New quadratic baryon mass relations

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(Received 17 June 1997; revised manuscript received 21 August 1997)

By assuming the existence of (quasi)linear baryon Regge trajectories, we derive new quadratic Gell-Mann–Okubo-type baryon mass relations. These relations are used to predict the masses of the charmed baryons absent from the baryon summary table so far, in good agreement with the predictions of many other approaches. [S0556-2821(97)06223-1]

PACS number(s): 12.40.Yx, 11.55.Jy, 12.40.Nn, 14.20.Lq

The investigation of the properties of hadrons containing heavy quarks is of great interest for understanding the dynamics of the quark-gluon interaction. Recently predictions about the heavy baryon mass spectrum have become a subject of increasing interest [1–11], due to current experimental activity of several groups at CERN [12], Fermilab [13], and the Cornell Electron Storage Ring (CESR) [14,15] aimed at the discovery of the baryons so far absent from the baryon summary table [16]. Recently, for the CERN Large Hadron Collider (LHC), B factories, and the Fermilab Tevatron with high luminosity, several experiments have been proposed in which a detailed study of heavy baryons can be performed. In this connection, an accurate theoretical prediction for the baryon mass spectrum becomes a guide for experimentalists. To calculate the heavy baryon mass spectrum, potential models [1,17–22], nonrelativistic quark models [23–25], relativistic quark models [6], bag models [26–29], lattice QCD [30–32], QCD spectral sum rules [33], heavy quark effective theory [11,34–36], chiral perturbation theory [2], chiral quark model [9], SU(4) skyrmion model [37], group theoretical [10,38,39], variational [40], and other approaches [3–5,7,8,41–45] are widely used.

The charm baryon masses measured to date are[1] [16]

$$\Lambda_c = 2285 \ \ \text{MeV},$$

$$\Sigma_c = 2453 \pm 1 \ \ \text{MeV},$$

$$\Xi_c = 2468 \pm 2 \ \ \text{MeV},$$

$$\Omega_c = 2704 \pm 4 \ \ \text{MeV},$$

$$\Sigma_c^* = 2521 \pm 4 \ \ \text{MeV},$$

$$\Xi_c^* = 2644 \pm 2 \ \ \text{MeV}.$$

An observation of the $\Xi_c' = 2563 \pm 15$ MeV was reported by the WA89 Collaboration [12]. The Ω_c^*, as well as double- and triple-charmed baryons, have not yet been observed.

Almost all very recent calculations very consistently predict the mass of the Ξ_c' to be around 2580 MeV [2,4,5,7,8,11] (see also [20,42,44]). Similarly, the mass of the Ω_c^* is very consistently predicted to be around 2770 MeV [2–5,7,9,11] (see also [1,21,26,30,42]). Predictions for the doubly and triply charmed baryon masses are less definite.

Here we wish to extend the approach based on the assumption of (quasi)linearity of the Regge trajectories of heavy hadrons in the low-energy region, initiated in our previous papers for heavy mesons [47,48], to baryons. We shall show that new quadratic Gell-Mann–Okubo-type baryon mass relations can be obtained and used to predict the missing charmed baryon masses. As we shall see, the predicted masses are in good agreement with the results of many other approaches, which should add confidence to an experimental focus on the predicted ranges.

The Regge poles (the singularities in the scattering amplitude), first introduced in the 1960s, manifest themselves in both the direct channel as resonances and the crossed channel as exchanged particles (Reggeons). It is known from the study of the analytic properties of the scattering amplitude $A(E,z)$ $[d\sigma/d\Omega = |A(E,z)|^2$; here, E is the energy and $z = \cos\theta$ the scattering angle] in the complex angular momentum (ℓ) plane that a physical resonance appears when $\text{Re}\,\ell(E)$ passes near a non-negative integer, and if the Regge pole moves to ever-increasing ℓ values in the complex ℓ plane as the energy E is increased, it generates a tower of high-spin states which is said to belong to a given Regge trajectory, for both mesons and baryons [49]. This fact is borne out by experiment: In the direct channel, one sees such trajectories of mesons going up to $\ell = 6$ and of baryons up to $\ell = 15/2$ (for the leading Δ trajectory) [16]. These trajectories are remarkably linear and approximately parallel; i.e., the angular momentum is a linear function of the square of the particle mass.

