Σ_c and Λ_c magnetic moments from QCD spectral sum rules

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The QCD spectral sum rules in the presence of the external electromagnetic field $F_{\mu\nu}$ are used to calculate the magnetic moments of Σ_c and Λ_c. Our results are $\mu_{\Sigma_c^{++}}=(2.1\pm0.3)\mu_N$, $\mu_{\Sigma_c^{+}}=(0.6\pm0.1)\mu_N$, $\mu_{\Sigma_c^{0}}=(-1.6\pm0.2)\mu_N$, and $\mu_{\Lambda_c}=(0.15\pm0.05)\mu_N$. [S0556-2821(97)00723-6]

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QCD spectral sum rules (QSSRs) [1] have been successful in extracting the masses and coupling constants of low-lying mesons and baryons. In this approach the nonperturbative effects are taken into account through various condensates in the QCD vacuum. As shown in [2,3] the light baryon masses are determined by the chiral symmetry-breaking quark condensate. In the infinite heavy quark mass limit the QSSR was first used to evaluate the heavy baryon mass [4]. The QSSRs with a finite heavy quark mass were treated in [5,6]. Recently the QSSR was employed in the framework of the heavy quark effective theory [7–11].

The baryon magnetic moment, like the baryon mass, is another important static quantity. Ioffe and Smilga [12] and, independently, Balitsky and Yung [13] extracted the nucleon magnetic moment treating the electromagnetic field $F_{\mu\nu}$ as an external field in the QSSR approach. They found that the nucleon magnetic moment is essentially related to the quark condensate and three susceptibilities χ, κ, and ξ. Later on the octet baryon magnetic moments [15] were obtained in a similar manner. In this work we shall employ the same approach to calculate the magnetic moments of Σ_c and Λ_c.

We shall consider the two-point correlator $\Pi_{\Sigma_c}(p)$ in the presence of an external electromagnetic field $F_{\alpha\beta}$:

$$\Pi_{\Sigma_c}(p)=i\int d^4x\langle 0|T\{\eta_{\Sigma_c}(x),\overline{\eta_{\Sigma_c}}(0)\}|0\rangle_{F_{\mu\nu}}e^{ip\cdot x}$$
$$=\Pi_0(p)+\Pi_{\mu\nu}(p)F^{\mu\nu}+\cdots \tag{1}$$

where $\Pi_0(p)$ is the polarization operator without the external field $F_{\mu\nu}$. The η_{Σ_c} in Eq. (1) is the interpolating current with Σ_c quantum numbers:

$$\eta_{\Sigma_c}(x)=\epsilon^{abc}\{[u^{aT}(x)Cc^b(x)]u^c(x)$$
$$-[u^{aT}(x)C\gamma_5c^b(x)]\gamma_5u^c(x)\}, \tag{2}$$

and

$$\langle 0|\eta_{\Sigma_c}(0)|\Sigma_c\rangle=\lambda_{\Sigma_c}\nu_{\Sigma_c}(p), \tag{3}$$

where $u^a(x)$ and $c^b(x)$ is the up and charm quark field, λ_{Σ_c} is the overlap amplitude of the interpolating current with the baryon state, and the ν_{Σ_c} is the Dirac spinor of the heavy baryon.

Three different tensor structures contribute to $\Pi_{\mu\nu}(p)$,

$$\Pi_{\mu\nu}(p)=\Pi(p)(\sigma_{\mu\nu}\hat{p}+\hat{p}\sigma_{\mu\nu})+\Pi_1(p)i(p_\mu\gamma_\nu-p_\nu\gamma_\mu)\hat{p}$$
$$+\Pi_2(p)\sigma_{\mu\nu}. \tag{4}$$

As in the original QSSR analysis of nucleon magnetic moment [12], we shall consider the first tensor structure $(\sigma_{\mu\nu}\hat{p}+\hat{p}\sigma_{\mu\nu})$.

The presence of the external field $F_{\mu\nu}$ will induce three new condensates in the QCD vacuum [12]:

$$\langle 0|\overline{q}\sigma_{\mu\nu}q|0\rangle_{F_{\mu\nu}}=e_q\chi F_{\mu\nu}\langle 0|\overline{q}q|0\rangle,$$

$$g_s\langle 0|\overline{q}\frac{\lambda^n}{2}G^n_{\mu\nu}q|0\rangle_{F_{\mu\nu}}=e_q\kappa F_{\mu\nu}\langle 0|\overline{q}q|0\rangle, \tag{5}$$

$$g_s\epsilon^{\mu\nu\lambda\sigma}\langle 0|\overline{q}\gamma_5\frac{\lambda^n}{2}G^n_{\lambda\sigma}q|0\rangle_{F_{\mu\nu}}=ie_q\xi F^{\mu\nu}\langle 0|\overline{q}q|0\rangle,$$

where q refers to the up and down quark, e_q is the quark charge. The χ, κ, and ξ in Eq. (5) are the quark condensate susceptibilities, which have been the subject of various studies [12–15]. Their values employed by different groups are consistent with each other. We shall adopt the values $\chi=-4.5\ \mathrm{GeV}^{-2}$, $\kappa=0.4$, $\xi=-0.8$.

