

$B \rightarrow K \eta'$ decay as a unique probe of the η' meson

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A theory of the $B \rightarrow K \eta'$ decay is proposed. It is based on the Cabibbo-favored $b \rightarrow \bar{c}s$ process followed by a direct materialization of the $\bar{c}c$ pair into the η' . This mechanism works due to a nonvalence Zweig-rule-violating c -quark component of the η' , which is unique to its very special nature. This nonperturbative “intrinsic charm” content of the η' is evaluated using the operator product expansion and QCD low-energy theorems. Our results are consistent with an unexpectedly large $B(B \rightarrow K \eta') \approx 7.8 \times 10^{-5}$ recently announced by CLEO. [S0556-2821(97)04521-9]

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I. INTRODUCTION

This paper suggests a theory of the $B \rightarrow K \eta'$ decay which may shed a new light on properties of the η' meson. Our study is motivated by recent results of the CLEO Collaboration [1] which has announced an unexpectedly large branching ratio

$$B(B \rightarrow K \eta') = (7.8_{-2.2}^{+2.7} \pm 1.0) \times 10^{-5}. \quad (1)$$

Little thought is needed to realize that this number is in severe contradiction with a standard view of the process at the quark level as a decay of the b quark into the light quarks which could be suggested as soon as the η' is usually considered to be a SU(3) singlet meson made of the u , d and s quarks (see Sec. II for more detail). This result may not seem too surprising if one remembers the well-known fact that the quark content of the η' is undistinguishable from the gluon one due to the axial anomaly. On the other hand, in the weak decay the b quark proceeds more strongly to the $\bar{c}cs$ system due to the Cabibbo enhancement of the latter in comparison to the $\bar{u}us$ state. Since a pair of c quarks can easily convert to gluons, one can suggest the following scenario of the $B \rightarrow K \eta'$ decay. The b quark proceeds into the s -quark and c -quark pair, while the latter directly materializes into the η' via a nonvalence “intrinsic charm” c -quark component of the η' which exists due to virtual $\bar{c}c \leftrightarrow$ gluons transitions. An immediate objection to this proposal which can come to one’s mind is that this process is expected to bring a very small contribution to the decay width as soon as it obviously violates the Zweig rule. We will argue that though the scenario we suggest is indeed Zweig rule violating, it nevertheless can explain the data. The reason is that we actually deal here with a situation where the Zweig rule itself is badly broken down. As will be discussed in detail in Sec. IV, both regularities and sources of breaking down the Zweig rule are nowadays well classified and studied. In particular, it is 100% violated for pseudoscalar mesons including the η' . In effect, we find that an extent to which the Zweig rule is

broken down in the problem at hand suffices to reconcile the theory with the data (1). The uniqueness of η' is in both a possibility to evaluate this effect and its very large magnitude. The decay $B \rightarrow K \eta'$ serves as a probe of the “intrinsic charm” content of the η' . On the contrary to how it may sound, the mechanism of violating the Zweig rule in the η' is purely nonperturbative. To be honest, we have to note that the accuracy of our result is rather low, of the order of factor 2 in the amplitude. It is important, however, that a main source of uncertainty in our approach is well localized and related to the poor knowledge of a particular vacuum condensate. Therefore the theoretical precision can be considerably improved in the future. We should stress that in contrast to a recent proposal [2] on importance of the axial anomaly in the closely related inclusive $B \rightarrow \eta' X_s$ decay, it does not play any role in our mechanism. On the contrary, the anomaly is exactly cancelled in the operator product expansion (OPE) in powers of m_c^{-1} for a c -quark bilinear operator [see Eqs. (16), (17) below], which is a starting point of our approach to the problem. In a sense, we therefore deal with a “postanomalous” effect which, of course, is suppressed by the parameter $1/m_c^2$. However, in the real world $m_c \approx 1.25$ GeV is not far from a hadronic ~ 1 GeV scale and, as will be shown below, the effect of the charmed loop is very large numerically. On the other hand, methods applied in our study closely parallel those developed earlier in a study of the famous U(1) problem (whose key ingredient is just the axial anomaly), and will be explained in the course of our presentation.

Our strategy consists of a few steps. We start in Sec. II with the standard approach to the $B \rightarrow K \eta'$ decay and demonstrate that its prediction is about two orders of magnitude smaller than the experimental number (1). We then propose in Sec. III an alternative gluon mechanism and explain our method for calculation of a crucial quantity of our consideration which is the matrix element $\langle 0 | \bar{c} \gamma_\mu \gamma_5 c | \eta' \rangle$. Using the data (1) as an input, we calculate an “experimental” value of this matrix element. To calculate the same quantity theoretically, we first reduce it by using the OPE to the matrix element of a pseudoscalar three gluon operator $\langle 0 | G \tilde{G} G | \eta' \rangle$. The latter object is further related to a particular correlation function of gluon currents extending ideas

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originally suggested by Witten [3] and Veneziano [4] in their approach to the U(1) problem. This correlation function is next calculated in Sec. IV in terms of a vacuum expectation value of the three gluon operator $\langle g^3 G^3 \rangle$ by using QCD low-energy theorems. We also discuss there physics responsible for breaking down the Zweig rule. In Sec. V we estimate the latter vacuum condensate and finally obtain a theoretical prediction for the matrix element of interest $\langle 0 | \bar{c} \gamma_\mu \gamma_5 c | \eta' \rangle$. We compare this number with an ‘‘experimental’’ value found from the observed data (1) and find a satisfactory agreement between them. This demonstrates that the gluon mechanism indeed explains the data with a reservation for uncertainty of our results. A final Sec. VI contains our conclusions.

II. THE STANDARD APPROACH TO $B \rightarrow K \eta'$

In this section we estimate a width of the $B \rightarrow K \eta'$ decay assuming that the η' meson is made exclusively of light quarks. In this case the relevant terms in the effective non-leptonic Hamiltonian are

$$H_{\delta B=1} = \frac{G_F}{\sqrt{2}} \left[V_{ub} V_{us}^* (c_1 O_1 + c_2 O_2) - V_{tb} V_{ts}^* \sum_{i=3}^{12} c_i O_i \right] + \text{H.c.} \quad (2)$$

Here O_i are defined as [we use the notation $L_\mu = \gamma_\mu(1 - \gamma_5)$, $R_\mu = \gamma_\mu(1 + \gamma_5)$]

$$\begin{aligned} O_1 &= \bar{s}^i L_\mu u^j \bar{u}^j L_\mu b^i, \quad O_2 = \bar{s} L_\mu u u \bar{u} L_\mu b, \\ O_{3(5)} &= \bar{s} L_\mu b \sum_q \bar{q} L_\mu (R_\mu) q, \\ O_{4(6)} &= \bar{s}^i L_\mu b^j \sum_q \bar{q}^j L_\mu (R_\mu) q^i, \\ O_{7(9)} &= \frac{3}{2} \bar{s} L_\mu b \sum_q e_q \bar{q} R_\mu (L_\mu) q, \\ O_{8(10)} &= \frac{3}{2} \bar{s}^i L_\mu b^j \sum_q e_q \bar{q}^j R_\mu (L_\mu) q^i, \\ O_{11} &= \frac{g}{32\pi^2} m_b \bar{s} (1 + \gamma_5) \sigma G b, \\ O_{12} &= \frac{e}{32\pi^2} m_b \bar{s} (1 + \gamma_5) \sigma F b, \end{aligned} \quad (3)$$

where i, j are the color indices and q is any of the the u, d, s, c quarks. $G_{\mu\nu} = G_{\mu\nu}^a t^a$ and $F_{\mu\nu}$ are the gluon and photon field strength tensors. O_1 and O_2 are the tree level operators, while O_{3-6} and O_{7-10} are the gluon and electroweak penguin operators, respectively. $O_{11,12}$ are the magnetic penguins. The Wilson coefficients $c_i = c_i(\mu)$ depend on the renormalization scale μ and to the next-to-leading order [5] [for $\alpha_s(m_Z) = 0.118$, $\alpha_{em}(m_Z) = 1/128$, $m_t = 176$ GeV, $\mu = 5$ GeV] are given by the set [5–7]

$$\begin{aligned} c_1 &= -0.3125, \quad c_2 = 1.1502, \quad c_3 = 0.0174, \quad c_4 = -0.0373, \\ c_5 &= 0.0104, \quad c_6 = -0.0459, \quad c_7 = 1.398 \times 10^{-5}, \\ c_8 &= 3.919 \times 10^{-4}, \quad c_9 = -0.0103, \quad c_{10} = 1.987 \times 10^{-3}, \\ c_{11} &= -0.299, \quad c_{12} = -0.634. \end{aligned} \quad (4)$$

