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Primordial black hole constraints in cosmologies with early matter domination

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Moduli fields, a natural prediction of any supergravity and superstring-inspired supersymmetry theory, may lead to a prolonged period of matter domination in the early universe. This can be observationally viable provided the moduli decay early enough to avoid harming nucleosynthesis. If primordial black holes form, they would be expected to do so before or during this matter-dominated era. We examine the extent to which the standard primordial black hole constraints are weakened in such a cosmology. Permitted mass fractions of black holes at formation are of order 10^{-8}, rather than the usual 10^{-20} or so. If the black holes form from density perturbations with a power-law spectrum, its spectral index is limited to $n \lesssim 1.3$, rather than the $n \lesssim 1.25$ obtained in the standard cosmology. [S0556-2821(97)03424-3]

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I. INTRODUCTION

Although a substantial amount of work has been carried out on the assumption of a ''standard'' cosmology, in which the universe proceeds from an early period of inflation through reheating to radiation domination and finally to matter domination in the recent past, there is no direct evidence supporting this picture until the relatively late epoch at which nucleosynthesis occurs. Recently, this standard picture has been questioned and some alternative cosmologies discussed. An example is ''thermal inflation'' [1,2], a short second period of inflation at lower energy scales which does not generate interesting density perturbations, but which may resolve additional relic density problems not solved by the original inflationary period.

Several cosmological constraints are sensitive to whatever assumption is made for the entire cosmological evolution; axion cosmology is one such situation [1], and the constraints originating from primordial black holes (PBHs) is another. Recently, the latter was reinvestigated for cosmologies with thermal inflation, showing that the standard constraints [3,4] on the formation density of PBHs would weaken quite markedly [5].

Another possible modification to the standard cosmology is the addition of a prolonged period of matter domination, induced by a slow-decaying massive particle. In $N=1$ supergravity models [6], supersymmetry (SUSY) is broken in some hidden sector and the gravitational strength force plays the role of messenger by transmitting SUSY breaking down to the visible sector. In these models there often exist scalar fields with masses of the order of the weak scale and gravitational strength coupling to the ordinary matter. If at early epochs one of these fields is sitting far from the minimum of its potential with an amplitude of order of the Planck scale, the coherent oscillations about the minimum will eventually dominate the energy density of the universe. These fields will then behave as nonrelativistic matter, and decay at very late times. The presence of these slow-decaying massive particles is predicted not only in some specific classes of supergravity models, but in almost all theories in which supersymmetry is broken at an intermediate scale. In string models, massless fields exist in all known string ground states and parametrize the continuous ground state degeneracies characteristic of supersymmetric theories. These fields are massless to all orders in perturbation theory, and get their mass, of order a TeV, from the same nonperturbative mechanism which breaks SUSY. Being coupled to the ordinary matter only by gravitational strength couplings, a long lifetime results. Possible examples are the dilaton of string theory and the massless gauge singlets of string compactifications, and they go generically under the name of moduli. Under natural assumptions on the couplings, one finds that the reheating temperature after the moduli decay is too low to allow standard nucleosynthesis. Therefore, moduli are generally far *too* good at giving a period of matter domination, lasting beyond the epoch of nucleosynthesis and destroying this crucial success of the standard cosmology [7,8].

Many attempts have been made to resolve this cosmological moduli problem [9]. However, all of them require new phenomena to occur on the cosmological side as well as in the theory of supersymmetry breaking. It is not our intention, in this paper, to propose another solution to the moduli problem, but rather to study the extent to which the modular cosmology may affect the standard primordial black hole constraints.

We will therefore assume that the moduli are somewhat more massive than the supersymmetry breaking mass terms of the sfermions $\widetilde{m}=(10^2-10^3)$ GeV, and that their decay can be just early enough. This assumption might appear unusual since it is generally taken for granted that moduli masses are of the same order of magnitude as sfermion masses. We believe that this assumption is not entirely justified. Recent developments in the context of the field-theory limit of superstrings have shown that the perennial problem of dilaton stabilization at a value compatible with a weak coupling regime may be solved by nonperturbative corrections to the Kähler potential. In this context it is therefore possible to determine with some accuracy the soft supersymmetry breaking parameters and the moduli masses and it turns out that the latter may be as large as $10^3\ \widetilde{m}$ without incurring excessive unnaturalness [10]. The same result has recently been obtained in the context of M theory where the communication of supersymmetry breaking arises radiatively by gravitational interactions and, more importantly, no fine-tuning is involved [11].

