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Nonleptonic two-body charmed meson decays in an effective model for their semileptonic decays

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We analyze $D\to PV$, $D\to PP$, and $D\to VV$ decays within a model developed to describe the semileptonic decays $D\to Vl\nu_l$ and $D\to Pl\nu_l$. This model combines the heavy quark effective Lagrangian and chiral perturbation theory. We determine amplitudes for decays in which the direct weak annihilation of the initial D meson is absent or negligible, and in which the final state interactions are small. This analysis reduces the arbitrariness in the choice of model parameters. The calculated decay widths are in good agreement with the experimental results. [S0556-2821(97)01323-4]

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I. INTRODUCTION

The nonleptonic D meson decays are challenging to understand theoretically [1–14]. The short distance effects are now well understood [15,16], but the nonperturbative techniques required for the evaluation of certain matrix elements are based on the approximate models. Often the factorization approximation is used [17,18,9–12]. The amplitude for the nonleptonic weak decay is then considered as a sum of the ‘‘spectator’’ contribution (Fig. 1) and the ‘‘annihilation’’ contribution, the direct annihilation of the initial heavy meson (Fig. 2). In the determination of the ‘‘spectator’’ contribution one uses the knowledge of the hadronic matrix elements calculated in D meson semileptonic decays.

Recently we have developed a model for the semileptonic decays $D\to Vl\nu_l$ and $D\to Pl\nu_l$, where P and V are light $J^P=0^-$ and 1^- mesons, respectively [19]. This model combines heavy quark effective theory (HQET) and the chiral Lagrangians. HQET is valid at a small recoil momentum [20,21] and can give definite predictions for heavy to light ($D\to V$ or $D\to P$) semileptonic decays in the kinematic region with large momentum transfer q^2 to the lepton pair. Unfortunately, it cannot predict the q^2 dependence of the form factors [20,21]. For these reasons, we have modified the Lagrangian for heavy and light pseudoscalar and vector mesons given by HQET and chiral symmetry [20]. Our model [19] gives a natural explanation of the pole-type form factors in the whole q^2 range, and it determines which form factors have a pole-type or a constant behavior, confirming the results of the QCD sum rules analysis [22]. To demonstrate that this model works well, we have calculated the decay widths in all measured charm meson semileptonic decays [19]. The model parameters were determined by the experimental values of two measured semileptonic decay widths. The predictions of the model are in good agreement with the remaining experimental data on semileptonic decays.

Another problem in the analysis of nonleptonic D meson decays is the final state interactions (FSI's) [9–12,17,18]. These arise from the interference of different isospin states or the presence of intermediate resonances, and both spectator and annihilation amplitudes can be affected. The FSI's are especially important for the annihilation contribution, which can often be successfully described by the dominance of nearby scalar or pseudoscalar resonances [9–12]. The effective model developed to describe the $D\to V(P)l\nu_l$ decay widths [19] contains only light vector and pseudosca-

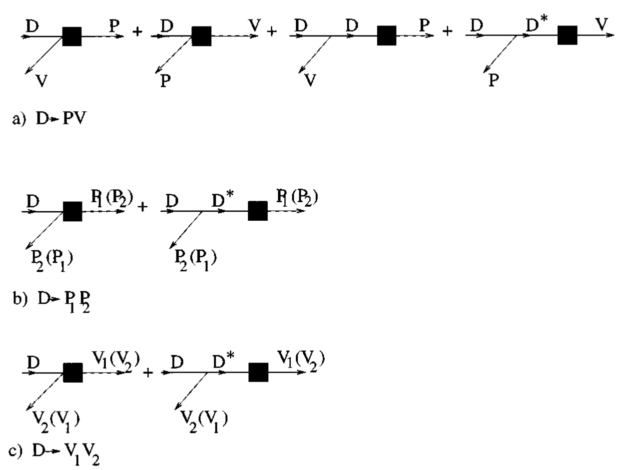

FIG. 1. Spectator contributions to nonleptonic two-body D meson decay: (a) $D\to PV$, (b) $D\to P_1P_2$, and (c) $D\to V_1V_2$. The black boxes represent the effective weak interaction and P and V are light psudoscalar and vector mesons, resepctively.

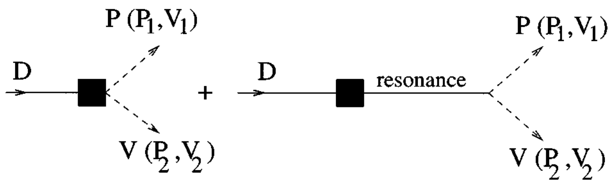

FIG. 2. Annihilation contributions to nonleptonic two-body D meson decays. The black box represents the effective weak interaction.

lar final states and, therefore, is not applicable to the annihilation amplitudes. Consequently, in the present paper we only apply this effective model to analyze those $D\to PV$, $D\to PP$, and $D\to VV$ decays in which the annihilation amplitude is absent or negligible. Other FSI's might arise as a result of elastic or inelastic rescattering. In this case, the two-body nonleptonic D meson decay amplitudes can be written in terms of isospin amplitudes and strong interaction phases [3]. As usual, we assume that the important contributions to FSI's are included in these phases. In fact, we will avoid the effects of the FSI's strong interaction phases by considering only the D meson decay modes in which the final state involves only a single isospin. Our analysis then includes the decays $D^+\to\overline{K^{*0}}\pi^+$, $D^+\to\rho^+\overline{K^0}$, $D^+\to\overline{K^0}\pi^+$, $D^+\to\overline{K^{*0}}\rho^+$, $D^+\to\Phi\pi^+$, $D_s^+\to\Phi\pi^+$, $D_s^+\to\Phi\rho^+$, $D^0\to\Phi\omega^0$, $D^0\to\Phi\eta$, $D^+\to\rho^+\eta(\eta')$, and $D^0\to\omega^0\eta(\eta')$.