Introducing the transition form factor

$$\begin{aligned} \langle K(q) | \bar{s} \gamma_\mu b | B(p+q) \rangle &= 2q_\mu f_+(m_{\eta'}^2) \\ &+ p_\mu [f_+(m_{\eta'}^2) + f_-(m_{\eta'}^2)] \end{aligned} \quad (5)$$

and the η' residue [8] (the chiral limit $m_q = 0$ is implied)

$$\begin{aligned} \langle \eta' | \bar{q} \gamma_\mu \gamma_5 q | 0 \rangle &= -i \frac{1}{\sqrt{3}} f_{\eta'} p_\mu, \quad \frac{1}{\sqrt{3}} f_{\eta'} = (0.5 - 0.8) \frac{f_\pi}{\sqrt{3}} \\ &\simeq 0.04 \text{ GeV}, \end{aligned} \quad (6)$$

and neglecting for the moment $m_{\eta'}, m_K$ in comparison to m_B , the magnetic penguins O_{11} , O_{12} , and $O(1/m_b, 1/N)$ terms in the factorized matrix elements of penguin operators, we obtain the following estimate for the amplitude of interest (here N stands for the number of colors):

$$\begin{aligned} \langle K \eta' | H_W | B \rangle &\simeq \frac{G_F}{\sqrt{2}} 2i(pq) f_+(m_{\eta'}^2) \frac{f_{\eta'}}{\sqrt{3}} \left[V_{ub} V_{us}^* \left(c_1 + \frac{c_2}{N} \right) \right. \\ &\left. - V_{tb} V_{ts}^* \left(3c_3 + c_4 - 3c_5 + \frac{3}{2} e_s c_{10} \right) \right], \end{aligned} \quad (7)$$

where, in particular, we have omitted left-right penguin contributions, which are suppressed by $1/m_b$. In our opinion, this procedure is much better than an alternative one, where only a subset of $1/m_b$ corrections is retained. Furthermore, it is well known that the factorization does not work in non-leptonic B decays. Effects due to a nonfactorizability are usually taken into account in a phenomenological manner by the substitution $c_1 + c_2/N \rightarrow a_1$ with $a_1 \simeq 0.25$ obtained by a global fit of the data on nonleptonic B decays [9]. Using this number in Eq. (7), we end up with¹

$$B(B \rightarrow K \eta') \simeq 1 \times 10^{-7} \quad (8)$$

which is by two orders of magnitude smaller than the experimental result [1]. It is easy to see that this small value is a consequence of a small residue of the η' supplemented with the Cabbibo suppression of the $b \rightarrow u$ transition. An account

¹We disagree with [10] where much larger $B(B \rightarrow K \eta') \simeq 3 \times 10^{-5}$ was proposed. In our opinion, this large width came as a result of an incorrect assignment of absorptive parts to matrix elements of penguin operators, which, by definition of the OPE, are not there. In particular, it follows (see p. 2187) from the formulas given in [10] that this decay width becomes infinite in the chiral SU(3) limit. In fact, at the level of penguin contributions the decays $B \rightarrow K \eta'$ and $B \rightarrow K \phi$ are just identical and, assuming that factorization works reasonably well, it is simply impossible to obtain anything substantially different from our estimate (9).

of penguin contributions, as can be seen from Eq. (7), does not help much. Indeed, neglecting for simplicity the tree level $b \rightarrow u$ transition, we can obtain an estimate for the ratio of the decay width of the process of interest to the width of the decay $B \rightarrow K \phi$:

$$\frac{\Gamma(B \rightarrow K \eta')}{\Gamma(B \rightarrow K \phi)} \simeq \frac{|\langle \eta' | \bar{s} \gamma_\mu \gamma_5 s | 0 \rangle \langle K | \bar{s} \gamma_\mu b | B \rangle|^2}{|\langle \phi | \bar{s} \gamma_\mu s | 0 \rangle \langle K | \bar{s} \gamma_\mu b | B \rangle|^2} = \frac{1}{3} \left(\frac{f_{\eta'}}{f_\phi} \right)^2 \simeq 2.5 \times 10^{-2}, \quad (9)$$

where we have used the definition $\langle \phi | \bar{s} \gamma_\mu s | 0 \rangle = \varepsilon_{\mu\phi} f_\phi m_\phi$ with $f_\phi \simeq 240$ MeV known experimentally from the $\phi \rightarrow e^+ e^-$ decay (ε_μ stands for the polarization vector of the ϕ meson). As $B(B \rightarrow K \phi) \simeq 1 \times 10^{-5}$ [6], we obtain a very small magnitude $B(B \rightarrow K \eta') \simeq 2.5 \times 10^{-7}$ in reasonable agreement with Eq. (8). It is now obvious that corrections due to a nonfactorizability of penguin operators, magnetic penguins contributions,² as well as $O(1/m_b, 1/N)$ terms which have been omitted in Eq. (7), cannot substantially change the estimate (8). We therefore conclude that the image of the η' meson as the SU(3) singlet quark state, made exclusively of the u, d, s quarks, is not adequate to the problem at hand. To avoid possible misunderstanding, we should note that the axial anomaly is in fact taken into account in the above mechanism. However, its role there is merely to fix the residue of the quark current (6) into the η' .

III. THE GLUON MECHANISM IN $B \rightarrow K \eta'$

Here we suggest an alternative mechanism for the $B \rightarrow K \eta'$ decay which is based on the well known fact that the η' is a very special meson strongly coupled to the gluons. Therefore, the process of interest can be mediated by the $b \rightarrow c$ decay followed by a conversion of the c quarks into gluons. This means that the matrix element

$$\langle 0 | \bar{c} \gamma_\mu \gamma_5 c | \eta'(p) \rangle = i f_{\eta'}^{(c)} p_\mu \quad (10)$$

is nonzero due to the $c \rightarrow$ gluons transitions. Of course, since one deals here with virtual c quarks, this matrix element is suppressed by the $1/m_c^2$ factor. On the other hand, the c quark is not very heavy and, taken together with the Cabbibo enhancement of the $b \rightarrow c$ transition in comparison to $b \rightarrow u$, the suggested scenario of the $B \rightarrow K \eta'$ can be expected to successfully compete with the standard one described in Sec. II. Actually, this gluon mechanism will be argued to dominate the decay. To get a feeling of how large the residue (10) must be in order to explain the data (1), we reverse the arguments and estimate this quantity ‘‘experimentally’’ under assumption that the proposed gluon mechanism exhausts the $B \rightarrow K \eta'$ decay. A corresponding number is easy to calculate. In the factorization approximation the amplitude takes the form

²It has been argued that the magnetic penguin operator O_{11} enhances the branching ratio for the $b \rightarrow s \phi$ decay by 20–30% [11]. We expect a similar (or, anyway, not larger) effect of this operator for the $B \rightarrow K \eta'$ decay.

$$M = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_1 \langle \eta'(p) | \bar{c} \gamma_\mu \gamma_5 c | 0 \rangle \langle K(q) | \bar{s} \gamma_\mu b | B(p+q) \rangle \quad (11)$$

(here $a_1 \simeq 0.25$, see Sec. II). For the $B \rightarrow K$ transition form factor (5) we use the dipole formula

$$f_+(p^2) = \frac{f_+(0)}{1 - p^2/m_*^2} \quad (12)$$

with $f_+(0) \simeq 0.32$, $m_* \simeq 5$ GeV [12,13]. Calculating now the decay width, we obtain numerically the branching ratio in terms of the residue $f_{\eta'}^{(c)}$ defined in Eq. (10),

$$B(B \rightarrow K \eta') \simeq 3.92 \times 10^{-3} \left(\frac{f_{\eta'}^{(c)}}{1 \text{ GeV}} \right)^2, \quad (13)$$

which together with the data (1) implies the ‘‘experimental’’ value [we use the central value of the branching ratio (1)]

$$f_{\eta'}^{(c)} \simeq 140 \text{ MeV (exp)}. \quad (14)$$

This number may seem to be too large for the proposed mechanism to work as it is only a few times smaller than the analogously normalized residue $\langle 0 | \bar{c} \gamma_\mu \gamma_5 c | \eta_c(p) \rangle = i f_{\eta_c} p_\mu$ with $f_{\eta_c} \simeq 400$ MeV known from the $\eta_c \rightarrow 2 \gamma$ decay. However, as will be argued in the rest of this paper, the theory is able to produce such a large residue $f_{\eta'}^{(c)}$. In effect, the gluon mechanism completely overplays the standard one by two orders of magnitude in the decay width, and reconciles a theoretical prediction for the $B \rightarrow K \eta'$ decay with the data.

