Generic consequences of a supersymmetric U(1) gauge factor at the TeV scale

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We consider an arbitrary supersymmetric U(1) gauge factor at the TeV scale, under which the two Higgs superfields $H_{1,2}$ of the standard model are nontrivial. We assume that there is a singlet superfield S such that H_1H_2S is an allowed term in the superpotential. We discuss first the generic consequences of this hypothesis on the structure of the two-doublet Higgs sector at the electroweak energy scale, as well as Z-Z' mixing and the neutralino sector. We then assume the existence of a grand unified symmetry and universal soft supersymmetry-breaking terms at that scale. We further assume that the additional U(1) is broken radiatively via a superpotential term of the form hh^cS, where h and h^c are exotic color-triplet fields which appear in E_6 models. We show that the U(1)-breaking scale and the parameter $\tan\beta \equiv v_2/v_1$ are then both predicted as functions of the H_1H_2S coupling. [S0556-2821(97)06323-6]

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I. INTRODUCTION

If supersymmetry is broken at the TeV energy scale and the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry is not extended, then supersymmetry protects the theory from nondecoupling contributions of physics above a TeV [1], and we get the minimal supersymmetric standard model (MSSM) [2]. However, if the gauge symmetry is extended also at the TeV energy scale and it breaks down to that of the standard model together with the supersymmetry, there will be in general new important phenomenological consequences, not only at the TeV scale, which is of course obvious, but also at the 100 GeV scale, which may not be as obvious [3–5]. In fact, as will be shown in this paper, the parameters of the two scales may also be related if the universality of soft supersymmetry-breaking terms is assumed.

A particularly interesting extension of the MSSM is the inclusion of an extra U(1) factor at the TeV energy scale. The motivation for this could be theoretical. If the standard model is embedded in a larger symmetry group of rank greater than 4, such as SO(10) (rank 5) or E_6 (rank 6), then an extra U(1) gauge factor is very possible. This is especially true for the supersymmetric E_6 model [6,7] based on the $E_8 \times E_8$ heterotic string. Specifically, if only flux loops are invoked [8] to break E_6 down to $SU(3)_C \times SU(2)_L \times U(1)_Y$, then a specific extra U(1) [conventionally known as $U(1)_\eta$] is obtained. Remarkably, $U(1)_\eta$ is also phenomenologically implicated [9] by the experimental $R_b \equiv \Gamma(Z \to b\overline{b})/\Gamma(Z \to \text{hadrons})$ excess. Another possible clue is the totality of neutrino-oscillation experiments (solar, atmospheric, and laboratory) which suggest that there are at least four neutrinos. This has been shown [10] to have a natural explanation in terms of the E_6 superstring model with a specific U(1) called $U(1)_N$.

In Sec. II, we consider a generic extra supersymmetric U(1) gauge factor at the TeV energy scale with two doublet superfields $H_{1,2}$ and a singlet superfield S such that H_1H_2S is an allowed term in the superpotential. [Note that if S has a nonzero charge under the aditional U(1), as is the case if the scalar component of S is to acquire a nonzero vacuum expectation value (VEV) so as to break this U(1), then above this breaking scale, no μH_1H_2 superpotential term exists. This is a possible resolution of the so-called μ problem in the MSSM where the magnitude of this term is unspecified.] We then derive its nondecoupling effects on the two-doublet Higgs sector at the 100 GeV scale. Although these have been discussed previously [3–5] in specific models, we present here the most general analysis. We show that the upper bound on the mass of the lighter of the two neutral scalar bosons exceeds that of the MSSM and increases as a function of the U(1) gauge coupling. Our results are summarized in Fig. 1 which includes previous upper bounds as specific isolated points.

In Sec. III, we specialize to a class of $U(1)_\alpha$ models derivable from E_6, of which the $U(1)_\eta$ and $U(1)_N$ models are special cases. This material is not new, but rather to establish notation and to facilitate the discussion of new results in subsequent sections. We also make contact with Sec. II here in Eqs. (26)–(29).

In Sec. IV, we discuss how the new Z' mixes with the standard Z in the general case, and formulate the effects in terms of the oblique parameters $\epsilon_{1,2,3}$ or S,T,U in the $U(1)_\alpha$ models. We also discuss the generic neutralino sector. Analogous discussions were given previously only for the $U(1)_N$ model [5].

In Sec. V, we present our main results. We show how supersymmetric scalar masses are affected by the extra D terms from $U(1)_\alpha$. Combining this with the results of Secs. II and III, and assuming universal soft supersymmetry-breaking terms at the grand-unification scale, we show that there is a relationship between the $U(1)_\alpha$ vacuum expectation value (which we require to be in the TeV range) and the well-known parameter $\tan\beta \equiv v_2/v_1$ used in the MSSM. This is achieved by the simple observation that the parameters m_1^2, m_2^2, and m_{12}^2 of the two-doublet Higgs potential must be matched with their derived values from the renormalization-group evolution of the appropriate quantities at the grand-unification energy scale, as given by Eqs. (64)–(69).

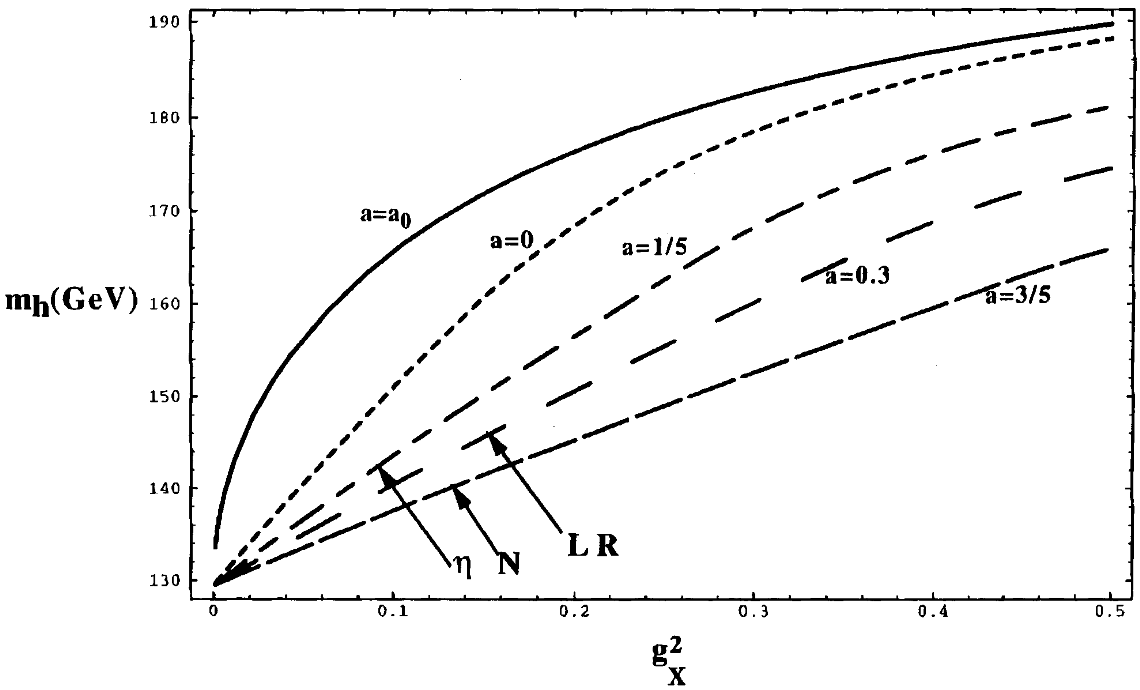


FIG. 1. The upper bound on the lighter Higgs boson mass m_h as a function of g_X^2 for various values of a. In all cases, we find the allowed value of $f=f_0$ that maximizes m_h. In the top curve, we find the pair $f=f_0$ and $a=a_0$ that maximizes m_h whereas the value of a is held fixed as labeled for the other curves. The points corresponding to the η, N, and exotic left-right models, described in Sec. III, are marked by arrows.

Finally, in Sec. VI, we have some concluding remarks.