Keeping in mind experimental evidence, let us assume, as

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[1]For Σ_c^* we take the uncertainty weighted average of the results of Ref. [46], $2530 \pm 5 \pm 5$ MeV, and the most recent results by CLEO [15], 2518.6 ± 2.2 MeV.

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in [47,48], the (quasi)linear form of Regge trajectories for baryons with identical J^P quantum numbers (i.e., belonging to a common multiplet). Then for the states with orbital momentum ℓ one has (i,j,k stand for the corresponding flavor content)

$$\ell=\alpha'_{kii}m^2_{kii}\ +a_{kii}(0),$$

$$\ell=\alpha'_{kji}m^2_{kji}\ +a_{kji}(0),$$

$$\ell=\alpha'_{kjj}m^2_{kjj}+a_{kjj}(0).$$

In the following, we restrict ourselves to positive parity baryons states alone (viz., $\frac{1}{2}^+$ and $\frac{3}{2}^+$ multiplets). In terms of the nonrelativistic quark model, for these states $\ell=\frac{1}{2}+L$, where L is orbital angular momentum of one of the quarks about the remaining pair of quarks (for negative parity states, one has, respectively, $\ell=\frac{3}{2}+L$).

Using now the relation among the intercepts [50–52],

$$a_{kii}(0)+a_{kjj}(0)=2a_{kji}(0), \tag{1}$$

one obtains, from the above relations,

$$\alpha'_{kii}m^2_{kii}+\alpha'_{kjj}m^2_{kjj}=2\alpha'_{kji}m^2_{kji}\,. \tag{2}$$

In order to eliminate the Regge slopes from this formula, we need a relation among the slopes. Two such relations exist,

$$\alpha'_{kii}\cdot\alpha'_{kjj}=(\alpha'_{kji})^2, \tag{3}$$

which follows from the factorization of residues of the t-channel poles [53–55], and

$$\frac{1}{\alpha'_{kii}}+\frac{1}{\alpha'_{kjj}}=\frac{2}{\alpha'_{kji}}, \tag{4}$$

which may be derived by generalizing the corresponding relation for quarkonia based on topological expansion and the $q\bar{q}$-string picture [52] to the case of a baryon viewed as a quark-diquark-string object[2] [59].

For light baryons (and small differences in the α' values), there is no essential difference between these two relations; viz., for $\alpha'_{kji}=\alpha'_{kii}/(1+x)$, $x\ll 1$, Eq. (4) gives $\alpha'_{kjj}=\alpha'_{kii}/(1+2x)$, whereas Eq. (3) gives $\alpha'_{kjj}=\alpha'_{kii}/(1+x)^2\approx\alpha'/(1+2x)$, i.e., essentially the same result to order x^2. However, for heavy baryons (and expected large differences from the α' values for the light baryons) these relations are incompatible; e.g., for $\alpha'_{kji}=\alpha'_{kii}/2$, Eq. (3) will give $\alpha'_{kjj}=\alpha'_{kii}/4$, whereas from Eq. (4), $\alpha'_{kjj}=\alpha'_{kii}/3$. One therefore has to choose between these relations in order to proceed further. Here, as in [47,48], we use Eq. (4), since it is much more consistent with Eq. (2) than is Eq. (3), which we tested by using measured light quark baryon masses in Eq. (2). Kosenko and Tutik [41] used the relation (3) and obtained much higher values for the charmed baryon masses than the measured ones (e.g., $\Omega_c=2788$ MeV) and those predicted by most other approaches (see Table I). The reason for this is that lower values for the Regge slopes, as illustrated by the example above, lead to higher values for the masses. We shall justify our choice of Eq. (4) in more detail in a separate publication [59].

Here we make the following change in the notation: the subscripts which stand for the flavor content of the baryon in Eqs. (1)–(4) are replaced by the numbers of the corresponding quarks (n stands for the number of the nonstrange quarks, etc.):

$$\alpha'_{n,s,c}\,,\quad a_{n,s,c}(0);$$

e.g.,

$$a_{nnn}(0)\equiv a_{3,0,0}(0),\quad a_{snn}(0)\equiv a_{2,1,0}(0),$$

$$a_{csn}(0)\equiv a_{1,1,1}(0),\quad \text{etc.}$$

It is easy to see that the following relations solve Eqs. (1) and (4), respectively:

$$a^\star_{n,s,c}(0)=a^\star(0)-\lambda^\star_s s-\lambda^\star_c c,\quad a^\star(0)\equiv a^\star_{3,0,0}(0), \tag{5}$$