We now proceed to a theoretical calculation of the η' residue of the charmed axial-vector current (10). Making use of the anomaly equation, we obtain, from Eq. (10),

$$f_{\eta'}^{(c)} = \frac{1}{m_{\eta'}^2} \left\langle 0 \left| 2m_c \bar{c} i \gamma_5 c + \frac{\alpha_s}{4\pi} G_{\mu\nu} \tilde{G}_{\mu\nu} \right| \eta' \right\rangle. \quad (15)$$

Since the c quark is heavier than the η' , it cannot contribute the matrix element (15) on a valence level. It does, however, contribute when propagating in a loop. The c quark in the loop is subject to external gluon fields populating the η' . A technical tool which allows us to evaluate the corresponding contribution to the matrix element (15) is the operator product expansion in inverse powers of the c -quark mass (the heavy quark expansion):

$$2m_c \bar{c} i \gamma_5 c = -\frac{\alpha_s}{4\pi} G_{\mu\nu} \tilde{G}_{\mu\nu} - \frac{1}{16\pi^2 m_c^2} g^3 f^{abc} G_{\mu\nu}^a \tilde{G}_{\nu\alpha}^b G_{\alpha\mu}^c + \dots \quad (16)$$

A detailed derivation of Eq. (16) is given in the Appendix, while here we restrict ourselves to a few comments. The first term in the right-hand side of Eq. (16) is the usual anomaly term with the opposite sign. This sign can be easily understood if one remembers that the anomaly term corresponds to a subtraction of the Pauli-Villars regulator from the naive divergence $2m_q \bar{q} i \gamma_5 q$ of the axial-vector current $\bar{q} \gamma_\mu \gamma_5 q$. On the other hand, the Pauli-Villars contribution is a special case of the heavy quark expansion (16) with the strict limit

$M_R = \infty$. The cancellation of leading terms $\sim G\tilde{G}$ which do not depend on m_c is in agreement with the intuitive idea that heavy quarks cannot contribute matrix elements over light hadrons³ in the limit $m_Q \rightarrow \infty$. The second term in Eq. (16) is the gluon operator of the lowest (after the anomaly term) dimension $d=6$. We omit one more $d=6$ operator $(D_\mu G_{\mu\nu})(D_\alpha G_{\alpha\nu})$ which can be related to a four-quark operator using the equation of motion. One can show that a contribution of this operator is suppressed both parametrically (in N and α_s) and numerically. The ellipsis in Eq. (16) stands for higher dimensional operators which we do not address here. To this accuracy we therefore obtain

$$f_{\eta'}^{(c)} = -\frac{1}{16\pi^2 m_\eta^2} \frac{1}{m_c^2} \langle 0 | g^3 f^{abc} G_{\mu\nu}^a \tilde{G}_{\nu\alpha}^b G_{\alpha\mu}^c | \eta' \rangle. \quad (17)$$

We note that the gluon operator in Eq. (17) correspond to a normalization point $\mu^2 \simeq m_c^2$.

We have thus reduced the problem to a calculation of the matrix element of the purely gluonic operator (17). Apart from a trivial rescaling of the normalization point, this matrix element is essentially defined by low-energy physics on a scale ~ 1 GeV. One could therefore think that we did not make any progress at all as matrix elements of gluon operators are usually not easy to calculate. The situation with the η' is, however, exceptional, and the matrix element (17) is amenable to a theoretical study. We will now show that it can be evaluated making use of the large N line of reasoning along with the property $m_\eta^2 \sim 1/N$ in close analogy with the way Witten addressed a very similar matrix element $\langle 0 | G\tilde{G} | \eta' \rangle$. The fact that $m_\eta^2 \sim 1/N$ was established by Witten [3] a long time ago in connection to the celebrated U(1) problem. Witten's objective was to understand within the large N approach how massless quarks are able to bring the correlation function of the topological density

$$T(p) = i \int dx e^{ipx} \left\langle 0 \left| T \left\{ \frac{\alpha_s}{4\pi} G_{\mu\nu} \tilde{G}_{\mu\nu}(x) \times \frac{\alpha_s}{4\pi} G_{\mu\nu} \tilde{G}_{\mu\nu}(0) \right\} \right| 0 \right\rangle \quad (18)$$

³A well-known example is the problem of a light particle mass: in the conformal anomaly equation

$$m_\eta^2 = \langle \eta' | \theta_{\mu\mu} | \eta' \rangle = \left\langle \eta' \left| \sum_q m_q \bar{q}q \right| \eta' \right\rangle + \frac{\beta(g^2)}{2g} \langle \eta' | G_{\mu\nu}^2 | \eta' \rangle$$

(where $\theta_{\mu\mu}$ is the trace of the momentum-energy tensor and the sum is taken over all quark flavors) the heavy quark can only contribute when propagating in a loop, and its contribution cancels a corresponding contribution of the heavy quark to the β function in the second term. Thus, the c quark does not contribute the η' mass in the limit $m_c \rightarrow \infty$.

at zero momentum $p \rightarrow 0$ down to zero as required by the chiral anomaly, if quark loops are suppressed by a power of N , and thus apparently do not show up to leading order in $1/N$. To lowest order in $1/N$ the two-point function (18) is given by sums over one-hadron intermediate states:

$$T(p) = \left[\sum_{\text{glueballs}} \frac{a_n^2}{M_n^2 - p^2} + \text{subtractions} \right] + \frac{1}{N} \sum_{\text{mesons}} \frac{c_n^2}{m_n^2 - p^2}, \quad (19)$$

where M_n, a_n and $m_n, N^{-1/2}c_n$ stand for the masses and residues of the n th glueball and meson states, respectively. Here $a_n, c_n = O(N^0)$, and moreover, to lowest order in $1/N$, the glueball residues a_n do not depend on whether massless quarks are present in the Lagrangian or not. Therefore the first term in Eq. (19) to leading order in $1/N$ corresponds to the two-point function in pure Yang-Mills theory (gluodynamics). The crucial observation made by Witten [3] and Veneziano [4] was that while a cancellation of the two terms in Eq. (19) is not possible at generic $p^2 \neq 0$, it can happen at $p^2 = 0$, if there is a single meson with $m^2 \sim 1/N$, which would then cancel the whole sum over glueballs together with subtraction terms in Eq. (19). Witten and Veneziano further identified this meson with the η' since the latter is the lightest flavor singlet pseudoscalar state in nature.

The reason we have repeated at length the argument due to Witten and Veneziano is that, as is easy to see, it carries practically without a word of alteration over any nondiagonal correlation function of the topological density $(\alpha_s/4\pi) G_{\mu\nu} \tilde{G}_{\mu\nu}$ and arbitrary gluon operator with the O^{-+} quantum numbers. Choosing for such the three gluon operator defining the matrix element (17), we obtain the relation

$$\begin{aligned} & \langle 0 | g^3 f^{abc} G_{\mu\nu}^a \tilde{G}_{\nu\alpha}^b G_{\alpha\mu}^c | \eta' \rangle \frac{1}{m_\eta^2} \left\langle \eta' \left| \frac{\alpha_s}{4\pi} G_{\mu\nu} \tilde{G}_{\mu\nu} \right| 0 \right\rangle \\ &= -i \int dx \left\langle 0 \left| T \left\{ g^3 f^{abc} G_{\mu\nu}^a \tilde{G}_{\nu\alpha}^b G_{\alpha\mu}^c(x) \times \frac{\alpha_s}{4\pi} G_{\mu\nu} \tilde{G}_{\mu\nu}(0) \right\} \right| 0_{\text{YM}} + O\left(\frac{1}{N}\right) \right\rangle \end{aligned} \quad (20)$$

where the subscript YM means that the correlation function in Eq. (20) refers to pure Yang-Mills theory. Its calculation will be addressed in the next section. Here we would like to mention that, as follows from Eq. (20), the residue of interest $\langle 0 | g^3 G\tilde{G}G | \eta' \rangle = O(N^{-1/2})$.

