Time-reversal violation in β decay in the standard model

P. Herczeg
Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545

I. B. Khriplovich
Budker Institute of Nuclear Physics, 630090 Novosibirsk, Russia

(Received 21 January 1997)

We estimate the size of time-reversal violation in β decay due to the Kobayashi-Maskawa phase δ, and the θ term $\mathcal{L}_\theta$. We find that the contribution of δ to the D and R correlations is not likely to be larger than of the order of $10^{-12}|a|(|s_2s_3s_\delta|/10^{-3})$ and $10^{-14}|a|(|s_2s_3s_\delta|/10^{-3})$, respectively, where $|a|\simeq 1$ for neutron and ^{19}Ne decay. For the contribution of $\mathcal{L}_\theta$ to D and R we conclude that it is not likely to be larger than of the order of $10^{-14}|a|(|\theta|/3\times 10^{-10})Q/(m_n-m_p)$ and $10^{-12}|a|(|\theta|/3\times 10^{-10})$, respectively (the present experimental limits on $s_2s_3s_\delta$ and θ are $|s_2s_3s_\delta|\lesssim 10^{-3}$ and $|\theta|\lesssim 3\times 10^{-10}$), where Q is the energy release in the decay. For both δ and $\mathcal{L}_\theta$ the D and R coefficients are dominated by long-distance contributions. The small values of D and R in the standard model compared to the best experimental limit on T-odd correlations (10^{-3} for D in ^{19}Ne decay) give potentially a wide window in which T-violating interactions beyond the standard model can be searched for. However, the range in this window that one will be able to exploit depends on the level at which the contributions of the final-state interactions can be kept under control. [S0556-2821(97)03513-3]

PACS number(s): 11.30.Er, 12.15.Ji, 23.40.−s, 24.80.+y

I. INTRODUCTION

CP violation has been observed so far only in the neutral kaon system. The origin of this effect is still unknown. The most economical explanation is that it is due to the Kobayashi-Maskawa phase δ [1] in the standard model (SM) [2]. Alternatively, it may be a manifestation of a CP-violating interaction in an extension of the SM. The most suitable observables to probe for new types of CP-violating interactions are those for which the contribution from δ is suppressed. One such observable (assuming that CPT invariance holds, as is the case in gauge theories) is the electric dipole moment of the neutron (d_n). A nonzero d_n due to δ appears only in second order in the weak interaction, and only if the strong interaction also participates. The theoretical prediction for this contribution is $d_n \simeq 10^{-31}$–10^{-32} e cm [3], to be compared with the present experimental limit $|(d_n)_{\text{expt}}|<10^{-25}$ e cm [4]. Thus, one has a window of about six orders of magnitude in which CP-violating interactions other than those in the weak interaction can be further searched for. Among the possibilities is the θ term $\mathcal{L}_\theta$ in the QCD Lagrangian [5], which is the second source of CP violation in the SM. The θ term gives rise to a d_n, and the best constraint for θ follows from the present experimental limit on d_n.

In this paper we investigate the size of time-reversal violation (T) in β decay in the SM. As is the case with d_n, T-violating effects in β decay (and in other semileptonic processes) due to δ vanish in first order in the weak interaction [6]. The strength of the δ-induced T-violating $n\to pe^-\overline{\nu}_e$ interaction is expected to be, therefore, many orders of magnitude below the best present experimental limit ($\simeq 10^{-3}G_F$) on such interactions. This is so also for the contributions from $\mathcal{L}_\theta$, which gives rise to T violation in β decay through the interference of the $G_F\theta$ and the weak amplitudes. A problem in β decay, which is not present for d_n, is associated with the final-state interactions. T-violating interactions are searched for in β decay through their contributions to T-odd correlations. Such correlations can receive contributions not only from T-violating interactions, but also from the T-invariant interactions among the particles produced in the decay. This is a contribution of the order of 10^{-5} to the D correlation in neutron decay (see Sec. II), and generally a larger one elsewhere. CP-violating effects are free of such contributions, but their study in β decay would require comparisons of correlations in neutron and antineutron decay (or in nucleus and antinucleus decay). Thus the range in the window between the present experimental limit and the SM contribution that one will be able to exploit to search for CP-violating interactions beyond the SM [7] depends not only on how much it will be possible to improve the sensitivities of the experiments, but also on the level at which it will be still possible to separate the contributions from the final-state interactions.

In the next section we estimate the contribution of δ to the T-odd D and R correlations. In Sec. III we do the same for $\mathcal{L}_\theta$. In Sec. IV we summarize our conclusions.

II. KOBAYASHI-MASKAWA CP VIOLATION

In the SM the matrix element describing the decay $n\to pe^-\overline{\nu}_e$ is given in first order in the weak interaction and neglecting isospin-breaking effects by [8]

$$M_\beta^{(+)}=\frac{g^2}{8m_W^2}\,V_{ud}\overline{u_e}\gamma_\lambda(1-\gamma_5)u_\nu\overline{u_p}\left(g_V\gamma^\lambda+\frac{ig_M}{2m_N}\,\sigma^{\lambda\rho}q_\rho -g_A\gamma^\lambda\gamma_5-\frac{g_P}{2m_N}\,\gamma_5q^\lambda\right), \tag{1}$$

where V_{ud} is the ud element of the Kobayashi-Maskawa (KM) matrix, $m_N=\frac{1}{2}(m_n+m_p)$, $q_\rho=(p_p-p_n)_\rho$, $g_V=1$, $g_A \simeq 1.26$, $g_M=3.7$, and $g_P \simeq 4m_N^2 g_A/m_\pi^2=228$ [9]. The amplitude (1) is CP invariant (the superscript on M_β indicates $CP=+1$). CP-violating contributions to $n \rightarrow pe^- \overline{\nu_e}$ due to the KM phase δ appear first in second order in the weak interaction, through diagrams involving the flavor-nonconserving nonleptonic weak interaction. Since in such diagrams the leptonic current remains intact, the most general form of the CP-violating $n \rightarrow pe^- \overline{\nu_e}$ amplitude is

$$\begin{aligned} M_\beta^{(-)} = \frac{g^2}{8m_W^2} V_{ud}\overline{u_e}\gamma_\lambda(1-\gamma_5)u_\nu \overline{u_p}\bigg(& g_V^{(2)}\gamma^\lambda + \frac{ig_M^{(2)}}{2m_N}\sigma^{\lambda\rho}q_\rho \\ & + \frac{g_S^{(2)}}{2m_N}q^\lambda - g_A^{(2)}\gamma^\lambda\gamma_5 - \frac{g_P^{(2)}}{2m_N}\gamma_5 q^\lambda \\ & - \frac{ig_T^{(2)}}{2m_N}\sigma^{\lambda\rho}\gamma_5 q_\rho\bigg)u_n, \end{aligned} \tag{2}$$

where the constants $g_k^{(2)}$ $(k=V,\ldots,T)$ are pure imaginary. We shall neglect always the CP-invariant components of the second-order weak contributions, since these are tiny relative to the contributions from $M_\beta^{(+)}$.

Experimental information on T violation in β decay is available on the coefficients D and R of the correlations $\langle \vec{J}\rangle\cdot\vec{p}_e\times\vec{p}_\nu/JE_eE_\nu$ and $\vec{\sigma}\cdot\langle\vec{J}\rangle\times\vec{p}_e/JE_e$ ($\vec{\sigma}$ is the electron spin, $\vec{J}$ is the nuclear spin), respectively. We shall write $D=D_t+D_f, R=R_t+R_f$, where D_t, R_t represent the T-violating contributions, and D_f, R_f are the T-invariant contributions due to the final-state interactions. The D coefficient is sensitive to V,A interactions, and the R coefficient to scalar- and (nonderivative) tensor-type interactions.