The determination of the moduli masses is, therefore, intimately connected to other relevant issues such as how and at which scale supersymmetry breaking takes place and what the mechanism for dilaton stabilization is. It is intriguing that some progress has recently been made along these lines, indicating that moduli might be harmless as far as primordial nucleosynthesis is concerned. Yet, modular cosmology may have other effects. The cosmological sequence in such a model is as follows. At a high temperature the moduli come to dominate and the universe begins an epoch of matter domination. During this period, the radiation field actually cools some way below the nucleosynthesis scale (about 10^{-3} GeV), but the moduli decay while the energy density is still high enough to permit thermalization slightly above the nucleosynthesis temperature. In this picture, baryogenesis must be caused by the decay of the moduli, rather than at the electroweak transition [12].

II. THE MODULI-DOMINATED EPOCH

In hidden-sector models, supersymmetry breaking is conveyed to the low-energy visible sector through Planck scale suppressed interactions. In nonrenormalizable hidden-sector models, supersymmetry breaking vanishes in the limit $m_{\rm Pl}\to\infty$, $m_{\rm Pl}$ being the Planck mass. Since the potential for a generic moduli field ϕ is generated through the same physics associated to supersymmetry breaking, its potential takes the form

$$V(\phi)=m_{3/2}^2 M_{\rm Pl}^2 \mathcal{V}(|\phi|/M_{\rm Pl}), \tag{1}$$

where $M_{\rm Pl}=m_{\rm Pl}/\sqrt{8\pi}$ is the reduced Planck mass and $m_{3/2}\sim 1$ TeV is the gravitino mass. We note that $\mathcal{V}$ is generally a polynomial function of its argument.

The potential for this dangerous direction vanishes in the flat-space limit since $m_{3/2}\to 0$ in that limit. As mentioned in the Introduction, excitations around the zero-temperature minimum ϕ_0 of the potential have a mass $m_\phi=\mathcal{O}(10)\ m_{3/2}$.

Moduli fields are expected to be initially shifted from their zero-temperature minimum due to the effect of thermal fluctuations or of quantum fluctuations during inflation [13]. Another source of the shift might be the fact that the moduli couplings to the inflaton generally modify, during inflation, the properties of the effective potential. Moduli usually acquire a mass squared of the order of H^2, where $H\sim 10^{13}$ GeV is the Hubble parameter during the inflationary stage, and the value of the minimum of the potential may be shifted [14]. The shift produced by such effects may be as large as $m_{\rm Pl}$.

Although the form of the potential is not known, for our purposes one may just consider oscillations around the minimum with initial amplitude ϕ_i and take $V(\phi)\simeq m_\phi^2\phi^2/2$. When the Hubble parameter H reaches a value $H\sim m_\phi$, the scalar field starts oscillating coherently around the minimum of the potential. This happens when the temperature of the universe is $T_i\sim\sqrt{m_\phi m_{\rm Pl}}$ (in the case in which the universe is radiation dominated at that epoch).

The initial energy stored in the oscillations $\rho_i\sim m_\phi^2\phi_i^2$ redshifts like matter and can eventually dominate the energy density. When it does so depends on ϕ_i. If ϕ_i is of order $m_{\rm Pl}$, then the moduli dominate immediately, while if ϕ_i is smaller, radiation domination will continue for a while before the moduli come to dominate, or, in extreme cases, the moduli may decay before they dominate the energy density. In hidden-sector models, moduli couple to other fields only through Planck suppressed interactions. Examples of such fields are the dilaton and the compactification moduli of string theory or, in general, for any gauge singlet field responsible for SUSY breaking. There are several types of Planck suppressed couplings the moduli might have with ordinary matter, but all of them lead to the same estimate of the decay width[1]

$$\Gamma_\phi\sim\frac{m_\phi^3}{m_{\rm Pl}^2}. \tag{2}$$

The condition for the moduli to dominate the energy density of universe when they decay (which will be at the epoch $H\simeq\Gamma_\phi$) is that their initial value satisfies

$$\phi_i \gtrsim 10^{-8}\left(\frac{m_\phi}{1\ \text{TeV}}\right)^{1/2} m_{\rm Pl}. \tag{3}$$

At the decay time the radiation fluid has temperature

$$T_{\rm dec}\sim m_\phi^{11/6}\phi_i^{-2/3}m_{\rm Pl}^{-1/6}. \tag{4}$$

The decay products of the moduli will thermalize, reheating the universe up to a temperature[2]

$$T_{\rm reh}\sim\frac{m_\phi^{3/2}}{m_{\rm Pl}^{1/2}}\sim 3\times 10^{-4}\left(\frac{m_\phi}{10^2\ \text{GeV}}\right)^{3/2}\ \text{MeV}. \tag{5}$$

Notice that the reheating temperature is independent of ϕ_i, provided that the universe is dominated by the moduli energy density when decays start.