IV. QCD LOW-ENERGY THEOREMS

The two-point function (20) is a new unknown quantity which has to be evaluated in order to estimate the matrix element (10). Analogously to the diagonal correlation function of the topological density (18), it vanishes in the perturbation theory to all orders in α_s , since the topological density is a total derivative whose matrix elements are all zero at

the perturbative level. Therefore the correlation function (20) can only be nonzero due to nonperturbative contributions. The goal of this section is to estimate Eq. (20).

The idea of how to study correlation functions of gluonic currents with $O^{+(-)}$ quantum number [such as our Eq. (20)] was suggested long ago [8], and we would like to shortly repeat it here. It has been known for a long time that in channels with scalar or pseudoscalar quantum numbers leading contributions to correlation functions of composite operators are related not to standard vacuum condensates, but rather to the so-called direct instantons [8]. The motivation to introduce this object into the theory was a very strong indication that the standard QCD sum rules [14] with standard power corrections due to local vacuum condensates are not able to describe the $O^{+(-)}$ channels. In other words, OPE does not reproduce there a scale of phenomena which is exactly known from elsewhere. A source of this effect was found in existence of direct instantons.

The meaning of this object is best explained if one considers first a two-point function of (pseudo)scalar gluon currents at large Euclidean momentum Q^2 . In this case a leading nonperturbative contribution is obtained when the momentum Q is transferred as a whole to a second vertex by a vacuum field (this is allowed by quantum numbers of the current) which therefore must be of small size $\rho \sim 1/Q$. Such a situation corresponds to a small coupling regime, in which the quasiclassical approximation becomes accurate. The vacuum field is therefore classical; it is the famous Belavin-Polyakov-Schwarz-Tyupkin (BPST) instanton [17]. Thus, a recipe for the calculation of the direct instanton contribution at large Q^2 is simple: the gluon field in the current must be substituted by the instanton. The integral over the instanton size is then dominated by small $\rho \sim 1/Q$. However, going down to a resonance region $Q^2 \sim$ a few GeV^2 , this simple picture breaks down [8]—instantons start to interact strongly with each other and large size vacuum fields, and the one instanton (or, the dilute instanton gas) approximation stops making sense. A consistent calculation of instanton contribution in this case becomes a complicated problem which requires going beyond the dilute instanton gas approximation and taking into account instanton interactions, e.g., in a form suggested by the instanton liquid vacuum model of Shuryak and Diakonov-Petrov (see [15,16], and references therein). We shall not proceed with this approach which is basically a specific model of the QCD vacuum. Rather, we will follow an alternative method which was proposed by Novikov, Shifman, Vainshtein, and Zakharov (NSVZ) [8]. It makes use of a strong assumption that though a vacuum field transferring a small momentum Q resembles the original undeformed BPST instanton only a little, it nevertheless retains the (anti-)self-duality property of the latter,

$$G_{\mu\nu}^a = \pm \tilde{G}_{\mu\nu}^a, \quad (21)$$

in absence of massless quarks, i.e., in Yang-Mills (YM) theory (where instanton transitions are not suppressed by fermion zero modes). This conjecture has been supported by explicit calculations of next-to-leading (after direct instantons) nonperturbative corrections for two-point functions of the scalar G^2 and pseudoscalar $G\tilde{G}$ currents at moderate Q^2 . Up to an overall sign, they turn out the same (the direct

instanton contributions are identical in both channels by definition). Therefore, a further extrapolation of the self-duality selection rule (21) to even lower $Q^2 \approx 0$ is expected to be correct at least 100% accurate. In fact, a phenomenologically successful mass formula for the η' derived in Ref. [8] by using the selection rule (21) implies that an actual accuracy of this approximations is of the order of 50%. This mass formula was obtained⁴ by relating the residue of the η' to the value of the two-point function of topological density in YM theory at $Q^2=0$, while the latter was evaluated using the selection rule (21) and a low-energy theorem [see Eq. (22) below]. As a result, $m_{\eta'}^2$ was found proportional to the gluon condensate $\langle g^2 G^2 \rangle$ in YM theory.

Following the same logic, we therefore assume that the self-duality selection rule (21) can also be applied to the correlation function of interest (20). If this is the case, the value of the latter is fixed by the low-energy theorem [8]

$$i \int dx \left\langle 0 \left| T \left\{ O(x) \frac{\alpha_s}{4\pi} G^2(0) \right\} \right| 0 \right\rangle_{\text{YM}} = \frac{2d}{b} \langle O \rangle_{\text{YM}}. \quad (22)$$

Here $O(x)$ is arbitrary color singlet local operator of canonical dimension d made of gluons and $b=11/3N$ stands for the first coefficient of the β function in pure Yang-Mills theory. As a derivation of the fundamental relation (22) is rather simple, for the sake of completeness we would like to remind the reader of it here. One starts with a redefinition of the gluon field

$$\bar{G}_{\mu\nu} \equiv g_0 G_{\mu\nu}, \quad (23)$$

where g_0 is the bare coupling constant of QCD defined at the cutoff scale M_0 . Then the path integral representation immediately yields the relation

$$i \int dx \langle 0 | T \{ O(x) \bar{G}^2(0) \} | 0 \rangle = - \frac{d}{d(1/4g_0^2)} \langle O \rangle. \quad (24)$$

On the other hand, the renormalizability and the dimensional transmutation phenomenon in a massless theory (either QCD with massless quarks or gluodynamics) ensure that

$$\langle O \rangle = \text{const} \times \left[M_0 \exp \left(- \frac{8\pi^2}{bg_0^2} \right) \right]^d \quad (25)$$

with the choice $b=11/3N-2/3n_f$ (where n_f is a number of flavors) or $b=11/3N$, respectively. Finally, performing the differentiation yields the low-energy theorem (22). More accurate derivation (which gives the same final result) including a regularization of ultraviolet divergences in Eq. (22) can be found in [8]. Note that by definition perturbative contributions are always subtracted in vacuum condensates such as

⁴Somewhat differently from Witten's arguments, this was done without an explicit reference to the large N picture, but rather using the fact that the η' is light on a characteristic mass scale in the 0^- channel, and therefore its inclusion is a local $Q^2 \approx 0$ effect in the momentum space which must nullify the two-point function (18) in full QCD.

$\langle O \rangle$. Using now the low-energy theorem (22) for the particular choice $O = g^3 G^3$, we obtain

$$f_{\eta'}^{(c)} \simeq \frac{3}{4\pi^2 b} \frac{1}{m_c^2} \frac{\langle g^3 G^3 \rangle_{\text{YM}}}{\langle 0 | (\alpha_s/4\pi) G_{\mu\nu} \tilde{G}_{\mu\nu} | \eta' \rangle}. \quad (26)$$

