_{\alpha\beta}$ of Eq. (2.3) to be consistent with the reality condition (2.4).] We also use

$$\psi\psi\equiv\psi^\alpha\psi_\alpha,\quad \overline{\psi\psi}\equiv\overline{\psi_\alpha}\,\overline{\psi^\alpha},\quad \psi\overline{\psi}\equiv\psi_\alpha\overline{\psi^\alpha}=\psi^\alpha\overline{\psi_\alpha}.$$

The symmetric γ matrices are

$$\sigma^1_{\alpha\beta}=i\mathbf{1},\quad \sigma^2_{\alpha\beta}=\tau^3,\quad \sigma^3_{\alpha\beta}=\tau^1, \tag{A3}$$

where τ are the Pauli matrices. They satisfy the algebra

$$(\sigma^i\sigma^j)_{\alpha\beta}=\delta^{ij}\epsilon_{\alpha\beta}+i\epsilon^{ijk}\sigma^k_{\alpha\beta} \tag{A4}$$

and the reality condition

$$(\sigma^i_{\alpha\beta})^*\equiv\sigma^{i\alpha\beta}=\epsilon^{\alpha\gamma}\sigma^i_{\gamma\delta}\epsilon^{\delta\beta}. \tag{A5}$$

2. Sp(2) spinors

We give the decomposition of Sp(2) spinors and γ matrices in terms of the corresponding Sp(1) quantities, which is used to write the action and supersymmetry variations in a Spin(5) form. Unless otherwise noted, all conventions are similar to those used above. Written in terms of Sp(1) spinors, the Sp(2) ones are

$$\eta_\alpha=\begin{pmatrix}\lambda_\alpha\\ \psi_\alpha\end{pmatrix},\quad \overline{\eta^\alpha}=\begin{pmatrix}\overline{\lambda^\alpha}\\ \psi^\alpha\end{pmatrix}. \tag{A6}$$

Indices are raised lowered and contracted using the metric

$$J_{\alpha\beta}=\begin{pmatrix}0 & \epsilon\\ \epsilon & 0\end{pmatrix}, \tag{A7}$$

with ϵ being the Sp(1) metric. The antisymmetric γ matrices are taken to be

$$\gamma^i_{\alpha\beta}=\begin{pmatrix}0 & \sigma^i\\ -\sigma^i & 0\end{pmatrix},\quad i=1,2,3,$$

$$\gamma^4_{\alpha\beta}=\begin{pmatrix}i\epsilon & 0\\ 0 & -i\epsilon\end{pmatrix},$$

$$\gamma^5_{\alpha\beta}=\begin{pmatrix}-\epsilon & 0\\ 0 & -\epsilon\end{pmatrix}, \tag{A8}$$

with the reality condition being

$$(\gamma^1_{\alpha\beta})^*\equiv\gamma^{i\alpha\beta}=-J^{\alpha\gamma}\gamma^i_{\gamma\delta}J^{\delta\beta}. \tag{A9}$$

[2]The isometry group of a generic $K3$ surface is trivial. However, the moduli spaces of the probe theories are usually noncompact pieces of the entire surface. In this case the hyper-Kähler metric can have a nontrivial isometry group.

APPENDIX B: LAGRANGIAN AND SUSY VARIATIONS

$\mathcal{N}=4$ Lagrangian:

$$
\begin{aligned}
&\frac{1}{4}\,K_{\Sigma^i\Sigma^i}[\dot{x}^j\dot{x}^j+i(\bar{\lambda}\dot{\lambda}+\lambda\dot{\bar{\lambda}}+D^2)]-K_{\Phi\bar{\Phi}}[\dot{\Phi}\dot{\bar{\Phi}}+i(\bar{\psi}\dot{\psi}+\psi\dot{\bar{\psi}})+FF^*]+\frac{1}{2}\,\dot{x}^j(K_{\Sigma^i\Sigma^i\Phi}\psi\sigma^j\lambda+K_{\Sigma^i\Sigma^i\bar{\Phi}}\bar{\psi}\sigma^j\bar{\lambda})+\dot{x}^i\epsilon^{ijk}\\
&\times\left(-\frac{1}{4}\,K_{\Sigma^l\Sigma^l\Sigma^k}\lambda\sigma^j\bar{\lambda}+K_{\Phi\bar{\Phi}\Sigma^k}\psi\sigma^j\bar{\psi}\right)+i\dot{\bar{\Phi}}\left(\frac{1}{4}\,K_{\Sigma^i\Sigma^i\bar{\Phi}}\lambda\bar{\lambda}-K_{\Phi\bar{\Phi}\bar{\Phi}}\psi\bar{\psi}-iK_{\Phi\bar{\Phi}\Sigma^i}\psi\sigma^i\lambda\right)-i\dot{\Phi}\left(\frac{1}{4}\,K_{\Sigma^i\Sigma^i\Phi}\lambda\bar{\lambda}-K_{\Phi\bar{\Phi}\Phi}\psi\bar{\psi}\right.\\
&\left.+iK_{\Phi\bar{\Phi}\Sigma^i}\bar{\psi}\sigma^i\bar{\lambda}\right)+D\left(\frac{1}{4}\,K_{\Sigma^l\Sigma^l\Sigma^i}\lambda\sigma^i\bar{\lambda}+K_{\Phi\bar{\Phi}\Sigma^i}\psi\sigma^i\bar{\psi}+\frac{i}{2}\,(K_{\Sigma^i\Sigma^i\Phi}\psi\lambda+K_{\Sigma^i\Sigma^i\bar{\Phi}}\overline{\psi\lambda})\right)+F\left(-\frac{1}{4}\,K_{\Sigma^i\Sigma^i\Phi}\lambda\lambda-K_{\Phi\bar{\Phi}\bar{\Phi}}\overline{\psi\psi}\right.\\
&\left.+iK_{\Phi\bar{\Phi}\Sigma^i}\lambda\sigma^i\bar{\psi}\right)+F^*\left(\frac{1}{4}\,K_{\Sigma^i\Sigma^i\bar{\Phi}}\overline{\lambda\lambda}+K_{\Phi\bar{\Phi}\Phi}\psi\psi-iK_{\Phi\bar{\Phi}\Sigma^i}\psi\sigma^i\bar{\lambda}\right)+\frac{1}{4}\,(K_{\Sigma^i\Sigma^i\bar{\Phi}\Phi}\overline{\lambda\lambda}\;\overline{\psi\psi}+K_{\Sigma^i\Sigma^i\Phi\Phi}\lambda\lambda\,\psi\psi)+K_{\Phi\bar{\Phi}\Phi\bar{\Phi}}\overline{\psi\psi}\psi\psi\\
&+\frac{1}{16}\,K_{\Sigma^l\Sigma^l\Sigma^i\Sigma^i}\overline{\lambda\lambda}\lambda\lambda-K_{\Phi\bar{\Phi}\Sigma^i\Sigma^j}\psi\sigma^i\bar{\lambda}\lambda\sigma^j\bar{\psi}-\frac{i}{4}\,(K_{\Sigma^i\Sigma^i\Sigma^j\bar{\Phi}}\lambda\sigma^j\bar{\psi}\,\overline{\lambda\lambda}+K_{\Sigma^i\Sigma^i\Sigma_j\Phi}\psi\sigma^j\bar{\lambda}\lambda\lambda)-i(K_{\Phi\bar{\Phi}\Sigma^i\Phi}\lambda\sigma^i\bar{\psi}\psi\psi\\
&+K_{\Phi\bar{\Phi}\Sigma^i\bar{\Phi}}\psi\sigma^i\bar{\lambda}\,\overline{\psi\psi}).
\end{aligned}
\tag{B1}
$$

The $\mathcal{N}=8$ Lagrangian[3] follows from the $\mathcal{N}=4$ one by making use of Eq. (3.7):

$$
\begin{aligned}
\mathcal{L}=&-f[\dot{x}^i\dot{x}^i+\dot{\Phi}\dot{\bar{\Phi}}+i(\bar{\lambda}\dot{\lambda}+\lambda\dot{\bar{\lambda}}+\bar{\psi}\dot{\psi}+\psi\dot{\bar{\psi}})]+\dot{x}^if_{,k}\epsilon^{ijk}(\psi\sigma^j\bar{\psi}+\lambda\sigma^i\bar{\lambda})-2\dot{x}^i(f_{,\Phi}\psi\sigma^i\lambda+f_{,\bar{\Phi}}\bar{\psi}\sigma^i\bar{\lambda})+\dot{\Phi}(f_{,i}\bar{\psi}\sigma^i\bar{\lambda}+if_{,\Phi}\psi\bar{\psi}\\
&+if_{,\bar{\Phi}}\lambda\bar{\lambda})+\dot{\bar{\Phi}}(f_{,i}\psi\sigma^i\lambda-if_{,\Phi}\psi\bar{\psi}-if_{,\bar{\Phi}}\lambda\bar{\lambda})+if_{,i\Phi}(\lambda\sigma^i\bar{\psi}\,\overline{\lambda\lambda}-\psi\sigma^i\bar{\lambda}\,\overline{\psi\psi})+if_{,i\bar{\Phi}}(\psi\sigma^i\bar{\lambda}\lambda\lambda-\lambda\sigma^i\bar{\psi}\psi\psi)-f_{,\Phi\Phi}\lambda\lambda\,\psi\psi\\
&-f_{,\bar{\Phi}\bar{\Phi}}\overline{\lambda\lambda}\;\overline{\psi\psi}-f_{,ij}\lambda\sigma^i\bar{\lambda}+f_{,\Phi\bar{\Phi}}\psi\psi\overline{\psi\psi}-\frac{1}{4}\,f_{,ii}\lambda\lambda\overline{\lambda\lambda}-fD^2+D[f_{,i}(\psi\sigma^i\bar{\psi}-\lambda\sigma^i\bar{\lambda})-2\,f_{,\Phi}\psi\lambda-2\,f_{,\bar{\Phi}}\overline{\psi\lambda}]-fFF^*\\
&+F(f_{,\Phi}\lambda\lambda-f_{,\bar{\Phi}}\overline{\psi\psi}+if_{,i}\lambda\sigma^i\bar{\psi})-F^*(f_{,\bar{\Phi}}\overline{\lambda\lambda}-f_{,\Phi}\psi\psi+if_{,i}\psi\sigma^i\bar{\lambda}).
\end{aligned}
\tag{B2}
$$

