New mass relations for heavy quarkonia

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By assuming the existence of (quasi)linear Regge trajectories for heavy quarkonia in the low energy region, we derive a new meson mass relation which shows good agreement with experiment for both charmed and b-flavored mesons. We show that for this relation to avoid depending on the values of the Regge slopes of these (quasi)linear trajectories, it must be of sixth power in meson masses. It may be reduced to a quadratic Gell-Mann–Okubo-type formula by fitting the values of the slopes. For charmed mesons, such a formula holds with an accuracy of $\sim$ 1%, and is in qualitative agreement with the relation obtained previously by the application of the linear mass spectrum to an SU(4) meson multiplet. [S0556-2821(97)05723-8]

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The generalization of the standard SU(3) Gell-Mann–Okubo mass formula [1] to higher symmetry groups, e.g., SU(4) and SU(5), became a natural subject of investigation after the discovery of the fourth and fifth quark flavors in the mid 1970s [2]. Attempts have been made in the literature to derive such a formula, either quadratic or linear in mass, by (a) using group theoretical methods [3–5], (b) generalizing the perturbative treatment of $U(3)\times U(3)$ chiral symmetry breaking and the corresponding Gell-Mann–Oakes–Renner relation [6] to $U(4)\times U(4)$ [7,8], (c) assuming the asymptotic realization of SU(4) symmetry in the algebra $[A_\alpha,A_\beta]=if_{\alpha\beta\gamma}V_\gamma$ (where V_α and A_β are vector and axial-vector charges, respectively) [9], (d) extending the Weinberg spectral function sum rules [10] to accommodate the higher symmetry-breaking effects [11], and (e) applying alternative methods, such as the linear mass spectrum for meson multiplets[1] [12,13]. In the following[2], η_n, η_s, η_c, η_b, K, D_n, D_s, B_n, B_s, and B_c stand for the masses of the $n\bar{n}$ ($n\equiv u$ or d), $s\bar{s}$, $c\bar{c}$, $b\bar{b}$, $s\bar{n}$, $c\bar{n}$, $c\bar{s}$, $b\bar{n}$, $b\bar{s}$, and $b\bar{c}$ mesons, respectively,[3] unless otherwise specified. The linear mass relations

$$D_n=\frac{\eta_n+\eta_c}{2},\quad D_s=\frac{\eta_s+\eta_c}{2}, \tag{1}$$

$$B_n=\frac{\eta_n+\eta_b}{2},\quad B_s=\frac{\eta_s+\eta_b}{2},\quad B_c=\frac{\eta_c+\eta_b}{2} \tag{2}$$

found in [5,8], although perhaps justified for vector mesons, since a vector meson mass is given approximately by a sum of the corresponding constituent quark masses,

$$m(i\bar{j})\simeq m(i)+m(j)$$

[in fact, for vector mesons, the relations (1) and (2) hold with an accuracy of up to $\sim$4%], are expected to fail for other meson multiplets, as confirmed by direct comparison with experiment. Similarly, the quadratic mass relation

$$D_s^2-D_n^2=K^2-\eta_n^2 \tag{3}$$

obtained in Ref. [7] by generalizing the SU(3) Gell-Mann–Oakes–Renner relations [6] to include the D_n and D_s mesons,

$$\frac{\eta_n^2}{2n}=\frac{K^2}{n+s}=\frac{D_n^2}{n+c}=\frac{D_s^2}{s+c} \tag{4}$$

[and therefore $D_s^2-D_n^2=K^2-\eta_n^2\propto(s-n)$, also found in Refs. [4,9,11]], does not agree with experiment. For pseudoscalar mesons, for example, with $\eta_n=\pi$, one has (in GeV^2) 0.388 for the left-hand side (LHS) of Eq. (3) vs 0.226 for the RHS. For vector mesons, the corresponding quantities are 0.424 vs 0.199, with about 100% discrepancy. The reason that the relation (3) does not hold is apparently due to the impossibility of perturbative treatment of $U(4)\times U(4)$ symmetry breaking, as a generalization of that of $U(3)\times U(3)$, due to very large bare mass of the c quark as compared to those of the u, d, and s quarks. In Ref. [13] by the application of the linear spectrum to the SU(4) meson multiplet, the following relation was obtained:

$$12\overline{D^2}=7\,\eta_0^2+5\,\eta_c^2, \tag{5}$$

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[1]Here, we speak of the linear spectrum over the additive quantum numbers I_3 and Y of a multiplet taking proper account of degeneracy, not (directly) make use of linear Regge trajectories.