This is the main result of this section. Coming back to our original definition (10), we see that we have related the residue of the charmed axial-vector current into η' with an apparently completely unrelated quantity which is the value of the cubic gluon condensate in pure Yang-Mills theory (the matrix element of the topological density is known [8], $\langle 0 | (\alpha_s/4\pi) G \tilde{G} | \eta' \rangle \simeq 0.04 \text{ GeV}^3$). This object will be addressed below, while here we would like to end up this section with a discussion of one important conceptual point. It is well known that usually nondiagonal transitions between quarks and gluons or between quarks of different flavors are suppressed—this is the famous Zweig rule. A most popular theoretical explanation of this phenomenon exploits the large N argumentation: in this limit all nondiagonal in flavors two-point functions are suppressed by powers of $1/N$ relatively to diagonal ones. Thus, at $N \rightarrow \infty$ any mixing dies off. On the other hand, the Zweig rule is strongly violated in the scalar and pseudoscalar $0^{+(-)}$ channels. A well known example of this violation is provided by the pseudoscalar meson nonet: while in the vector channel the ρ and ω mesons are almost pure mass eigenstates of the broken flavor $\text{SU}(3)$ $\rho \sim (\bar{u}u - \bar{d}d)$, $\omega \sim \bar{s}s$ and the ρ - ω mixing is small, nothing similar is observed in the pseudoscalar nonet. There the η is predominantly the octet $\eta \sim (\bar{u}u + \bar{d}d - 2\bar{s}s)$, which means that the mixing is 100%. The theoretical explanation [8] as to why the Zweig rule is violated for quark or gluon currents with the $0^{+(-)}$ quantum numbers is that in these channels there are direct instantons which are able to convert quarks into gluons and vice versa $\bar{q}q \leftrightarrow gg$ at the classical level without any suppression. Literally speaking, all factors ensuring a smallness drop out: powers of the coupling constant disappear since the instanton field is strong, $G_{\mu\nu} \sim 1/g$, and geometrical loop factors such as $1/(16\pi^2)$ do not arise because there are no loops. One of the striking examples of such is a conversion of gluons into photons [8]: while naively the amplitude

$$\left\langle 0 \left| -\frac{b\alpha_s}{8\pi} G^2 \right| 2\gamma \right\rangle = O\left(\left(\frac{\alpha_s}{\pi} \right)^2 \frac{\alpha}{\pi} \right), \quad (27)$$

in fact it is only $O(\alpha/\pi)$ according to a strict low-energy theorem [8] which reads

$$\left\langle 0 \left| \frac{\beta(\alpha_s)}{4\alpha_s} G^2 \right| \gamma(k_1) \gamma(k_2) \right\rangle = \frac{\alpha}{3\pi} N n_f \langle Q_q^2 \rangle F_{\mu\nu}^{(1)} F_{\mu\nu}^{(2)}. \quad (28)$$

(Here $\langle Q_q^2 \rangle$ is the mean quark electric charge and $F_{\mu\nu}^{(i)} = k_\mu^{(i)} \varepsilon_\nu^{(i)} - k_\nu^{(i)} \varepsilon_\mu^{(i)}$, $i=1,2$ stands for the fields strength of a plane wave.) We would like to note that this process is rather similar to our case. The gluons proceed to photons through a loop of the c quark, but the perturbatively expected suppression factor $(\alpha_s/\pi)^2$ does not occur.

A natural question to ask now is does all this mean that the large N picture is strongly violated in the $J^P = 0^{+(-)}$ channels? A related question is whether the experimental fact that $m_{\eta'}^2/m_\rho^2 > 1$ despite that $m_{\eta'}^2 \sim 1/N$, $m_\rho^2 = O(N^0)$ implies such a violation. The answer to both questions is no. Moreover, neither the direct instantons, nor the low-energy theorems are at variance with the large N picture. The point is that, as was explained in Ref. [8], N is the dimensionless parameter, and true mass relations look rather similar to $m_{\eta'}^2 = M^2/N$, where M^2 is some mass scale. It is usually tacitly assumed within the large N reasoning that this mass scale is universal for all hadrons. This is not the case, as was demonstrated by NSVZ [8]: the mass scale M^2 is not universal but determined by the quantum numbers of the channel considered. More concretely, this mass is set by a scale at which the asymptotic freedom is violated in the particular channel. The latter depends drastically on whether direct instantons are allowed (which is the case in the $0^{+(-)}$ channels) or not. If they are there, an interaction of external current with vacuum fields is very strong, and the asymptotic freedom breaks down at very small distances, i.e., a characteristic mass scale in this channel is not the typical hadron mass $\sim m_\rho^2$, but rather much higher. It is therefore clear that the second of the questions posed above is not properly formulated: the mass of the η' should be compared not with m_ρ^2 , but to a characteristic mass in the O^- channel which is $\sim 15 \text{ GeV}^2$ [8]. This fact is the ‘‘experimental’’ evidence that the $1/N$ argumentation is quite accurate for η' , and $1/N$ terms, omitted in Eq. (20), are small *in comparison to both explicitly written terms*. At the same time, the above consideration explains why the effects which we discuss are not negligibly small numerically: despite the fact that the matrix element $\langle 0 | g^3 G \tilde{G} G | \eta' \rangle \sim N^{-1/2}$, a large dimensional parameter in front of $N^{-1/2}$ is able to make it large in reality.

V. HOW LARGE IS $\langle g^3 G^3 \rangle$ IN PURE GLUODYNAMICS

We are now returning to the main subject of our consideration. Our task has reduced, according to Eq. (26), to a determination of the cubic gluon condensate $\langle g^3 G^3 \rangle_{\text{YM}}$ in pure YM theory. Note that this quantity does not have to (and in fact does not) coincide with the cubic condensate $\langle g^3 G^3 \rangle$ in the real world. While for the latter there exists a semiphenomenological estimate [18]

$$\langle g^3 G^3 \rangle = (0.06 - 0.1) \text{ GeV}^6 \quad (29)$$

obtained within the QCD sum rules approach, it is of no direct use in Eq. (26). Unfortunately, we are not aware of any method (except, perhaps, the lattice approach) which could reliably calculate $\langle g^3 G^3 \rangle_{\text{YM}}$ to an accuracy of, say, 20%. Because of this uncertainty, we are unable to get a theoretical prediction for $B(B \rightarrow K \eta')$ with a precision comparable to the experimental one. What instead will be argued in this section is that different estimates of the value $\langle g^3 G^3 \rangle_{\text{YM}}$ enable one to claim that the large number (1) is within the realms of our current understanding of *nonperturbative* QCD. We believe that this statement is interesting by itself in view of a failure of the standard approach to this problem (see Sec. II). It will be shown below that

$\langle g^3 G^3 \rangle_{\text{YM}} > \langle g^3 G^3 \rangle_{\text{QCD}}$ (this result follows directly from the theory and has a status of theorem), and moreover numerically

$$\langle g^3 G^3 \rangle_{\text{YM}} = (0.4 - 1.4) \text{ GeV}^6. \quad (30)$$

The first tool we are going to use is again the low-energy theorem (22). Let us recall how a very similar question on a value of the condensate $\langle g^2 G^2 \rangle_{\text{YM}}$ was addressed in the classical paper [8]. First of all, we note that this condensate corresponds to an imaginary world in which all quarks are very heavy. This world could be obtained from the real one when the masses of light u , d , and s quarks are smoothly drawn up to some large value m_q . By the decoupling theorem, this mass must not be very large—once it reaches the confinement scale $\mu \approx 200$ MeV, the heavy quarks decouple and do not influence any more the gluon condensate. On the other hand, the confinement scale μ is not too far from the s -quark mass $m_s \approx 150$ MeV. Therefore, one can expect that a value of $\langle g^2 G^2 \rangle$ in a world with $m_u = m_d = m_s \equiv m_q \approx 150$ MeV would give a reasonable estimate for the YM condensate $\langle g^2 G^2 \rangle_{\text{YM}}$. In the linear approximation in m_u, m_d we need to know the derivative

$$\frac{d}{dm_q} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle = -i \int dx \left\langle 0 \left| T \left\{ \frac{\alpha_s}{\pi} G^2(0) \bar{q}q(x) \right\} \right| 0 \right\rangle. \quad (31)$$

The latter two-point function is fixed by the low-energy theorem (22) to be proportional to the quark condensate known from elsewhere. Since the quark condensate is negative, it follows that $(d/dm_q) \langle g^2 G^2 \rangle > 0$. In this way NSVZ have obtained an estimate

$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\text{YM}} = (2-3) \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\text{QCD}}. \quad (32)$$

As has been argued in [8], the sign of effect $\langle g^2 G^2 \rangle_{\text{YM}} > \langle g^2 G^2 \rangle_{\text{QCD}}$ is in perfect agreement with the instanton picture. Indeed, raising the quark masses diminishes the chiral suppression of instantons and therefore increases $\langle G^2 \rangle$.