In $M_\beta^{(-)}$ [Eq. (2)] tensor-type couplings are absent, and the only source of scalar-type amplitudes is the $g_S^{(2)}$ term, which can be written in the form

$$M_{\beta,S}^{(-)}=C_S\overline{u_e}(1-\gamma_5)u_\nu\overline{u_p}u_n, \tag{3}$$

with $C_S=(g_S^{(2)}m_e/2m_N)(g^2V_{ud}/8m_W^2)$. Thus all the contributions from scalar-type couplings are proportional to m_e. We shall neglect the $g_P^{(2)}$ term (which is equivalent to a pseudoscalar-type amplitude), since it is suppressed for nonrelativistic nucleons. The contributions of $g_T^{(2)}$ and $g_M^{(2)}$ to the D coefficient are suppressed by the factors $Q/2m_N \simeq 7\times10^{-4}Q/(m_n-m_p)$ and $(E_e-E_\nu)/2m_N \simeq 5\times10^{-5}$, respectively, where Q is the energy release in the decay [10]. Inspection shows that these form factors do not contribute to the R coefficient. Thus $g_T^{(2)}$ and $g_M^{(2)}$ need to be considered only when larger contributions to D_t are absent.

T-violating effects in $n\rightarrow pe^-\overline{\nu_e}$ arise from the interference of the amplitudes (1) and (2). Neglecting in D_t the contributions from the $g_W^{(2)}$ and $g_T^{(2)}$ terms, D_t and R_t due to the amplitude (2) are given by [11]

$$D_t \simeq a\ \mathrm{Im}\,\frac{1}{2}\left(\frac{g_V^{(2)}}{g_V}-\frac{g_A^{(2)}}{g_A}\right), \tag{4}$$

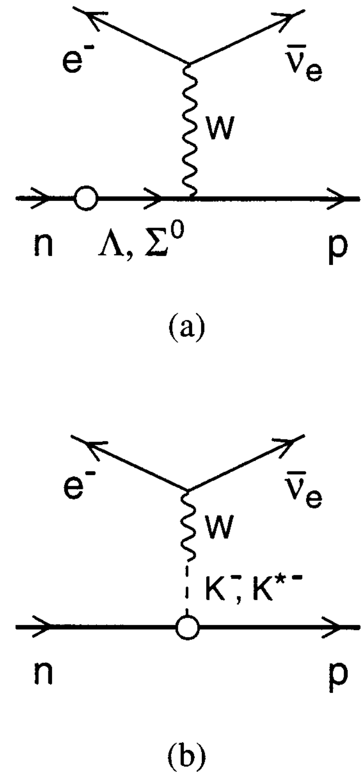


FIG. 1. Hadronic tree diagrams contributing to the T-violating $n\rightarrow pe^-\overline{\nu_e}$ amplitude in the Kobayashi-Maskawa model. The circle represents the nonleptonic weak interaction.

$$R_t \simeq -a\,\frac{m_e}{2m_N}\,\mathrm{Im}\,\frac{g_S^{(2)}}{2g_V}, \tag{5}$$

where the constant a is given by

$$a=4\,\delta_{JJ'}\left(\frac{J}{J+1}\right)^{1/2}\frac{g_Vg_AM_FM_{\mathrm{GT}}}{g_V^2|M_F|^2+g_A^2|M_{\mathrm{GT}}|^2}. \tag{6}$$

In Eqs. (4) and (5) M_F and M_{GT} are the Fermi and Gamow-Teller nuclear matrix elements, J and J' are the angular momenta of the parent and the daughter nucleus. For ^{19}Ne and n decay $a=-1.03$ and $a=0.87$, respectively. The best limit on D_t/a comes from ^{19}Ne decay. The experimental value $D=(0.1\pm0.6)\times10^{-3}$ [12] implies

$$|D_t/a|<1.1\times10^{-3} \quad (90\%\ \ \mathrm{C.L.}). \tag{7}$$

The contribution D_f of the electromagnetic final-state interactions has been estimated for this case to be $\sim 2\times10^{-4}p_e/(p_e)_{\max}$ [13]. The present experimental result for D in n decay is $D=(-0.5\pm1.4)\times10^{-3}$ [14]. D_f for n decay is smaller than that for ^{19}Ne decay by an order of magnitude [13]. New experiments to search for T violation in n decay [15] and ^{19}Ne decay [16] are under way, aiming at improving the limit (7) by about an order of magnitude.

The best limit on $\mathrm{Im}\,C_S$ from β decay is $10^{-1}G_F$ [17], deduced from the experimental value of a T-even observable. A considerably more stringent limit ($\simeq 10^{-4}G_F$) on $\mathrm{Im}\,C_S$ follows from the experimental bound on the P- and T-violating tensor electron-nucleon interaction [18].

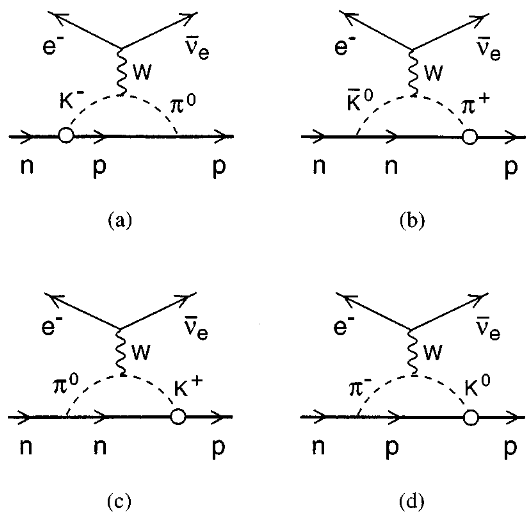


FIG. 2. One-loop hadronic diagrams contributing to the T-violating $n\to pe^-\overline{\nu_e}$ amplitude in the Kobayashi-Maskawa model. The circle represents the nonleptonic weak interaction.

From the diagrams contributing to the amplitude (2) we shall consider the diagrams with the lightest single-baryon and the lightest spin-zero and spin-one single-meson intermediate states (Fig. 1), the one-loop hadronic diagrams in Fig. 2, and the short-distance contributions shown in Fig. 3. It is reasonable to expect that the largest contributions to D_t and R_t will be among the contributions from these diagrams.

1. Contributions of single-hadron intermediate states

We shall consider first diagram (a) in Fig. 1. The lightest contributing baryon intermediate states are the Λ and Σ^0. The source of CP violation is the CP-violating component ($H^{(-)}$) of the $\Delta S=1$ nonleptonic weak Hamiltonian H. For the P-conserving component $H_+^{(-)}$ of $H^{(-)}$ the Λ and Σ^0 contributions have the distinguishing feature that they are enhanced by a denominator containing the SU(3)-breaking mass difference [see Eqs. (10) and (27) below]. The P-violating part $H_-^{(-)}$ of $H^{(-)}$ need not be considered, since the contributions of $H_-^{(-)}$ are suppressed relative to the contributions of $H_+^{(-)}$ by the factor $(m_B-m_n)/(m_B+m_n)$ ($B=\Lambda,\Sigma^0$), which eliminates the above-mentioned enhancement. Also, the form factors in the matrix elements of the octet part of $H_-^{(-)}$ (which dominates) between the states of the baryon octet vanish in the limit of SU(3) symmetry [19].

The $n\to B$ matrix element of $H_+^{(-)}$ is described by the amplitude $a_B^{(-)}$, defined by [20]

$$\langle B|H_+^{(-)}|n\rangle = a_B^{(-)}\overline{u_B}u_n \quad (B=\Lambda,\Sigma^0). \tag{8}$$

To evaluate the diagram we need yet the matrix element

$$M_\beta^B=\frac{g^2}{8m_W^2}V_{us}\overline{u_e}\gamma_\lambda(1-\gamma_5)u_\nu\overline{u_p}\Bigg(g_V^B\gamma^\lambda+\frac{ig_M^B}{m_B+m_p}\sigma^{\lambda\rho}q_\rho + \frac{g_S^B}{m_B+m_p}q^\lambda-g_A^B\gamma^\lambda\gamma_5-\frac{g_P^B}{m_B+m_p}\gamma_5q^\lambda - \frac{ig_T^B}{m_B+m_p}\sigma^{\lambda\rho}\gamma_5q_\rho\Bigg), \tag{9}$$

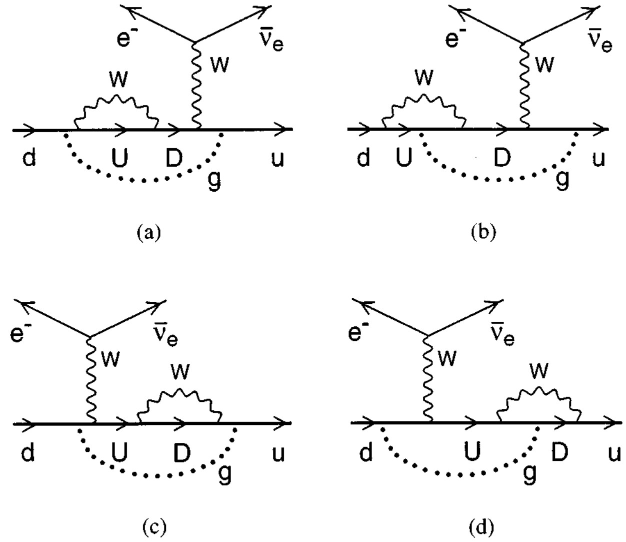


FIG. 3. The lowest-order short-distance contributions to the T-violating $n\to pe^-\overline{\nu_e}$ amplitude in the Kobayashi-Maskawa model.

which describes the $B\to pe^-\overline{\nu_e}$ vertex.