To evaluate the spectator graphs for nonleptonic decays (Fig. 1) we use the form factors for the $D\to V$ and $D\to P$ weak decays, calculated for the semileptonic decays [19]. This explores how well their particular q^2 behavior also explains the nonleptonic decay amplitudes. At the same time the analysis of the nonleptonic decays enables us to choose between different solutions for the model parameters found in the semileptonic decays, determining the set of the solutions which is in the best agreement with the experimental results for the nonleptonic decay widths. Moreover, we obtain a value for the parameter β, which can not be determined from the semileptonic decay alone, but enters in the nonleptonic decays.

TABLE I. The pole masses and decay constants in GeV.

H	m_H	f_H	P	m_P	f_P	V	m_V	f_V
D	1.87	0.21 ± 0.04	π	0.14	0.13	ρ	0.77	0.216
D_s	1.97	0.24 ± 0.04	K	0.50	0.16	K^*	0.89	0.216
D^*	2.01	0.21 ± 0.04	η	0.55	0.13 ± 0.008	ω	0.78	0.156
D_s^*	2.11	0.24 ± 0.04	η'	0.96	0.11 ± 0.007	Φ	1.02	0.233

The paper is organized as follows. In Sec. II we present the effective Lagrangian for heavy and light pseudoscalar and vector mesons, determined by the requirements of HQET and chiral symmetry, and we briefly review the results previously obtained for the $D\to Vl\nu_l$, $D\to Pl\nu_l$ decays [19]. In Sec. III we analyze the nonleptonic decay widths. Finally, a short summary of the results is given in Sec. IV.

II. HQET AND CHPT LAGRANGIAN FOR $D\to V(P)l\nu$

We incorporate in our Lagrangian both the heavy flavor SU(2) symmetry [23,24] and the $SU(3)_L\times SU(3)_R$ chiral symmetry, spontaneously broken to the diagonal $SU(3)_V$ [25], which can be used for the description of heavy and light pseudoscalar and vector mesons. A similar Lagrangian, but without the light vector octet, was first introduced by Wise [21], Burdman and Donoghue [26], and Yan *et al.* [27]. It was then generalized with the inclusion of light vector mesons in [1,20,28].

The light degrees of freedom are described by the 3×3 Hermitian matrices

$$\Pi=\begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}}+\frac{\eta_0}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}}+\frac{\eta_0}{\sqrt{3}} & K^0 \\ K^- & \overline{K^0} & -\frac{2}{\sqrt{6}}\eta_8+\frac{\eta_0}{\sqrt{3}} \end{pmatrix} \tag{1}$$

and

$$\rho_\mu=\begin{pmatrix} \frac{\rho_\mu^0+\omega_\mu}{\sqrt{2}} & \rho_\mu^+ & K_\mu^{*+} \\ \rho_\mu^- & \frac{-\rho_\mu^0+\omega_\mu}{\sqrt{2}} & K_\mu^{*0} \\ K_\mu^{*-} & \overline{K^{*0}}_\mu & \Phi_\mu \end{pmatrix} \tag{2}$$

for the pseudoscalar and vector mesons, respectively. The mass eigenstates are defined by $\eta=\eta_8\cos\theta_P-\eta_0\sin\theta_P$ and $\eta'=\eta_8\sin\theta_P+\eta_0\cos\theta_P$, where $\theta_P=(-20\pm5)°$ [30] is the η-η' mixing angle. The matrices (1) and (2) are conveniently written in terms of

$$u=\exp\left(\frac{i\Pi}{f}\right), \tag{3}$$

where f is the pseudoscalar decay constant and

TABLE II. The pole mesons and the constants w_V, K_V, w_P, and K_P for the Cabibbo-allowed and Cabibbo-suppressed $D\to VP$ decays. Here $c=\cos\theta_C$ and $s=\sin\theta_C$ and θ_C is the Cabibbo angle. The $f_{1\mathrm{mix}}$, $f'_{1\mathrm{mix}}$, $f_{2\mathrm{mix}}$, and $f'_{2\mathrm{mix}}$ are functions of the η-η' mixing angle θ_P and decay constants f_η, $f_{\eta'}$ given in Eq. (31).

H	V	P	H'	H'^*	w_V	K_V	w_P	K_P
D^+	$\overline{K^*}^0$	π^+	D_s^+	D^{*0}	a_1c^2	1	a_2c^2	1
D^+	ρ^+	$\overline{K^0}$	D^0	D_s^{*+}	a_2c^2	1	a_1c^2	1
D_s^+	Φ	π^+	D_s^+		a_1c^2	1	0	0
D^+	Φ	π^+		D^{*0}	0	0	a_2sc	1
D^0	Φ	π^0		D^{*0}	0	0	a_2sc	$1/\sqrt{2}$
D_s^+	ρ^+	η		D_s^{*+}	0	0	a_1c^2	$f_{2\mathrm{mix}}$
D_s^+	ρ^+	η'		D_s^{*+}	0	0	a_1c^2	$f'_{2\mathrm{mix}}$
D^+	ρ^+	η	D^0	D^{*+}	$a_2sc(f_{2\mathrm{mix}}-f_{1\mathrm{mix}})$	1	$-a_1sc$	$f_{1\mathrm{mix}}$
D^+	ρ^+	η'	D^0	D^{*+}	$a_2sc(f'_{2\mathrm{mix}}-f'_{1\mathrm{mix}})$	1	$-a_1sc$	$f'_{1\mathrm{mix}}$
D^0	Φ	η		D^{*0}	0	0	a_2sc	$f_{1\mathrm{mix}}$
D^0	Φ	η'		D^{*0}	0	0	a_2sc	$f'_{1\mathrm{mix}}$
D^0	ω	η	D^0	D^{*0}	$a_2sc(f_{1\mathrm{mix}}-f_{2\mathrm{mix}})$	$1/\sqrt{2}$	a_2sc	$f_{1\mathrm{mix}}/\sqrt{2}$
D^0	ω	η'	D^0	D^{*0}	$a_2sc(f'_{1\mathrm{mix}}-f'_{2\mathrm{mix}})$	$1/\sqrt{2}$	a_2sc	$f'_{1\mathrm{mix}}/\sqrt{2}$