[1] Among the moduli interactions, one may also find anharmonic type terms if some other heavy field ψ is present in the hidden sector [15]. In this case the energy stored in the moduli oscillations may be initially released via the anharmonic couplings which transfer it to the ψ oscillations, which rapidly decay. It is therefore important to emphasize that the presence of additional anharmonic couplings may considerably affect the reheating process and bring some changes to the scenario investigated in this paper. For sake of simplicity we will assume from now on that the moduli field is not coupled to some extra heavy field.

[2] This description is a little bit misleading since the temperature during most of the reheating period decreases [16]. Indeed, reheating starts at the epoch when $\Gamma_\phi\rho_\phi$ becomes comparable to the radiation energy density times the Hubble expansion rate $\rho_{\rm R}H$. This occurs when the thermal bath has the temperature $T=m_\phi^{13/10}\phi_i^{2/5}m_{\rm Pl}^{-7/10}$. From this time on, the temperature of radiation falls less rapidly than it would if moduli were not decaying.

The decay products of ϕ will destroy the ^{4}He and D nuclei, and thus successful nucleosynthesis predictions, unless $T_{\rm reh}$ is larger than about 1 MeV. If the moduli field has mass 10^2 GeV, $T_{\rm reh}$ is well below the energy scale of nucleosynthesis, but if instead one assumes $m_\phi \sim 10^4$ GeV, then the reheat temperature becomes comparable and it may be possible to thermalize to a high enough temperature for standard nucleosynthesis to proceed. In the case $\phi_i \sim m_{\rm Pl}$ where the moduli dominate as soon as they begin to oscillate, this corresponds to an expansion of the universe during matter domination by a factor of around $(m_{\rm Pl}/m_\phi)^{4/3} \sim 10^{20}$, a very prolonged period indeed.

III. PRIMORDIAL BLACK HOLE CONSTRAINTS

A. Formation density constraints

In a radiation-dominated universe at temperature T, the horizon mass is given roughly by

$$M_H \simeq 10^{18}\ {\rm g}\left(\frac{10^7\ {\rm GeV}}{T}\right)^2. \tag{6}$$

PBHs of a given mass are expected to form around the time when that mass equals the horizon mass; production of smaller black holes is suppressed as pressure prevents the collapse of any density perturbation. In a matter-dominated universe, formation may occur on scales below the horizon mass, as we discuss later.

The lifetime of the black hole can be parametrized as [17,4]

$$\tau_{\rm evap} = \frac{9\times 10^{-27}}{f(M)}\left(\frac{M}{1\ {\rm g}}\right)^3 {\rm sec}, \tag{7}$$

where $f(M)$ depends on the number of particle species which can be emitted and is normalized to one for holes which emit only massless particles. A black hole of initial mass around 5×10^{14} g would be evaporating at the present epoch, while masses around 10^{10} g would be evaporating at nucleosynthesis. Those lighter black holes may form early on in the period of moduli domination, or even before it if its onset is delayed.

We denote the fraction of the density of the universe in black holes of a given mass as β, with β_i denoting the initial density at formation. The ratio of the PBH density to the density in other forms is denoted $\alpha \equiv \beta/(1-\beta)$.

The various limits which can be placed on the PBH density are well known [3,4]; we shall use the compilation given in Ref. [5]. There are a range of constraints from effects of evaporation, while for more massive black holes, $M \gtrsim 10^{15}$ g, there is a limit comes from their contribution to the present density parameter. The formation of PBHs with $M > 10^{30}$ g is tightly constrained due to the absence of spectral distortions in the cosmic microwave background (CMB) [18,20]. An additional, less secure, constraint arises if one assumes that evaporation leaves behind a Planck mass relic [19].