$$\frac{1}{\alpha'_{\star;n,s,c}}=\frac{1}{\alpha'_\star}+\gamma^\star_s s+\gamma^\star_c c,\quad \alpha'_\star\equiv\alpha'_{\star;3,0,0}\,,\quad n+s+c=3, \tag{6}$$

where the sub- and superscript $\star$'s allow for possible differences between multiplets (such as $\frac{1}{2}^+$ octet and $\frac{3}{2}^+$ decuplet). This particularly simple choice of the solution to Eqs. (1) and (4) (i.e., linearity in the Regge intercepts and inverse slopes) corresponds to a minimal number of free parameters (four: two λ's and two γ's), and is therefore most suitable for our present purposes.

It then follows from Eqs. (6) that

$$\alpha'_\Lambda=\alpha'_\Sigma=\frac{\alpha'_N}{1+\gamma^N_s\alpha'_N}, \tag{7}$$

$$\alpha'_\Xi=\frac{\alpha'_N}{1+2\gamma^N_s\alpha'_N}, \tag{8}$$

$$\alpha'_{\Lambda_c}=\alpha'_{\Sigma_c}=\frac{\alpha'_N}{1+\gamma^N_c\alpha'_N}, \tag{9}$$

$$\alpha'_{\Xi_c}=\alpha'_{\Xi'_c}=\frac{\alpha'_N}{1+(\gamma^N_s+\gamma^N_c)\alpha'_N}, \tag{10}$$

[2]This structure is known to be responsible for the slopes of baryon trajectories being equal to those of meson trajectories [56–58].

TABLE I. Simultaneous fit to the vector meson, octet baryon, and decuplet baryon spectra, through the relations $\rho^2=K^{*2}/(1+x)-\lambda_s^\rho=\phi^2/(1+2x)-2\lambda_s^\rho$, $N^2=\Sigma'^2/(1+x)-\lambda_s^N=\Xi^2/(1+2x)-2\lambda_s^N$, and $\Delta^2=\Sigma^{*2}/(1+x)-\lambda_s^\Delta=\Xi^{*2}/(1+2x)-2\lambda_s^\Delta=\Omega^2/(1+3x)-3\lambda_s^\Delta$, as compared to the measured value $K^{*0}=896$ MeV, $\phi=1019$ MeV, $\Sigma'^2=1.290\pm0.003$ GeV^2, $\Xi=1318\pm3$ MeV, $\Sigma^*=1385\pm2$ MeV, $\Xi^*=1533.5\pm1.5$ MeV, and $\Omega=1672.5$ MeV. The input parameters are $\rho=769$ MeV, $N=939$ MeV, and $\Delta=1232$ MeV. λ's are measured in GeV^2.

x	λ_s^ρ	λ_s^N	λ_s^Δ	K^*	ϕ	Σ'^2	Ξ	Σ^*	Ξ^*	Ω
0	0.219	0.422	0.420	900	1015	1.304	1314	1392	1536	1667
0.010	0.209	0.407	0.395	899	1015	1.302	1315	1390	1534	1669
0.020	0.201	0.392	0.371	899	1016	1.299	1316	1388	1533	1670
0.030	0.192	0.377	0.348	898	1017	1.296	1317	1386	1532	1671
0.040	0.183	0.363	0.326	897	1017	1.295	1318	1385	1531	1672
0.050	0.175	0.350	0.305	897	1018	1.293	1319	1383	1530	1673
0.060	0.167	0.336	0.285	897	1018	1.291	1319	1382	1529	1673
0.070	0.159	0.323	0.266	896	1018	1.289	1320	1382	1529	1674
0.080	0.152	0.310	0.249	896	1019	1.287	1320	1381	1529	1675
0.100	0.137	0.286	0.215	895	1019	1.284	1321	1381	1529	1677
0.150	0.104	0.232	0.140	894	1019	1.281	1323	1381	1529	1676
0.200	0.075	0.185	0.077	894	1019	1.280	1324	1383	1530	1673
0.277	0.038	0.122	0	896	1018	1.282	1323	1392	1535	1667

$$\alpha'_{\Omega_c}=\frac{\alpha'_N}{1+(2\gamma_s^N+\gamma_c^N)\alpha'_N}, \tag{11}$$