At the phenomenological level we have

$$\mathrm{Im}\Pi(s)=\frac{1}{4}\mu_{\Sigma_c}\lambda^2_{\Sigma_c}\delta'(s-m^2_{\Sigma_c})+C\delta(s-m^2_{\Sigma_c})+\mathrm{Im}\Pi^{\mathrm{pert}}(s)\,\theta(s-t_{\Sigma_c}), \tag{6}$$

where the first term corresponds to the Σ_c magnetic moment and is of the double pole. The second term comes from the transition $\Sigma_c \rightarrow$ excited states and is of single pole. The third term is the usual continuum contribution and t_{Σ_c} is the continuum threshold. The single-pole transition term does not damp out after Borel transform. Therefore it should be explicitly included in the QSSR analysis.

Within an operator product expansion we obtain, to lowest order of α_s and for condensates up to dimension six at the quark level,

$$\mathrm{Im}\Pi(s)=\ \mathrm{Im}\Pi^{\ \mathrm{pert}}(s)+\ \mathrm{Im}\Pi^{\mathrm{np}}(s), \tag{7}$$

$$\mathrm{Im}\Pi^{\mathrm{pert}}(s)=\frac{e_u}{8}s\left(1-\frac{m_c^2}{s}\right)^3. \tag{8}$$

Making a Borel tranform of $\Pi(p)$ and transferring the continuum contribution to the left-hand side, we obtain

$$\begin{aligned} M_B^2\int_{m_c^2}^{t_{\Sigma_c}}&\mathrm{Im}\Pi^{\mathrm{pert}}(s)e^{-(s/M_B^2)}ds L^{-4/9}\\ &+\left\{-\frac{1}{12}\chi e_u a^2 M_B^2(1-e^{-(t_{\Sigma_c}-m_c^2/M_B^2)})L^{-(16/27)}\right.\\ &-\frac{1}{24}e_c a^2-\frac{1}{36}e_u a^2\left(\frac{m_c^2}{M_B^2}+1\right)\\ &+\frac{1}{96}\chi m_0^2 e_u a^2\left(2\frac{m_c^2}{M_B^2}+1\right)L^{-(10/9)}-\frac{1}{72}\kappa e_u a^2\\ &\left.\times\left(2\frac{m_c^2}{M_B^2}-1\right)-\frac{1}{36}\xi e_u a^2\left(\frac{m_c^2}{M_B^2}+1\right)\right\}e^{-(m_c^2/M_B^2)}L^{4/9}\\ =\ &\frac{1}{4}(2\pi)^4\lambda^2_{\Sigma_c}e^{-(m^2_{\Sigma_c}/M_B^2)}\mu_{\Sigma_c}(1+CM_B^2), \end{aligned} \tag{9}$$

where $a=-(2\pi)^2\langle 0|\bar{q}q|0\rangle=0.55\ \mathrm{GeV}^3$, $am_0^2=(2\pi)^2 g_s\langle 0|\bar{q}\sigma Gq|0\rangle$, $m_0^2=0.8\ \mathrm{GeV}^2$, $q=u,d$, $L=\ln(10M_B)/\ln(5)$. C is the unknown constant to be determined from the sum rule, which parametrizes the transition contribution. We have checked that in the chiral limit $m_c\rightarrow 0$, our result reproduces the sum rule for nucleon magnetic moment [12].

For the charm quark mass we use $m_c=1.47\pm 0.1$ GeV. We adopt the estimated continuum threshold and overlapping amplititude in the QSSR analysis [5,6,9–11,16]. $t_{\Sigma_c}=10\ \mathrm{GeV}^2$ and $(2\pi)^4\lambda^2_{\Sigma_c}=0.8\pm 0.1\ \mathrm{GeV}^6$. For the Σ_c mass we may either use the predictions in the QSSR analysis or the experimental value [22], $m_{\Sigma_c}=2.455$ GeV.