[2]Here η_n stands for the masses of both isovector and isoscalar $n\bar{n}$ states which coincide on a naive quark model level.

[3]Since these designations apply to all spin states, vector mesons will be confusingly labeled as η's. We ask the reader to bear with us in this in the interest of minimizing notation.

where $\overline{D}$ is the average mass of the D_n and D_s states [which are mass degenerate when SU(4) flavor symmetry is broken down to SU(3) by $m(c)\neq m(s)=m(n)$], and η_0 is the mass average of the corresponding SU(3) multiplet (which is also mass degenerate in this case). As shown in Ref. [13], this relation holds with an accuracy of up to $\sim$ 5% for all well-established SU(4) meson multiplets.

It is well known that the hadrons composed of light (u,d,s) quarks populate linear Regge trajectories; i.e., the square of the mass of a state with orbital momentum ℓ is proportional to ℓ: $M^2(\ell)=\ell/\alpha'+$ const, where the slope α' depends weakly on the flavor content of the states lying on the corresponding trajectory:

$$\alpha'_{n\bar{n}}\simeq 0.88 \ \text{GeV}^{-2}, \quad \alpha'_{s\bar{n}}\simeq 0.84 \ \text{GeV}^{-2},$$

$$\alpha'_{s\bar{s}}\simeq 0.80 \ \text{GeV}^{-2}. \tag{6}$$

In contrast, the data on the properties of Regge trajectories of hadrons containing heavy quarks are almost nonexistent at the present time, although it is established [14] that the slope of the trajectories decreases with increasing quark mass [as seen in Eqs. (6)] in the mass region of the lowest excitations. This is due to an increasing (with mass) contribution of the color Coulomb interaction, leading to a curvature of the trajectory near the ground state. However, as the analyses show [14–16], in the asymptotic regime of the highest excitations, the trajectories of both light and heavy quarkonia are linear and have the same slope $\alpha'\simeq 0.9$ GeV^{-2}, in agreement with natural expectations from the string model.

Knowledge of Regge trajectories in the scattering region, i.e., at $t<0$, and of the intercepts $a(0)$ and slopes α' is also useful for many nonspectral purposes, for example, in the recombination [17] and fragmentation [18] models. Therefore, as pointed out in Ref. [14], the slopes and intercepts of the Regge trajectories are the fundamental constants of hadron dynamics, perhaps generally more important than the mass of any particular state. Thus, not only the derivation of a mass relation but also the determination of the parameters $a(0)$ and α' of heavy quarkonia is of great importance, since they afford opportunities for better understanding of the dynamics of the strong interactions in the processes of production of charmed and beauty hadrons at high energies.

Here we apply Regge phenomenology for the derivation of a mass formula for the SU(4) meson multiplet, by assuming the (quasi)linear form of Regge trajectories for heavy quarkonia with slopes which are generally different from (less than) the standard one, $\alpha'\simeq 0.9$ GeV^{-2}. We show that for the formula to avoid depending on the values of the slopes, it must be of sixth power in meson masses. It may be reduced to a quadratic Gell-Mann–Okubo–type relation, by fitting the values of the slopes, which is in qualitative agreement with Eq. (5).

Let us assume the (quasi)linear form of Regge trajectories for hadrons with identical J^{PC} quantum numbers (i.e., belonging to a common multiplet). Then for the states with orbital momentum ℓ one has

$$\ell=\alpha'_{i\bar{i}}m^2_{i\bar{i}}+a_{i\bar{i}}(0),$$

$$\ell=\alpha'_{j\bar{i}}m^2_{j\bar{i}}+a_{j\bar{i}}(0),$$

$$\ell=\alpha'_{j\bar{j}}m^2_{j\bar{j}}+a_{j\bar{j}}(0).$$

We now use the following relation among the intercepts:

$$a_{i\bar{i}}(0)+a_{j\bar{j}}(0)=2a_{j\bar{i}}(0). \tag{7}$$

This relation was first derived for $u(d)$ and s quarks in the dual-resonance model [19]. It is satisfied in two-dimensional QCD [20], the dual-analytic model [21], and the quark bremsstrahlung model [22]. Also, it saturates inequalities for Regge trajectories [23] which follow from the s-channel unitarity condition.