$$\hat{\rho}_\mu = i\frac{g_V}{\sqrt{2}}\rho_\mu, \tag{4}$$

where $g_V=5.9$ is given by the values of the vector masses since we assume the exact vector dominance [19]. Introducing the vector and axial vector currents $\mathcal{V}_\mu=\frac{1}{2}(u^\dagger\partial_\mu u+u\partial_\mu u^\dagger)$ and $\mathcal{A}_\mu=\frac{1}{2}(u^\dagger\partial_\mu u-u\partial_\mu u^\dagger)$ and the gauge field tensor $F_{\mu\nu}(\hat{\rho})=\partial_\mu\hat{\rho}_\nu-\partial_\nu\hat{\rho}_\mu+[\hat{\rho}_\mu,\hat{\rho}_\nu]$, the light meson part of the strong Lagrangian can be written as

$$\mathcal{L}_{\mathrm{light}}=-\frac{f^2}{2}\{\mathrm{tr}(\mathcal{A}_\mu\mathcal{A}^\mu)+2\ \mathrm{tr}[(\mathcal{V}_\mu-\hat{\rho}_\mu)^2]\} + \frac{1}{2g_V^2}\mathrm{tr}[F_{\mu\nu}(\hat{\rho})F^{\mu\nu}(\hat{\rho})]. \tag{5}$$

Both the heavy pseudoscalar and the heavy vector mesons are incorporated in the 4×4 matrix,

$$H_a=\tfrac{1}{2}(1+\not{v})(D^*_{a\mu}\gamma^\mu-D_a\gamma_5), \tag{6}$$

where $a=1,2,3$ is the $\mathrm{SU}(3)_V$ index of the light flavors and $D^*_{a\mu}$ and D_a annihilate a spin-1 and spin-0 heavy meson $c\overline{q_a}$ of velocity v, respectively. They have a mass dimension 3/2 instead of the usual 1, so that the Lagrangian is explicitly mass independent in the heavy quark limit $m_c\to\infty$. Defining

$$\overline{H_a}=\gamma^0H_a^\dagger\gamma^0=(D^{*\dagger}_{a\mu}\gamma^\mu+D_a^\dagger\gamma_5)\tfrac{1}{2}(1+\not{v}), \tag{7}$$

we can write the leading order strong Lagrangian as

$$\begin{aligned}\mathcal{L}_{\mathrm{even}}=\mathcal{L}_{\mathrm{light}}&+i\mathrm{Tr}[H_av_\mu(\partial^\mu+\mathcal{V}^\mu)\overline{H_a}]\\&+ig\mathrm{Tr}[H_b\gamma_\mu\gamma_5(\mathcal{A}^\mu)_{ba}\overline{H_a}]\\&+i\beta\mathrm{Tr}[H_bv_\mu(\mathcal{V}^\mu-\hat{\rho}^\mu)_{ba}\overline{H_a}]\\&+\frac{\beta^2}{4f^2}\mathrm{Tr}(\overline{H_b}H_a\overline{H_a}H_b).\end{aligned} \tag{8}$$

This Lagrangian contains two unknown parameters g and β, which are not determined by symmetry arguments, and must be determined empirically. This is the most general even-parity Lagrangian of leading order in the heavy quark mass ($m_Q\to\infty$) and the chiral symmetry limit ($m_q\to0$ and the minimal number of derivatives).

TABLE III. The pole mesons and the constants w_1, $K_{V(1)}$, w_2, and $K_{V(2)}$ for the Cabibbo-allowed and Cabibbo-suppressed $D\to V_{(1)}V_{(2)}$ decays. Here $c=\cos\theta_C$ and $s=\sin\theta_C$ and θ_C is the Cabibbo angle.

H	V_1	V_2	H'^*_1	H'^*_2	w_1	$K_{V(1)}$	w_2	$K_{V(2)}$
D^+	$\overline{K^*}^0$	ρ^+	D_s^{*+}	D^{*0}	a_1c^2	1	a_2c^2	1
D_s	ρ^+	Φ	D_s^{*+}		a_1c^2	1	0	0
D^0	ρ^0	Φ	D^{*0}		a_2sc	$1/\sqrt{2}$	0	0
D^+	ρ^+	Φ	D^{*0}		a_2sc	1	0	0
D^0	ω	Φ	D^{*0}		a_2sc	$1/\sqrt{2}$	0	0

TABLE IV. Four possible solutions for the model parameters as determined by the $D^+ \to \overline{K^{*0}} l^+ \nu_l$ data.

	λ [GeV^{-1}]	α_1 [GeV$^{1/2}$]	α_2 [GeV$^{1/2}$]
Set I	-0.34 ± 0.07	-0.14 ± 0.01	-0.83 ± 0.04
Set II	-0.34 ± 0.07	-0.14 ± 0.01	-0.10 ± 0.03
Set III	-0.74 ± 0.14	-0.064 ± 0.007	-0.60 ± 0.03
Set IV	-0.74 ± 0.14	-0.064 ± 0.007	$+0.18 \pm 0.03$

We will also need the odd-parity Lagrangian for the heavy meson sector. The lowest order contribution to this Lagrangian is given by

$$\mathcal{L}_{\rm odd} = i\lambda \,\mathrm{Tr}[H_a \sigma_{\mu\nu} F^{\mu\nu}(\hat{\rho})_{ab} \overline{H_b}]. \tag{9}$$

The parameter λ is free, but we know that this term is of the order $1/\Lambda_\chi$ with Λ_χ being the chiral perturbation theory scale [29].