II. TREE-LEVEL NONDECOUPLING AT THE 100 GeV SCALE

As the U(1) gauge factor is broken together with the supersymmetry at the TeV scale, the resulting heavy scalar particles have nondecoupling contributions to the interactions of the light scalar particles [1]. Consequently, the two-doublet Higgs structure is of a more general form than that of the minimal supersymmetric standard model. Previous specific examples have been given [3–5]. Here we present the most general analysis. We denote the scalar components of H_1, H_2, and S as $\tilde{\Phi}_1$, Φ_2, and χ, respectively. Under $SU(3)_C\times SU(2)_L\times U(1)_Y\times U(1)_X$, we then have

$$\tilde{\Phi}_1=\begin{pmatrix}\overline{\phi_1^0}\\ -\phi_1^-\end{pmatrix}\sim\left(1,2,-\frac{1}{2};-a\right), \tag{1}$$

$$\Phi_2=\begin{pmatrix}\phi_2^+\\ \phi_2^0\end{pmatrix}\sim\left(1,2,\frac{1}{2};-1+a\right), \tag{2}$$

$$\chi=\chi^0\sim(1,1,0;1), \tag{3}$$

where each last entry is the arbitrary assignment of that scalar multiplet under the extra $U(1)_X$ with coupling g_x, assuming of course that the superpotential has the term fH_1H_2S. The corresponding scalar potential contains thus

$$V_F=f^2[(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)+(\Phi_1^\dagger\Phi_1+\Phi_2^\dagger\Phi_2)\overline{\chi}\chi], \tag{4}$$

and, from the gauge interactions,

$$\begin{aligned}V_D=&\frac{1}{8}g_2^2[(\Phi_1^\dagger\Phi_1)^2+(\Phi_2^\dagger\Phi_2)^2+2(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)\\ &-4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)]+\frac{1}{8}g_1^2[-\Phi_1^\dagger\Phi_1+\Phi_2^\dagger\Phi_2]^2\\ &+\frac{1}{2}g_x^2[-a\Phi_1^\dagger\Phi_1-(1-a)\Phi_2^\dagger\Phi_2+\overline{\chi}\chi]^2.\end{aligned} \tag{5}$$

Let $\langle\chi\rangle=u$; then, $\sqrt{2}\mathrm{Re}\chi$ is a physical scalar boson with $m^2=2g_x^2u^2$, and the $(\Phi_1^\dagger\Phi_1)\sqrt{2}\mathrm{Re}\chi$ coupling is $\sqrt{2}u(f^2-g_x^2a)$. Hence the effective $(\Phi_1^\dagger\Phi_1)^2$ coupling λ_1 is given by

$$\begin{aligned}\lambda_1&=\frac{1}{4}(g_1^2+g_2^2)+g_x^2a^2-\frac{2(f^2-g_x^2a)^2}{2g_x^2}\\ &=\frac{1}{4}(g_1^2+g_2^2)+2af^2-\frac{f^4}{g_x^2}.\end{aligned} \tag{6}$$

Similarly,

$$\lambda_2=\frac{1}{4}(g_1^2+g_2^2)+2(1-a)f^2-\frac{f^4}{g_x^2}, \tag{7}$$

$$\lambda_3=-\frac{1}{4}g_1^2+\frac{1}{4}g_2^2+f^2-\frac{f^4}{g_x^2}, \tag{8}$$

$$\lambda_4=-\frac{1}{2}g_2^2+f^2, \tag{9}$$

where the two-doublet Higgs potential has the generic form

$$V=m_1^2\Phi_1^\dagger\Phi_1+m_2^2\Phi_2^\dagger\Phi_2+m_{12}^2(\Phi_1^\dagger\Phi_2+\Phi_2^\dagger\Phi_1)+\frac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2+\frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2+\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)+\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1). \quad (10)$$

From Eqs. (6)–(9), it is clear that the MSSM is recovered in the limit $f=0$. [Note that $m_{12}^2\neq 0$ only after U(1) symmetry breaking and it would be proportional to f if universal soft supersymmetry breaking is assumed.] Let $\langle\phi_{1,2}^0\rangle\equiv v_{1,2}$, $\tan\beta\equiv v_2/v_1$, and $v^2\equiv v_1^2+v_2^2$; then, this V has an upper bound on the lighter of the two neutral scalar bosons given by

$$(m_h^2)_{\max}=2v^2[\lambda_1\cos^4\beta+\lambda_2\sin^4\beta+2(\lambda_3+\lambda_4)\sin^2\beta\cos^2\beta]+\epsilon, \quad (11)$$

where we have added the radiative correction [11] due to the t quark and its supersymmetric scalar partners, i.e.,

$$\epsilon=\frac{3g_2^2m_t^4}{8\pi^2M_W^2}\ln\left(1+\frac{\tilde{m}^2}{m_t^2}\right). \quad (12)$$

We note also that the soft supersymmetry-breaking term $fA_f\Phi_1^\dagger\Phi_2\chi+$H.c. (from which we obtain $m_{12}^2=fA_fu$) also contributes to λ_4 and generates some additional quartic scalar couplings. However, we assume here that fA_f/g_x^2u is small, because we are mainly interested in the case where the electroweak Higgs sector has two relatively light doublets and not just one light doublet. Using Eqs. (6)–(9), we obtain

$$(m_h^2)_{\max}=M_Z^2\cos^22\beta+\epsilon+\frac{f^2}{\sqrt{2}G_F}\left[A-\frac{f^2}{g_x^2}\right], \quad (13)$$

where

$$A=\frac{3}{2}+(2a-1)\cos2\beta-\frac{1}{2}\cos^22\beta. \quad (14)$$

If $A>0$, then the MSSM bound can be exceeded. However, f^2 is still constrained from the requirement that V be bounded from below. We note here that although V_F of Eq. (4) and V_D of Eq. (5) are non-negative for any value of f, V of Eq. (10) is not automatically bounded from below. This simply means that if f is too large, the minimum of the original potential only breaks the extra U(1) but not the electroweak gauge symmetry. Given g_x and a, we can vary $\cos 2\beta$ and f to find the largest numerical value of m_h. We show in Fig. 1 this upper bound on m_h as a function of g_x^2 for several specific values of a. The value a_0 is chosen in the top curve to maximize m_h for a given value of g_x^2. This upper bound increases as g_x^2 increases. However, it is reasonable to assume that g_x cannot be too large. In fact, in the specific models to be discussed in the next section, $g_x^2<0.16$. As shown in Fig. 1, even for $g_x^2=0.5$, the upper bound is only about 190 GeV.

It should be mentioned that an upper bound on m_h has been previously obtained [12] assuming that there is no extra U(1) at the supersymmetry-breaking scale. However, the same proof also goes through with an extra U(1). We improve on Ref. [12] in this case by computing exactly how the off-diagonal nondecoupling terms affect the upper bound on m_h, resulting in Fig. 1 as shown. If fA_f/g_x^2u is not small as we have assumed, then the reduction to V of Eq. (10) is not valid [13].

III. U(1) GAUGE FACTOR FROM E(6)

As already mentioned in the Introduction, an extra supersymmetric U(1) gauge factor at the TeV scale is a very viable possibility from the spontaneous breakdown of E_6. Consider the following sequential reduction:

$$E_6\to SO(10)[\times U(1)_\psi], \quad (15)$$

$$SO(10)\to SU(5)[\times U(1)_\chi], \quad (16)$$

$$SU(5)\to SU(3)_C\times SU(2)_L[\times U(1)_Y]. \quad (17)$$

At each step, a U(1) gauge factor may or may not appear, depending on the details of the symmetry breaking. Assuming that a single extra U(1) survives down to the TeV energy scale, then it is generally given by a linear combination of $U(1)_\psi$ and $U(1)_\chi$ which we will call $U(1)_\alpha$.