With Eq. (7), one obtains, from the above three relations,

$$\alpha'_{i\bar{i}}m^2_{i\bar{i}}+\alpha'_{j\bar{j}}m^2_{j\bar{j}}=2\alpha'_{j\bar{i}}m^2_{j\bar{i}}. \tag{8}$$

In order to eliminate the Regge slopes from this formula, we need a relation among the slopes. Two such relations have been proposed in the literature,

$$\alpha'_{i\bar{i}}\cdot\alpha'_{j\bar{j}}=(\alpha'_{j\bar{i}})^2, \tag{9}$$

which follows from the factorization of residues of the t-channel poles [25,26], and

$$\frac{1}{\alpha'_{i\bar{i}}}+\frac{1}{\alpha'_{j\bar{j}}}=\frac{2}{\alpha'_{j\bar{i}}}, \tag{10}$$

based on topological expansion and the $q\bar{q}$-string picture of hadrons [24].

For light quarkonia (and small differences in the α' values), there is no essential difference between these two relations; viz., for $\alpha'_{j\bar{i}}=\alpha'_{i\bar{i}}/(1+x)$, $x\ll 1$, Eq. (10) gives $\alpha'_{j\bar{j}}=\alpha'_{i\bar{i}}/(1+2x)$, whereas Eq. (9) gives $\alpha'_{j\bar{j}}=\alpha'_{i\bar{i}}/(1+x)^2\approx\alpha'/(1+2x)$, i.e., essentially the same result to order x^2. However, for heavy quarkonia (and expected large differences from the α' values for the light quarkonia) these relations are incompatible; e.g., for $\alpha'_{j\bar{i}}=\alpha'_{i\bar{i}}/2$, Eq. (9) will give $\alpha'_{j\bar{j}}=\alpha'_{i\bar{i}}/4$, whereas Eq. (10) will give $\alpha'_{j\bar{j}}=\alpha'_{i\bar{i}}/3$. One has therefore to discriminate between these relations in order to proceed further. Here we use Eq. (10), since it is much more consistent with Eq. (8) than is Eq. (9), which we tested by using measured quarkonia masses in Eq. (8). We shall justify this choice in more detail in a separate publication [27].

Since we are interested in SU(4) breaking, and since it simplifies the discussion, we take average slope in the light quark sector:

$$\alpha'_{n\bar{n}}\cong\alpha'_{s\bar{n}}\cong\alpha'_{s\bar{s}}\cong\alpha'\simeq 0.85 \ \text{GeV}^{-2}. \tag{11}$$

Equation (11) leads, through Eq. (10) with $i=u(d),s$ and $j=c$, to

$$\alpha'_{c\bar{n}}\cong\alpha'_{c\bar{s}}.$$

It then follows from the relations based on Eq. (8),

$$\alpha' \eta_n^2 + \alpha'_{c\bar{c}} \eta_c^2 = 2\alpha'_{c\bar{n}} D_n^2, \tag{12}$$

$$\alpha' \eta_s^2 + \alpha'_{c\bar{c}} \eta_c^2 = 2\alpha'_{c\bar{s}} D_s^2, \tag{13}$$

that

$$\alpha'_{c\bar{n}} \cong \alpha'_{c\bar{s}} = \alpha' \frac{(\eta_s^2 - \eta_n^2)}{2(D_s^2 - D_n^2)}, \tag{14}$$

$$\alpha'_{c\bar{c}} = \alpha' \left[\frac{(\eta_s^2 - \eta_n^2)}{(D_s^2 - D_n^2)} \frac{D_n^2}{\eta_c^2} - \frac{\eta_n^2}{\eta_c^2} \right]. \tag{15}$$

Using these values of the slopes in Eq. (10) with $i=n, \ j=c$, we obtain

$$(\eta_s^2 D_n^2 - \eta_n^2 D_s^2)(\eta_s^2 - \eta_n^2) + \eta_c^2 (D_s^2 - D_n^2)(\eta_s^2 - \eta_n^2) = 4(\eta_s^2 D_n^2 - \eta_n^2 D_s^2)(D_s^2 - D_n^2), \tag{16}$$