All these constraints are expressed as limits on the fraction of the mass of the universe in black holes at the present or at the time of evaporation. To constrain the initial mass fraction, one needs to assume a form for the entire cosmology back to the formation epoch, given by Eq. (6). Figure 1

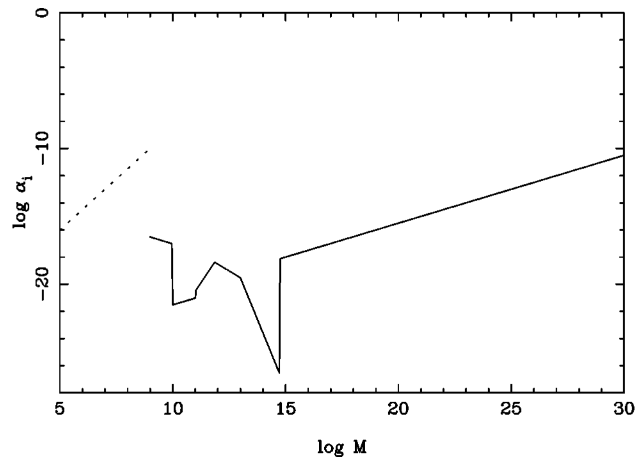

FIG. 1. The tightest limits on α_i, for the standard cosmology. The mass is in grams. The relic constraint is shown as a dotted line, emphasizing that it is not compulsory. The solid lines, from left to right, represent $n\bar{n}$ production at nucleosynthesis, deuterium destruction, He-4 spallation, entropy production, gamma ray production, and total density in PBHs; see Ref. [5] for details.

shows the result of carrying this out for the standard cosmology, where the universe was radiation dominated until very recently [5]. We see that the constraints are extremely tight; typically only something around 10^{-20} of the mass of the universe is permitted to form black holes in the standard cosmology.

We now turn to our main purpose, examining the change in the constraints induced by a period of moduli domination. In order to attain a reheating temperature of $T_{\rm reh} \sim 10^{-3}$ GeV, so that nucleosynthesis can proceed, we require $m_\phi \sim 2\times 10^4$ GeV. In the extreme case of $\phi_i \sim m_{\rm Pl}$, the moduli begin to oscillate at temperature $T_{\rm MD} \sim 2\times 10^{11}$ GeV, since

$$\rho_i = m_\phi^2 m_{\rm Pl}^2 = \frac{\pi^2}{30} g_\star^{\rm MD} T_{\rm MD}^4. \tag{8}$$

Here $g_\star^{\rm MD} \sim 250$ is the number of degrees of freedom in the minimal supersymmetric standard model. In the following, we will consider two scenarios: one in which $\phi_i \sim m_{\rm Pl}$ and moduli domination begins immediately, and an intermediate scenario where $\phi_i \sim 10^{-4}\, m_{\rm Pl}$ and there is a delay before moduli domination commences.

1. Immediate moduli domination

From Eq. (6), PBHs in the mass range $2\times 10^9\ {\rm g} \leqslant M \leqslant 2\times 10^{38}$ g are formed during the moduli-dominated era. During moduli-domination, the PBHs constitute a constant fraction of the total energy density. The time of the decay of the moduli $t_{\rm dec}$ and of the reheating of the subsequent thermalized fluid $t_{\rm reh}$ can be taken as the same, giving a PBH density at reheating of

$$\left(\frac{\rho_{\rm PBH}}{\rho_{\rm rad}}\right)_{\rm reh} = \left(\frac{\rho_{\rm PBH}}{\rho_{\rm mod}}\right)_{\rm dec} = \left(\frac{\rho_{\rm PBH}}{\rho_{\rm mod}}\right)_i \equiv \alpha_i . \tag{9}$$

Here $\rho_{\rm rad}$ and $\rho_{\rm mod}$ are the energy densities in radiation and moduli, respectively. Therefore, for PBHs formed during moduli domination and surviving beyond it (which is all those of interest), we have

$$\left(\frac{\rho_{\rm PBH}}{\rho_{\rm rad}}\right)_{\rm evap} \equiv \alpha_{\rm evap} = \frac{\beta_i}{1-\beta_i}\frac{T_{\rm reh}}{T_{\rm evap}}. \tag{10}$$