In our calculation of the D meson semileptonic decays to leading order in both $1/M$ and the chiral expansion we previously showed that the weak current is [19]

$$\begin{aligned} J_a^\mu = & \frac{1}{2} i \alpha \mathrm{Tr}[\gamma^\mu (1-\gamma_5) H_b u_{ba}^\dagger] \\ & + \alpha_1 \mathrm{Tr}[\gamma_5 H_b (\hat{\rho}^\mu - \mathcal{V}^\mu)_{bc} u_{ca}^\dagger] \\ & + \alpha_2 \mathrm{Tr}[\gamma^\mu \gamma_5 H_b v_\alpha (\hat{\rho}^\alpha - \mathcal{V}^\alpha)_{bc} u_{ca}^\dagger] + \cdots, \end{aligned} \tag{10}$$

where $\alpha = f_D \sqrt{m_D}$ [21]. The α_1 term was first considered in [20]. We found [19] that the α_2 gives a contribution of the same order in $1/M$ and the chiral expansion as the term proportional to α_1.

The $H \to V$ and $H \to P$ current matrix elements can be quite generally written as

TABLE V. The pole mesons and the constants w_1, $K_{P(1)}$, w_2, and $K_{P(2)}$ for the $D \to P_{(1)} P_{(2)}$ decay. Here $c = \cos\theta_C$ and $s = \sin\theta_C$ and θ_C is the Cabibbo angle.

H	P_1	P_2	H'^*_1	H'^*_2	w_1	$K_{P(1)}$	w_2	$K_{P(2)}$
D^+	$\overline{K^0}$	π^+	D_s^{*+}	D^{*0}	$a_1 c^2$	1	$a_2 c^2$	1

$$\begin{aligned} \langle V_{(i)}(\epsilon, p') | (V-A)^\mu | H(p) \rangle \\ = & -\frac{2V^{(i)}(q^2)}{m_H + m_{V(i)}} \epsilon^{\mu\nu\alpha\beta} \epsilon^*_\nu p_\alpha p'_\beta - i \epsilon^* . q \frac{2 m_{V(i)}}{q^2} q_\mu A_0^{(i)} \\ & \times (q^2) + i (m_H + m_{V(i)}) \left(\epsilon^*_\mu - \frac{\epsilon^* \cdot q}{q^2} q_\mu \right) A_1^{(i)}(q^2) \\ & - \frac{i\epsilon^* \cdot q}{m_H + m_{V(i)}} \left[(p+p')_\mu - \frac{m_H^2 - m_{V(i)}^2}{q^2} q_\mu \right] A_2^{(i)}(q^2), \end{aligned} \tag{11}$$

and

$$\begin{aligned} \langle P_{(i)}(p') | (V-A)_\mu | H(p) \rangle \\ = & \left[(p+p')_\mu - \frac{m_H^2 - m_{P(i)}^2}{q^2} q_\mu \right] F_1^{(i)}(q^2) \\ & + \frac{m_H^2 - m_{P(i)}^2}{q^2} q_\mu F_0^{(i)}(q^2), \end{aligned} \tag{12}$$

where $q = p - p'$ is the exchanged momentum and the index (i) specifies the particular final meson, P or V. In order that these matrix elements be finite at $q^2 = 0$, the form factors must satisfy the relations

$$A_0(0) + \frac{m_H + m_V}{2m_V} A_1(0) - \frac{m_H - m_V}{2m_V} A_2(0) = 0, \tag{13}$$

$$F_1(0) = F_0(0), \tag{14}$$

and, therefore, are not free parameters.

TABLE VI. The braching ratios for the decays that depend only on the parameter g. The second and third columns give the predictions for the two possible values $g_<$ and $g_>$, while the fourth column gives the experimental braching ratios [30]. The theoretical error bars are due to the uncertainty of the parameter g.

	$\mathcal{B}_{\rm th}$[%] $g = g_< = -0.96 \pm 0.18$	$\mathcal{B}_{\rm th}$[%] $g = g_> = 0.15 \pm 0.08$	$\mathcal{B}_{\rm exp}$[%]
$D^+ \to \Phi \pi^+$	0.60 ± 0.41	0.40 ± 0.12	0.61 ± 0.06
$D_s^+ \to \rho^+ \eta$	9.1 ± 7.2	9.0 ± 2.5	10.3 ± 3.2
$D_s^+ \to \rho^+ \eta'$	4.5 ± 3.0	4.5 ± 1.3	12.0 ± 4.5
$D^+ \to \overline{K^0} \pi^+$	4.23 ± 2.2	2.2 ± 0.7	2.74 ± 0.29
$D^0 \to \Phi \eta$	0.02 ± 0.02	0.018 ± 0.005	< 0.28
$D^0 \to \Phi \pi^0$	0.08 ± 0.52	0.07 ± 0.02	< 0.14