which is a new mass relation for the SU(4) meson multiplet. To test Eq. (16), we use the four well-established multiplets[4] [28]: (1) $1\,^1S_0$ $J^{PC}=0^{-+}$, $m(\pi)=0.138$ GeV, $m(\eta_s)=0.686$ GeV, $m(D)=1.868$ GeV, $m(D_s)=1.969$ GeV, $m(\eta_c)=2.980$ GeV; (2) $1\,^3S_1$ $J^{PC}=1^{--}$, $m(\rho)=0.769$ GeV, $m(\phi)=1.019$ GeV, $m(D^*)=2.009$ GeV, $m(D_s^*)=2.112$ GeV, $m(J/\psi)=3.097$ GeV; (3) $1\,^1P_1$ $J^{PC}=1^{+-}$, $m(b_1)=1.231$ GeV, $m(h_1')=1.380$ GeV, $m(D\_1)=2.422$ GeV, $m(D_{s1})=2.535$ GeV, $m(h_c(1P))=3.526$ GeV; (4) $1\,^3P_2$ $J^{PC}=2^{++}$, $m(a_2)=1.318$ GeV, $m(f_2')=1.525$ GeV, $m(D_2^*)=2.459$ GeV, $m(D_{s2}^*)=2.573$ GeV, $m(\chi_{c2}(1P))=3.556$ GeV.

We rewrite Eq. (16) in the form

$$\eta_c^2 = \frac{(\eta_s^2 D_n^2 - \eta_n^2 D_s^2)[4(D_s^2 - D_n^2) - (\eta_s^2 - \eta_n^2)]}{(\eta_s^2 - \eta_n^2)(D_s^2 - D_n^2)}. \tag{17}$$

We shall test the relation by comparing the values of η_c given by Eq. (17) with those established by experiment, using the known masses of the remaining states, for the four multiplets.

(1) $1\,^1S_0$ $J^{PC}=0^{-+}$. One obtains 3.137 GeV for the value of $m(\eta_c)$ vs experimentally established value 2.980 GeV; in this case the accuracy of Eq. (16) is $\sim$ 5%.

(2) $1\,^3S_1$ $J^{PC}=1^{--}$. In this case one obtains 3.200 GeV vs 3.097 GeV, as the value of $m(J/\psi)$; the accuracy is $\sim$ 3.3%.

(3) $1\,^1P_1$ $J^{PC}=1^{+-}$. Now one obtains 3.490 GeV vs 3.526 GeV, as the value of $m(h_c(1P))$; the accuracy is $\sim$ 1.0%.

(4) $1\,^3P_2$ $J^{PC}=2^{++}$. In this case one has 3.598 GeV vs 3.556 GeV, as the value of $m(\chi_{c2}(1P))$; the accuracy is $\sim$ 1.2%.

One sees that the formula (16) holds with a high accuracy for all four well-established meson multiplets. The major contribution to the discrepancy between our formula result and experiment is the approximation (11). For higher excited states the trajectories become more accurately parallel, and the approximation (11) and subsequent relation $\alpha'_{c\bar{n}} = \alpha'_{c\bar{s}}$ [Eq. (13)] become more exact. As shown above, the formula (16) does hold with improving accuracy as one proceeds to higher spin multiplets.

A possible additional reason for the discrepancy is the curvature of the η_c trajectory near $\ell=0$ since the mass of the η_c is lower than expected from a linear extrapolation. Similarly, if one tries, apart from its Goldstone nature, to fit the pion to the trajectory on which the $b_1(1231)$ and $\pi_2(1670)$ lie $[\ell=0.85M^2(\ell)-0.30]$, extrapolation down to $\ell=0$ gives $m(\pi)\simeq 0.6$ GeV, much higher than the physical value $m(\pi)=0.138$ GeV.

We note that a relation based on Eq. (9) for the slopes which is also of sixth power in meson masses,

$$4D_n^2(D_s^2 - D_n^2)(\eta_s^2 - \eta_n^2) = \eta_c^2(\eta_s^2 - \eta_n^2)^2 + 4\eta_n^2(D_s^2 - D_n^2)^2,$$

holds for the four multiplets with an accuracy of 15–20 %. The reason for such a large discrepancy with experiment is a lower value for the slope of the charmonia trajectory given by Eq. (9), as compared to that given by Eq. (10), leading to higher values for the charmonia masses.