Since the duration of the matter-dominated period is negligible compared with the PBH lifetime we can use the Freidmann equation to relate the time and temperature at evaporation:

$$t_{\rm evap} = 0.4\left(\frac{m_{\rm Pl}}{T_{\rm evap}}\right)^2 t_{\rm Pl}. \tag{11}$$

This expression for $t_{\rm evap}$ can be equated with Eq. (7), taking $f(M)=1$ since for PBHs with $M > 4\times 10^9$ g the evaporation temperature is sufficiently low ($<10^{-4}$ GeV) that only massless particles are emitted, to give

$$\frac{T_{\rm reh}}{T_{\rm evap}} = 8\times 10^{-21}\left(\frac{M}{m_{\rm Pl}}\right)^{3/2}, \tag{12}$$

so that

$$\frac{\beta_i}{1-\beta_i} = 1\times 10^{20}\left(\frac{m_{\rm Pl}}{M}\right)^{3/2}\alpha_{\rm evap}. \tag{13}$$

The gravitational constraints, which require that the present-day densities of PBHs and relics do not exceed the maximum values set by the age and expansion rate of the universe, are

$$\Omega_{\rm PBH,eq} = \left(\frac{\rho_{\rm PBH}}{\rho_{\rm rad}}\right)_{\rm eq} = \frac{\beta_i}{1-\beta_i}\frac{T_{\rm reh}}{T_{\rm eq}} < 1 \tag{14}$$

$$\Omega_{\rm rel,eq} = \left(\frac{\rho_{\rm PBH}}{\rho_{\rm rad}}\right)_{\rm eq} = \frac{m_{\rm Pl}}{M}\frac{\beta_i}{1-\beta_i}\frac{T_{\rm reh}}{T_{\rm eq}} < 1, \tag{15}$$

where eq indicates the epoch of matter-radiation equality in the universe's recent past, after which the density of PBHs or relics, relative to the critical density, remains constant. In the case of PBHs with $M > 2\times 10^{38}$ g, formed after moduli domination, the requirement that the present-day density of PBHs does not exceed the limit above is obviously the same as in the standard evolution of the universe. The PBHs formed before moduli domination are sufficiently light $M \leqslant 10^9$ g that only the relic constraint $\Omega_{\rm rel,eq} < 1$ applies to them, where

$$\Omega_{\rm rel,eq} = \left(\frac{\rho_{\rm PBH}}{\rho_{\rm rad}}\right)_{\rm eq} = \frac{m_{\rm Pl}}{M}\frac{\beta_i}{1-\beta_i}\frac{T_{\rm reh}}{T_{\rm eq}}\frac{T_i}{T_{\rm MD}} < 1, \tag{16}$$

and in fact it turns out that large initial mass fractions of PBHs $\beta_i \sim 1$ are allowed. The various limits on the initial mass fraction of PBHs are illustrated in Fig. 2.

2. Delayed moduli domination

An initial value $\phi_i \sim m_{\rm Pl}$ is the most natural, but it is not impossible for it to be smaller and this leads to a shorter period of moduli domination. As an example, we take $\phi_i \sim 10^{-4} m_\phi$ so that moduli domination commences when the energy stored in the oscillations of the moduli field becomes greater than that of the radiation

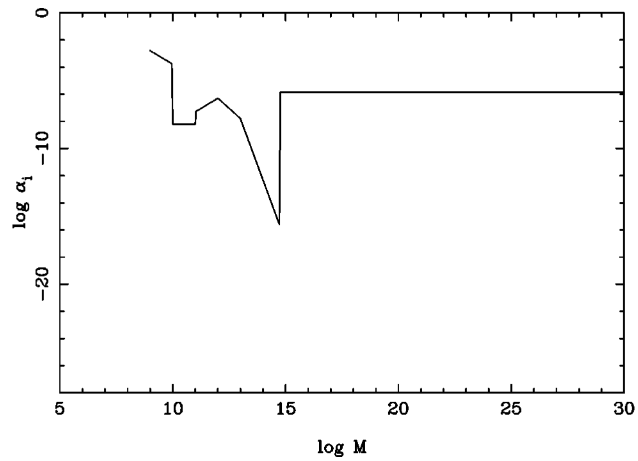

FIG. 2. The tightest limits on the initial mass fraction of PBHs α_i if moduli domination commences immediately. The mass is in grams. The rightmost line, indicating the density constraint, continues horizontally until $M \sim 10^{38}$ g; PBHs more massive than this form after moduli domination and the standard constraint $\alpha_i < 10^{-19}\sqrt{M/10^{15}\ \rm g}$ then applies. The constraints are the same as in Fig. 1.

$$\frac{\rho_\phi}{\rho_{\rm rad}} = \frac{\phi_i^2}{m_{\rm Pl}^2}\frac{2\times 10^{11}\ \rm GeV}{T_{\rm MD}} > 1, \tag{17}$$

at temperature $T_{\rm MD} = 2\times 10^3$ GeV.