We shall estimate first the contributions of the g_V^B and g_A^B terms in Eq. (9). These induce a $g_V^{(2)}$ and a $g_A^{(2)}$ term in the matrix element (2), given by

$$g_k^{(2)}=(g_k^{(2)})_B\simeq-\frac{V_{us}}{V_{ud}}\frac{ia_B^{(-)}}{m_B-m_n}g_k^B \quad (B=\Lambda,\Sigma^0;k=V,A). \tag{10}$$

The corresponding coefficient D_t [Eq. (4)] is given by

$$(D_t/a)_{\Lambda+\Sigma}\simeq-\frac{V_{us}}{V_{ud}}\frac{1}{2}\left[\frac{a_\Lambda^{(-)}}{m_\Lambda-m_n}\Delta_\Lambda+\frac{a_\Sigma^{(-)}}{m_{\Sigma^0}-m_n}\Delta_\Sigma\right], \tag{11}$$

where a has been defined in Eq. (6), and

$$\Delta_\Lambda\equiv(g_V^\Lambda/g_V)-(g_A^\Lambda/g_A)\simeq-\sqrt{2/3}D/(F+D)\simeq-0.52, \tag{12}$$

$$\Delta_\Sigma\equiv(g_V^\Sigma/g_V)-(g_A^\Sigma/g_A)\simeq-\sqrt{2}D/(F+D)\simeq-0.90. \tag{13}$$

In Eqs. (12) and (13) we used for g_k^Λ and g_k^Σ ($k=V,A$) the values $g_V^\Lambda=\sqrt{\frac{3}{2}}$, $g_A^\Lambda=(1/\sqrt{6})(3F+D)$, $g_V^\Sigma=-(1/\sqrt{2})$, and $g_A^\Sigma=(1/\sqrt{2})(F-D)$, which follow from SU(3) symmetry, and $F=-0.460$, $D=-0.800$ [21].

Since CP violation requires the participation of quarks heavier than the c quark, the main contribution to the amplitudes $a_B^{(-)}$ comes from the penguin diagrams [22,23]. An estimate of $a_B^{(-)}$ can be made using the effective $\Delta S=1$ nonleptonic Hamiltonian [22–24]

$$H=\frac{G_F}{2\sqrt{2}}\,V_{ud}^{*}V_{us}\sum_{k=1}^{6}\,c_kO_k\,, \tag{14}$$

where the O_k's are local operators of dimension six, and the c_k's are the Wilson coefficients. The operator basis we employ is the one introduced in Ref. [22]. In the matrix elements $\langle B|H_+^{(-)}|n\rangle$ it is sufficient to keep the contributions from O_1, O_5, and O_6: the matrix elements $\langle B|O_k|n\rangle$ ($i=2,3,4$) vanish in models where the baryons are composed of three quarks [22,25] and, therefore, they are expected to be small. For the matrix element of O_6 we shall use the relation $\langle B|O_6|n\rangle=-\frac{3}{8}\langle B|O_5|n\rangle$ [25], which holds in the same models. It follows that the amplitudes $a_B^{(-)}$ are given by

$$a_B^{(-)}\simeq\omega_{B,1}\ \mathrm{Im}\ c_1+\omega_{B,5}\ \mathrm{Im}\ c_5'\,. \tag{15}$$

In Eq. (15) $c_5'=c_5-\frac{3}{8}c_6$, and $\omega_{B,k}$ is defined by

$$\overline{u_B}\omega_{B,k}u_n=\left\langle B\left|\frac{G_F}{2\sqrt{2}}\ V_{ud}^{*}V_{us}i(O_k)+\right|n\right\rangle, \tag{16}$$

where $(O_k)_+$ is the parity-conserving part of the operator O_k.

For the Wilson coefficients we shall use the values obtained in a recent calculation in Ref. [26]. For $\Lambda_{\mathrm{QCD}}=\Lambda_{\overline{\mathrm{MS}}}^{(4)}=0.3$ GeV (where $\overline{\mathrm{MS}}$ denotes the modified minimal subtraction scheme), and μ(the renormalization scale)$=1$ GeV, they are [27]

$$\mathrm{Im}\ c_1=0.090\ \mathrm{Im}\ \tau, \tag{17}$$

$$\mathrm{Im}\ c_5=-0.102\ \mathrm{Im}\ \tau, \tag{18}$$

$$\mathrm{Im}\ c_6=-0.044\ \mathrm{Im}\ \tau, \tag{19}$$

where

$$\mathrm{Im}\ \tau=-\mathrm{Im}(V_{td}^{*}V_{ts}/V_{ud}^{*}V_{us})=(c_2/c_1c_3)s_2s_3s_\delta\simeq s_2s_3s_\delta\,. \tag{20}$$

In Eq. (15) the O_5 contribution is expected to dominate, since while both Im c_1 and Im c_5' are of the order of α_s, the vacuum saturation part of $\langle B|O_5|n\rangle$ is enhanced by the factor $m_\pi^2/(m_u+m_d)(m_s-m_d)$ [28]. It turns out, however, that this enhancement is not effective in the D_t coefficient: inspection shows that the sum of the two terms in the square brackets in Eq. (11) vanishes for the vacuum saturation parts of $\langle\Lambda|(O_5)_+|n\rangle$ and $\langle\Sigma^0|(O_5)_+|n\rangle$ [29]. D_t is, therefore, determined by $\langle B|(O_1)|n\rangle$ ($B=\Lambda,\Sigma^0$) and the remaining (nonfactorization) part $\langle B|(O_5)_+|n\rangle_r$ of $\langle B|(O_5)+|n\rangle$.

An order of magnitude estimate of $(a_B^{(-)})_r$ [$=a_B^{(-)}$ with $\omega_{B,5}$ replaced by $(\omega_{B,5})_r$] can be obtained from the experimental values $(A_B)_{\mathrm{expt}}$ of the corresponding s-wave hyperon-decay amplitudes A_B. This is derived as follows. The single-baryon matrix elements a_B of H_+ are related to A_B by the soft-pion relation $a_B=-i\sqrt{2}f\pi A_B$. Assuming that the contribution of O_1 to the CP-invariant matrix element $a_B^{(+)}$ is of the order of $(a_B^{(+)})_{\mathrm{expt}}$ [30], the matrix element $\omega_{B,1}$ is of the order of $\sqrt{2}f\pi(A_B)_{\mathrm{expt}}$ (since Re c_1 is of the order of unity). Now we observe that as $(\omega_{B,5})_r$ is not enhanced, the matrix elements $|\omega_{B,1}|$ and $|(\omega_{B,5})_r|$ should be of comparable size. Since one has also $|\mathrm{Im}\ c_1|\simeq|\mathrm{Im}\ c_5'|$ [see Eqs. (17)–(19)], we obtain $|(a_B^{(-)})_r|\sim|\omega_{B,1}\ \mathrm{Im}\ c_1|\sim|\sqrt{2}f\pi(A_B)_{\mathrm{expt}}\ \mathrm{Im}\ c_1|$. This estimate yields $|(a_\Lambda^{(-)})_r|$, $|(a_\Sigma^{(-)})_r|\simeq4\times10^{-6}$ Im τ MeV.