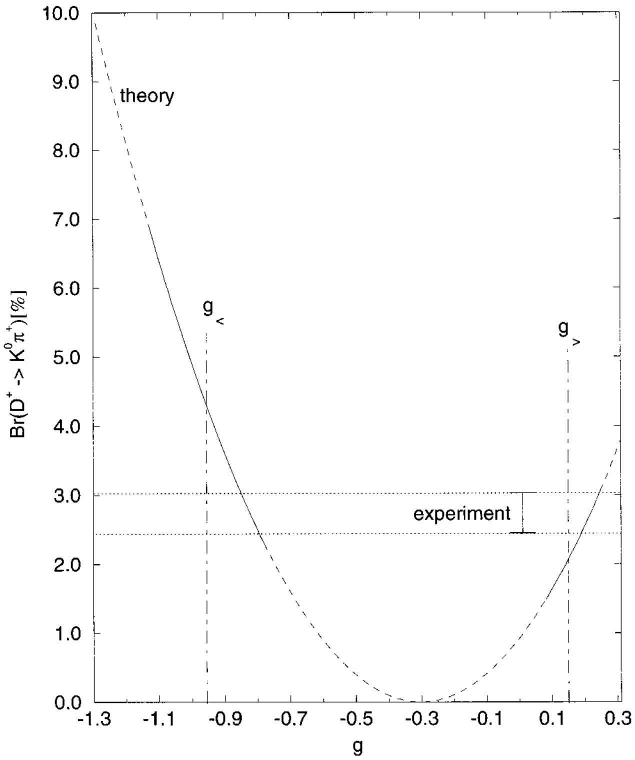

FIG. 3. The branching ratio for $D^+ \to \overline{K^0}\pi^+$ dependance on g. The solid parts of the dashed line indicate the allowed ranges of $g_<$ and $g_>$.

In order to extrapolate the amplitude from the zero recoil point to the rest of the allowed kinematical region we have made a very simple, physically motivated, assumption: *The vertices do not change significantly, while the propagators of the off-shell heavy mesons are given by the full propagators* $1/(p^2 - m^2)$ *instead of the HQET propagators* $1/(2mv \cdot k)$ [19]. With these assumptions we are able to incorporate the following features: the HQET prediction almost exactly at the maximum q^2, a natural explanation for the pole-type form factors when appropriate, and predictions of flat q^2 behavior for the form factors A_1 and A_2, which has been confirmed in the QCD sum rule analysis of [22].

Finally, we include SU(3) symmetry breaking by using the physical masses and decay constants shown in Table I. The decay constants for the η and η' were taken from [31], for the light vector mesons from [11], and for the D mesons from [32–34].

The relevant form factors for $D \to V$ decays defined in Eq. (11) calculated in our model [19] are

$$\frac{1}{K_{V(i)}} V^{(i)}(q^2) = (m_H + m_{V(i)}) \times \left(2\frac{m_{H'*(i)}}{m_H}\right)^{1/2} \frac{m_{H'*(i)}}{q^2 - m^2_{H'*(i)}} f_{H'*(i)} \lambda \frac{g_V}{\sqrt{2}}, \tag{15}$$

$$\frac{1}{K_{V(i)}} A_0^{(i)}(q^2) = \left[\frac{1}{m_{V(i)}} \left(\frac{m_{H'(i)}}{m_H}\right)^{1/2} \frac{q^2}{q^2 - m^2_{H'(i)}} f_{H'(i)} \beta + \sqrt{\frac{m_H}{m_{V(i)}}}\alpha_1 - \frac{1}{2} \times \frac{q^2 + m_H^2 - m_{V(i)}^2}{m_H^2} \sqrt{\frac{m_H}{m_{V(i)}}}\alpha_2 \right] \frac{g_V}{\sqrt{2}}, \tag{16}$$

$$\frac{1}{K_{V(i)}} A_1^{(i)}(q^2) = -2\sqrt{\frac{m_H}{m_H + m_{V(i)}}}\alpha_1 \frac{g_V}{\sqrt{2}}, \tag{17}$$

(18)

and

$$\frac{1}{K_{V(i)}} A_2^{(i)}(q^2) = \left[-\frac{m_H + m_{V(i)}}{m_H\sqrt{m_H}}\alpha_2 \right] \frac{g_V}{\sqrt{2}}, \tag{19}$$

where the pole mesons and the constants $K_{V(i)}$, which contribute to the corresponding processes $D \to PV$ and $D \to V_{(1)}V_{(2)}$, are given in Tables II and III, respectively.

We determined the three parameters (λ, α_1, α_2) in [19] using the three measured values of helicity amplitudes $\Gamma/\Gamma_{\text{tot}} = 0.048 \pm 0.004$, $\Gamma_L/\Gamma_T = 1.23 \pm 0.13$, and $\Gamma_+/\Gamma_- =$

TABLE VII. The braching ratios for the decays that depend only on the set of parameters α_1, α_2, and λ with $\beta = 0$. The second, third, fourth, and fifth columns give the predictions for sets I, II, III, and IV, while the sixth column gives the experimental braching ratios [30]. The theoretical error bars are due to the uncertainty in parameters α_1, α_2, and λ.

	$\mathcal{B}_{\text{th}}$[%] set I	$\mathcal{B}_{\text{th}}$[%] set II	$\mathcal{B}_{\text{th}}$[%] set III	$\mathcal{B}_{\text{th}}$[%] set IV	$\mathcal{B}_{\text{exp}}$[%]
$D_s^+ \to \Phi\pi^+$	5.6 ± 0.3	2.2 ± 0.1	5.1 ± 0.3	3.5 ± 1.0	3.6 ± 0.9
$D_s^+ \to \Phi\rho^+$	4.4 ± 0.8	7.5 ± 1.0	3.5 ± 1.1	5.0 ± 1.5	6.7 ± 2.3
$D^0 \to \Phi\rho^0$	0.029 ± 0.005	0.038 ± 0.007	0.012 ± 0.004	0.017 ± 0.005	0.11 ± 0.03
$D^+ \to \overline{K^{*0}}\rho^+$	2.9 ± 0.4	5.2 ± 0.7	2.7 ± 1.1	3.8 ± 1.4	2.1 ± 1.4
$D^+ \to \Phi\rho^+$	0.14 ± 0.03	0.19 ± 0.03	0.06 ± 0.02	0.085 ± 0.03	< 1.5
$D^0 \to \Phi\omega$	0.028 ± 0.004	0.036 ± 0.004	0.011 ± 0.004	0.015 ± 0.004	< 0.21

0.16 ± 0.04 for the process $D^+\to\overline{K^{*0}}l^+\nu_l$, taken from the Particle Data Group average of all the data [30]. The parameter β could not be determined from this decay rate, since $A_0(q^2)$ cannot be observed in the semileptonic decays.