Proceeding analogously, we write

$$\frac{d}{dm_q} \langle g^3 G^3 \rangle = -i \int dx \langle 0 | T \{ g^3 G^3(0) \bar{q}q(x) \} | 0 \rangle. \quad (33)$$

The difference from the case of quadratic gluon condensate (31) is that the two-point function does not coincide with the low-energy theorem as it was in Eq. (31). Therefore, its value is not known exactly. Nevertheless, the low-energy theorem (22) can still be used to *estimate* the correlation function (33). To this end, consider the relation (22) in QCD with $b = 11/3N - 2/3n_f$ for three different operators

$$\begin{aligned} i \int dx \left\langle 0 \left| T \left\{ \frac{\alpha_s}{4\pi} G^2(x) \frac{\alpha_s}{4\pi} G^2(0) \right\} \right| 0 \right\rangle &= \frac{2}{b} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle, \\ i \int dx \left\langle 0 \left| T \left\{ \bar{q}q(x) \frac{\alpha_s}{4\pi} G^2(0) \right\} \right| 0 \right\rangle &= \frac{6}{b} \langle \bar{q}q \rangle, \\ i \int dx \left\langle 0 \left| T \left\{ g^3 G^3(x) \frac{\alpha_s}{4\pi} G^2(0) \right\} \right| 0 \right\rangle &= \frac{12}{b} \langle g^3 G^3 \rangle. \end{aligned} \quad (34)$$

We now assume that these low-energy theorems are saturated by some effective glueball state σ with $m_\sigma \sim 1$ GeV. It

should be stressed that we do not insist on existence of a real narrow glueball resonance with such mass. Actually, introducing such a (fictitious?) glueball amounts to an effective description of the physics of 0^+ channel. Analogous methods have been used in a similar in spirit problem of a strange content of the nucleon [19] which also deals with the 0^+ channel. Note that the glueball mass drops out in the final result (37). Introducing the residues

$$\left\langle 0 \left| \frac{\alpha_s}{4\pi} G^2 \right| \sigma \right\rangle = \lambda_1, \quad \langle 0 | \bar{q}q | \sigma \rangle = \lambda_2, \quad \langle 0 | g^3 G^3 | \sigma \rangle = \lambda_3 \quad (35)$$

we put Eqs. (34) in the form

$$\begin{aligned} \frac{\lambda_1^2}{m_\sigma^2} &= \frac{2}{b} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle, \\ \frac{\lambda_1 \lambda_2}{m_\sigma^2} &= \frac{6}{b} \langle \bar{q}q \rangle, \\ \frac{\lambda_3 \lambda_1}{m_\sigma^2} &= \frac{12}{b} \langle g^3 G^3 \rangle. \end{aligned} \quad (36)$$

Using these equations and assuming the same scalar glueball dominance in Eq. (33), we obtain

$$\frac{d}{dm_q} \langle g^3 G^3 \rangle = - \frac{\lambda_3 \lambda_2}{m_\sigma^2} = - \frac{36}{b} \frac{\langle \bar{q}q \rangle \langle g^3 G^3 \rangle}{\langle (\alpha_s/\pi) G^2 \rangle}. \quad (37)$$

As $\langle \bar{q}q \rangle < 0$, the sign of the derivative is fixed: $(d/dm_q) \langle g^3 G^3 \rangle > 0$. Moreover, while an expected accuracy of the estimate (37) is of the order of 100%, the sign is entirely model independent and in fact is fixed by the positivity of spectral densities in Eqs. (34). By the same token as for $(d/dm_q) \langle g^2 G^2 \rangle$, this sign agrees with expectations based on the instanton picture of the vacuum. Numerically, using the values⁵ $\langle \bar{q}q \rangle \approx -0.017 \text{ GeV}^3$, $\langle (\alpha_s/\pi) G^2 \rangle \approx 0.012 \text{ GeV}^4$, in the linear approximation in m_q Eq. (37) yields, for $m_q = 150$ MeV,

$$\langle g^3 G^3 \rangle_{\text{YM}} \approx 4.4 \langle g^3 G^3 \rangle_{\text{QCD}} = (0.26 - 0.44) \text{ GeV}^6, \quad (38)$$

where we have used the estimate (29). Note that the effect of going from QCD to gluodynamics is larger for the cubic condensate (38) than for the quadratic one (32). This fact is a direct consequence of the low-energy theorem (34), and can also be easily understood on dimensional grounds since $\langle g^3 G^3 \rangle \sim \langle g^2 G^2 \rangle^{3/2}$, increasing $\langle g^2 G^2 \rangle$ by a factor ~ 2.5

⁵The number for the quark condensate corresponds to the normalization point 1 GeV². Here we would like to point out that the normalization point in our Eq. (26) is $\mu^2 \approx m_c^2$, while the low-energy theorems refer, strictly speaking, to much lower $\mu \approx 500$ MeV. In view of a large numerical uncertainty of our results we neglect this perturbative evolution. Still, one has to bear in mind that the large anomalous dimension of the G^3 operator $\gamma = -18$ [20] is working in the same direction: taking it into account is only able to enhance our final result (30) or the estimate (38) by a factor of 2.

[see Eq. (32)] yields an increase of $\langle g^3 G^3 \rangle$ by $(2.5)^{3/2} \simeq 4$ in comparison to its value in QCD. Equation (38) constitutes our first estimate for a value of the cubic gluon condensate in pure YM theory which is based on the low-energy theorems (34) and the semiphenomenological information on its value in real world (29).

Another possible source of information on the value $\langle g^3 G^3 \rangle_{\text{YM}}$ is the instanton liquid vacuum model (see [15,16]). Two basic parameters of this model are average instanton size ρ_c and interinstanton separation R . The latter parameter is chosen such that to reproduce the phenomenological value of the gluon condensate $\langle g^2 G^2 \rangle$ using the fact that each instanton contributes a fixed amount $\int dx g^2 G^2 = 32\pi^2$ to this quantity. This yields the number $R \simeq 1/200 \text{ MeV}^{-1}$ for the phenomenological value (in QCD) $\langle (\alpha_s/\pi) G^2 \rangle \simeq 0.012 \text{ GeV}^4$. This number provides the upper limit for the instanton density in QCD as it implies that the entire gluon condensate is due to instantons. On the other hand, the ratio ρ_c/R does not depend on the value of condensate and is fixed dynamically to be $\rho_c/R \simeq 1/3$. A value of the cubic condensate calculated in this model was found to be essentially larger than the semiphenomenological number (29):

$$\frac{\langle g^3 G^3 \rangle}{\langle g^2 G^2 \rangle} = \frac{12}{5\rho_c^2} \simeq 0.9 \text{ GeV}^2, \quad \langle g^3 G^3 \rangle \simeq 0.4 \text{ GeV}^6 \quad (39)$$

(the formula for $\langle g^3 G^3 \rangle$ in terms of the instanton radius ρ_c was first established in [14]). We would like to make the following comment in reference to the result (39). For our purposes it is more suitable to discuss the instanton vacuum picture not in QCD, but in pure YM theory. We note that the instanton vacuum is more simple in gluodynamics than in QCD because of absence of the chiral suppression of instantons. In this case the interinstanton separation must be chosen to fit the gluon condensate $\langle g^2 G^2 \rangle$ in YM theory, which is according to Eq. (32) larger than the corresponding number in QCD. On the other hand, for gluodynamics the ratio $\rho_c/R \simeq 1/3$ remains the same. Using Eq. (39) and the estimate (32), we obtain

$$\langle g^3 G^3 \rangle_{\text{YM}} \simeq 1.7 \text{ GeV}^6, \quad (40)$$

which is a few times larger than our first estimate (38). We feel that this result provides the upper estimate for the quantity of interest, and the true answer for $\langle g^3 G^3 \rangle$ lies somewhere in between the two numbers (38) and (40).

Finally, we would like to discuss information on vacuum condensates, which is available from lattice simulations. The quadratic gluon condensate in YM theory on the lattice was reported to be

$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle = \begin{cases} 0.15 \text{ GeV}^4 & \text{SU}(2)[21] \\ 0.10 \text{ GeV}^4 & \text{SU}(3)[22] \end{cases}$$

We do not feel qualified enough to discuss a precision of these calculations. In particular, it is not very clear (at least to us) whether the large scale separation is accurately performed to make possible a comparison with the SVZ definition. Still, it seems undoubtful that these results point in the same direction as Eq. (32): the quadratic gluon condensate in YM theory is essentially larger than in QCD. As for the

cubic condensate, though a corresponding Monte Carlo data does exist, it is rather difficult to extract from it a value of the nonperturbative part of $\langle g^3 G^3 \rangle$ [23]. An estimate of this quantity can still be obtained in an indirect way as the lattice simulations of instantons suggest⁶ that the average size of instantons in gluodynamics on the lattice is approximately 1/400 MeV which is a bit larger than the value predicted by the instanton liquid model. In this case Eq. (39) together with the above value of the quadratic condensate yields the estimate [for the SU(3) color group]

$$\langle g^3 G^3 \rangle_{\text{YM}} \simeq 1.5 \text{ GeV}^6, \quad (41)$$

which is numerically close to Eq. (40). Comparing finally all three estimates (38), (40), and (41), we suggest that while Eq. (38) presumably gives a lower bound for the number of interest, Eqs. (40) and (41) seem to set up an upper limit with a possible short distance enhancement. A reasonable compromise yields our final estimate given above by Eq. (30).