Under the maximal subgroup $SU(3)_C\times SU(3)_L\times SU(3)_R$, the fundamental representation of E_6 is given by

$$\mathbf{27}=(3,3,1)+(3^*,1,3^*)+(1,3^*,3). \quad (18)$$

Under the subgroup $SU(5)\times U(1)_\psi\times U(1)_\chi$, we then have

$$\mathbf{27}=(10;1,-1)[(u,d),u^c,e^c]+(5^*;1,3)[d^c,(\nu_e,e)]+(1;1,-5)[N]+(5;-2,2)[h,(E^c,N_E^c)]+(5^*;-2,-2)[h^c,(\nu_E,E)]+(1;4,0)[S], \quad (19)$$

where the U(1) charges refer to $2\sqrt{6}Q_\psi$ and $2\sqrt{10}Q_\chi$. Note that the known quarks and leptons are contained in $(10;1,-1)$ and $(5^*;1,3)$, and the two Higgs scalar doublets are represented by (ν_E,E) and (E^c,N_E^c). Let

$$Q_\alpha=Q_\psi\cos\alpha-Q_\chi\sin\alpha; \quad (20)$$

then, the η model [7,9] is obtained with $\tan\alpha=\sqrt{3/5}$ and we have

$$\mathbf{27}=(10;2)+(5^*;-1)+(1;5)+(5;-4)+(5^*;-1)+(1;5), \quad (21)$$

where $2\sqrt{15}Q_\eta$ is denoted. The N model [10] is obtained with $\tan\alpha=-1/\sqrt{15}$, resulting in

$$\mathbf{27}=(10;1)+(5^*;2)+(1;0)+(5;-2)+(5^*;-3)+(1;5), \quad (22)$$

where $2\sqrt{10}Q_N$ is denoted. This model is so called because the superfield N has $Q_N=0$. It allows S to be a naturally light singlet neutrino and is ideally suited to explain the totality of all neutrino-oscillation experiments, i.e., solar [14], atmospheric [15], and laboratory [16]. It is also a natural consequence of an alternative SO(10) decomposition [17] of E_6, i.e.,

$$\mathbf{16}=[(u,d),u^c,e^c;h^c,(\nu_E,E);S],\tag{23}$$

$$\mathbf{10}=[h,(E^c,N_E^c);d^c,(\nu_e,e)],\tag{24}$$

$$\mathbf{1}=[N],\tag{25}$$

which differs from the conventional assignment by how the SU(5) multiplets are embedded.

Identifying $\tilde{\Phi}_1$, Φ_2, and χ with the scalar components of (ν_E,E), (E^c,N_E^c), and S of which we can choose one copy of each via a discrete symmetry [10] to be the ones with VEV's, we see that the general anaylsis of the previous section is applicable for this class of U(1)-extended models. [Of course, more than one copy of (ν_E,E), (E^c,N_E^c), or S could have VEV's, but that would lead to a much less constrained scenario.] Assuming that $\mathrm{U}(1)_\alpha$ is normalized in the same way as $\mathrm{U}(1)_Y$, we find it to be a very good approximation [5] to have $g_\alpha^2=(5/3)g_1^2$. We then obtain, for the η model,

$$g_x^2=\frac{25}{36}g_1^2,\quad a=\frac{1}{5},\tag{26}$$

and, for the N model,

$$g_x^2=\frac{25}{24}g_1^2,\quad a=\frac{3}{5},\tag{27}$$

whereas, in the exotic left-right model [3,17],

$$g_x^2=\frac{(g_1^2+g_2^2)(1-\sin^2\theta_W)^2}{4(1-2\sin^2\theta_W)},\quad a=\tan^2\theta_W.\tag{28}$$

These three specific points have been singled out in Fig. 1. Furthermore, when we take the squark masses to be about 1 TeV we find the largest numerical value of m_h in the $\mathrm{U}(1)_\alpha$ models to be about 142 GeV, as compared to 128 GeV in the MSSM, and it is achieved with

$$\tan\alpha=-\frac{2\sqrt{3/5}\cos2\beta}{3-\cos^2 2\beta},\tag{29}$$

which is possible in the η model, i.e., $\alpha=\sqrt{3/5}$ and $\cos2\beta=-1$.

IV. Z-Z' AND NEUTRALINO SECTORS

The part of the Lagrangian containing the interaction of $\Phi_{1,2}$ and χ with the vector gauge bosons $A_i (i=1,2,3)$, B, and Z' belonging to the gauge factors $\mathrm{SU}(2)_L$, $\mathrm{U}(1)_Y$, and $\mathrm{U}(1)_X$, respectively, is given by

$$\begin{aligned}\mathcal{L}=&\left|\left(\partial^\mu-\frac{ig_2}{2}\tau_iA_i^\mu+\frac{ig_1}{2}B^\mu+ig_xaZ'^\mu\right)\tilde{\Phi}_1\right|^2\\&+\left|\left(\partial^\mu-\frac{ig_2}{2}\tau_iA_i^\mu-\frac{ig_1}{2}B^\mu+ig_x(1-a)Z'^\mu\right)\Phi_2\right|^2\\&+|(\partial^\mu-ig_xZ'^\mu)\chi|^2,\end{aligned}\tag{30}$$

where τ_i are the usual 2×2 Pauli matrices. With the definition $Z\equiv(g_2A_3-g_1B)/g_Z$, where $g_Z\equiv\sqrt{g_1^2+g_2^2}$, the mass-squared matrix spanning Z and Z' is given by

$$\mathcal{M}_{Z,Z'}^2=\begin{bmatrix}(1/2)g_Z^2(v_1^2+v_2^2) & g_Zg_x[-av_1^2+(1-a)v_2^2]\\ g_Zg_x[-av_1^2+(1-a)v_2^2] & 2g_x^2[u^2+a^2v_1^2+(1-a)^2v_2^2]\end{bmatrix}.\tag{31}$$

Let the mass eigenstates of the Z-Z' system be

$$Z_1=Z\cos\theta+Z'\sin\theta,\quad Z_2=-Z\sin\theta+Z'\cos\theta;\tag{32}$$

then, the experimentally observed neutral gauge boson is identified in this model as Z_1, with the mass given by

$$M_{Z_1}^2\equiv M_Z^2\simeq\frac{1}{2}g_Z^2v^2\left[1-(\sin^2\beta-a)^2\frac{v^2}{u^2}\right]\tag{33}$$

and

$$\theta\simeq-\frac{g_Z}{2g_x}(\sin^2\beta-a)\frac{v^2}{u^2}.\tag{34}$$

Note that Z_2 has essentially the same mass as the physical scalar boson $\sqrt{2}\mathrm{Re}\chi$ discussed earlier.

So far, our discussion of the Z-Z' sector is completely general. However, in order to make contact with experiment, we have to specify how Z' interacts with the known quarks and leptons. In the class of $\mathrm{U}(1)_\alpha$ models from E_6, all such couplings are determined. In particular, we have

$$g_x=\sqrt{\frac{2}{3}}g_\alpha\cos\alpha,\quad a=\frac{1}{2}\left(1-\sqrt{\frac{3}{5}}\tan\alpha\right).\tag{35}$$

Using the leptonic widths and forward-backward asymmetries of Z_1 decay, the deviations from the standard model are conveniently parametrized [18]:

$$\epsilon_1=\left[\sin^4\beta-\frac{1}{4}\left(1-\sqrt{\frac{3}{5}}\tan\alpha\right)^2\right]\frac{v^2}{u^2}\simeq\alpha_{\mathrm{em}}T,\tag{36}$$

$$\epsilon_2=\frac{1}{4}(3-\sqrt{15}\tan\alpha)\left[\sin^2\beta-\frac{1}{2}\left(1-\sqrt{\frac{3}{5}}\tan\alpha\right)\right]\frac{v^2}{u^2}\simeq-\frac{\alpha_{\rm em}U}{4\sin^2\theta_W}, \tag{37}$$

$$\epsilon_3=\frac{1}{4}\left[1-3\sqrt{\frac{3}{5}}\tan\alpha+\frac{1}{2\sin^2\theta_W}\left(1+\sqrt{\frac{3}{5}}\tan\alpha\right)\right]\times\left[\sin^2\beta-\frac{1}{2}\left(1-\sqrt{\frac{3}{5}}\tan\alpha\right)\right]\frac{v^2}{u^2}\simeq\frac{\alpha_{\rm em}S}{4\sin^2\theta_W}. \tag{38}$$

Since the experimental errors on these quantities are fractions of a percent, $u\sim$ TeV is allowed.