$$\alpha'_{\Xi_{cc}}=\frac{\alpha'_N}{1+2\gamma_c^N\alpha'_N}, \tag{12}$$

$$\alpha'_{\Omega_{cc}}=\frac{\alpha'_N}{1+(\gamma_s^N+2\gamma_c^N)\alpha'_N}, \tag{13}$$

where we use $\star=N$ to represent the $\frac{1}{2}^+$ multiplet and with $\star=\Delta$ to represent the $\frac{3}{2}^+$ multiplet,

$$\alpha'_{\Sigma^*}=\frac{\alpha'_\Delta}{1+\gamma_s^\Delta\alpha'_\Delta}, \tag{14}$$

$$\alpha'_{\Xi^*}=\frac{\alpha'_\Delta}{1+2\gamma_s^\Delta\alpha'_\Delta}, \tag{15}$$

$$\alpha'_{\Omega}=\frac{\alpha'_\Delta}{1+3\gamma_s^\Delta\alpha'_\Delta}, \tag{16}$$

$$\alpha'_{\Sigma_c^*}=\frac{\alpha'_\Delta}{1+\gamma_c^\Delta\alpha'_\Delta}, \tag{17}$$

TABLE II. Comparison of predictions for the charmed baryon masses not measured so far (in MeV): Potential models [1,17–22], chiral perturbation theory [2], relativistic quark model [6], chiral quark model [9], heavy quark effective theory [11,35,36], nonrelativistic quark models [23,25], bag models [26–29], lattice QCD [30,32], QCD spectral sum rules [33], SU(4) Skyrmion model [37], group theoretical models [38,39], variational approach [40], and other models [3–5,7,8,41–45].

Reference	Ξ_c'	Ξ_{cc}	Ω_{cc}	Ω_c^*	Ξ_{cc}^*	Ω_{cc}^*	Ω_{ccc}
Present work	2569 ± 6	3610 ± 3	3804 ± 8	2767 ± 7	3735 ± 17	3850 ± 25	4930 ± 45
[1]			3737	2760		3797	4787
[2]	2579			2768			
[3]				2771			
[4]	2580 ± 20	3660 ± 70	3740 ± 80	2770 ± 30	3740 ± 70	3820 ± 80	
[5]	2582	3676	3787	2775	3746	3851	
[6]		3660	3760		3810	3890	
[7]	2580 ± 10			2770 ± 10			
[8]	2583 ± 3						
[9]	2593			2765			
[11]	2581 ± 2			2761 ± 5			
[17]	2510	3550	3730	2720	3610	3770	4810
[18]	2532			2780			5026
[19]	2566	3605	3732	2836	3680	3801	4793
[20]	2579	3645	3824		3733		4837
[21]	2558	3613	3703	2775	3741	3835	4797
[22]		3710			3750		4923
[23]	2590			2805			
[25]	2608			2822			
[26]	2530	3511	3664	2764	3630	3764	4747

TABLE II. (*Continued*).

Reference	Ξ_c'	Ξ_{cc}	Ω_{cc}	Ω_c^*	Ξ_{cc}^*	Ω_{cc}^*	Ω_{ccc}
[27]							5040
[28]	2500			2710			
[29]	2467			2659			
[30]				2767 ± 35			
[32]	2570^{+6+6}_{-3-6}			2660^{+5+6}_{-3-7}			
[33]		3630 ± 50	3720 ± 50		3735 ± 50	3840 ± 50	
[35]		3742			3811		
[36]	2570	3610	3710	2740	3680	3760	4730
[37]	2596	3752	3934	2811	3793	3964	5127
[38]	2600	3725	3915	2811	3783	3953	5106
[39]	2690	3700	3960	2810	3768	3931	5019
[40]	2578	3614			3731		
[41]	2616	3837	4036				
[42]	2583			2772			
[43]	2542	3710	3852	2798	3781	3923	5048
[44]	2578	3661	3785	2782	3732	3856	4895
[45]	2584	3758	3861				

$$\alpha'_{\Xi_c^*}=\frac{\alpha'_\Delta}{1+(\gamma_s^\Delta+\gamma_c^\Delta)\alpha'_\Delta}, \tag{18}$$

$$\alpha'_{\Omega_c^*}=\frac{\alpha'_\Delta}{1+(2\gamma_s^\Delta+\gamma_c^\Delta)\alpha'_\Delta}, \tag{19}$$

$$\alpha'_{\Xi_{cc}^*}=\frac{\alpha'_\Delta}{1+2\gamma_c^\Delta\alpha'_\Delta}, \tag{20}$$

$$\alpha'_{\Omega_{cc}^*}=\frac{\alpha'_\Delta}{1+(\gamma_s^\Delta+2\gamma_c^\Delta)\alpha'_\Delta}, \tag{21}$$

$$\alpha'_{\Omega_{ccc}}=\frac{\alpha'_\Delta}{1+3\gamma_c^\Delta\alpha'_\Delta}. \tag{22}$$