All the matrix elements needed for D_t have been calculated in the MIT bag model [25], obtaining $\omega_{\Lambda,1}=(-4.9\times10^{-8})\sqrt{2}f_\pi$, $(\omega_{\Lambda,5})_r=(6.5\times10^{-8})\sqrt{2}f_\pi$, $\omega_{\Sigma,1}=(1/\sqrt{2})\times(11.9\times10^{-8})\sqrt{2}f_\pi$, and $(\omega_{\Sigma,5})_r=(1/\sqrt{2})(-3.0\times10^{-8})\sqrt{2}f_\pi$, where $f_\pi(\simeq131$ MeV) is the pion-decay constant. Using these matrix elements and the Wilson coefficients in Eqs. (17)–(19), we find from Eq. (15) for the nonfactorization part $(a_B^{(-)})_r$ of $a_B^{(-)}$ ($B=\Lambda,\Sigma^0$) [31]

$$(a_\Lambda^{(-)})_r\simeq-1.8\times10^{-6}\ \mathrm{Im}\ \tau\ \ \mathrm{MeV}, \tag{21}$$

$$(a_\Sigma^{(-)})_r\simeq1.7\times10^{-6}\ \mathrm{Im}\ \tau\ \ \mathrm{MeV}. \tag{22}$$

The values (21) and (22) are not far from the values obtained above from the estimate using the experimental s-wave hyperon-decay amplitudes.

From Eqs. (13)–(15), (21), and (22) we obtain $(D_t/a)_\Lambda\simeq-6\times10^{-10}$ Im τ, $(D_t/a)_\Sigma\simeq7\times10^{-10}$ Im τ, so that

$$(D_t/a)_{\Lambda+\Sigma}\simeq10^{-10}\ \mathrm{Im}\ \tau. \tag{23}$$

With the upper limit [26]

$$|\mathrm{Im}\ \tau|\lesssim10^{-3} \tag{24}$$

deduced from the experimental value of ϵ (Im τ is in the range $3\times10^{-4}\lesssim\mathrm{Im}\ \tau\lesssim10^{-3}$ if δ accounts for ϵ), Eq. (23) yields

$$|(D_t/a)_{\Lambda+\Sigma}|\lesssim10^{-13}. \tag{25}$$

The theoretical uncertainties in the result (25) come mainly from the theoretical uncertainties in the amplitudes $a_B^{(-)}$. The mismatch between the scale ($\mu=1$ GeV) at which the Wilson coefficients are given and the scale (0.2–0.4 GeV) at which the matrix elements of the operators were evaluated is not expected to be a serious problem for the imaginary parts of the Wilson coefficients, since these do not depend strongly on μ (they originate predominantly from the region in momentum space between m_t and m_c). The theoretical uncertainties in the matrix elements of the operators are difficult to assess. These are likely to be large enough to allow values for which the cancellation between $(D_t/a)_\Lambda$ and $(D_t/a)_\Sigma$ in Eq. (11) is weaker, or absent. The size of $(D_t/a)_{\Lambda+\Sigma}$ may be, therefore, comparable to the size of $(D_t/a)_\Lambda$ or $(D_t/a)_\Sigma$. Consequently, a safer conclusion for $(D_t/a)_{\Lambda+\Sigma}$ than Eq. (25) is the limit

$$|(D_t/a)_{\Lambda+\Sigma}|\lesssim10^{-12}. \tag{26}$$

The contributions to D_t from g_M^B and g_T^B, which induce a $g_M^{(2)}$ and $g_T^{(2)}$ term in the matrix element (2), are negligible relative to the contribution (23) [32].

The constants g_P^B can be ignored, since they give rise to a $g_P^{(2)}$ term. The constants g_S^B induce a contribution to $g_S^{(2)}$, given by

$$(g_S^{(2)})_{\Lambda+\Sigma}=\sum_{B=\Lambda,\Sigma^0}\frac{V_{us}}{V_{ud}}\frac{ia_B^{(-)}}{m_B-m_n}g_S^B\frac{m_p+m_n}{m_B+m_p}. \quad (27)$$

For g_S^B [which is induced in Eq. (9) by SU(3) breaking] a bag model estimate gives $g_S^B \simeq 0.3 g_V^B$ [9]. With these values, and taking for $a_\Lambda^{(-)}$ and $a_\Sigma^{(-)}$ the vacuum saturation values $(a_\Lambda^{(-)})_{\rm vs}\simeq -7\times 10^{-6}$ Im τ MeV, $(a_\Sigma^{(-)})_{\rm vs}\simeq 6\times 10^{-6}$ Im τ MeV (see Ref. [31]), we obtain $(g_S^{(2)})_\Lambda \simeq 3\times 10^{-9}$ Im τ, $(g_S^{(2)})_\Sigma \simeq -10^{-9}$ Im τ. The corresponding R_t coefficient is

$$(R_t/a)_{\Lambda+\Sigma}\simeq -3\times 10^{-13}\ \mathrm{Im}\ \tau, \quad (28)$$

so that

$$|(R_t/a)_{\Lambda+\Sigma}|\lesssim 3\times 10^{-16}. \quad (29)$$

Diagram (b) in Fig. 1 with the K^- intermediate state is a further source of a scalar type $n\to pe^-\overline{\nu_e}$ amplitude. Neglecting the parity-conserving part of $H^{(-)}$ (since it gives rise to a pseudoscalar-type β-decay coupling), the CP-violating $n\to pe^-\overline{\nu_e}$ interaction from the K^- diagram is of the form (3) with

$$C_S=(C_S)_K=i\,\frac{V_{us}}{V_{ud}}A_{K^-}^{(-)}\frac{f_K m_e}{m_K^2}\left(\frac{g^2}{8m_W^2}V_{ud}\right), \quad (30)$$

where $f_K\simeq 1.23 f_\pi$ is the K^--decay constant, and the amplitude $A_{K^-}^{(-)}$ is defined by

$$\langle K^- p|H_-^{(-)}|n\rangle=\overline{u_n}iA_{K^-}^{(-)}u_p. \quad (31)$$

We shall estimate $A_{K^-}^{(-)}$ using vacuum saturation. In this approximation only the operators O_5 and O_6 contribute. We find

$$A_{K^-}^{(-)}=-\frac{G_F}{2\sqrt{2}}V_{ud}V_{us}^*\frac{32}{9}f_S\frac{f_K m_K^2}{m_u+m_s}\left(\mathrm{Im}\ c_5+\tfrac{3}{16}\ \mathrm{Im}\ c_6\right), \quad (32)$$

where $f_S\equiv f_S(0)$ is defined by

$$\langle p|\overline{u}d|n\rangle=f_S(q^2)\overline{u_p}u_n. \quad (33)$$

Using the values of Im c_5 and Im c_6 given in Eqs. (18) and (19), $f_S=0.6$ [33], and $m_s=175$ MeV, $m_u=5.1$ MeV [34], we obtain

$$A_{K^-}^{(-)}\simeq 5\times 10^{-8}\ \mathrm{Im}\ \tau, \quad (34)$$

and from Eqs. (30), (5), and (24)

$$|(R_t/a)_K|\lesssim 2\times 10^{-15}. \quad (35)$$

The lightest vector meson that can contribute is the $K^{*-}(892)$ [see diagram (b) in Fig. 2]. The form factors h_V and h_A at the γ_λ and the $\gamma_\lambda\gamma_5$ terms in the $\langle pK^*|H^{(-)}|n\rangle$ matrix element give a contribution

$$g_j^{(2)}\simeq i\,\frac{V_{us}}{V_{ud}}h_j\frac{\sqrt{2}}{f_{K^*}}\quad (j=V,A) \quad (36)$$

to $g_V^{(2)}$ and $g_A^{(2)}$. In Eq. (36) f_{K^*} is the K^*-W coupling constant. Using $f_{K^*}\simeq f_\rho \simeq 5.5$, and assuming that h_V and h_A are of the same order of magnitude as $A_{K^-}^{(-)}$ [Eq. (34)], we obtain $|g_V^{(2)}|$, $|g_A^{(2)}|\sim 3\times 10^{-9}|\mathrm{Im}\ \tau|$ and, therefore, possibly

$$(D_t/a)_{K^*}\simeq 2\times 10^{-9}\ \mathrm{Im}\ \tau. \quad (37)$$

A scalar coupling with

$$g_S^{(2)}\simeq i\,\frac{V_{us}}{V_{ud}}h_S\frac{\sqrt{2}}{f_{K^*}} \quad (38)$$

is induced by the form factor h_S at the $q^\lambda/2m_N$ term in $\langle pK^*|H^{(-)}|n\rangle$. For $h_S\simeq A_{K^-}^{(-)}$ one would have

$$(R_t/a)_{K^*}\simeq -4\times 10^{-13}\ \mathrm{Im}\ \tau. \quad (39)$$

We note that in the vacuum saturation approximation D_t vanishes, since $\langle pK^*|H^{(-)}|n\rangle$ is proportional to $\langle p|\overline{u}\gamma_\lambda(1-\gamma_5)d|n\rangle$. A scalar-type $n\to pe^-\overline{\nu_e}$ amplitude is not induced in this approximation either, since in $\langle p|\overline{u}\gamma_\lambda(1-\gamma_5)d|n\rangle$ the induced scalar form factor is absent.