In the MSSM, there are four neutralinos (two gauge fermions and two Higgs fermions) which mix in a well-known 4×4 mass matrix [19]. Here we have six neutralinos: the gauginos of $U(1)_Y$ and the third component of $SU(2)_L$, the Higgsinos of $\overline{\phi_1^0}$ and ϕ_2^0, the $U(1)_X$ gaugino, and the χ Higgsino. The corresponding mass matrix is then given by

$$\mathcal{M}_{\mathcal{N}}=\begin{bmatrix} M_1 & 0 & -g_1v_1/\sqrt{2} & g_1v_2/\sqrt{2} & 0 & 0\\ 0 & M_2 & g_2v_1/\sqrt{2} & -g_2v_2/\sqrt{2} & 0 & 0\\ -g_1v_1/\sqrt{2} & g_2v_1/\sqrt{2} & 0 & fu & -g_xav_1\sqrt{2} & fv_2\\ g_1v_2/\sqrt{2} & -g_2v_2/\sqrt{2} & fu & 0 & -g_x(1-a)v_2\sqrt{2} & fv_1\\ 0 & 0 & -g_xav_1\sqrt{2} & -g_x(1-a)v_2\sqrt{2} & M_x & g_xu\sqrt{2}\\ 0 & 0 & fv_2 & fv_1 & g_xu\sqrt{2} & 0 \end{bmatrix}, \tag{39}$$

where $M_{1,x,2}$ are allowed U(1) and SU(2) gauge-invariant Majorana mass terms which break the supersymmetry softly. Note that without the last two rows and columns, the above matrix does reduce to that of the MSSM if fu is identified with $-\mu$. However, the μ parameter in the MSSM is unconstrained, whereas here fu is bounded and f itself appears in the Higgs potential.

Since g_xu should be of order TeV, the neutralino mass matrix $\mathcal{M}_N$ reduces to either a 4×4 or 2×2 matrix, depending on whether f is much less than g_x or not. In the former case, it reduces to that of the MSSM but with the stipulation that the μ parameter must be small, i.e., of order 100 GeV. This means that the two gauginos mix significantly with the two Higgsinos and the lightest supersymmetric particle (LSP) is likely to have non-negligible components from all four states. In the latter case, the effective 2×2 mass matrix becomes

$$\mathcal{M}'_{\mathcal{N}}=\begin{bmatrix} M_1+g_1^2v_1v_2/fu & -g_1g_2v_1v_2/fu\\ -g_1g_2v_1v_2/fu & M_2+g_2^2v_1v_2/fu\end{bmatrix}. \tag{40}$$

Since v_1v_2/u is small, the mass eigenstates of $\mathcal{M}'_{\mathcal{N}}$ are approximately the gauginos $\tilde{B}$ and $\tilde{W}_3$, with masses M_1 and M_2, respectively. In supergravity models with uniform gaugino masses at the grand unified theory (GUT) breaking scale,

$$M_1=\frac{5g_1^2}{3g_2^2}M_2\simeq0.5M_2; \tag{41}$$

hence, $\tilde{B}$ would be the LSP, which makes it a good candidate for cold dark matter.

V. SUPERSYMMETRIC SCALAR MASSES

The spontaneous breaking of the additional U(1) gauge factor at the TeV scale is not possible without also breaking the supersymmetry [10]. As a reasonable and predictive procedure, we will adopt the common hypothesis that soft supersymmetry-breaking operators appear at the GUT scale as the result of a hidden sector which is linked to the observable sector only through gravity. Hence these terms will be assumed to be universal, i.e., of the same magnitude for all fields.

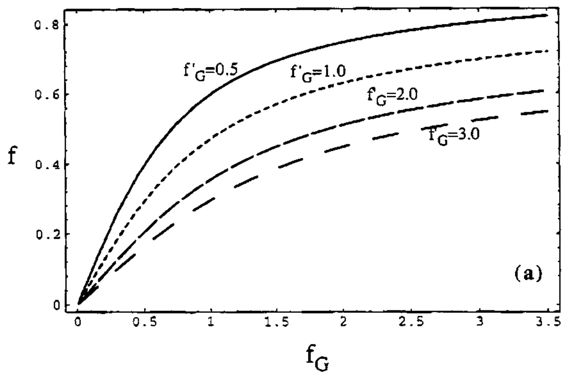
Consider now the masses of the supersymmetric scalar partners of the quarks and leptons:

$$m_B^2=m_0^2+m_R^2+m_F^2+m_D^2, \tag{42}$$

where m_0 is a universal soft supersymmetry-breaking mass at the GUT scale, m_R^2 is a correction generated by the renormalization-group equations running from the GUT scale down to the TeV scale, m_F is the explicit mass of the fermion partner, and m_D^2 is a term induced by gauge symmetry breaking with rank reduction and can be expressed in terms of the gauge-boson masses. In the MSSM, m_D^2 is of order M_Z^2 and does not change m_B significantly. In the $U(1)_\alpha$-extended model, m_D^2 is of order $M_{Z'}^2$ and will affect m_B in a nontrivial way. For example, in the case of ordinary quarks and leptons,

$$\Delta m_D^2(10;1,-1)=\frac{1}{8}M_{Z'}^2\left(1+\sqrt{\frac{3}{5}}\tan\alpha\right), \tag{43}$$

$$\Delta m_D^2(5^*;1,3)=\frac{1}{8}M_{Z'}^2\left(1-3\sqrt{\frac{3}{5}}\tan\alpha\right). \tag{44}$$



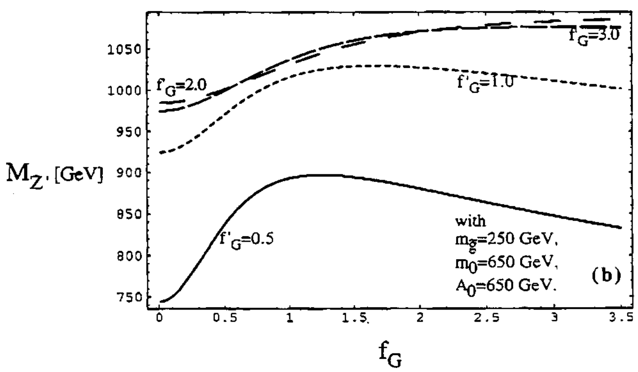


FIG. 2. (a) The parameter f at 1 TeV as a function of $f_G = f(M_G)$ for models with an extra U(1) originating from E_6. In descending order, the curves represent $f'_G = f'(M_G) = 0.5$, 1.0, 2.0, and 3.0. (b) The mass $M_{Z'}$ as a function of f_G with the same values $m_{\tilde{g}} = 250$ GeV, $A_0 = 650$ GeV, and $m_0 = 650$ GeV for different curves with the values of f'_G as in (a).

This would have important consequences on the experimental search of supersymmetric particles. In fact, if m_F is not too large, it is possible for the exotic scalars (which may be interpreted as leptoquarks depending on their Yukawa couplings) to be lighter than the usual scalar quarks and leptons. We have already discussed this issue in Ref. [20].

Assuming Eq. (42), we first consider the spontaneous breaking of $U(1)_\alpha$, i.e., $\langle \chi \rangle = u$, which requires m_χ^2 to be negative. This may be achieved by considering the superpotential

$$W = f H_1 H_2 S + f' h h^c S + \lambda_t H_2 Q_3 t^c \tag{45}$$

(where we have omitted the rest of the MSSM Yukawa couplings), together with the trilinear soft supersymmetry-breaking terms

$$V_{\mathrm{soft}} = f A_f \Phi_1^\dagger \Phi_2 \chi + f' A_{f'} \tilde{h} \tilde{h}^c \chi + \lambda_t A_t \Phi_2 \tilde{Q}_3 \tilde{t}^c, \tag{46}$$

along with the soft supersymmetry-breaking scalar masses. Starting with a wide range of given values of m_0, the universal gaugino mass $m_{1/2}$, and the universal trilinear massive parameter A_0 at the GUT scale, we find that m_χ^2 does indeed turn negative near the TeV energy scale for many typical values of f and f'. An example of this is given in Fig. 2. The evolution of m_χ^2 is mostly driven by f', but f also contributes primarily through its direct effect on $A_{f'}$. From the negative value of m_χ^2 at the TeV scale, we then obtain the predicted mass of Z', i.e., $M_{Z'} = (-2m_\chi^2)^{1/2}$, which is also the mass of the physical scalar boson $\sqrt{2}\mathrm{Re}\chi$. However, as we will discuss shortly, the mass of the Z' so obtained must also be consistent with the desired electroweak symmetry-breaking conditions.