The model parameters appear linearly in the form factors (15)–(19), and so the polarized decay rates Γ_0, Γ_+, and Γ_- are quadratic functions of them. For this reason there are eight sets of solutions for the three parameters $(\lambda,\alpha_1,\alpha_2)$. It was found from the analysis of the strong decays $D^*\to D\pi$ and electromagnetic decays $D^*\to D\gamma$ [28] that the parameter λ has the same sign as the parameter λ', which describes the contribution of the magnetic moment of the heavy (charm) quark. In the heavy quark limit we have $\lambda'=-1/(6m_c)$. Assuming that the finite mass effects are not so large as to change the sign, we find that $\lambda<0$. Therefore only four solutions remain. They are shown in Table IV.

The calculated branching ratios and polarization variables for the other semileptonic decays of the type $D\to V$ are in agreement with all the known experimental data [19].

In our approach the form factors for $D\to P$ decays are given by [19]

$$\frac{1}{K_{P(i)}}F_1^{(i)}(q^2)=\frac{1}{f_{P(i)}}\left(-\frac{f_H}{2}+gf_{H'*(i)}\times\frac{m_{H'*(i)}\sqrt{m_Hm_{H'*(i)}}}{q^2-m^2_{H'*(i)}}\right), \qquad (20)$$

$$\frac{1}{K_{P(i)}}F_0^{(i)}(q^2)=\frac{1}{f_{P(i)}}\left(-\frac{f_H}{2}-gf_{H'*(i)}\sqrt{\frac{m_H}{m_{H'*(i)}}}+\frac{q^2}{m_H^2-m^2_{P(i)}}\times\left[\frac{-f_H}{2}+gf_{H'*(i)}\sqrt{\frac{m_H}{m_{H'*(i)}}}\right]\right), \qquad (21)$$

where the pole mesons and the constants $K_{P(i)}$, which contribute to the corresponding processes $D\to PV$ and $D\to P_{(1)}P_{(2)}$, are given in Tables II and V, respectively. We neglected the lepton mass, and so the form factor F_0, which multiplies q^μ, did not contribute to the decay width.

Using the best known experimental branching ratio $\mathcal{B}[D^0\to K^-l^+\nu_l]=(3.68\pm0.21)\%$ [30], we found two solutions for g:

$$\text{solution 1}: g\equiv g_>=0.15\pm0.08;$$

$$\text{solution 2}: g\equiv g_<=-0.96\pm0.18. \qquad (22)$$

The quoted error for $g_>$ is mainly due to the uncertainty in the value f_D, while the quoted error for $g_<$ is mainly due to the uncertainty in $f_{D_s^*}$. Unfortunately we were not able to choose between the two possible solutions for g in Eqs. (22).

TABLE VIII. The predicted (column 2) and measured [30] (column 3) branching ratios. The theoretical predictions are calculated for the optimal choice of the parameters: $g=0.15\pm0.08$, $\beta=3.5\pm3$, and set I (Table IV). The theoretical error bars are due to the uncertainty in parameters g, β, α_1, α_2, and λ.

Decay	$\mathcal{B}_{\rm th}[\%]$	$\mathcal{B}_{\rm expt}[\%]$
$D^+\to\overline{K^{*0}}\pi^+$	2.4 ± 1.2	1.92 ± 0.19
$D^+\to\rho^+\overline{K^0}$	6.6 ± 3.0	6.6 ± 2.5
$D^+\to\Phi\pi^+$	0.40 ± 0.12	0.61 ± 0.06
$D_s^+\to\Phi\pi^+$	5.4 ± 0.5	3.6 ± 0.9
$D_s^+\to\rho^+\eta$	9.0 ± 2.5	10.3 ± 3.2
$D_s^+\to\rho^+\eta'$	4.5 ± 1.3	12.0 ± 4.5
$D^+\to\overline{K^0}\pi^+$	2.2 ± 0.7	2.74 ± 0.29
$D_s^+\to\Phi\rho^+$	4.4 ± 0.8	6.7 ± 2.3
$D^0\to\Phi\rho^0$	0.029 ± 0.005	0.11 ± 0.03
$D^+\to\overline{K^{*0}}\rho^+$	2.9 ± 0.4	2.1 ± 1.4
$D^+\to\rho^+\eta$	$0.05\pm^{0.9}_{0.05}$	<1.2
$D^+\to\rho^+\eta'$	$0.02\pm^{0.2}_{0.02}$	<1.5
$D^0\to\Phi\eta$	0.018 ± 0.005	<0.28
$D^0\to\omega\eta$	0.09 ± 0.03	–
$D^0\to\omega\eta'$	0.015 ± 0.015	–
$D^0\to\Phi\pi^0$	0.07 ± 0.02	<0.14
$D^+\to\Phi\rho^+$	0.14 ± 0.03	<1.5
$D^0\to\Phi\omega$	0.028 ± 0.004	<0.21

III. NONLEPTONIC DECAYS

The effective Hamiltonian for charm decays is given by

$$H_w=\frac{G_F}{\sqrt{2}}V_{ci}V^*_{uj}\{a_1(\overline{u}\Gamma_\mu q_j)(\overline{q}_i\Gamma^\mu c)+a_2(\overline{u}\Gamma_\mu c)(\overline{q}_i\Gamma^\mu q_j)\}, \qquad (23)$$

where $V_{qq'}$ is an element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, i and j stand for d or s quark flavors, $\Gamma_\mu=\gamma_\mu(1-\gamma^5)$, and a_1 and a_2 are the Wilson coefficients:

$$a_1=1.26\pm0.04, \quad a_2=-0.51\pm0.05. \qquad (24)$$

These values are taken from [15,17,18,5] and they are in agreement with the next-to-leading order calculation [16]. The factorization approach in two-body nonleptonic decays means one can write the amplitude in the form

$$\langle AB|\overline{q_i}\Gamma_\mu q_j\overline{q_k}\Gamma^\mu c|D\rangle=\langle A|\overline{q_i}\Gamma_\mu q_j|0\rangle\langle B|\overline{q_k}\Gamma^\mu c|D\rangle+\langle B|\overline{q_i}\Gamma_\mu q_j|0\rangle\langle A|\overline{q_k}\Gamma^\mu c|D\rangle+\langle AB|\overline{q_i}\Gamma_\mu q_j|0\rangle\langle 0|\overline{q_k}\Gamma^\mu c|D\rangle. \tag{25}$$