We emphasize that the formula (16) does not depend on the *values* of the Regge slopes, but only on the relation between them, Eq. (10), which justifies its use in both the low energy region where the slopes are different and the high energy region where all the slopes coincide. In the latter case, as follows from Eqs. (12) and (13), $\eta_s^2 - \eta_n^2 = 2(D_s^2 - D_n^2)$, and Eq. (17) reduces to

$$\eta_n^2 + \eta_c^2 = 2D_n^2, \tag{18}$$

consistent with Eq. (12) in this limit. One may also find from Eqs. (12) and (13) with equal slopes, and the standard SU(3) Gell-Mann–Okubo relation

$$\eta_n^2 + \eta_s^2 = 2K^2, \tag{19}$$

that Eq. (3) also holds in this limit.

The entire analysis may, of course, be repeated with B_n, B_s, and η_b in place of D_n, D_s, η_c, respectively, and will lead [assuming Eq. (10)] to a relation similar to (16):

$$(\eta_s^2 B_n^2 - \eta_n^2 B_s^2)(\eta_s^2 - \eta_n^2) + \eta_b^2 (B_s^2 - B_n^2)(\eta_s^2 - \eta_n^2) = 4(\eta_s^2 B_n^2 - \eta_n^2 B_s^2)(B_s^2 - B_n^2), \tag{20}$$

which is a new mass relation for the SU(5) meson multiplet. We can test this relation for vector mesons since the masses of all of the beauty states involved are established experimentally only for vector mesons [28]: $m(B^*)=5.325$ GeV, $m(B_s^*)=5.416$ GeV, and $m(\Upsilon(1S))=9.460$ GeV. The formula (20) yields $m(\Upsilon(1S))=9.791$ GeV, within $\sim$ 3.5% of

[4]We use the values $\eta_n=\pi=0.138$ GeV and $\eta_s=0.686$ GeV for pseudoscalar mesons, the latter following from the standard SU(3) Gell-Mann–Okubo formula which we arrive at below [Eq. (19)] applied to pseudoscalar multiplet: $\eta_s^2=2K^2-\pi^2$. We also use the masses of the isovector and isoscalar mostly octet states as the values of η_n and η_s, respectively, in Eqs. (16) and (17) for the remaining three multiplets.

the physical value, and an accuracy which is very similar to that for charmed vector mesons.

Let us now discuss the question of the generalization of the standard SU(3) Gell-Mann–Okubo mass formula which is quadratic in mass to the case of heavier quarkonia. We shall continue to assume the validity of Eq. (11) and introduce $x>0$ through the relation

$$\alpha'_{c\bar{n}}=\alpha'_{c\bar{s}}=\frac{\alpha'}{1+x}. \tag{21}$$

It then follows from Eq. (10) that

$$\alpha'_{c\bar{c}}=\frac{\alpha'}{1+2x}, \tag{22}$$

and one obtains, from Eqs. (12) and (13),

$$(1+x)(\eta_n^2+\eta_s^2)+\frac{2(1+x)}{1+2x}\,\eta_c^2=2(D_n^2+D_s^2). \tag{23}$$

Results of the calculations of the Regge slopes of heavy quarkonia in Refs. [26], $\alpha'_{c\bar{n}}/\alpha'\simeq\alpha'_{c\bar{s}}/\alpha'\simeq 0.73$, $\alpha'_{c\bar{c}}/\alpha'\simeq 0.58$, and [24,29] $\alpha'_{c\bar{c}}\simeq 0.5$ GeV^{-2}, support the value

$$x\cong 0.355. \tag{24}$$

With this x, it follows from Eq. (23) and the standard SU(3) Gell-Mann–Okubo formula (19) that

$$8.13K^2+4.75\eta_c^2=6(D_n^2+D_s^2). \tag{25}$$

Thus, the new sixth order mass relations may be accurately reduced to quadratic ones by use of specific values for the Regge slopes.

For the four well-established meson multiplets, the formula (23) gives in its LHS and RHS, respectively[5] (in GeV^2):

(1) $1\ ^1S_0\ J^{PC}=0^{-+}$, 44.17 vs 44.20;
(2) $1\ ^3S_1\ J^{PC}=1^{--}$, 52.03 vs 50.98;
(3) $1\ ^1P_1\ J^{PC}=1^{+-}$, 73.63 vs 73.75;
(4) $1\ ^3P_2\ J^{PC}=2^{++}$, 76.67 vs 76.00.