We may further improve the numerical analysis by taking into account the renormalization group evolutions of the sum rule (9) through the anomalous dimensions of the condensates and currents. The working interval of the Borel mass M_B^2 for the sum rule (9) is $1.7\ \mathrm{GeV}^2\leqslant M_B^2\leqslant 2.5\ \mathrm{GeV}^2$

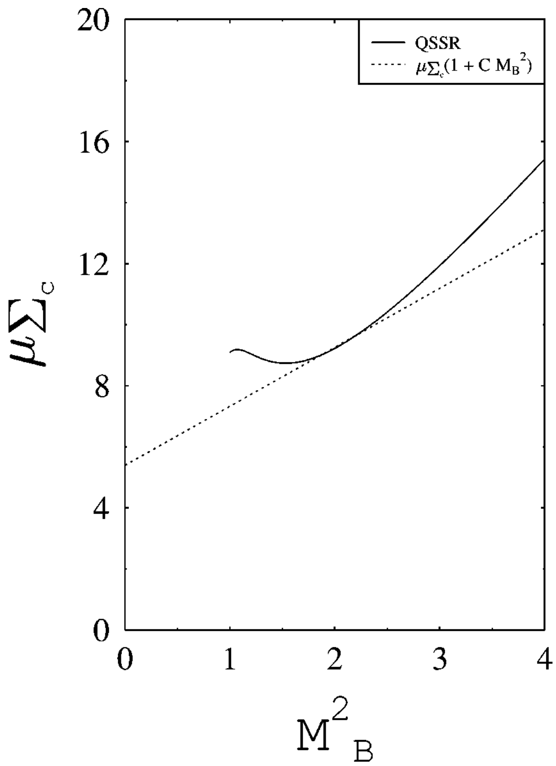

FIG. 1. The Borel mass dependence of the Σ_c^{++} magnetic moment for the continuum threshold $t_{\Sigma_c}=10\ \mathrm{GeV}^2$. The solid curve is the QCD sum rule prediction for $\mu_{\Sigma_c^{++}}$. The dotted line is a straight-line approximation. The intersect with Y axis is the Σ_c magnetic moment in unit of $e/2m_{\Sigma_c}$.

where both the continuum contribution and power corrections are controllable [5,6]. Moving the factor $\frac{1}{4}(2\pi)^4\lambda^2_{\Sigma_c}e^{-(m^2_{\Sigma_c}/M_B^2)}$ on the right-hand side to the left and fitting the new sum rules with a straight-line approximation we may extract the Σ_c magnetic moment. We show the Borel mass dependence of the new sum rule and the fitting straight line in Fig. 1 for the continuum threshold $t_{\Sigma_c}=10\ \mathrm{GeV}^2$.

It can be seen in Fig. 1 that the nondiagonal transition contribution is important, though it is not dominant in the working interval of the Borel mass $1.7\ \mathrm{GeV}^2\leqslant M_B^2\leqslant 2.5\ \mathrm{GeV}^2$. The sum rule is insensitive to the susceptibilities κ and ξ due to their small values. Their contributions are less than 5%. When χ varies from $-4.5\ \mathrm{GeV}^{-2}$ to $-3.5\ \mathrm{GeV}^{-2}$ or to $-5.5\ \mathrm{GeV}^{-2}$, the sum rules change within 10%.

Our final result is $\mu_{\Sigma_c^{++}}=(5.4\pm 0.5)\,e/2m_{\Sigma_c}$, where $e/2m_B$ is a natural unit in QSSR analysis of the baryon magnetic moment. By replacing e_u in Eq. (9) with $(e_u+e_d)/2$ or e_d we arrive at the magnetic moments for the other Σ_c multiplets, $\mu_{\Sigma_c^+}=(0.6\pm 0.1)\,e/2m_{\Sigma_c}$ and $\mu_{\Sigma_c^0}=(-4.2\pm 0.4)\,e/2m_{\Sigma_c}$. In units of nuclear magneton $\mu_{\Sigma_c^{++}}=(2.1\pm 0.3)\mu_N$, $\mu_{\Sigma_c^+}=(0.23\pm 0.03)\mu_N$, and $\mu_{\Sigma_c^0}=(-1.6\pm 0.2)\mu_N$.