In our calculations we take into account only the first two contributions. The last one is the annihilation contribution (Fig. 2), which is absent or negligible in the particular decay modes we consider. In other decays this contribution was found to be rather important [17,18,11,4]. It was pointed out in [17,18,10,12] that the simple dominance by the lightest scalar or pseudoscalar mesons in $\langle AB|\overline{q_i}\Gamma_\mu q_j|0\rangle$ cannot explain the rather large contribution present in some of the nonleptonic decays, which we will not consider. Our model [19], being rather poor in the number of resonances, is applicable to the analysis of the spectator amplitudes, but not the annihilation contributions.

We will use the following definitions of the light meson and the heavy meson couplings:

$$\langle P(p)|j_\mu|0\rangle=-if_P p_\mu, \tag{26}$$

$$\langle V(p,\epsilon^*)|j_\mu|0\rangle=m_V f_V\epsilon^*_\mu, \tag{27}$$

$$\langle 0|j_\mu|D(P)\rangle=-if_D m_D v_\mu, \tag{28}$$

$$\langle 0|j_\mu|D^*(\epsilon,P)\rangle=im_{D*}f_{D*}\epsilon_\mu. \tag{29}$$

Then using Eqs. (11) and (12) we can write the amplitude for the nonleptonic decay $D\to PV$ processes [Fig. 1(a)] as

$$M(D(p)\to PV(\epsilon^*))=\frac{G_F}{\sqrt{2}}\epsilon^*\cdot p2m_V[-w_V K_V f_P A_0(m_P^2)+w_P K_P f_V F_1(m_V^2)]. \tag{30}$$

The factors w_V, w_P, K_V, and K_P are given in Table II, while the masses and decay constants are given in Table I. In the cases when the η and η' mesons are in the final state the factors K_V and K_P depend on the η-η' mixing angle θ_P and decay constants f_η and $f_{\eta'}$ through the functions $f_{1\mathrm{mix}}$, $f'_{1\mathrm{mix}}$, $f_{2\mathrm{mix}}$, and $f'_{2\mathrm{mix}}$ defined by

$$f_{1\mathrm{mix}}=\frac{f_\eta}{\sqrt{8}}\left[\frac{1+c^2}{f_\eta}+\frac{sc}{f_{\eta'}}\right],$$

$$f'_{1\mathrm{mix}}=\frac{f_{\eta'}}{\sqrt{8}}\left[\frac{sc}{f_\eta}+\frac{1+s^2}{f_{\eta'}}\right],$$

$$f_{2\mathrm{mix}}=\frac{f_\eta}{\sqrt{8}}\left[\frac{1-5c^2}{f_\eta}-\frac{5sc}{f_{\eta'}}\right],$$

$$f'_{2\mathrm{mix}}=\frac{f_{\eta'}}{\sqrt{8}}\left[\frac{-5sc}{f_\eta}+\frac{1-5s^2}{f_{\eta'}}\right], \tag{31}$$

where $s=\sin\theta_P$ and $c=\cos\theta_P$.

In Fig. 1(b) we show the contributions to the decay $D\to P_1P_2$, which leads to the amplitude

$$M(D(p)\to P_{(1)}P_{(2)})=\frac{G_F}{\sqrt{2}}[-iw_1K_{P(1)}f_{P(2)}(m_H^2-m_{P(1)}^2)F_0^{(1)}(m_{P(2)}^2)-iw_2K_{P(2)}f_{P(1)}(m_H^2-m_{P(2)}^2)F_0^{(2)}(m_{P(1)}^2)]. \tag{32}$$

The factors w_1, w_2, $K_{P(1)}$, and $K_{P(2)}$ are presented in Table V.

Finally, we find the $D\to V_{(1)}V_{(2)}$ decay amplitude [Fig. 1(c)] to be

$$\begin{aligned}M(D(p)\to V_{(1)}(p_1,\epsilon_1),V_{(2)}(p_2,\epsilon_2))=\frac{G_F}{\sqrt{2}}\Bigg(&w_1K_{V(1)}f_{V(2)}m_{V(2)}\epsilon_{2\mu}\Bigg[-\frac{2V^{(1)}(m_{V(2)}^2)}{m_H+m_{V(1)}}\varepsilon^{\mu\nu\alpha\beta}\epsilon^*_{1\nu}p_\alpha p_{1\beta}+i(m_H+m_{V(1)})A_1^{(1)}\\&\times(m_{V(2)}^2)\epsilon_1^{\mu*}-i\frac{A_2^{(1)}(m_{V(2)}^2)}{m_H+m_{V(1)}}\epsilon_1^*\cdot p_{V2}(p+p_{V1})^\mu\Bigg]+w_2K_{V(2)}f_{V(1)}m_{V(1)}\\&\times\epsilon_{1\mu}\Bigg[-\frac{2V^{(2)}(m_{V(1)}^2)}{m_H+m_{V(2)}}\varepsilon^{\mu\nu\alpha\beta}\epsilon^*_{2\nu}p_\alpha p_{2\beta}+i(m_H+m_{V(2)})A_1^{(2)}(m_{V(1)}^2)\\&\times\epsilon_2^{\mu*}-i\frac{A_2^{(2)}(m_{V(1)}^2)}{m_H+m_{V(2)}}\epsilon_2^*\cdot p_{V1}(p+p_{V2})^\mu\Bigg]\Bigg).\end{aligned} \tag{33}$$

The factors w_1, w_2, $K_{V(1)}$, and $K_{V(2)}$ for $D\to V_{(1)}V_{(2)}$ processes are given in Table III.