We assume that the top quark's pole mass is 175 GeV and that, at 1 TeV, $\alpha_s = 0.1$, which corresponds to $\alpha_s(M_Z) \approx 0.12$. We will also assume that at the TeV scale and above, the particle content of the model is that of three complete **27**'s of

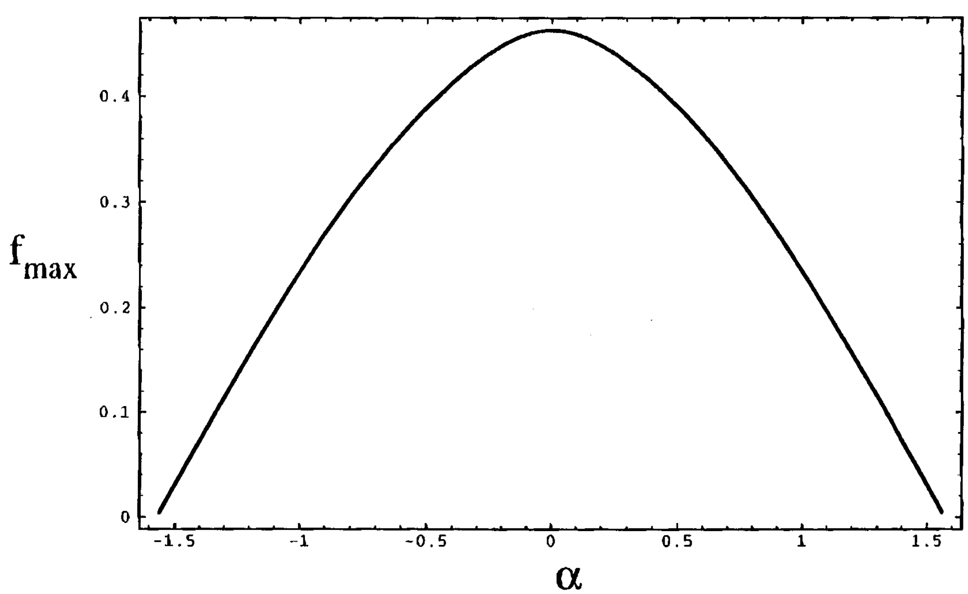


FIG. 3. The maximum value of $f = f_{\max}$ for which the Higgs potential is bounded from below as a function of α, defined in Eq. (20).

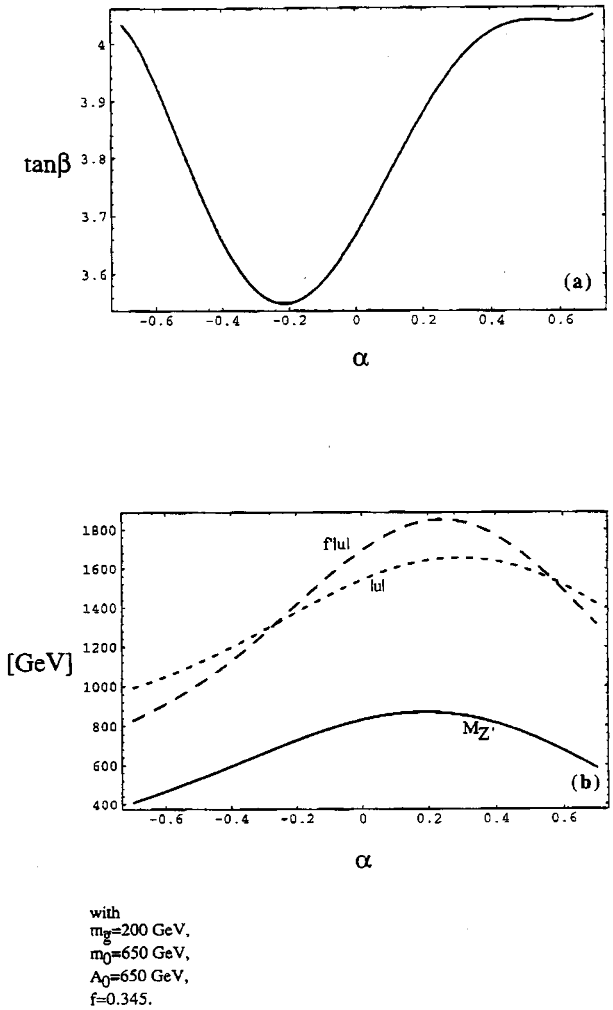


FIG. 4. (a) $\tan\beta$ as a function of α for $f=0.345$ and $m_{\tilde{g}}=200$ GeV, $A_0=650$ GeV, and $m_0=650$ GeV. (b) $M_{Z'}$ (solid line), $|u|$ (short-dashed line), and $f'|u|$ (long-dashed line) as a function of α for the same values of input parameters as in (a).

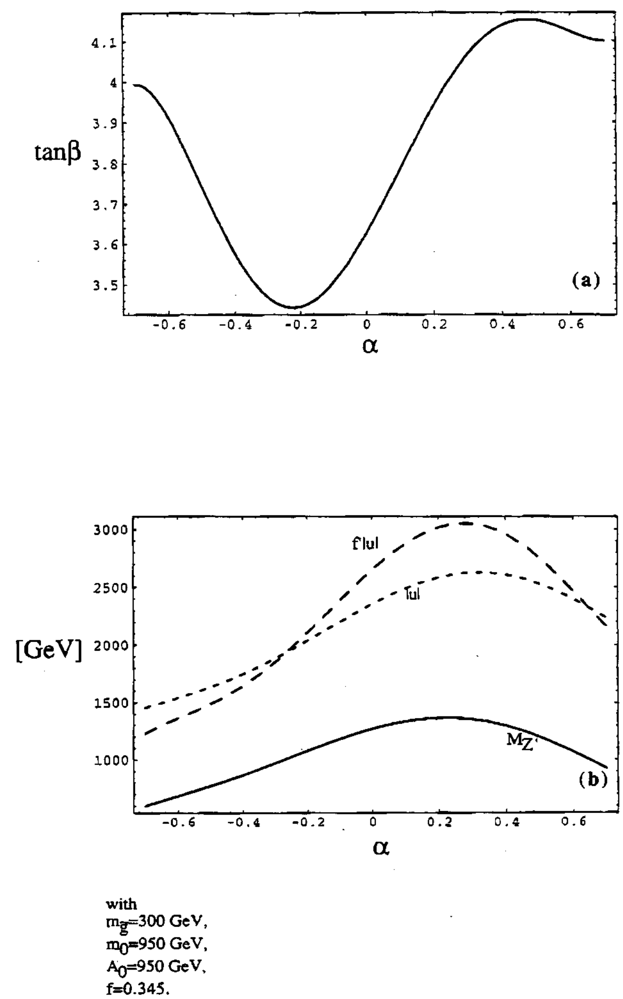


FIG. 5. (a) $\tan\beta$ as a function of α for $f=0.345$ and $m_{\tilde{g}}=300$ GeV, $A_0=950$ GeV, and $m_0=650$ GeV. (b) $M_{Z'}$ (solid line), $|u|$ (short-dashed line), and $f'|u|$ (long-dashed line) as a function of α for the same values of input parameters as in (a).

E_6 and some additional field content so as to achieve gauge-coupling unification. The additional field content could be near the unification scale and hence provide threshold corrections that allow the gauge couplings to unify, perhaps even at the string compactification scale. Another possibility is to add an anomaly-free pair of $SU(2)_L$ doublet fields so as to mimic gauge coupling unification in the MSSM. Such an example is discussed for the $\alpha=N$ model of Ref. [8]. This model has the same unification scale as is possible in the MSSM. In calculating the gauge-coupling β functions, we will in fact assume the field content of that model, but the choice of additional matter fields or threshold corrections to bring about gauge-coupling unification has no significant effect on our calculation. The fact that such models have three complete **27**'s has the noteworthy implication that the gauge coupling at the unification scale is approximately the strong coupling. The reason is that with three copies of h and h^c, the β function for α_s is zero in one loop above the TeV scale. Similarly, the gluino mass also does not evolve in this approximation.