From Eq. (6), PBHs with $M < 2\times 10^{25}$ g are formed in the radiation-dominated period before the moduli domination commences. Their energy density, relative to that in other forms, varies as T^{-1} initially then remains constant during moduli domination. It then increases as T^{-1} during the subsequent radiation domination so that

$$\left(\frac{\rho_{\rm PBH}}{\rho_{\rm rad}}\right)_{\rm evap} \equiv \alpha_{\rm evap} = \frac{\beta_i}{1-\beta_i}\frac{T_i}{T_{\rm MD}}\frac{T_{\rm reh}}{T_{\rm evap}}, \tag{18}$$

leading to

$$\frac{\beta_i}{1-\beta_i} = 2\times 10^5 \frac{m_{\rm Pl}}{M}\alpha_{\rm evap}. \tag{19}$$

Similarly for the gravitational constraints

$$\Omega_{\rm PBH,eq} = \left(\frac{\rho_{\rm PBH}}{\rho_{\rm rad}}\right)_{\rm eq} = \frac{\beta_i}{1-\beta_i}\frac{T_i}{T_{\rm MD}}\frac{T_{\rm reh}}{T_{\rm eq}} < 1 \tag{20}$$

$$\Omega_{\rm rel,eq} = \left(\frac{\rho_{\rm rel}}{\rho_{\rm rad}}\right)_{\rm eq} = \frac{m_{\rm Pl}}{M}\frac{\beta_i}{1-\beta_i}\frac{T_i}{T_{\rm MD}}\frac{T_{\rm reh}}{T_{\rm eq}} < 1. \tag{21}$$

For PBHs with $M > 2\times 10^{25}$ g, formed after moduli domination commences, the gravitational constraint is the same as when moduli domination starts immediately at $T = 2\times 10^{11}$ GeV. The various limits on the initial mass fraction of PBHs in this case are illustrated in Fig. 3.

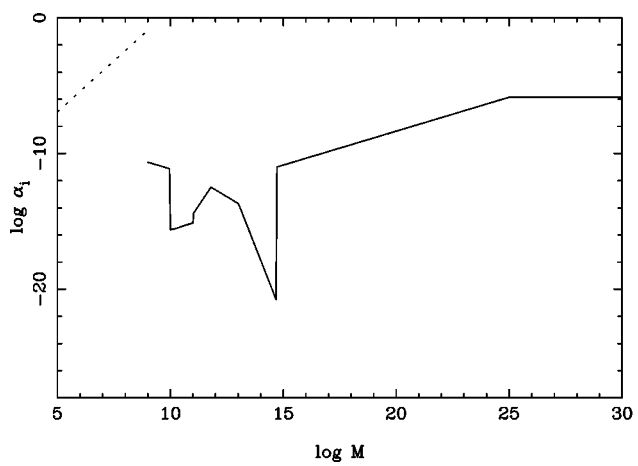

FIG. 3. The tightest limits on the initial mass fraction of PBHs α_i if $\phi_i \sim 10^{-4} m_{\rm Pl}$ and moduli domination is delayed. The mass is in grams. For $M > 2\times 10^{25}$ g the limits are the same as when moduli domination commences immediately. The constraints are the same as in Fig. 1.

B. Density perturbation constraints

PBH formation can be used to constrain the spectral index of the density perturbation spectrum [20,21,5]. To do this, we assume an initial spectrum which is a power law across the entire range of scales from the PBH scale up to the present horizon scale. Constraints can then be placed on the spectral index n of those perturbations. To do this, we need to consider the variation of $\sigma_{\rm hor}(M)$, the mass variance evaluated at horizon crossing [5]. During matter domination $\sigma_{\rm hor}(M) \propto M^{(1-n)/6}$, whereas during radiation domination $\sigma_{\rm hor}(M) \propto M^{(1-n)/4}$; the different scalings arise because during matter-domination the comoving mass density is conserved but during radiation-domination it decreases so that mass scales enter the horizon more quickly [5].