We are now in a position to estimate the principal input in Eq. (11) which is the residue $f_{\eta'}^{(c)}$ of the charmed axial-vector current into the η' , and which was the main object of our consideration in this paper. Using Eqs. (26) and (30), we obtain the following answer for this parameter:

$$f_{\eta'}^{(c)} = (50 - 180) \text{ MeV}. \quad (42)$$

Note that literally the ‘‘experimental’’ number (14) corresponds to the value of the condensate $\langle g^3 G^3 \rangle_{\text{YM}} \simeq 1 \text{ GeV}^6$ which is about the midpoint of our prediction (30). Given the accuracy of our result (42), we thus conclude that the gluon mechanism seems to be sufficient to describe the data (1). Unfortunately, we are currently unable to improve our estimate (42), where the main source of uncertainty is due to a poor knowledge of the cubic condensate in YM theory (30). Some ways to do this will be discussed in the next section.

VI. CONCLUSIONS

In this paper we proposed a theory of the $B \rightarrow K \eta'$ decay. We showed that at the quark level this process proceeds via the $b \rightarrow \bar{c} c s$ weak decay followed by a conversion of the c -quark pair directly into the η' which is possible due to a presence of a nonvalence Zweig rule violating ‘‘intrinsic charm’’ component of the η' wave function. We have found that a mechanism of breaking down the Zweig rule in our case is of a purely nonperturbative origin. We have further evaluated a most important ingredient of the factorized $B \rightarrow K \eta'$ amplitude, which is the matrix element of the charmed current $\langle 0 | \bar{c} \gamma_\mu \gamma_5 c | \eta' \rangle$, using a combination of the operator product expansion technique, large N approach, and QCD low-energy theorems. Our results demonstrate that the proposed mechanism is likely to exhaust an extremely large branching ratio measured by the CLEO Collaboration, with a certain reservation for a poor accuracy of our final answer (42). We do not pretend to have made a numerically reliable calculation of the matrix element $\langle 0 | \bar{c} \gamma_\mu \gamma_5 c | \eta'(p) \rangle$

⁶E. V. Shuryak (private communication). See also [16].

$=if_{\eta'}^{(c)}p_\mu$ which in our opinion determines the $B \rightarrow K \eta'$ decay width. We have rather presented a semiquantitative (but parametrically well established) picture which demonstrates a close relation between this matrix element and properties of the vacuum of QCD (or, more accurately, YM theory). Though the obtained result looks unexpectedly large, it has a good explanation within QCD, and is related to strong fluctuations in vacuum $0^{+(-)}$ channels. The main source of uncertainty in our approach is well localized and related to a value of the cubic condensate $\langle g^3 G^3 \rangle_{\text{YM}}$ which is currently not known with enough precision. Yet, the exact low-energy theorems indicate that this matrix element is quite large. The instanton vacuum model and lattice calculations also seem to favor a large value of this condensate. In this reference, more refined calculations of this quantity are highly welcome. In particular, it would be very interesting if this condensate could be reliably extracted from lattice simulations. An alternative way which can be suggested to improve the determination of the condensate $\langle g^3 G^3 \rangle_{\text{YM}}$ is akin to the idea of the QCD sum rule approach [14]. If we had other physical processes which essentially depend on the same cubic condensate, it could be then fixed ‘‘phenomenologically’’ once and forever with an expected consistency between theoretical predictions for different physical amplitudes. We plan to return to these issues elsewhere.

We would like to emphasize that the conclusion on a large Zweig rule violating c -quark component of the η' certainly goes beyond the particular example of the $B \rightarrow K \eta'$ decay and may well be important in other physical processes. We repeat that there are two basic reasons for a large magnitude of the residue $f_{\eta'}^{(c)}$: (1) the c -quark mass is not too far from the hadron scale 1 GeV and (2) the Zweig rule is badly broken down in the $0^{+(-)}$ channels. While there is nothing particularly special (except for its numerical effect) about the first factor, the second one is specific to the unique nature and quantum numbers of the η' . Therefore, we do not expect that any other than the η' light particle could yield a similar contribution to the B decay. In a more general content, there is an increasing evidence for importance of nonvalence Zweig rule violating components in hadrons. We remind the reader that the problem of the strange quarks in the nucleon (the so-called $\pi N \sigma$ term) is resolved [19] (see also [24]) within physics which is very similar to that discussed in this paper. Furthermore, there are many other examples where ‘‘intrinsic’’ nonvalence configurations (including, in particular, the ‘‘intrinsic charm’’ hadron components) seem important (see, e.g., [25] and [26]). All these examples unambiguously demonstrate that nonvalence components of the hadron can be sizeable. In QCD terms such a situation means that a corresponding matrix element has a nonperturbative origin without the naively expected α_s/π suppression. We have shown in this paper that this experimentally testable physics is amenable to theoretical control. In this respect, the η' from $B \rightarrow \eta'$ decays is an excellent laboratory for the study of fundamental properties of strongly interacting QCD.

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APPENDIX

The purpose of this appendix is to derive Eq. (16) in the text. A convenient machinery for such a class of problem was invented by Schwinger a long time ago [27]. The Schwinger technique allows us in many instances (e.g., when one is interested in a short distance expansion) to operate with a propagator in external field without a specification of the field. The result has a form of expansion in powers of the field and its derivatives. The relevant object in our problem is the c -quark propagator at short distances $\sim 1/m_c$, and the expansion in the external gluon field amounts to a representation of the propagator in a form of series in powers of Λ_{QCD}/m_c which is a sort of OPE. We refer the interested reader to the pedagogical technical review [28] for more detail and relevant references.

The Schwinger operator approach is based on a realization of commutation relations of the coordinate X_μ and momentum P_μ ($P_\mu = iD_\mu$, where D_μ is the covariant derivative) operators

$$\begin{aligned} [P_\mu, X_\nu] &= i g_{\mu\nu}, \\ [P_\mu, P_\nu] &= i g G_{\mu\nu}^a T^a, \end{aligned} \quad (\text{A1})$$

where T^a are the generators of the color group. One introduces in the coordinate space a formal complete set of states $|x\rangle$ as the eigenstates of the coordinate operator X_μ :

$$\begin{aligned} X_\mu |x\rangle &= x_\mu |x\rangle, \\ \langle y|x\rangle &= \delta(x-y), \\ \int dx |x\rangle \langle x| &= 1, \end{aligned} \quad (\text{A2})$$

while in this basis the momentum operator P_μ acts as the covariant derivative

$$\langle y|P_\mu|x\rangle = \left(i \frac{\partial}{\partial x_\mu} + g A_\mu^a(x) T^a \right) \delta(x-y). \quad (\text{A3})$$

In these notations we have to evaluate the expression

$$\bar{c} i \gamma_5 c = \langle x | \text{Tr} \{ \gamma_5 (\not{P} - m)^{-1} \} | x \rangle, \quad (\text{A4})$$

where m is the c -quark mass and Tr denotes the trace over both color and Lorentz indices. Using a resolution of unity

$$1 = (\not{P} + m)^{-1} (\not{P} + m)$$

and the formula $[\sigma_{\mu\nu} = i/2(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)]$

$$\not{P}^2 = P^2 + \frac{1}{2} \sigma_{\mu\nu} G_{\mu\nu}^a T^a,$$

we expand Eq. (A4) in powers of $\sigma G \equiv \sigma_{\mu\nu} G_{\mu\nu}^a T^a$:

$$\begin{aligned} \bar{c}i\gamma_5c &= m \left\langle x \left| \text{Tr} \left\{ \gamma_5 \frac{g^2}{4} \frac{1}{(P^2-m^2)^2} (\sigma G) \frac{1}{P^2-m^2} (\sigma G) - \frac{g^3}{8} \frac{1}{(P^2-m^2)^2} (\sigma G) \frac{1}{P^2-m^2} (\sigma G) \frac{1}{P^2-m^2} (\sigma G) + \dots \right\} \right| x \right\rangle \\ &\equiv D_1 + D_2 + \dots, \end{aligned} \tag{A5}$$

where the ellipsis stands for higher power terms in the external field expansion. It is convenient to start the calculation with the second term in Eq. (A5). It explicitly contains the gluon field $G_{\mu\nu}$ to the third power, and therefore to our accuracy one can neglect the noncommutativity of the operators P_α and $G_{\mu\nu}$. Thus

$$D_2 = -\frac{g^3}{8} m \text{Tr} \left\{ \frac{1}{(P^2-m^2)^4} \gamma_5 (\sigma G)^3 \right\}. \tag{A6}$$