Defining

$$\mathcal{D}\equiv 8\pi^2 \frac{d}{d\ln\mu} \tag{47}$$

(where μ is the scale), the relevant renormalization-group equations are

$$\mathcal{D}\ln\lambda_t^2=-\sum_i c_i^{(t)}g_i^2+6\lambda_t^2+f^2, \tag{48}$$

$$\mathcal{D}\ln f^2=-\sum_i c_i^{(f)}g_i^2+3\lambda_t^2+4f^2+3f'^2, \tag{49}$$

$$\mathcal{D}\ln f'^2=-\sum_i c_i^{(f')}g_i^2+3f^2+5f'^2 \tag{50}$$

for the Yukawa couplings,

$$\mathcal{D}A_t=\sum_i c_i^{(t)}g_i^2M_i+6\lambda_t^2A_t+f^2A_f, \tag{51}$$

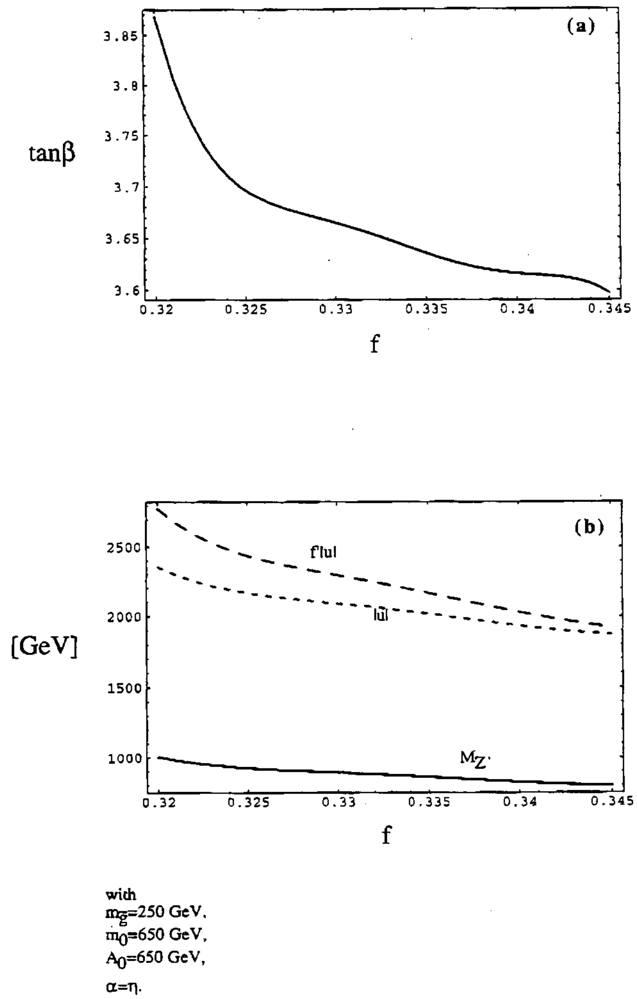


FIG. 6. (a) $\tan\beta$ as a function of f for $\alpha=\eta$ and $m_{\tilde{g}}=250$ GeV, $A_0=650$ GeV, and $m_0=650$ GeV. (b) $M_{Z'}$ (solid line), $|u|$ (short-dashed line), and $f'|u|$ (long-dashed line) as a function of f for the same values of input parameters as in (a).

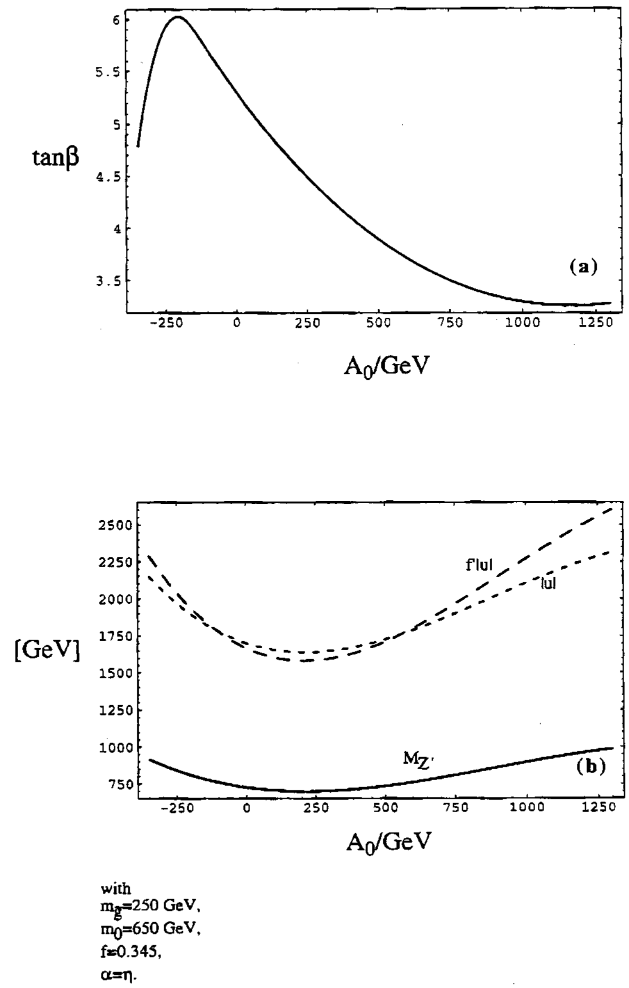


FIG. 7. (a) $\tan\beta$ as a function of A_0 for $\alpha=\eta$ and $f=0.345$, $m_{\tilde{g}}=250$ GeV, and $m_0=650$ GeV. (b) $M_{Z'}$ (solid line), $|u|$ (short-dashed line), and $f'|u|$ (long-dashed line) as a function of A_0 for the same values of input parameters as in (a).

$$\mathcal{D}A_f=\sum_i c_i^{(f)}g_i^2M_i+3\lambda_t^2A_t+4f^2A_f+3f'^2A_{f'}, \quad (52)$$

$$\mathcal{D}A_{f'}=\sum_i c_i^{(f')}g_i^2M_i+3f^2A_f+5f'^2A_{f'} \quad (53)$$

for the trilinear scalar parameters A_i, and

$$\mathcal{D}m_S^2=-\sum_i c_i^{(S)}g_i^2+2f^2X_f+3f'^2X_{f'}, \quad (54)$$

$$\mathcal{D}m_h^2=-\sum_i c_i^{(h)}g_i^2+f'^2X_{f'}, \quad (55)$$

$$\mathcal{D}m_{h^c}^2=-\sum_i c_i^{(h^c)}g_i^2+f'^2X_{f'}, \quad (56)$$

$$\mathcal{D}m_{\Phi_1}^2=-\sum_i c_i^{(\Phi_1)}g_i^2+f^2X_f, \quad (57)$$

$$\mathcal{D}m_{\Phi_2}^2=-\sum_i c_i^{(\Phi_2)}g_i^2+3\lambda_2^2X_t+f^2X_f, \quad (58)$$

$$\mathcal{D}m_{Q_3}^2=-\sum_i c_i^{(Q_3)}g_i^2+\lambda_2^2X_t, \quad (59)$$

$$\mathcal{D}m_{t^c}^2=-\sum_i c_i^{(t^c)}g_i^2+\lambda_2^2X_t, \quad (60)$$

where we have defined

$$X_t\equiv m_{Q_3}^2+m_{t^c}^2+m_{\Phi_2}^2+A_t^2, \quad (61)$$

$$X_f\equiv m_S^2+m_{\Phi_1}^2+m_{\Phi_2}^2+A_f^2, \quad (62)$$

$$X_{f'}\equiv m_S^2+m_h^2+m_{h^c}^2+A_{f'}^2, \quad (63)$$

and the coefficients $c_i^{(\mathrm{field})}$ have the obvious values. Further, the gaugino mass M_i scales the same as α_i. These equations are modified in an obvious manner if $\tan\beta$ is large enough that λ_b and λ_τ cannot be ignored or if there is more than one