In the case of PBHs formed during matter domination the standard scenario for PBH formation no longer holds. It has been shown [22] that perturbation growth during coherent scalar field oscillation behaves in exactly the same way as in a dust universe, provided, as here, that the oscillations are very rapid compared to other timescales in the problem. PBH formation during such a matter-dominated period was considered in Ref. [20]. Because there is no pressure, it is now possible for PBHs to form well within the horizon, but in order to do so the initial perturbation must be sufficiently spherical as gravitational collapse is unstable to aspherical growth. PBHs are formed from typically-sized density fluctuations [of order $\sigma(M)$] which grow as $t^{2/3}$ after entering the horizon until they are of order 1 and then collapse. Therefore PBHs of mass M take longer to form during matter domination than during radiation domination;[3] however, this delay is negligible compared with the decay time, as discussed previously. Also the formation rate, which is given by [23]

$$\beta(M) \approx 2\times 10^{-2} \sigma^{13/2}(M), \tag{22}$$

and the mass fraction α remain constant during matter domination and therefore their values are unaffected by the delay.

For PBHs with $M > 2\times 10^{9}$ g, formed during moduli domination, we have

$$\sigma_{\rm hor}(M) = \sigma_{\rm hor}(M_0) \times \left(\frac{M_{\rm eq}}{M_0}\right)^{(1-n)/6} \left(\frac{M_{\rm dec}}{M_{\rm eq}}\right)^{(1-n)/4} \left(\frac{M_{\rm hor}}{M_{\rm dec}}\right)^{(1-n)/6}, \tag{23}$$

where $M_0 \simeq 10^{56}$ g is the present horizon mass. This simplifies to

$$\sigma_{\rm hor}(M) = \sigma_{\rm hor}(M_0) \left(\frac{M}{M_0}\right)^{(1-n)/6} \left(\frac{M_{\rm dec}}{M_{\rm eq}}\right)^{(1-n)/12}, \tag{24}$$

and since during radiation domination $M_{\rm H} \propto T^{-2}$

$$\sigma_{\rm hor}(M) = \sigma_{\rm hor}(M_0) \left(\frac{M}{M_0}\right)^{(1-n)/6} \left(\frac{T_{\rm eq}}{T_{\rm dec}}\right)^{(1-n)/6} = \sigma_{\rm hor}(M_0) \left(1.4\times 10^{-6} \frac{M}{M_0}\right)^{(1-n)/6}, \tag{25}$$

for masses M forming during moduli domination.

The lightest holes that can form are determined by the reheating temperature after the original period of inflation which is responsible for generating the density perturbations. The minimum mass is then given by Eq. (6). Normally, the tightest constraint on n comes from the lightest PBHs. We use the method outlined in [5], but using the expressions for $\sigma(M)$ and $\beta(M)$ given above, to obtain the constraints.

For immediate moduli domination, we find the tightest limit to be $n < 1.23$ from the deuterium constraint evaluated at $M \sim 10^{10}$ g, although all the constraints due to the evaporation of PBHs require $n < 1.32$. The limit from the present-day density of PBHs is tightest at $M \sim 5\times 10^{14}$ g giving $n < 1.30$. Relics do not constrain n, since even very large initial PBH abundances β_i will be diluted away.

For our example case of delayed moduli domination, the most constraining PBHs are formed during the radiation-dominated era before moduli domination commences. For them $\sigma(M)$ has a different form:

$$\sigma_{\rm hor}(M) = \sigma_{\rm hor}(M_0) \left(\frac{M_{\rm eq}}{M_0}\right)^{(1-n)/6} \left(\frac{M_{\rm dec}}{M_{\rm eq}}\right)^{(1-n)/4} \times \left(\frac{M_{\rm MD}}{M_{\rm dec}}\right)^{(1-n)/6} \left(\frac{M}{M_{\rm MD}}\right)^{(1-n)/4}, \tag{26}$$

which simplifies to

$$\sigma_{\rm hor}(M) = \sigma_{\rm hor}(M_0) \left(10^{5} \frac{M}{M_0}\right)^{(1-n)/4}, \tag{27}$$

[3]In common with other authors, we have not considered perturbations which enter the horizon during radiation domination without forming black holes, but which survive into matter domination where they can collapse. These might provide additional constraints.