Consider first the $J^P=\frac{3}{2}^+$ baryons. Introduce, for simplicity,

$$x\equiv\gamma_s^\Delta\alpha'_\Delta,\quad y\equiv\gamma_c^\Delta\alpha'_\Delta. \tag{23}$$

It then follows from Eqs. (5), (6), and (14)–(22) that

$$\Delta^2=\frac{\Sigma^{*2}}{1+x}-\lambda_s^\Delta=\frac{\Xi^{*2}}{1+2x}-2\lambda_s^\Delta=\frac{\Omega^2}{1+3x}-3\lambda_s^\Delta=\frac{\Sigma_c^{*2}}{1+y}-\lambda_c^\Delta=\frac{\Xi_c^{*2}}{1+x+y}-\lambda_s^\Delta-\lambda_c^\Delta=\frac{\Omega_c^{*2}}{1+2x+y}-2\lambda_s^\Delta-\lambda_c^\Delta=\frac{\Xi_{cc}^{*2}}{1+2y}-2\lambda_c^\Delta$$

$$=\frac{\Omega_{cc}^{*2}}{1+x+2y}-\lambda_s^\Delta-2\lambda_c^\Delta=\frac{\Omega_{ccc}^2}{1+3y}-3\lambda_c^\Delta. \qquad (24)$$

Note that there are four unknown parameters for each multiplet. By eliminating them, i.e., x, y, λ_s^Δ, and λ_c^Δ, from the above nine equalities, we can obtain five relations for baryon masses: e.g.,

$$\Omega^2-\Delta^2=3(\Xi^{*2}-\Sigma^{*2}), \qquad (25)$$

$$\Omega_{ccc}^2-\Delta^2=3(\Xi_{cc}^{*2}-\Sigma_c^{*2}), \qquad (26)$$

$$\Omega_{ccc}^2-\Omega^2=3(\Omega_{cc}^{*2}-\Omega_c^{*2}), \qquad (27)$$

$$(\Sigma_c^{*2}-\Delta^2)+(\Omega_c^{*2}-\Xi^{*2})=2(\Xi_c^{*2}-\Sigma^{*2}), \qquad (28)$$

$$(\Omega_{cc}^{*2}-\Xi_{cc}^{*2})+(\Sigma^{*2}-\Delta^2)=2(\Xi_c^{*2}-\Sigma_c^{*2}). \qquad (29)$$

However, just four of them are linearly independent, because of an invariance of the nine equalities under simultaneous permutation $(x\leftrightarrow y, \quad \lambda_s\leftrightarrow\lambda_c)$.

Here only Eq. (25) can be tested, since Eqs. (26)–(29) contain the baryon masses not measured so far. For Eq. (25), one obtains (in GeV^2) 1.280 ± 0.005 vs 1.300 ± 0.030, taking the electromagnetic mass splittings as a measure of the uncertainty (since electromagnetic corrections are not included in our analysis), with an accuracy of $\sim1.5\%$.

The analysis may be easily repeated for the $J^P=\frac{1}{2}^+$ baryons, leading to the following two independent mass relations:

$$(\Sigma_c'^2-N^2)+(\Omega_c^2-\Xi^2)=2(\tilde{\Xi}_c^2-\Sigma'^2), \qquad (30)$$

$$(\Omega_{cc}^2-\Xi_{cc}^2)+(\Sigma'^2-N^2)=2(\tilde{\Xi}_c^2-\Sigma_c'^2), \qquad (31)$$

where

$$\Sigma'^2\equiv a\Lambda^2+(1-a)\Sigma^2, \qquad (32)$$

$$\Sigma_c'^2\equiv b\Lambda_c^2+(1-b)\Sigma_c^2, \qquad (33)$$

$$\tilde{\Xi}_c^2\equiv c\Xi_c^2+(1-c)\Xi_c'^2 \qquad (34)$$

are introduced to distinguish between the states having the same flavor content and J^P quantum numbers, and a, b, and c are not known *a priori.* In order to establish the values of a, b, and c, we use the following relation for the intercepts of the $\frac{1}{2}^+$ baryon trajectories in the noncharmed sector [60],