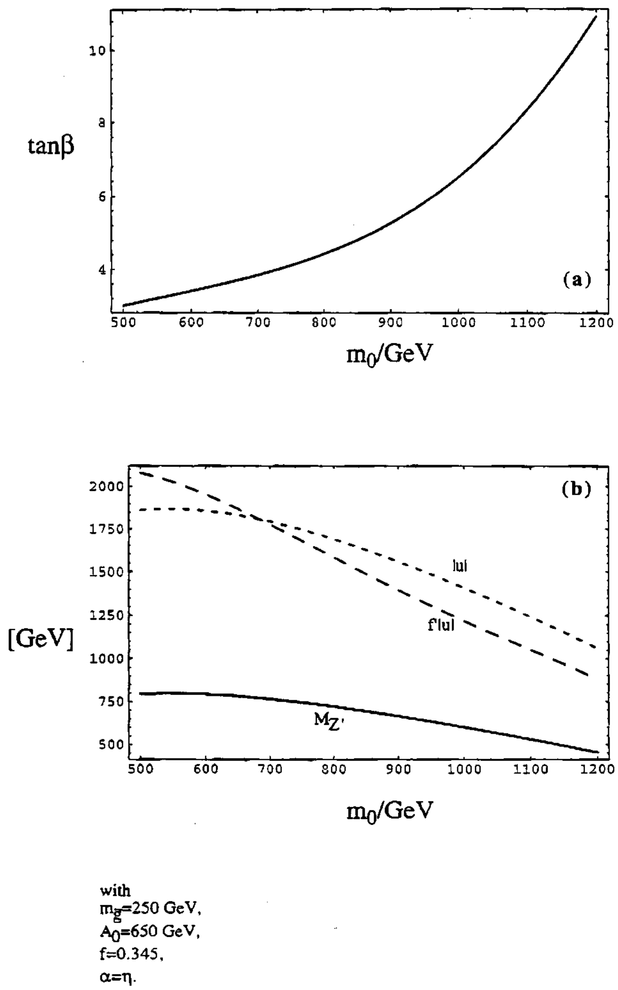


FIG. 8. (a) $\tan\beta$ as a function of m_0 for $\alpha=\eta$ and $f=0.345$, $m_{\tilde{g}}=250$ GeV, and $A_0=650$ GeV. (b) $M_{Z'}$ (solid line), $|u|$ (short-dashed line), and $f'|u|$ (long-dashed line) as a function of m_0 for the same values of input parameters as in (a).

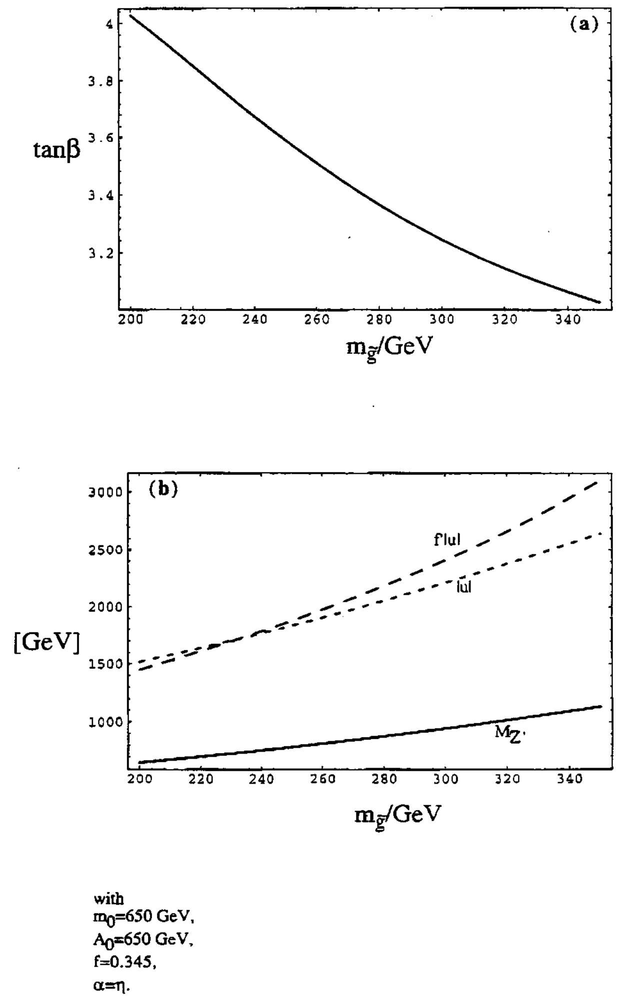


FIG. 9. (a) $\tan\beta$ as a function of $m_{\tilde{g}}$ for $\alpha=\eta$ and $f=0.345$, $A_0=650$ GeV, and $m_0=650$ GeV. (b) $M_{Z'}$ (solid line), $|u|$ (short-dashed line), and $f'|u|$ (long-dashed line) as a function of m_0 for the same values of input parameters as in (a).

sizable coupling serving the purpose of f', which is certainly possible since we have three copies of h and h^c in these models.

A very important outcome of Eq. (42) is that the $\mathrm{U}(1)_\alpha$ and electroweak symmetry breakings are related. To see this, go back to the two-doublet Higgs potential V of Eq. (10). Using Eqs. (6)–(9) and Eq. (35), we can express the parameters m_{12}^2, m_1^2, and m_2^2 in terms of the mass of the pseudoscalar boson m_A, and $\tan\beta$:

$$m_{12}^2=-m_A^2\sin\beta\cos\beta, \tag{64}$$

$$m_1^2=m_A^2\sin^2\beta-\frac{1}{2}M_Z^2\cos 2\beta-\frac{2f^2}{g_Z^2}M_Z^2\times\left[2\sin^2\beta+\left(1-\sqrt{\frac{3}{5}}\tan\alpha\right)\cos^2\beta-\frac{3f^2}{2\cos^2\alpha g_\alpha^2}\right], \tag{65}$$

$$m_2^2=m_A^2\cos^2\beta+\frac{1}{2}M_Z^2\cos 2\beta-\frac{2f^2}{g_Z^2}M_Z^2\times\left[2\cos^2\beta+\left(1+\sqrt{\frac{3}{5}}\tan\alpha\right)\sin^2\beta-\frac{3f^2}{2\cos^2\alpha g_\alpha^2}\right]. \tag{66}$$

On the other hand, using Eq. (42), we have

$$m_{12}^2=fA_fu, \tag{67}$$

$$m_1^2=m_0^2+m_{R1}^2+f^2u^2-\frac{1}{4}\left(1-\sqrt{\frac{3}{5}}\tan\alpha\right)M_{Z'}^2, \tag{68}$$

$$m_2^2=m_0^2+m_{R2}^2+f^2u^2-\frac{1}{4}\left(1+\sqrt{\frac{3}{5}}\tan\alpha\right)M_{Z'}^2, \tag{69}$$

where m_{R1}^2 and m_{R2}^2 differ in that λ_t (the Yukawa coupling of Φ_2 to the t quark) contributes to the latter but not to the former. Both depend on m_0, $m_{1/2}$, A_0, and the various gauge couplings g_i, as well as f and f'. Matching Eqs. (64)–(66)

with Eqs. (67)–(69) allows us to determine u and $\tan\beta$ for a given set of parameters at the grand-unification scale.

We will now briefly discuss our method for finding u and $\tan\beta$ for a given set of universal soft supersymmetry-breaking parameters $m_{\tilde{g}}$, m_0, and A_0 at the GUT scale and the Yukawa coupling f, when such a solution exists. First, we guess a value for $\tan\beta$ so as to choose a value for λ_t. We then form a table $[M_{Z'}, m_{R1}^2, m_{R2}^2, A_f](f')$ for many very closely spaced values of f' extending up to where $f'(M_G)$ reaches its perturbative limit. By ''closely spaced values of f','' we mean that between two consecutive entries in the table, none of the four parameters differs by more than 1%. Second, we guess a value for $M_{Z'}$ which lies within the range in the table, so as to choose m_{R1}^2, m_{R2}^2, and A_f from the entry of the table which has $M_{Z'}$ closest to this value. Third, we equate the right-hand sides of Eq. (65) + Eq. (66) and Eq. (68) + Eq. (69) to solve for m_A^2 as a linear function of u^2 and $\cos^2\beta$. Fourth, using the previous result for m_A^2 we equate the right-hand sides of Eqs. (65), (66), (68), and (69) to solve for u^2 as a function of $\cos^2\beta$ of the form of a linear function divided by another linear function. Fifth, using the expressions from the previous two steps we equate the right-hand sides of Eq. (64) with that of Eq. (67) and solve numerically for $\cos^2\beta$, and hence $\tan\beta$, by first searching for a root close to the value corresponding to our original guess for $\tan\beta$. In doing this fifth step, one needs to choose $fu>0$ or $fu<0$ analogous to $\mu>0$ or $\mu<0$ in the MSSM, and then check that the solution is consistent with $m_A^2 = -fA_f u/\sin\beta\cos\beta > 0$. In fact, taking all Yukawa couplings and $\tan\beta$ to be positive as well as our convention for the trilinear coupling parameters A_i, solutions exist only for $u<0$. Next, if a solution to these steps has been found, we start the entire cycle over using the values for $\tan\beta$ and $M_{Z'}$ just calculated as the new ''guessed'' values. This iteration is continued until the predicted $\tan\beta$ and $M_{Z'}$ become fixed to a reasonable accuracy (we demand about 5% accuracy). This process can be speeded up by adding a sixth step to the cycle which repeats the third through fifth steps until the prediction for $\tan\beta$ and $M_{Z'}$ become fixed for the table found in the second step of the cycle.