In order to avoid the strong interaction final state effects in the interference between different final isospin states we analyze decays in which the final state involves only a single isospin. This occurs when there is an isospin-0 particle in the final state (ω, Φ, η, η'), or when a final state has the maximal third component of the isospin; for example, $D^+\to\overline{K^{*0}}\pi^+$, $D^+\to\rho^+\overline{K^{*0}}$, $D^+\to\overline{K^0}\pi^+$, and $D^+\to\overline{K^{*0}}\rho^+$ with $|I,I_3\rangle=|3/2,3/2\rangle$).

Our analysis of semileptonic decays $D\to V(P)l\nu_l$ [19] left some ambiguity in the choice of the model parameters: There are two values of g ($g_<$, and $g_>$), Eqs. (22), and four solutions for the parameters (λ, α_1, α_2) (Table IV). The calculated nonleptonic decay amplitudes depend on the choice of these parameters. However, although the uncertainties are quite large, they are mostly due to the calculated errors in $g_<$ and $g_>$, Eqs. (22), which is in turn due to the uncertainty in f_D and $f_{D_s^*}$: The only parameter that is not constrained by the semileptonic decay data is the parameter β in the form factor A_0, but the predictions for the nonleptonic decay rates are not very sensitive to β. From Eqs. (30) and (16) it can easily be seen that β appears multiplied by m_P^2 in the $D\to PV$ decay width and is only significant for the decays $D\to PV$, where P is K, η, or η'.

First we discuss the results for the decay amplitudes which depend only on the form factors F_0 and F_1 and consequently only on the parameter g, namely, $D^+\to\overline{K^0}\pi^+$, $D^+\to\Phi\pi^+$, $D_s^+\to\rho^+\eta(\eta')$, $D^0\to\Phi\eta$, and $D^0\to\Phi\pi^0$. The predicted branching ratios for the two different values $g_<$ and $g_>$ are given in Table VI. The comparison with the experimental data in Table VI does not exclude either of the values for g, $g_<$, or $g_>$. For example, Fig. 3 presents the dependence of the branching ratio for $D^+\to\overline{K^0}\pi^+$ on the parameter g to illustrate that the uncertainty in the calculation depends sensitively on the uncertainty in the value g. However, the calculated rates shown in Table VIII, below, do agree with the experimental data though the errors are quite large, except perhaps for the decay $D_s^+\to\rho^+\eta'$.

Next, we summarize the results obtained for the decays which depend only on the form factors V, A_0, A_1, and A_2, and consequently only on the parameters (λ,α_1,α_2), namely, $D_s^+\to\Phi\pi^+$, $D_s^+\to\Phi\rho^+$, $D^0\to\Phi\rho^0$, and $D^+\to\overline{K^{*0}}\rho^+$. The decay $D_s^+\to\Phi\pi^+$ depends also on the parameter β, but this dependence is very slight, since the light pseudoscalar meson in the final state is a π. The branching ratios for sets I, II, III, and IV in Table IV with $\beta=0$ are shown in Table VII. The results for all sets are in rather good agreement with the experimental data, with the exception of $D^0\to\Phi\rho^0$, which we do not understand.

In addition to the above two types of nonleptonic decays, there are two measured branching ratios for $D^+\to\overline{K^{*0}}\pi^+$ and $D^+\to\rho^+\overline{K^0}$. Their decay amplitudes depend on both g and the parameters λ, α_1, and α_2. The branching ratio for $D^+\to\overline{K^{*0}}\pi^+$, which is not sensitive to β since the π mass is small, excludes the parameter $g_<$, sets II and IV, and prefers

$$g=g_>=0.15\pm0.08 \quad \text{and} \quad \text{set I (Table IV)}.$$

From the $D^+\to\rho^+\overline{K^0}$ decay, which has K pseudoscalar meson in the final state, one can then estimate the parameter β. Unfortunately, this decay has a considerable experimental error, $B=(6.6\pm2.5)\%$ [30], which results in large error in β:

$$\beta=3.5\pm3. \tag{34}$$

The predictions for the branching ratios for the possible decays are presented in Table VIII assuming set I for λ, α_1 and α_2, $g=g_>=0.15\pm0.08$, and $\beta=3.5\pm3$. The quoted errors are due to the uncertainties in the model parameters, mainly g.

IV. SUMMARY

We have calculated the branching ratios for the nonleptonic decay modes $D\to PV$, $D\to P_1P_2$, and $D\to V_1V_2$ in which the annihilation contribution is absent or negligible, and in which the final state involves only a single isospin in order to avoid the effects of strong interaction phases. Factorization of the matrix elements was then assumed and we used the effective model developed to describe the semileptonic decays $D\to V(P)l\nu_l$ to calculate the nonleptonic matrix elements. We reproduced the experimental results for branching ratios for the $D^+\to\overline{K^{*0}}\pi^+$, $D^+\to\rho^+\overline{K^0}$, $D_s^+\to\Phi\pi^+$, $D_s^+\to\rho^+\eta$, $D^+\to\overline{K^0}\pi^+$, $D_s^+\to\Phi\rho^+$, and $D^+\to\overline{K^{*0}}\rho^+$ decays, albeit within substantial uncertainties. We also determined the set of parameters λ, α_1,α_2, and g, which gave the best agreement with the experimental results, and used this set of parameters to estimate the parameter β from the branching ratio for $D^+\to\rho^+\overline{K^0}$. We then made the predictions for a number of nonleptonic decay rates which have not yet been measured.

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