$$2[a_N(0)+a_\Xi(0)]=3a_\Lambda(0)+a_\Sigma(0), \qquad (35)$$

which has been subsequently generalized to the charmed sector by replacing the s quark by the c quark, as follows [41]:

$$2[a_N(0)+a_{\Xi_{cc}}(0)]=3a_{\Lambda_c}(0)+a_{\Sigma_c}(0). \qquad (36)$$

It then follows from the corresponding relations based on Eqs. (1) and (2) that, respectively,

$$\alpha_N'N^2+\alpha_\Xi'\Xi^2=2\alpha_{\Sigma'}'\left(\frac{3}{4}\Lambda^2+\frac{1}{4}\Sigma^2\right), \quad \alpha_{\Sigma'}'\equiv\alpha_\Lambda'=\alpha_\Sigma', \qquad (37)$$

$$\alpha_N'N^2+\alpha_{\Xi_{cc}}'\Xi_{cc}^2=2\alpha_{\Sigma_c'}'\left(\frac{3}{4}\Lambda_c^2+\frac{1}{4}\Sigma_c^2\right), \quad \alpha_{\Sigma_c'}'\equiv\alpha_{\Lambda_c}'=\alpha_{\Sigma_c}', \qquad (38)$$

and, therefore,

$$\Sigma'^2=\frac{3}{4}\Lambda^2+\frac{1}{4}\Sigma^2, \qquad (39)$$

$$\Sigma_c'^2=\frac{3}{4}\Lambda_c^2+\frac{1}{4}\Sigma_c^2; \qquad (40)$$

i.e., in the relations (32) and (33), $a=b=\frac{3}{4}$. It is also seen that the only parameter which is responsible for different weighting of the states having the same flavor content and J^P quantum numbers is the isospin of the state. Thus, since both Ξ_c and Ξ_c' have equal isospin $(I=\frac{1}{2})$, they should enter a mass relation with equal weights; i.e., in Eq. (34), $c=1/2$ and

$$\tilde{\Xi}_c^2=\frac{\Xi_c^2+\Xi_c'^2}{2}. \qquad (41)$$

Equations (25)–(31), with Eqs. (39)–(41), are new quadratic baryon mass relations. In the following, we shall make predictions for the baryon masses not measured so far using these relations.

For the $\frac{1}{2}^+$ baryons, in the approximation of equality of the slopes in the light quark sector, $\alpha_N'\cong\alpha'_{\Sigma'}\cong\alpha_\Xi'$ [i.e., $\gamma_s^N\alpha_N'\ll1$ in Eqs. (7) and (8)], it follows from Eq. (37) that

$$2(N^2+\Xi^2)\cong3\Lambda^2+\Sigma^2, \qquad (42)$$

which is a relation obtained by Oneda and Terasaki in the algebraic approach to hadronic physics [61] which holds with an accuracy of $\sim1.5\%$: (in GeV) 5.235 ± 0.015 vs 5.160 ± 0.010. A similar approximation for the $\frac{3}{2}^+$ baryons leads, through Eq. (2), to the relations

$$\Omega^2-\Xi^{*2}\cong\Xi^{*2}-\Sigma^{*2}\cong\Sigma^{*2}-\Delta^2, \qquad (43)$$

which have long been discussed in the literature [51,61–63] and hold with a high accuracy, as well as Eq. (42).

Note that the linear counterpart of Eq. (42), the standard Gell-Mann–Okubo mass formula for the octet baryons,

$$2(N+\Xi)=3\Lambda+\Sigma,$$

holds with better accuracy than Eq. (42): One has (in GeV) 4.514 ± 0.006 vs 4.540 ± 0.004, with $\sim 0.6\%$ accuracy. This is because Eq. (42) is not exact but given in the approximation of the equality of the slopes in the light quark sector which is the reason for its worse accuracy. As follows from our analysis given above, the only exact relation in the light quark sector is Eq. (25), whose accuracy (1.5%) is very similar to that of its linear counterpart

$$\Omega-\Delta=3(\Xi^*-\Sigma^*),$$

which is the only relation for the decuplet baryons to the second order in flavor symmetry breaking [64], which gives (in GeV) 0.440 ± 0.002 vs 0.446 ± 0.010, with 1.35% accuracy. We arrive therefore at the conclusion that both quadratic and linear mass relations are quite successful in the light quark sector. This situation is very similar to earlier symmetry-breaking models, often quite accurate in restricted domains, which cannot discriminate between quadratic and linear mass relations and, in fact, employ both.