Before we discuss our results, we remind the reader that f has a maximum possible value that comes from requiring that the Higgs potential be bounded from below and which depends on the additional U(1). We plot this maximum value $f_{\max}$ as a function of α [see Eq. (20)] in Fig. 3. In particular, the η model requires f to be less than about 0.35, whereas, for $\alpha=0$, f could be as great as 0.46. Note that as $|\alpha|$ approaces $\pi/2$, $f_{\max}$ approaches 0. From Fig. 2(a), one can see that if f is small enough so that $\alpha=\eta$ is allowed, then $f(M_G)$ will always be perturbatively small for a perturbatively valued f'. In our examples, we will only be interested in values of $f<0.35$.

In Fig. 4, we show the predicted values of $\tan\beta$ and $M_{Z'}$ as a function of α for $f=0.345$ and $m_{\tilde{g}}=200$ GeV, $A_0=650$ GeV, and $m_0=650$ GeV. In accordance with Fig. 3, we are only interested in showing $|\alpha|$ less than about 0.7. We have also plotted the magnitude $|u|$ of the VEV of the singlet Higgs boson χ and the mass $f'|u|$ of the exotic fermion $h(h^c)$. In Fig. 5, we show the similar situation for $f=0.345$ and $m_{\tilde{g}}=300$ GeV, $A_0=950$ GeV, and $m_0=950$ GeV. These two figures are quite similar except that the mass scale in Fig. 5(b) has been pushed up relative to that shown in Fig. 4(b). These choices of soft supersymmetry-breaking parameters are fairly typical in that generally we need m_0 to be at least twice as great as $m_{\tilde{g}}$ to find a solution. Further, if we want to have a solution for all α less than some value, A_0 must be positive and of order m_0.

In Figs. 6–9, we illustrate the effects of varying the parameters f, $m_{\tilde{g}}$, A_0, and m_0 for a fixed value of $\alpha=\eta$. We look at the solutions for $\tan\beta$ and $M_{Z'}$ (as well as $|u|$ and $f'|u|$) when the four input parameters are varied one at a time around the point $f=0.345$ and $m_{\tilde{g}}=250$ GeV, $A_0=650$ GeV, and $m_0=650$ GeV. Note from Fig. 6 that with decreasing f, $\tan\beta$ and $M_{Z'}$ both increase. We do not extend f above 0.345 so as to avoid the upper bound coming from Fig. 3. We find also that we cannot decrease f much below 0.32 for this example and still have a solution for the electroweak breaking. To use smaller values of f, one would have to increase the scale of the soft supersymmetry-breaking parameters. In Fig. 7, we look at the effect of varying A_0. The range of A_0 examined is restricted because any extension in either direction would require values of $M_{Z'}$ larger than can be reached via the $f'hh^cS$ term with f' within the perturbative regime. In Fig. 8, we vary m_0. Note that with increasing m_0, the predicted $\tan\beta$ increases significantly and $M_{Z'}$ decreases. In this example, increasing m_0 beyond 1200 GeV would predict an $M_{Z'}$ less than 500 GeV and a $\tan\beta$ greater than 10. The lower limit of 500 GeV for m_0 used here is due to the same reason as just given for the range of A_0 plotted in the previous figure. In Fig. 9, we show the effect of varying the gluino mass which is also here the GUT scale universal gaugino mass. With increasing gluino mass, $\tan\beta$ decreases while $M_{Z'}$ increases. The upper limit of 350 GeV used here for the gluino mass again corresponds to about the size of that parameter for this example where increasing it anymore would require values of $|u|$ larger than can be reached perturbatively through the renormalization-group equations. We find the general trends of Fig. 6–9 to be typical of other choices of parameter values where consistent solutions exist.

If m_0 is demanded to be less than about 1 TeV, then in general $\tan\beta<10$, where the b and τ Yukawa couplings are small enough not to contribute significantly to the renormalization-group equations. It is interesting to note that in contrast to the MSSM, where $m_1^2 - m_2^2 = -m_{R_2}^2(\lambda_t) = -(m_A^2 + m_Z^2)\cos 2\beta$, solutions with $\tan\beta<1$ in principle are possible here due to the TeV scale D terms. However, to have such a solution in practice with $m_t^{(\mathrm{pole})} \approx 175$ GeV means having $\lambda_t(m_t)$ greater than its fixed-point value of about 1.22 with $\alpha_G = \alpha_s(1\ \mathrm{TeV}) \approx 0.1$ where the gauge couplings run according to the additional exotic field content as we have chosen.

If $fA_f/g_x^2 u$, where $g_x^2 = (2/3) g_1^2 \cos^2\alpha$ is not small, then Eqs. (64)–(66) have additional contributions, but they are always suppressed by v^2/u^2 relative to $m_{12}^2 = fA_f u$, and hence our numerical results on $\tan\beta$ and $M_{Z'}$, etc., do not change appreciably. The corrections are only important if the masses and splittings of the two Higgs doublets are considered.

VI. CONCLUSIONS

We have shown in this paper that there are many interesting and important phenomenological consequences if we assume the existence of a supersymmetric U(1) gauge factor at the TeV energy scale. We assume that there is a Higgs superfield S which is a singlet under the standard gauge group but which transforms nontrivially under this extra U(1) so that it may break the latter spontaneously without breaking the former. We assume also that H_1H_2S is an allowed term in the superpotential. We then analyze the most general form of the Higgs potential and derive an upper limit on the lighter of the two neutral scalar Higgs bosons of the two-doublet Higgs sector as shown in Fig. 1. This generalizes the well-known case of the minimal supersymmetric standard model.

We then specialize to the case where this extra U(1) is derivable from a E_6 model with the particle content given by its fundamental **27** representation. We discuss the effect on Z-Z' mixing and the oblique parameters $\epsilon_{1,2,3}$, as well as the extended neutralino mass matrix. We then work out in detail the consequences for supersymmetric scalar masses. We note that the mere existence of a spontaneously broken U(1) gauge factor at the TeV scale implies new important corrections to these masses through the so-called D terms which are now dominated by $M_{Z'}^2$ instead of just M_Z^2 in the MSSM. This changes the entire supersymmetric scalar particle spectrum and should not be overlooked in future particle searches.

Assuming universal soft supersymmetry-breaking terms at the GUT scale, we match the electroweak breaking parameters with the corresponding ones from the U(1) breaking. Specifically, the values of m_1^2, m_2^2, and m_{12}^2 in the well-known two-doublet Higgs potential are constrained as shown by Eqs. (64)–(69). We then obtain consistent numerical solutions to these constraints and demonstrate how the U(1)-breaking scale and the parameter $\tan\beta \equiv v_2/v_1$ are related through the H_1H_2S coupling. Our results are presented in Figs. 2–9.

During the final stage of completing this manuscript, we became aware of Ref. [21], which also discusses electroweak symmetry breaking with an additional supersymmetric U(1) gauge factor, but the emphasis there is on the case $f'=0$. The case $f'\neq 0$ is also discussed there, but the conclusion is that whereas the breaking of the additional U(1) radiatively via the term $f'hh^cS$, already noted in Ref. [7], can be achieved with universal soft supersymmetry-breaking terms at the GUT scale, it does not work in the large trilinear coupling scenario. Our approach is essentially orthogonal. We concentrate on solutions where the U(1) scale is much larger than the electroweak scale. With the two scales being intimately related through the matching of Eqs. (64)–(66) with Eqs. (67)–(69), it is in fact highly nontrivial to find solutions which are consistent with this matching even with an arbitrary f'. We note also that our examples are models with complete E_6 particle content and, in our approximation, the Yukawa coupling f is bounded as shown in Fig. 3. In the more general case, the bound on f increases as the trilinear coupling increases.

ACKNOWLEDGMENTS

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