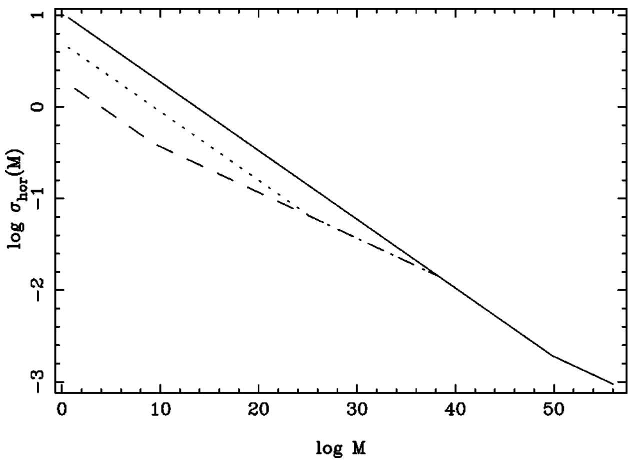

FIG. 4. The variation of $\sigma_{\rm hor}(M)$ with mass for the three scenarios considered; the solid, dotted, and dashed lines representing the standard cosmology, immediate moduli domination and delayed moduli domination respectively. The mass is in grams, and we take $n=1.3$ for illustrative purposes.

for our specific parameters. Since the PBHs of interest are formed during radiation domination the standard expression for β applies [4]:

$$\beta(M)\approx\sigma(M)\exp\left(-\frac{1}{18\sigma^2(M)}\right). \tag{28}$$

The tightest limit is now $n<1.26$ from the deuterium constraint evaluated at $M\sim10^{10}$ g, with all the constraints due to the evaporation of PBHs require $n<1.28$. The tightest limit from the present-day density of PBHs is $n<1.30$ at $M\sim5\times10^{14}$ g. The relic constraint may provide an even tighter limit if the reheat temperature after inflation is close to 10^{16} GeV; however, to avoid the gravitino problem in supersymmetric cosmologies requires a much lower reheat temperature: $T_{\rm RH}<(2-6)\times10^{9}$ GeV for $m_{3/2}=(1-10)$ TeV [24]. In Fig. 4 we illustrate the variation of $\sigma_{\rm hor}(M)$ in the standard cosmology, for immediate moduli domination and for our example case of delayed moduli domination.

The tightest constraint for immediate moduli domination is only slightly weaker than in the standard cosmology [5], where the tightest constraint (again deuterium) is $n<1.22$, with the evaporation constraints all giving $n<1.24$. The weakening is only small, since during matter domination PBHs form more readily so that to attain any particular value of β_i a smaller value of $\sigma(M)$, and hence n, is necessary. This reduces the effect of the larger value of β_i allowed due to the period of matter domination, and also leads to a larger spread in the limits on n from different sources. The tightest constraint is significantly weaker for delayed moduli domination, since in this case the most constraining PBHs are formed during radiation domination so that the main difference from the standard scenario is the larger values of β_i allowed.

IV. CONCLUSIONS

If there is a prolonged period of matter-domination by moduli in the early universe, it leads to a weakening of the constraint on density perturbations from primordial black hole formation. It again reminds us of the sensitivity of this bound to the entire assumed cosmological history. If the moduli dominate immediately, the fraction of the density of the universe permitted to go into PBHs becomes of order 10^{-8}, rather than the 10^{-20} or so which the standard cosmology requires. Delayed moduli domination leads to an intermediate constraint on those PBHs which form before moduli domination. This weakening is similar to that found [5] for the case where an extra period of inflation at low energies, known as thermal inflation, is assumed.

When expressed as a limit on the spectral index of a power-law density perturbation spectrum, we obtain $n\lesssim1.3$ for immediate moduli domination, rather than $n\lesssim1.25$ as in the standard cosmology. The weakening is similar to that from thermal inflation, which also led to $n\lesssim1.3$. Interestingly, the constraint can actually be weakest if moduli domination is delayed, because PBH formation is harder during radiation domination than moduli domination.

We end by noting that the assumption of Gaussian perturbations in the black hole formation calculation has recently been questioned by Bullock and Primack [25]. As shown in Ref. [5], in the most non-Gaussian case found by Bullock and Primack the constraint on n can be weakened further, by up to 0.05. However, the standard way of obtaining a power-law spectrum with $n>1$, hybrid inflation models, gives Gaussian perturbations.

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