2. One-loop hadronic diagrams

From this class of diagrams the ones with the lightest hadrons are the diagrams containing the K-π-N loop (see Fig. 2). Inspection of the G-parity behavior of the effective weak currents shows that these diagrams, and also similar loop diagrams involving other intermediate states, can give a CP-violating contribution only to the form factors $g_T^{(2)}$ and $g_S^{(2)}$ in Eq. (2). The contributions to $g_T^{(2)}$ (which come from $H_-^{(-)}$) and $g_S^{(2)}$ (which are generated by $H_+^{(-)}$) are, respectively, of the order of $(iV_{us}/V_{ud})(g_{\pi NN}/\pi^2)A_j^{(-)}$, and $(iV_{us}/V_{ud})(g_{\pi NN}/\pi^2)B_j^{(-)}$, where $A_j^{(-)}$ and $B_j^{(-)}$ ($j=K^\pm,K^0,\overline{K^0}$) are the parity-violating and the parity-conserving $n\to NK$ amplitudes.

Using the vacuum saturation value of the $A_{K^-}^{(-)}$ amplitude [Eq. (34)] to represent the size of the parity-violating $n\to NK$ amplitudes, we find that D_t/a in neutron decay is of the order of 10^{-15}–10^{-14}, which is negligible relative to the contribution of diagram (a) in Fig. 1. It is interesting to note that in the chiral SU(3) limit D_t receives contributions only from the diagrams in Fig. 2, and from analogous diagrams where the nucleon in the loop is replaced by its strange SU(3) partners. The reason is that these are the only diagrams containing terms proportional to the logarithm of the mass of the pseudoscalar meson [35].

The parity-conserving $n\to pK^-$ amplitude $B_{K^-}^{(-)}$ in the vacuum saturation approximation and at low q^2 is given by

$$B_{K^-}^{(-)}=\frac{2m_N g_A(q^2)m_\pi^2}{(m_u+m_d)(m_\pi^2-q^2)f_S}A_{K^-}^{(-)}, \quad (40)$$

where f_S is defined in Eq. (33), and where we used the partial conservation of axial-vector current (PCAC) prediction for the nucleon matrix element of the divergence of the axial-vector current $\langle p|\partial^\lambda A_\lambda|n\rangle$. For $q^2\simeq m_p^2$, which corresponds to the typical momenta in the loop, the matrix element $\langle p|\partial^\lambda A_\lambda|n\rangle$ is not known. If we assume that the form of this matrix element at low q^2 is a reasonable approximation for all q^2 [36], we obtain $B_{K^-}^{(-)} \simeq 3A_{K^-}^{(-)}$ for $q^2 \simeq m_p^2$, implying that the upper limit on this contribution to R_t/a is of the order of 10^{-14}. The amplitudes $B_{K^0}^{(-)}$ and $B_{\bar{K}^0}^{(-)}$ vanish in the chiral limit [37], but at $q^2 \simeq m_p^2$ could be comparable to $B_{K^-}^{(-)}$.

3. Short-distance contributions

We have to consider the transition $d \to u e^- \overline{\nu_e}$ in second order in the weak interaction. To have a CP-violating contribution three of the four W vertices must lie on the quark line. Note that on the quark line one can organize only a self-energy insertion. A proper W vertex is precluded by charge conservation. A self-energy insertion alone reduces ''on the mass shell'' (in the short-distance contributions we consider the quarks as free ones) to an uninteresting mass- and wave-function renormalization. To circumvent this a gluon propagator has to be included, and it must be attached in such a way that the diagram would become an irreducible one. The resulting diagrams are shown in Fig. 3.

Inspection of diagrams (a)–(d) in Fig. 3 shows that as long as we neglect the u- and d-quark masses the effective quark vertex has a $V-A$ structure, just as the first-order W-quark vertex. A CP-violating phase becomes, therefore, part of an unobservable overall phase.

An effective right-handed $(V+A)$ current with a CP-violating phase is generated by diagrams (a) and (c) only, and is proportional to the product $m_u m_d$. Chirality-changing terms in the effective current are generated by all four diagrams, and are linear in a light-quark mass.

Thus all short-distance contributions are strongly suppressed, being proportional to $m_u m_d$ or $m_{u,d}$ times momentum transfer. Moreover, these two-loop diagrams are suppressed further by geometrical factors.

The above general arguments can be confirmed by explicit calculation in the (purely theoretical) limit when all the quarks are light compared to the W. In this approximation only diagrams (b) and (d) with the well-known penguin block survive. The corresponding effective CP-odd quark-lepton β-decay interaction is given by

$$H_\beta^d = \frac{G_F}{\sqrt{2}} \overline{e} \gamma_\lambda (1-\gamma_5) \nu_e \overline{u} \gamma_\mu (1-\gamma_5) d \times \left\{ -i \frac{G_F}{\sqrt{2}} c_2 c_3 s_1^2 s_2 s_3 s_\delta \frac{\alpha_s}{54\pi^3} q^\lambda q^\mu \left[\ln^2 \frac{m_t^2}{m_c^2} + \ln^2 \frac{m_b^2}{m_c^2} - \ln^2 \frac{m_t^2}{m_b^2} + 2 \ln \frac{m_t^2}{m_c^2} \ln \frac{m_c^2}{m_s^2} \right] \right\}. \tag{41}$$

The interaction (41) can be written in the form of an effective scalar-pseudoscalar interaction, where the scalar-type and the pseudoscalar-type components are proportional to $m_e(m_d - m_u)$ and $m_e(m_d + m_u)$, respectively. For $m_t > m_W$ the scalar-pseudoscalar interaction is not expected to be significantly different from Eq. (41), except for some suppression due to the heavier t-quark mass. Thus we expect [taking $\alpha_s = 0.4$, and assuming that the factor with the logarithms is of the order of 100, as it is in Eq. (41)] the short-distance contribution to R_t to be

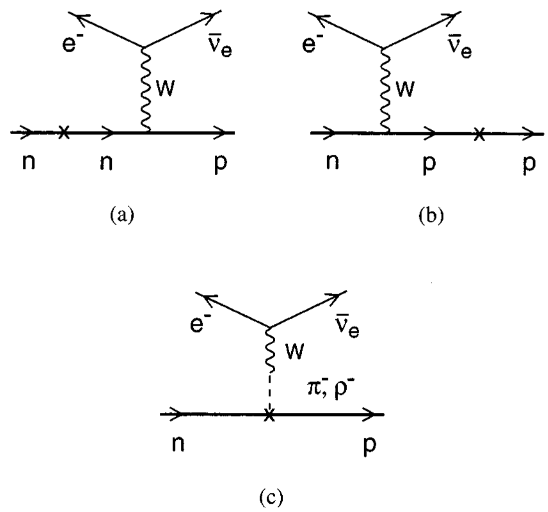


FIG. 4. Hadronic tree diagrams contributing to the T-violating $n \to p e^- \overline{\nu_e}$ amplitude induced by the θ term. The cross represents $\mathcal{L}_\theta$.

$$|(R_t/a)_{\mathrm{sd}}| \lesssim 10^{-17}. \tag{42}$$

For $m_t > m_W$ a $V+A$ interaction may also be induced. But even if this interaction would turn out to be stronger than the interaction (41) by an order of magnitude [what is conceivable, since it would be proportional to $m_u m_d$ rather than to $m_e(m_d - m_u)$], its contribution to D_t/a would still be only of the order of 10^{-16} and, therefore, negligible relative to the contribution of diagram (a) in Fig. 1.