Auxiliary fields are given in Spin(5) language using the results of Appendix A:

$$D=\frac{1}{2}\,f^{-1}f_{,i}\eta^\alpha\gamma^i_{\alpha\beta}\bar{\eta}^\beta,$$

$$F=\frac{i}{2}\,f^{-1}f_{,i}\bar{\eta}^\alpha\gamma^i_{\alpha\beta}\bar{\eta}^\beta,$$

$$F^*=\frac{i}{2}\,f^{-1}f_{,i}\eta^\alpha\gamma^i_{\alpha\beta}\eta^\beta. \tag{B3}$$

Manifest SUSY:

$$
\begin{gathered}
\delta x^i=-i(\epsilon\sigma^i\bar{\lambda}+\bar{\epsilon}\sigma^i\lambda),\\
\delta\lambda_\alpha=\dot{x}^i(\epsilon\sigma^i)_\alpha+iD\epsilon_\alpha,\quad \delta\bar{\lambda}_\alpha=\dot{x}^i(\bar{\epsilon}\sigma^i)_\alpha-iD\bar{\epsilon}_\alpha,\\
\delta D=\epsilon\dot{\bar{\lambda}}-\bar{\epsilon}\dot{\lambda},\\
\delta\Phi=2\epsilon\psi,\quad \delta\bar{\Phi}=-2\overline{\epsilon\psi},\\
\delta\psi_\alpha=-i\dot{\Phi}\bar{\epsilon}_\alpha+F\epsilon_\alpha,\quad \delta\bar{\psi}_\alpha=-i\dot{\bar{\Phi}}\epsilon_\alpha+F^*\bar{\epsilon}_\alpha,\\
\delta F=-2i\bar{\epsilon}\dot{\psi},\quad \delta F^*=-2i\epsilon\dot{\bar{\psi}}.
\end{gathered}
\tag{B4}
$$

Nonmanifest SUSY:

$$
\begin{gathered}
\delta x^i=-i(\epsilon\sigma^i\psi-\bar{\epsilon}\sigma^i\bar{\psi}),\\
\delta\lambda_\alpha=i\dot{\Phi}\epsilon_\alpha+F^*\bar{\epsilon}_\alpha,\quad \delta\bar{\lambda}_\alpha=-i\dot{\bar{\Phi}}\bar{\epsilon}_\alpha-F\epsilon_\alpha,\\
\delta D=\epsilon\dot{\psi}-\bar{\epsilon}\dot{\bar{\psi}},\\
\delta\Phi=-2\overline{\epsilon\lambda},\quad \delta\bar{\Phi}=2\epsilon\lambda,\\
\delta\psi_\alpha=\dot{x}^i(\bar{\epsilon}\sigma^i)_\alpha+iD\bar{\epsilon}_\alpha,\quad \delta\bar{\psi}_\alpha=-\dot{x}^i(\epsilon\sigma^i)_\alpha+iD\epsilon_\alpha,\\
\delta F=-2i\bar{\epsilon}\dot{\bar{\lambda}},\quad \delta F^*=-2i\epsilon\dot{\lambda}.
\end{gathered}
\tag{B5}
$$

[3]There is a sign ambiguity in expressions of the form $\psi\sigma^i\lambda$ or $\bar{\psi}\sigma^i\bar{\lambda}$, corresponding to lower or upper indices on the spinors. We universally take spinors with lower indices. No such ambiguity arises if one of the spinors is barred.

Spin(5) SUSY follow from Eqs. (B4) and (B5) using Eqs. (B3) and the results of Appendix A:

$$\delta x^i = i(\epsilon^\alpha \gamma^i_{\alpha\beta}\overline{\eta^\beta} - \overline{\epsilon^\alpha}\gamma^i_{\alpha\beta}\eta^\beta), \quad \delta\eta_\alpha = \dot{x}^i\gamma^i_{\alpha\beta}\epsilon^\beta + \frac{i}{2} f^{-1} f_{,i}(\eta^\gamma\gamma^i_{\gamma\delta}\overline{\eta^\delta}\epsilon_\alpha + \eta^\gamma\gamma^i_{\gamma\delta}\eta^\delta\overline{\epsilon_\alpha}),$$

$$\delta\overline{\eta_\alpha} = -\dot{x}^i\gamma^i_{\alpha\beta}\overline{\epsilon^\beta} - \frac{i}{2} f^{-1} f_{,i}(\eta^\gamma\gamma^i_{\gamma\delta}\overline{\eta^\delta\epsilon_\alpha} + \overline{\eta^\gamma}\gamma^i_{\gamma\delta}\overline{\eta^\delta}\epsilon_\alpha). \tag{B6}$$

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