One sees that the formula (25) holds at a 1% level for all four well-established meson multiplets, thus confirming the assumption on the (quasi)linearity of the Regge trajectories of heavy quarkonia in the low energy region. The formula (25) is in qualitative agreement with the relation (5) obtained by two of the present authors in Ref. [13] by the application of the linear mass spectrum to an SU(4) meson multiplet.

In our subsequent paper [31], we derive the new sixth power mass relations obtained in this paper, for pseudoscalar mesons, using current algebra. The newly derived formulas contain $\tilde{\eta}_c\equiv f_0/f\,\eta_c$ and $\tilde{\eta}_b\equiv f_0/f\,\eta_b$, in place of η_c and η_b in Eqs. (16) and (20), respectively, where $f=f_i$, $i=1,2,\ldots,15$, is the multiplet decay constant in the assumption of exact SU(4) [(u,d,s,c) and (u,d,s,b), respectively] flavor symmetry, and f_0 is the isoscalar singlet decay constant which is, in general, different from f, since SU(4) symmetry alone does not imply $f_0=f$, in addition to $f_1=f_2=\cdots=f_{15}=f$ [32]. With $f_0/f=1.06\pm 0.04$ obtained in [33] from the ratios of the experimentally measured widths, $\Gamma(\eta\rightarrow\gamma\gamma)/\Gamma(\pi^0\rightarrow\gamma\gamma)$, $\Gamma(\eta'\rightarrow\gamma\gamma)/\Gamma(\pi^0\rightarrow\gamma\gamma)$, the newly derived relation in the (u,d,s,c) sector gives [with $\tilde{\eta}_c=3.137$ GeV, as follows from Eq. (17)]

$$\eta_c=2.964\pm 0.112\ \ \text{GeV},$$

which is within the physical η_c mass of 2.98 GeV with an accuracy of better than 1%. It is obvious that the ratio $f_0/f\neq 1$, not taken into account in the framework discussed in this paper, is the only source of the 5% discrepancy between the value for the mass of the pseudoscalar $c\bar{c}$ state provided by Eq. (16) and the physical η_c mass.

Finally, we note that the derived Regge slopes in the charm sector are

$$\alpha'_{c\bar{n}}\simeq\alpha'_{c\bar{s}}\simeq 0.63\ \ \text{GeV}^{-2},\quad \alpha'_{c\bar{c}}\simeq 0.50\ \ \text{GeV}^{-2}.$$

[5]For the pseudovector nonet, we use the value $K_{1B}=1.339$ GeV which follows from the assumption on a 45° mixing between axial-vector and pseudovector nonets in the isodoublet channel [30].

[1] S. Okubo, Prog. Theor. Phys. **27**, 949 (1962); **28**, 24 (1962); M. Gell-Mann and Y. Ne'eman, *The Eightfold Way* (Benjamin, New York, 1964).

[2] J. J. Aubert *et al.*, Phys. Rev. Lett. **33**, 1404 (1974); J.-E. Augustin *et al.*, *ibid.* **33**, 1406 (1974); C. Bacci *et al.*, *ibid.* **33**, 1408 (1974); G. S. Abrams *et al.*, *ibid.* **33**, 1453 (1974); S. W. Herb *et al.*, *ibid.* **39**, 252 (1977).

[3] A. J. Macfarlane, J. Phys. G **1**, 601 (1975).

[4] S. Borchardt, V. S. Mathur, and S. Okubo, Phys. Rev. Lett. **34**, 38 (1975); V. S. Mathur, S. Okubo, and S. Borchardt, Phys. Rev. D **11**, 2572 (1976); H. Hayashi *et al.*, Ann. Phys. (N.Y.) **101**, 394 (1976).

[5] D. B. Lichtenberg, Lett. Nuovo Cimento **13**, 346 (1975); A. Kazi, G. Kramer, and D. H. Schiller, Acta Phys. Austriaca **45**, 65 (1976); D. H. Boal and R. Torgerson, Phys. Rev. D **15**, 327 (1977); D. H. Boal, *ibid.* **18**, 3446 (1978); S. Iwao, Lett. Nuovo Cimento **20**, 347 (1977); **21**, 239 (1978); **21**, 245 (1978); **22**, 192 (1978); S. A. Rashid, Indian J. Pure Appl. Phys. **19**, 172 (1981).

[6] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

[7] M. K. Gaillard, B. W. Lee, and J. L. Rosner, Rev. Mod. Phys. **47**, 277 (1975).