III. *CP* VIOLATION FROM THE θ TERM

The second source of CP violation in the SM is the θ term [5]

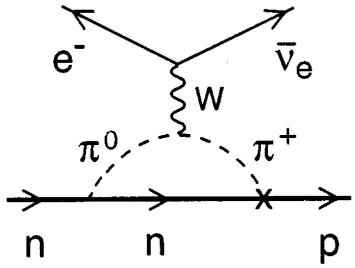
$$\mathcal{L}_\theta = -\theta \frac{g_s^2}{64\pi^2} \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu}^a F_{\alpha\beta}^a \tag{43}$$

in the QCD Lagrangian. The best limit on the parameter θ comes from the electric dipole moment of the neutron. The experimental limit on d_n [4] and the calculation of d_n in Ref. [38] yield

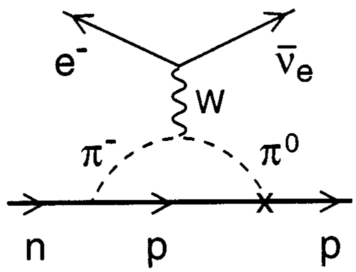
$$|\theta| \lesssim 3 \times 10^{-10}. \tag{44}$$

The most general CP-violating $n \to p e^- \overline{\nu_e}$ amplitude $M_\beta^{(-)}$ due to $\mathcal{L}_\theta$ is of the form (2), where we shall denote now the form factors by $g_k^\theta (k = V, \ldots, T)$. In the limit of isospin invariance the only form factors that can be induced by $\mathcal{L}_\theta$ for the $\overline{u}\gamma_\mu d$ and the $\overline{u}\gamma_\mu\gamma_5 d$ current are the second-class form factors [39] g_T^θ and g_S^θ, respectively. The reason is that $\mathcal{L}_\theta$ conserves G, but not P. In the following we shall neglect the contributions from g_M^θ and g_P^θ, since these are suppressed not only by isospin-breaking factors, but also kinematically.

To estimate the size of the coefficients D_t and R_t we shall consider the hadronic diagrams involving the lightest single-baryon, spin-zero meson, and spin-one meson intermediate states (Fig. 4), and the one-loop hadronic diagrams in Fig. 5.



(a)



(b)

FIG. 5. One-loop hadronic diagrams contributing to the T-violating $n\to pe^-\overline{\nu_e}$ amplitude induced by the θ term. The cross represents $\mathcal{L}_\theta$. There are two additional diagrams involving the same particles, which can be obtained from diagrams (a) and (b) by exchanging the strong and the $\mathcal{L}_\theta$ vertices.

The lightest single-baryon intermediate state that can contribute is the nucleon. The corresponding diagrams [(a) and (b) in Fig. 4] involve the $n\to pW^-$ vertex and the amplitude B_θ, defined by

$$\langle N(p')|\mathcal{L}_\theta|N(p)\rangle = B_\theta \overline{u_N}(p')\gamma_5 u_N(p). \tag{45}$$

For g_V^θ, g_A^θ, g_T^θ, and g_S^θ we obtain

$$g_V^\theta = i\,\frac{B_\theta}{2m_n}\,g_A\left(\frac{m_n-m_p}{m_p}\right), \tag{46}$$

$$g_A^\theta = i\,\frac{B_\theta}{2m_n}\,g_V\left(\frac{m_n-m_p}{m_p}\right), \tag{47}$$

$$g_T^\theta \simeq i\,\frac{B_\theta}{m_n}\,g_M, \tag{48}$$

$$g_S^\theta \simeq i\,\frac{B_\theta}{m_n}\,g_P. \tag{49}$$

The amplitude B_θ has been estimated in Ref. [40] assuming η' dominance for the nucleon matrix element of $\mathcal{L}_\theta$. This yielded

$$B_\theta \simeq 20\theta\,\frac{m_u m_d}{m_u+m_d} \simeq 70\theta \ \ \text{MeV}, \tag{50}$$

where we have used $m_u=5.1$ MeV, $m_d=9.3$ MeV (34). The value (50) corresponds to the value $g_{\eta' NN}=10$, used in Ref. [40] for the strong $\eta' NN$ coupling constant (the value quoted in Ref. [41]). An analysis in Ref. [42] indicates that $g_{\eta' NN}$ may not be this large. Using Eq. (50) the limit (44) implies

$$|B_\theta| \lesssim 2\times 10^{-8} \ \ \text{MeV}. \tag{51}$$

For the D_t coefficient in neutron decay we obtain [see Eq. (4) and Ref. [10]]

$$(D_t/a)_n \simeq \frac{B_\theta}{2m_n}\left[\left(\frac{g_A}{g_V}-\frac{g_V}{g_A}\right)\frac{m_n-m_p}{2m_p}+\left(\frac{g_M}{g_A}\right)\frac{m_n-m_p}{m_n+m_p}\right]. \tag{52}$$

In other β decays the second term in the square bracket in Eq. (52) is of the order of (g_M/g_A) $[|Q|/(m_n+m_p)]$, where Q is the energy release in the decay. The dominant contribution in Eq. (52) is the weak magnetism term. The limit (44) implies

$$|(D_t/a)_n| \lesssim 2\times 10^{-14}. \tag{53}$$

The R_t coefficient is relatively large, due to the large value of g_P. We obtain

$$(R_t/a)_n \simeq -\frac{B_\theta}{2m_n}\,\frac{m_e}{m_n+m_p}\,\frac{g_P}{g_V}, \tag{54}$$

and thus

$$|(R_t/a)_n| \lesssim 7\times 10^{-13}. \tag{55}$$

The pion intermediate state in diagram (c) of Fig. 4 generates a scalar-type CP-violating $n\to pe^-\overline{\nu_e}$ amplitude with

$$g_S^\theta = iA_\pi^{(-)}\,\frac{2m_N f_\pi}{m_\pi^2}, \tag{56}$$

where $A_\pi^{(-)}$ is defined by

$$\langle p\pi|\mathcal{L}_\theta|n\rangle = iA_\pi^{(-)}\overline{u_p}u_n. \tag{57}$$

Using the estimate $A_\pi^{(-)}\simeq\sqrt{2}(0.027)\theta$ [38] we obtain

$$|(R_t/a)_\pi| \lesssim 2\times 10^{-14}. \tag{58}$$

The lightest contributing vector meson is the ρ^- [see diagram (c) in Fig. 4]. Since for the ρ^- only $\overline{u}\gamma_\mu d$ is involved in the diagram, we have $g_V^\theta = g_M^\theta = g_S^\theta = 0$. For g_T^θ we obtain

$$g_T^\theta = h_T\,\frac{\sqrt{2}}{f_\rho}, \tag{59}$$

where h_T is defined by

$$\langle p\rho^-|\mathcal{L}_\theta|n\rangle = \overline{u_p}\,\frac{ih_T}{2m_N}\,\sigma^{\lambda\mu}\gamma_5 q_\mu u_n \epsilon_\lambda^{(\rho)}. \tag{60}$$

In Eq. (60) $\epsilon_\lambda^{(\rho)}$ is the polarization vector of the ρ. If h_T and $A_\pi^{(-)}$ are of comparable size, we would have

$$|(D_t/a)_\rho|\simeq \frac{1}{2}\,\frac{m_n-m_p}{2m_N}\,\frac{\sqrt{2}}{f_\rho}\,\frac{|h_T|}{g_A}\lesssim 8\times 10^{-16}. \qquad (61)$$

With isospin breaking included the matrix element $\langle p\rho^-|\mathcal{L}_\theta|n\rangle$ contains also a $\gamma_\lambda\gamma_5$ term. This may give a contribution to D_t which is comparable to the contribution in Eq. (61).

1. One-loop hadronic diagrams

From such diagrams the dominant contribution is expected to come from the πN intermediate state (see the diagrams in Fig. 5).