To the same accuracy the momentum operator P_μ in the denominator of Eq. (A6) is substituted by the c number p_μ , and the x integration is performed using the formula

$$\begin{aligned} \langle x | (P^2 - m^2)^{-1}_{A=0} | x \rangle &= \int \frac{d^4 p}{(2\pi)^4} (p^2 - m^2)^{-(n-1)} \\ &= \frac{(-1)^n m^{6-2n}}{16\pi^2 i (n-2)(n-3)}, \end{aligned}$$

which yields

$$D_2 = -\frac{ig^3}{2^8 \times 3 \times \pi^2 m^3} \text{Tr} \{ \gamma_5 (\sigma G)^3 \}. \tag{A7}$$

A calculation of the trace over the Lorentz indices gives

$$\text{Tr} \{ \gamma_5 (\sigma G)^3 \} = -2^5 \text{tr}_c G_{\mu\nu} \tilde{G}_{\nu\alpha} G_{\alpha\mu},$$

where tr_c stands for the trace over the color indices. The latter is

$$\text{tr}_c G_{\mu\nu} \tilde{G}_{\nu\alpha} G_{\alpha\mu} = \frac{i}{2} f^{abc} G_{\mu\nu}^a \tilde{G}_{\nu\alpha}^b G_{\alpha\mu}^c \equiv \frac{i}{2} G \tilde{G} G,$$

and finally we obtain

$$D_2 = -\frac{1}{16 \times 3 \times \pi^2 m^3} g^3 G \tilde{G} G. \tag{A8}$$

A calculation of the first term in Eq. (A5) is more tedious as now we have to take into account the noncommutativity of operators in order to evaluate this expression to the $G\tilde{G}$ accuracy. Using the identity

$$\begin{aligned} (\sigma G) \frac{1}{P^2-m^2} &= \frac{1}{P^2-m^2} (P^2-m^2) (\sigma G) \frac{1}{P^2-m^2} \\ &= \frac{1}{P^2-m^2} (\sigma G) \\ &\quad + \frac{1}{P^2-m^2} [P^2, (\sigma G)] \frac{1}{P^2-m^2}, \end{aligned} \tag{A9}$$

we write

$$\begin{aligned} D_1 &= \frac{g^2}{4} m \left\langle x \left| \text{Tr} \left\{ \gamma_5 \frac{1}{(P^2-m^2)^3} (\sigma G)^2 \right. \right. \right. \\ &\quad \left. \left. + \gamma_5 \frac{1}{(P^2-m^2)^3} [P^2, (\sigma G)] \frac{1}{P^2-m^2} (\sigma G) \right\} \right| x \right\rangle \\ &\equiv \Pi_1 + \Pi_2. \end{aligned} \tag{A10}$$

The first term in Eq. (A10) is readily calculated to yield

$$\Pi_1 = -\frac{g^2}{32 \times \pi^2 m} G_{\mu\nu} \tilde{G}_{\mu\nu}, \tag{A11}$$

while the second one needs more care. Let us perform first the trace over the Lorentz indices:

$$\begin{aligned} \Pi_2 &= -2ig^2 \left\langle x \left| \text{Tr} \left\{ \frac{1}{(P^2-m^2)^3} [P^2, G_{\mu\nu}] \right. \right. \right. \\ &\quad \left. \left. \times \frac{1}{P^2-m^2} \tilde{G}_{\mu\nu} \right\} \right| x \right\rangle. \end{aligned} \tag{A12}$$

Calculating the commutator

$$\begin{aligned} [P^2, G_{\mu\nu}] &= P_\lambda [P_\lambda, G_{\mu\nu}] + [P_\lambda, G_{\mu\nu}] P_\lambda \\ &= 2iP_\lambda (D_\lambda G_{\mu\nu}) + D^2 G_{\mu\nu}, \end{aligned}$$

we obtain

$$\begin{aligned}
\Pi_2 &= 4g^2 m \left\langle x \left| \text{Tr} \left\{ \frac{1}{(P^2 - m^2)^3} P_\lambda (D_\lambda G_{\mu\nu}) \frac{1}{P^2 - m^2} \tilde{G}_{\mu\nu} \right\} \right| x \right\rangle - 2ig^2 m \left\langle x \left| \text{Tr} \left\{ \frac{1}{(P^2 - m^2)^3} (D^2 G_{\mu\nu}) \frac{1}{P^2 - m^2} \tilde{G}_{\mu\nu} \right\} \right| x \right\rangle \\
&= 4g^2 m \left\langle x \left| \text{Tr} \left\{ \frac{1}{(P^2 - m^2)^4} P_\lambda (D_\lambda G_{\mu\nu}) \tilde{G}_{\mu\nu} \right\} \right| x \right\rangle + 4g^2 m \left\langle x \left| \text{Tr} \left\{ \frac{1}{(P^2 - m^2)^4} [P^2, P_\lambda (D_\lambda G_{\mu\nu})] \frac{1}{P^2 - m^2} \tilde{G}_{\mu\nu} \right\} \right| x \right\rangle \\
&\quad - 2ig^2 m \left\langle x \left| \text{Tr} \left\{ \frac{1}{(P^2 - m^2)^4} (D^2 G_{\mu\nu}) \tilde{G}_{\mu\nu} \right\} \right| x \right\rangle + \dots . \tag{A13}
\end{aligned}$$

One can easily see that the first term in this expression gives rise to higher dimensional operators and thus does not contribute to our accuracy. However, the second term does contain a needed power of the gluon field since

$$\begin{aligned}
[P^2, P_\lambda (D_\lambda G_{\mu\nu})] &= 2iP_\lambda P_\beta D_\beta D_\lambda G_{\mu\nu} + \dots \\
&= i\{P_\lambda, P_\beta\} D_\beta D_\lambda G_{\mu\nu} + \dots ,
\end{aligned}$$

which results in

$$\begin{aligned}
\Pi_2 &= 2ig^2 m \left\langle x \left| \text{Tr} \left\{ \frac{2}{(P^2 - m^2)^5} \{P_\lambda, P_\beta\} D_\beta D_\lambda G_{\mu\nu} \tilde{G}_{\mu\nu} \right. \right. \right. \\
&\quad \left. \left. \left. - \frac{1}{(P^2 - m^2)^4} (D^2 G_{\mu\nu}) \tilde{G}_{\mu\nu} \right\} \right| x \right\rangle . \tag{A14}
\end{aligned}$$

In the first term to our accuracy one can substitute $P_\mu \rightarrow p_\mu$. We further use the Bianki identity to evaluate

$$\begin{aligned}
(D^2 G_{\mu\nu}) \tilde{G}_{\mu\nu} &= D_\alpha (-D_\nu G_{\alpha\mu} - D_\mu G_{\nu\alpha}) \tilde{G}_{\mu\nu} \\
&= 2iG_{\alpha\mu} \tilde{G}_{\mu\nu} G_{\nu\alpha} \tag{A15}
\end{aligned}$$

and eventually obtain

$$\Pi_2 = -\frac{1}{32 \times 3 \times \pi^2 m^3} g^3 G \tilde{G} G . \tag{A16}$$

Finally, collecting together Eqs. (A8), (A11), and (A16) and multiplying the whole answer by $2m$, we arrive at

$$2m\bar{c}i\gamma_5 c = -\frac{\alpha_s}{4\pi} G \tilde{G} - \frac{1}{16\pi^2 m^2} g^3 G \tilde{G} G + \dots , \tag{A17}$$

which completes the proof of Eq. (16).

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