Similarly we can extract the magnetic moments of Λ_c with the following interpolating current:

$$\eta_{\Lambda_c}(x)=\epsilon^{abc}[u^{aT}(x)C\gamma_5 u^b(x)]c^c(x). \tag{10}$$

The final sum rule reads as follows:

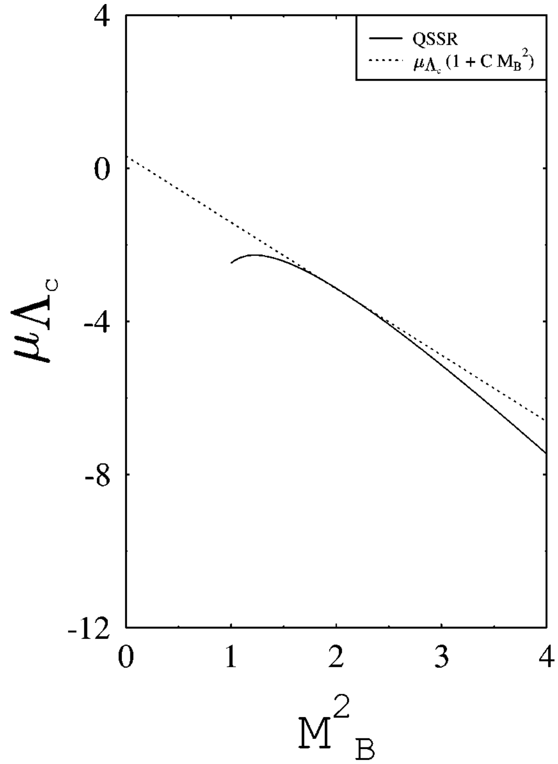

FIG. 2. The Borel mass dependence of μ_{Λ_c} and the fitting straight line.

$$\frac{3e_c}{16}M_B^2\int_{m_c^2}^{t_{\Lambda_c}}\left[\frac{m_c^2}{2}\left(1-\frac{m_c^4}{s^2}\right)-\frac{s}{3}\left(1-\frac{m_c^6}{s^3}\right)\right]e^{-(s/M_B^2)}ds\,L^{-(4/9)}$$
$$+\left\{-\frac{e_c}{24}a^2+\frac{e_u+e_d}{144}a^2\left(1-\frac{m_c^2}{M_B^2}\right)\right.$$
$$\left.+\frac{e_u+e_d}{576}\chi m_0^2a^2L^{-10/9}+\frac{e_u+e_d}{48}\kappa a^2\right\}e^{-(m_c^2/M_B^2)}L^{4/9}$$
$$=\frac{1}{4}(2\pi)^4\lambda_{\Lambda_c}^2e^{-(m_{\Lambda_c}^2/M_B^2)}\mu_{\Lambda_c}(1+CM_B^2), \qquad (11)$$

With the parameters [5,6,9–11,16] $t_{\Lambda_c}=10$ GeV^2, $(2\pi)^4\lambda_{\Lambda_c}^2=1.0\pm0.2$ GeV^6, and $m_{\Lambda_c}=2.285$ GeV, we get $\mu_{\Lambda_c}=(0.15\pm0.05)\mu_N$. The Borel dependence of μ_{Λ_c} and the fitting line is shown in Fig. 2. As in the analysis of the Σ_c magnetic moment, the straight line approximation is good.

It is not difficult to extend the same analysis to extract the magnetic moments of Σ_b and Λ_b. Yet the overlapping amplititudes λ_{Σ_b} and λ_{Λ_b} determined in the QSSR approach with finite bottom quark mass have large errors. Therefore we do not present numerical results here.

In summary, we have calculated the magnetic moment of Σ_c and Λ_c using the external field method in the QCD sum rules. There are no experimental data for the heavy baryon magnetic moments. Yet naive predictions have been made in the phenomenological models such as nonrelativistic quark model (NRQM) [17,18], bag model [19], and the Skyrme model [20,21]. Especially in the quark model the heavy baryon magnetic moments have a rather simple form. $\mu_{\Lambda_c}=\mu_c$, $\mu_{\Sigma_c^{++}}=\frac{8}{9}\mu_p-\frac{1}{3}\mu_c$, $\mu_{\Sigma_c^{+}}=\frac{2}{9}\mu_p-\frac{1}{3}\mu_c$, and $\mu_{\Sigma_c^0}=-\frac{4}{9}\mu_p-\frac{1}{3}\mu_c$. Our result of the Σ_c magnetic moment is in good agreement with the NRQM prediction. In the μ_{Λ_c} sum rule (11), the contribution from higher-dimension condensates is significant and comparable with the charm quark perturbative contribution numerically, since the light quark perturbative contribution and the induced light quark condensate vanishes. If we turn off all the higher-dimension nonperturbative contributions by setting the quark condensate $a=0$, we arrive at $\mu_{\Lambda_c}=0.35\mu_N$, which is very close to the NRQM prediction. Therefore the higher-dimension condensates lead to the possible deviation from the naive quark model result. It will be very interesting to measure μ_{Λ_c} experimentally.

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