The mass of the Ξ_c' can now be obtained from Eqs. (30) and (39)–(41). Using the measured masses of the states entering these relations, one finds

$$\Xi_c'=2569\pm 6 \ \ \text{MeV}. \tag{44}$$

The mass of the Ω_c^* is obtained from Eq. (28):

$$\Omega_c^*=2767\pm 7 \ \ \text{MeV}. \tag{45}$$

One sees that the value for the Ξ_c' mass, Eq. (44), lies within the interval provided by experiment [12]. Both Eqs. (44) and (45) are consistent with the values 2580 and 2770 MeV, respectively, predicted by almost all very recent calculations [2,4,5,7,11].

Now, we have two (independent) relations for the $\frac{3}{2}^+$ baryons, Eqs. (26) or (27) and (29), to make predictions for the three unknown masses of the Ξ_{cc}^*, Ω_{cc}^*, and Ω_{ccc}. Similarly, we have one relation for the $\frac{1}{2}^+$ baryons, Eq. (31), to make predictions for the two unknown masses of the Ξ_{cc} and Ω_{cc}. We need therefore two additional relations for each of the two multiplets. It should be noted that in our predictions we cannot rely upon a specific choice of the free parameters. Indeed, we have fitted the three, vector meson, octet baryon, and decuplet baryon mass spectra, simultaneously, by using a common value of x in Eq. (24) and similar relations for vector mesons and octet baryons for all three multiplets. Our results are shown in Table I (the calculation is completed when λ_s^Δ becomes zero first of the three λ's). It is clear that the three spectra in the light quark sector are reproduced very accurately for a rather wide range of the free parameters. Therefore, predictions for the charmed sector based on the numerical values of the free parameters in the light quark sector cannot be reliable, since different choices of these parameters would lead to different results for the charmed baryon masses [e.g., as clear from Eq. (24), the values of Ξ_c^*, Ω_c^*, Ξ_{cc}^*, and Ω_{cc}^* depend on both x and λ_s^Δ and are very sensitive to their variations]. Thus, the only way to reliable predictions for the charmed sector is to avoid depending on the numerical values of the slopes and intercepts (or, equivalently, the free parameters), and establish mass relations among the light and heavy baryons and use them for predicting the heavy baryon masses.

In order to obtain two additional relations (for each of the two multiplets), we shall use the approximation of equality of the slopes in the light quark sector referred to above. Indeed, as we have tested, the best simultaneous χ^2 fit to the three spectra in Table I corresponds to $x=0.051\ll 1$, and therefore the approximation of equality of the slopes in the light quark sector is completely justified. We note, however, that the χ^2 function is very shallow in the x and λ directions, which reflects the fact that many different choices of the free parameters will give good fits to the three spectra, as seen in Table I.

For the $\frac{1}{2}^+$ baryons, it then follows from Eqs. (7)–(10) (with $\gamma_s^N\alpha_N'\ll 1$) that

$$\alpha_\Xi'\cong\alpha_N', \quad \alpha_{\Sigma_c'}'\cong\alpha_{\Xi_c}'. \tag{46}$$

We now apply the procedure developed for mesons in [47] to baryons, using the following relations based on Eqs. (2) and (46),

$$\alpha_N'N^2+\alpha_{\Xi_{cc}}'\Xi_{cc}^2=2\alpha_{\Sigma_c'}'\Sigma_c'^2,$$

$$\alpha_N'\Xi^2+\alpha_{\Xi_{cc}}'\Xi_{cc}^2=2\alpha_{\Sigma_c'}'\tilde{\Xi}_c^2,$$

$$\frac{1}{\alpha_N'}+\frac{1}{\alpha_{\Xi_{cc}}'}=\frac{2}{\alpha_{\Sigma_c'}'},$$

and obtain a sixth power relation for the $\frac{1}{2}^+$ baryon masses:

$$(\Xi^2\Sigma_c'^2-N^2\tilde{\Xi}_c^2)(\Xi^2-N^2)+\Xi_{cc}^2(\tilde{\Xi}_c^2-\Sigma_c'^2)(\Xi^2-N^2)$$
$$=4(\Xi^2\Sigma_c'^2-N^2\tilde{\Xi}_c^2)(\tilde{\Xi}\_c^2-\Sigma_c'^2). \tag{47}$$