As the neutron electric dipole moment [38], $g_T^{(\theta)}$ induced by the πN intermediate state can be calculated rigorously in the chiral $m_\pi\to 0$ limit. Using the result for d_n in Ref. [38] and isospin symmetry, we obtain

$$g_T^\theta=\frac{A_\pi^{(-)}g_{\pi NN}}{\pi^2\sqrt{2}}\ln\frac{m_N}{m_\pi}, \qquad (62)$$

where $A_\pi^{(-)}$ has been defined in Eq. (57), and $g_{\pi NN}$ ($\simeq 13.5$) is the strong πNN coupling constant. Equation (62) and the limit (44) imply

$$|(D_t/a)_{\pi N}|\simeq \frac{1}{2}\,\frac{m_n-m_p}{m_n+m_p}\,\frac{|g_T^\theta|}{g_A}\lesssim 6\times 10^{-15}. \qquad (63)$$

There is no contribution to R_t, since only the vector current is involved in the $\pi\to\pi$ matrix element.

IV. CONCLUSIONS

In this paper we investigated the size of T violation in β decay due to the Kobayashi-Maskawa phase δ, and the θ term $\mathcal{L}_\theta$.

The Kobayashi-Maskawa phase gives rise to a T-violating β-decay amplitude only in second order in the weak interaction. We find that the contributions $(D_t)_\delta$ and $(R_t)_\delta$ of δ to the D and R correlation are not likely to be larger than of the order of $10^{-12}(\mathrm{Im}\,\tau/10^{-3})|a|$ and $10^{-14}(\mathrm{Im}\,\tau/10^{-3})|a|$, respectively, where the constant a has been defined in Eq. (6) ($|a|\simeq 1$ for neutron and ^{19}Ne decay). The parameter $\mathrm{Im}\,\tau$ ($\simeq s_2s_3s_\delta$) is in the range $3\times 10^{-4}\lesssim \mathrm{Im}\,\tau\lesssim 10^{-3}$ if δ accounts for ϵ. The largest contribution to D_t comes from the diagrams involving the Λ and Σ^0 intermediate states, and possibly from the diagram with the K^{*-} intermediate state. For $(R_t)_\delta$ the dominant contribution is most likely from the diagrams involving the $K-\pi-N$ loop.

The θ term gives rise to T violation in β decay through the interference of the amplitude of order $G_F\theta$ with the first-order weak amplitude. Since $\mathcal{L}_\theta$ can induce first-class form factors in the effective $n\to pW^-$ vertex only through isospin breaking, the contributions $(D_t)_\theta$ and $(R_t)_\theta$ of $\mathcal{L}_\theta$ to the T-odd D and R correlations are suppressed either kinematically, or by isospin-breaking factors (or both). We find that $(D_t)_\theta$ and $(R_t)_\theta$ are not likely to be larger than of the order of $10^{-14}(|\theta|/3\times 10^{-10})|a||Q|/(m_n-m_p)$ and $10^{-12}(|\theta|/3\times 10^{-10})|a|$, respectively (the present experimental upper limit on $|\theta|$ is 3×10^{-10}), where Q is the energy release in the decay. The largest contribution to both D_t and R_t comes from the single-nucleon intermediate state. For D_t the dominant part is the one given rise by the weak magnetism form factor, and for R_t the part generated by the induced pseudoscalar form factor. The contribution from the single-nucleon intermediate state involves the amplitude B_θ, which determines the nucleon matrix element of $\mathcal{L}_\theta$. The value of B_θ we used may involve large uncertainties, partly from the uncertainty in the value of the strong $\eta' NN$ coupling constant $g_{\eta' NN}$, and possibly also from other sources. For $g_{\eta' NN}$ we used the value $g_{\eta' NN}=10$. If $|g_{\eta' NN}|<3$, D_t would be dominated by the one-loop diagram with the πN intermediate state. The largest contribution to $(R_t)_\theta$ would still be the one from the single-nucleon intermediate state.

Our results for β decay can also be used for an evaluation of the contributions of δ and $\mathcal{L}_\theta$ to T-odd observables in muon capture [43]. A difference with respect to β decay is that the strength of the T-violating effective scalar- and pseudoscalar-type interactions is proportional to m_μ, rather than m_e. There is a similar enhancement for the contributions of the T-violating weak magnetism and pseudotensor couplings.

ACKNOWLEDGMENTS

We would like to thank Albert R. Young for his stimulating interest in the subject of this paper. The work of P.H. was supported by the United States Department of Energy. I.B.K. acknowledges support by the Russian Foundation for Basic Research through Grant No. 95-02-04436-a.

[1] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[2] The electroweak component of the standard model will be understood here to be the minimal version of the $SU(2)_L\times U(1)$ gauge theory, containing three families of leptons and quarks, one Higgs doublet, and only left-handed neutrinos.

[3] I. B. Khriplovich and A. R. Zhitnitsky, Phys. Lett. **109B**, 490 (1982). An estimate given in M. B. Gavela, A. Le Yaouanc, L. Oliver, O. Pène, J.-C. Raynal, and T. N. Pham, *ibid.* **109B**, 215 (1982), is an order of magnitude larger.

[4] I. S. Altarev *et al.*, Phys. Lett. B **276**, 242 (1992); K. F. Smith *et al.*, *ibid.* **234**, 191 (1990). We follow the treatment of these results in I. B. Khriplovich and S. K. Lamoreaux, *CP-Violation Without Strangeness* (Springer-Verlag, Berlin, in press).

[5] A. A. Belavin, A. M. Polyakov, A. S. Schwartz, and Yu. S. Tyupkin, Phys. Lett. **59B**, 85 (1975); V. N. Gribov (unpublished); G. 't Hooft, Phys. Rev. Lett. **37**, 8 (1976); R. Jackiw and C. Rebbi, *ibid.* **37**, 172 (1976); C. G. Callan, Jr., R. F. Dashen, and D. J. Gross, Phys. Lett. **63B**, 334 (1976).

[6] M. Kobayashi and T. Maskawa [1]. The reason for the absence of CP violation is that as a consequence of the $V-A$ structure of the weak currents a CP-violating phase associated with a

term in the semileptonic Hamiltonian describing a particular quark transition must be an overall phase, which has no observable consequences in first order in the weak interaction.

[7] In some extensions of the standard model where T violation in β decay appears already at the tree level, D_t/a and R_t/a [see Eqs. (4) and (5) in the text] can be as large as or close to the present limits on D/a and R/a. See P. Herczeg, Phys. Rev. D **28**, 200 (1983); in *Progress in Nuclear Physics*, edited by W-Y. Pauchy Hwang, S-C. Lee, C-E. Lee, and D. J. Ernst (North-Holland, New York, 1991), p. 171; in *Precision Tests of the Standard Electroweak Model*, edited by P. Langacker, Advanced Series on Directions in High Energy Physics Vol. 14 (World Scientific, Singapore, 1995), p. 786. The D coefficient in supersymmetric models has been considered in E. Christova and M. Fabbrichesi, Phys. Lett. B **315**, 113 (1993).

[8] Our metric and γ matrices are the same as in J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1963).

[9] For a review see J. F. Donoghue, E. Golowich, and B. Holstein, Phys. Rep. **131**, 319 (1986).

[10] G. A. Lobov, Yad. Fiz. **2**, 716 (1965) [Sov. J. Nucl. Phys. **2**, 512 (1966)].

[11] J. D. Jackson, S. B. Treiman, and H. W. Wyld, Jr., Phys. Rev. **106**, 517 (1957).

[12] A. L. Hallin, F. P. Calaprice, D. W. MacArthur, L. E. Piilonen, M. B. Schneider, and D. F. Schreiber, Phys. Rev. Lett. **52**, 337 (1984).

[13] C. G. Callan, Jr. and S. B. Treiman, Phys. Rev. **162**, 1494 (1967). L. B. Okun' and I. B. Khriplovich, Yad. Fiz. **6**, 1265 (1967) [Sov. J. Nucl. Phys. **6**, 919 (1968)]; J. C. Brodine, Phys. Rev. D **1**, 100 (1970); B. R. Holstein, Phys. Rev. C **28**, 342 (1983).

[14] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).

[15] For a review see E. Wasserman, in *Dark Matter in Cosmology, Clocks, and Tests of Fundamental Laws*, Proceedings of the XXX Rencontre de Moriond, edited by B. Guiderdoni, G. Greene, E. Hinds, and J. Trân Thanh Vân (Editions Frontieres, Gif-sur-Yvette, 1995), p. 613.