[8] C. Montonen, M. Roos, and N. Törnqvist, Lett. Nuovo Cimento **12**, 627 (1975).

[9] H. L. Hallock and S. Oneda, Phys. Rev. D **18**, 841 (1978); **19**, 347 (1979); **20**, 2932 (1979); M. Majewski and W. Tybor, Acta Phys. Pol. B **9**, 177 (1978).

[10] S. Weinberg, Phys. Rev. Lett. **18**, 507 (1967).

[11] B. Bagchi, V. P. Gautam, and A. Nandi, Lett. Nuovo Cimento **24**, 175 (1979); V. P. Gautam, B. Bagchi, and A. Nandy, J. Phys. G **5**, 885 (1979).

[12] L. Burakovsky and L. P. Horwitz, Nucl. Phys. **A609**, 585 (1996); **A614**, 373 (1997); Found. Phys. Lett. **9**, 561 (1996).
[13] L. Burakovsky and L. P. Horwitz, Found. Phys. Lett. **10**, 131 (1997).
[14] J.-L. Basdevant and S. Boukraa, Z. Phys. C **28**, 413 (1985); Ann. Phys. (Paris) **10**, 475 (1985).
[15] J. S. Kang and H. Schnitzer, Phys. Rev. D **12**, 841 (1975).
[16] C. Quigg and J. L. Rosner, Phys. Rep. **56**, 167 (1979).
[17] K. P. Das and R. C. Hwa, Phys. Lett. **68B**, 459 (1977); T. Tashiro *et al.*, Z. Phys. C **35**, 21 (1987); A. B. Batunin, B. B. Kiselyev, and A. K. Likhoded, Yad. Phys. **49**, 554 (1989) [Sov. J. Nucl. Phys. **49**, 346 (1989)].
[18] A. Capella and J. van Tran Thanh, Z. Phys. C **10**, 249 (1981); Phys. Lett. **114B**, 450 (1982); A. B. Kaidalov, *ibid.* **116B**, 459 (1982); A. B. Kaidalov and K. A. Ter-Martirosyan, Sov. J. Nucl. Phys. **39**, 979 (1984); A. B. Kaidalov and O. I. Piskounova, Z. Phys. C **30**, 145 (1986); G. I. Lykasov and M. N. Sergeenko, *ibid.* **52**, 635 (1991).
[19] K. Kawarabayashi, S. Kitakado, and H. Yabuki, Phys. Lett. **28B**, 432 (1969).
[20] R. C. Brower, J. Ellis, M. G. Schmidt, and J. H. Weis, Nucl. Phys. **B128**, 175 (1977).
[21] N. A. Kobylinsky, E. S. Martynov, and A. B. Prognimak, Ukr. Fiz. Zh. **24**, 969 (1979).
[22] V. V. Dixit and L. A. Balazs, Phys. Rev. D **20**, 816 (1979).
[23] K. Igi and S. Yazaki, Phys. Lett. **71B**, 158 (1977).
[24] A. B. Kaidalov, Z. Phys. C **12**, 63 (1982).
[25] J. Pasupathy, Phys. Rev. Lett. **37**, 1336 (1976); K. Igi, Phys. Lett. **66B**, 276 (1977); Phys. Rev. D **16**, 196 (1977).
[26] M. Kuroda and B.-L. Young, Phys. Rev. D **16**, 204 (1977).
[27] L. Burakovsky, T. Goldman, and L. P. Horwitz (in preparation).
[28] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).
[29] N.-P. Chang and C. A. Nelson, Phys. Rev. Lett. **35**, 1492 (1975); Phys. Rev. D **13**, 1345 (1976).
[30] G. M. Brandenburger *et al.*, Phys. Rev. Lett. **36**, 703 (1976); R. K. Carnegie *et al.*, Nucl. Phys. **B127**, 509 (1977); M. G. Bowler, J. Phys. G **3**, 775 (1977).
[31] L. Burakovsky and T. Goldman (in preparation).
[32] V. M. Aulchenko *et al.*, Phys. Lett. B **186**, 540 (1987).
[33] H. Kolanoski, in *Proceedings of the 1985 International Symposium on Lepton and Photon Interactions at High Energies*, Kyoto, 1985, edited by M. Konuma and K. Takahashi (Kyoto University, Research Institute of Fundamental Physics, Kyoto, Japan, 1986), p. 90; J. L. Rosner, *ibid.*, p. 448.