The same procedure applied for the $\frac{3}{2}^+$ baryons leads to a similar sixth power relation for the $\frac{3}{2}^+$ baryon masses:

$$(\Xi^{*2}\Sigma_c^{*2}-\Delta^2\Xi_c^{*2})(\Xi^{*2}-\Delta^2)+\Xi_{cc}^{*2}(\Xi_c^{*2}-\Sigma_c^{*2})$$
$$\times(\Xi^{*2}-\Delta^2)=4(\Xi^{*2}\Sigma_c^{*2}-\Delta^2\Xi_c^{*2})(\Xi_c^{*2}-\Sigma_c^{*2}). \tag{48}$$

Equations (47) and (48) yield the following values for the masses of the Ξ_{cc} and Ξ_{cc}^*:

$$\Xi_{cc}=3610\pm 3 \ \ \text{MeV}, \tag{49}$$

$$\Xi_{cc}^*=3735\pm 17 \ \ \text{MeV}. \tag{50}$$

The values for the masses of the Ω_{cc} and Ω_{cc}^* can now be obtained from Eqs. (29) and (31), respectively:

$$\Omega_{cc}=3804\pm 8 \ \ \text{MeV}, \tag{51}$$

$$\Omega_{cc}^*=3850\pm 25 \ \ \text{MeV}. \tag{52}$$

The remaining value for the Ω_{ccc} mass is obtained either from Eq. (26) or (27):

$$\Omega_{ccc}=\begin{cases}4930\pm45\ \text{ MeV} & \text{from Eq. (26)},\\ 4928\pm70\ \text{ MeV} & \text{from Eq. (27)}.\end{cases} \tag{53}$$

Both results are consistent, as they should be.

The effect on the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryon spectra of setting $x=0$ in Eq. (24) and corresponding relations for $\frac{1}{2}^+$ baryons is negligible ($\leqslant$ few MeV), except for the splitting between nonstrange and singly strange baryons [see Eqs. (42) and (43)]. Even in this case the absolute size of this splitting is small, and so the included error is not more than 2%. More significantly, this does not affect the multiply strange and charm states by more than 1%.

Our results are shown in Table II, together with the predictions of many other approaches. One sees that our predictions for the charmed baryon masses done in the Regge framework are in good agreement with those of different approaches. In particular, the predicted value for the Ξ_c' lies in the range provided by experiment [12] and is in close proximity to 2580 MeV, consistent with the very recent predictions [2,4,5,7,8,11]. The predicted value for the Ω_c^* mass is in close proximity to 2770 MeV, consistent with almost all very recent calculations [1–5,7,9,11].

As remarked by Kaidalov [52], the relations (2) and (4), on which our mass predictions are based, have such a structure such that a variation of α'_{kji} by 10–15 % leads only to about 1% change in the values of masses m_{kji}. Thus, although our calculation of the baryon masses in the double- and triple-charm sectors is based on the assumption of equality of the slopes in the light quark sector, we expect our results to be insensitive to any further adjustment of the values of these slopes.

Extension of the present framework to the bottom sector and predictions for the masses of the beauty baryons will be the subject of a separate publication.

We note (from Table II) with interest that our results are closest to those derived using a quark-diquark model [44]. Agreement between such a model and linear Regge trajectories is expected from both the QCD area law of the Wilson loop [57] and string approach [56]. We plan to investigate this further in the future.

Note added. After the paper was submitted for publication, the authors became aware of the preliminary CLEO results on the $\Xi_c'-\Xi_c$ mass difference from $\Xi_c^{+\prime}\to\Xi_c^+\gamma$, $\Xi_c^{0\prime}\to\Xi_c^0\gamma$ decay [65]:

$$\Xi_c^{+\prime}-\Xi_c^+=107.8\pm1.7\pm2.5\ \text{ MeV},$$

$$\Xi_c^{0\prime}-\Xi_c^0=107.0\pm1.4\pm2.5\ \text{ MeV}.$$

With $\Xi_c=2468\pm2$ MeV, these results lead to

$$\Xi_c'=2575\pm5\ \text{ MeV},$$

which is in good agreement with our prediction $\Xi_c'=2569\pm6$ MeV.

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