[16] For a review see A. R. Young *et al.*, in *Dark Matter in Cosmology, Clocks, and Tests of Fundamental Laws* [15], p. 539.

[17] E. G. Adelberger, Phys. Rev. Lett. **70**, 2856 (1993). The result of a search for the R correlation in ^{19}Ne decay [M. B. Schneider, F. P. Calaprice, A. L. Hallin, D. W. MacArthur, and D. F. Schreiber, *ibid.* **51**, 1239 (1983)] yields a weaker limit. The contribution of the final-state interactions to R in ^{19}Ne decay is $\sim 10^{-3}$ [P. Vogel and B. Werner, Nucl. Phys. **A404**, 345 (1983); B. Holstein, Phys. Rev. C **28**, 342 (1983)].

[18] I. B. Khriplovich, Nucl. Phys. **B352**, 385 (1991). The present bound on the coupling constant C_T of the P- and T-violating tensor electron-nucleon interaction used to derive this limit is $|C_T| < 1.3\times 10^{-8}$ [J. P. Jacobs, W. M. Klipstein, S. K. Lamoreaux, B. R. Heckel, and E. N. Fortson, Phys. Rev. A **52**, 3521 (1995)].

[19] This feature is analogous to the result of B. W. Lee and A. R. Swift, Phys. Rev. **136**, B228 (1964), which states that the $B\to B'$ matrix elements (where B, B' are states of the baryon octet) of a P-violating, CP-invariant Hamiltonian which transforms as the 6th component of an octet vanish in the limit of SU(3) symmetry. Note that the octet part of $H_-^{(-)}$ transforms as λ_7.

[20] Our phase conventions are such that the CP-violating amplitudes $a_B^{(-)}$ are real. Note also that M_B^β [Eq. (9)] is CP invariant.

[21] M. Roos, Phys. Lett. B **246**, 179 (1990).

[22] A. I. Vainshtein, V. I. Zakharov, and M. A. Shifman, Pis'ma Zh. Eksp. Teor. Fiz. **22**, 123 (1975) [JETP Lett. **22**, 55 (1975)]; Zh. Eksp. Teor. Fiz. **72**, 1275 (1977) [Sov. Phys. JETP **45**, 670 (1977)]; M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B120**, 316 (1977).

[23] F. J. Gilman and M. B. Wise, Phys. Lett. **83B**, 83 (1979); B. Guberina and R. D. Peccei, Nucl. Phys. **B163**, 289 (1980).

[24] For a review and further references see G. Buchalla, A. J. Buras, and M. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).

[25] J. F. Donoghue, E. Golowich, W. A. Ponce, and B. R. Holstein, Phys. Rev. D **21**, 186 (1980).

[26] A. J. Buras, M. Jamin, and M. E. Lautenbacher, Nucl. Phys. **B408**, 209 (1993).

[27] We used the leading-order results given in Appendix B, Table I of Ref. [26]. The values given in Eqs. (17)–(19) are for $\Lambda_{\overline{\rm MS}}^{(4)}=0.3$ GeV, $\mu=1$ GeV, and $m_t=130$ GeV. For $\Lambda_{\overline{\rm MS}}^{(4)}=0.2$ the Wilson coefficients, $a_B^{(-)}$ and D_t/a_D are smaller by a little. As noted in Ref. [26], the dependence of the imaginary parts of the Wilson coefficients on the value of the top quark mass is very weak.

[28] The enhancement of the vacuum saturation part of the O_5 contribution to $B\to B'\pi$ (and to nonleptonic kaon decays) was pointed out in Ref. [22]. The vacuum saturation contribution to $\langle B'|(O_5)_+|B\rangle$ is proportional to $\langle 0|\bar{d}d+\bar{s}s|0\rangle\langle B'|\bar{d}s|B\rangle$ and, therefore, contains the same enhancement factor [25]. In the vacuum saturation approximation $\langle B'|O_1|B\rangle$ vanishes.

[29] This can be shown by using in the factor $\langle B|\bar{d}s|n\rangle$ in the vacuum saturation part of $\langle B|(O_5)_+|n\rangle$ the divergence equation $\partial_\lambda \bar{d}\gamma_\lambda s = i(m_d-m_s)\bar{d}s$. One obtains then $\langle B|\bar{d}s|n\rangle = [(m_B-m_n)/(m_s-m_d)] f_V^B \bar{u}_n u_B$, where f_V^B is the vector form factor in $\langle B|\bar{d}\gamma_\lambda s|n\rangle$. Assuming SU(3) symmetry for $\langle B|\bar{d}\gamma_\lambda s|n\rangle$ (as we did for $\langle p|\bar{u}\gamma_\lambda s|B\rangle$ and $\langle p|\bar{u}\gamma_\lambda\gamma_5 s|B\rangle$) the stated result follows.

[30] This assumption is consistent with the existing calculations. See Vainshtein *et al.* [22], and J. F. Donoghue *et al.* [25].

[31] For comparison, the vacuum saturation part of $a_\Lambda^{(-)}$ is $(a_\Lambda^{(-)})_{\rm vs}\simeq(\omega_{\Lambda,5})_{\rm vs}{\rm Im}\, c_5'' = \frac{8}{9}\sqrt{3}m_\pi^2 V_{ud}^* V_{us}[f_\pi^2/(m_u+m_d)][(m_\Lambda-m_n)/(m_s-m_d)]\,{\rm Im}\, c_5'' \simeq [(5\times 10^{-7})\sqrt{2}f_\pi]{\rm Im}\, c_5''$, where $c_5''=c_5+\frac{3}{16}c_6$; similarly, $(a_\Sigma^{(-)})_{\rm vs}\simeq[(-4\times 10^{-7})\sqrt{2}f_\pi]{\rm Im}\, c_5''$; note that ${\rm Im}\, c_5''\simeq {\rm Im}\, c_5'$. We used $m_s=175$ MeV, $m_u=5.1$ MeV, and $m_d=9.3$ MeV (see Ref. [34] below).

[32] Using $g_T^B\simeq 0.3 g_A^B$ [9], the SU(3) symmetric values of g_M^B, and the vacuum saturation values of $a_B^{(-)}$ (see Ref. [31]), one obtains $D_t/a\simeq 2\times 10^{-13}$ Im τ and $D_t/a\simeq 6\times 10^{-13}$ Im τ for the contributions of g_M^B and g_T^B, respectively.

[33] S. L. Adler *et al.*, Phys. Rev. D **11**, 3309 (1975).

[34] H. Leutwyler, Phys. Lett. B **378**, 313 (1996).

[35] This is analogous to the behavior of the neutron electric dipole moment in the $m_\pi\to 0$ limit. See R. J. Crewther, P. Di Vecchia, G. Veneziano, and E. Witten, Phys. Lett. **88B**, 123 (1978); **91B**, 487(E) (1980).

[36] That this might be so was conjectured in C. H. Llewellyn Smith, Phys. Rep., Phys. Lett. **3C**, 261 (1972). See also S. L. Adler, Phys. Rev. D **12**, 3501 (1975).

[37] V. M. Khatsymovsky, I. B. Khriplovich, and A. S. Yelkhovsky, Ann. Phys. (N.Y.) **186**, 1 (1988); J. F. Donoghue, B. R. Holstein, and M. J. Musolf, Phys. Lett. B **196**, 196 (1987).
[38] R. J. Crewther *et al.* [35].
[39] ''First-class'' and ''second-class'' currents of a given parity have opposite *G* parities. See S. Weinberg, Phys. Rev. **112**, 1375 (1958).
[40] P. Cea and G. Nardulli, Phys. Lett. **144B**, 115 (1984).
[41] M. M. Nagels *et al.*, Nucl. Phys. **B147**, 189 (1979).
[42] W. Grein and P. Kroll, Nucl. Phys. **A338**, 332 (1980).
[43] No experiments have been carried out so far to search for *T* violation in muon capture. For a review of aspects of this field see, e.g., J. Deutsch, in *The Future of Muon Physics*, edited by K. Jungmann, V. W. Hughes, and G. zu Putlitz (Springer-Verlag, Berlin, 1992), p. 143. A study of *T* violation in muon capture in extensions of the standard model is in preparation by N. C. Mukhopadhyay and P. Herczeg.