T-violating muon polarization in $K^+\to\mu^+\nu\gamma$

C. H. Chen
Institute of Physics, Academia Sinica, Taipei, Taiwan, Republic of China

C. Q. Geng and C. C. Lih
Department of Physics, National Tsing Hua University, Hsinchu, Taiwan, Republic of China

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We study the T-violating transverse muon polarization in the decay of $K^+\to\mu^+\nu\gamma$ due to CP violation in theories beyond the standard model. We find that the polarization asymmetry could be large in some CP violation models and it may be detectable at the ongoing KEK experiment of E246 as well as the proposed BNL experiment. [S0556-2821(97)07223-8]

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I. INTRODUCTION

In the framework of local quantum field theories, with Lorentz invariance and the usual spin-statistics connection, time-reversal (T) violation implies CP violation (and vice versa), because of the CPT invariance of such theories. Experimentally, only CP violation has been observed so far and this only in the neutral kaon system. But the origin of this violation remains unclear. In the standard model, CP violation arises from a unique physical phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [1]. To ensure that this phase is indeed the source of CP violation or T violation, one needs to look for new processes, especially that outside K^0 system. It would be particularly interesting if the time reversal symmetry is directly violated in these processes, rather than inferring it as a consequence of CPT invariance.

Within kaon physics, the most interesting search of T violation would be to look for the component of the muon polarization normal to the decay plane, called transverse muon polarization (P_T), in the charged kaon decays such as $K^+\to\pi^0\mu^+\nu$ ($K^+_{\mu3}$) [2], $K^+\to\mu^+\nu\gamma$ ($K^+_{\mu2\gamma}$) [3], and $K^+\to\pi^+\mu^+\mu^-$ [4]. The polarization in $K^+_{\mu3}$ as well as that in $K^+_{\mu2\gamma}$ will be measured to a high accuracy at the ongoing KEK E246 experiment [5] and at a recently proposed BNL experiment [6], while for $K^+\to\pi^+\mu^+\mu^-$ that would be done in a future kaon factory [7].

In this paper, we concentrate on the radiative $K^+_{\mu2}$ decay. We will first give a general analysis on all components of the muon polarization and then present our estimations on the transverse component in various CP violation theories beyond the standard model. The transverse muon polarization in the decay of $K^+\to\mu^+\nu\gamma$ is related to the T odd triple correlation, i.e.,

$$P_T\propto\vec{s}_\mu\cdot(\vec{p}_\mu\times\vec{p}_\gamma), \tag{1}$$

where $\vec{s}_\mu$ is the muon spin vector and $\vec{p}_i$ ($i=\mu$ and γ) the momenta of the muon and photon in the rest frame of K^+, respectively. It is expected [8] that the CKM phase does not induce the polarization in Eq. (1). Therefore measuring the polarization could be a signature of physics beyond the standard model. There are many different sources that might give rise to the polarization. The most exciting ones are the weak CP violation from some kinds of nonstandard CP violation models. However, the electromagnetic interaction among the final state particle can also make a contribution, which is usually less interesting and could even hide the signals from the weak CP violation. We shall refer to the final-state-interaction (FSI) contribution as a theoretical background of that by the weak CP violation. It has been estimated that $P_T\sim10^{-3}$ and 10^{-6} in $K^0_{\mu3}$ ($K^0\to\pi^-\mu^+\nu$) [9] and $K^+_{\mu3}$ ($K^+\to\pi^0\mu^+\nu$) [10], due to the FSI effects at one and two-loop levels, respectively. For $K^+\to\mu^+\nu\gamma$, although there is only one charged final state particle like $K^+_{\mu3}$, FSI arises at one-loop diagrams because of the existence of the photon in the final states [11,12] and it is found that $P_T^{\rm FSI}(K^+\to\mu^+\nu\gamma)\sim10^{-3}$ in most of the decay allowed phase space [13]. To distinguish the real CP-violating effects from the FSIs, one has to explore various possible models with the polarization being larger than 10^{-3}.

The paper is organized as follows. In Sec. II, we carry out a general analysis of the muon polarization in $K^+\to\mu^+\nu\gamma$. In Sec. III, we study the CP-violating muon polarization effects in some extensions of the standard model. We give our concluding remarks in Sec. IV.

II. GENERAL ANALYSIS FOR MUON POLARIZATION

For a general investigation of the transverse muon polarization in $K^+\to\mu^+\nu\gamma$ decay including some new CP-violating sources, we first carry out the most general four-fermion interactions given by

$$\begin{aligned}\mathcal{L}=&-\frac{G_F}{\sqrt{2}}\sin\theta_C\,\bar{s}\,\gamma^\alpha(1-\gamma_5)u\,\bar{\nu}\,\gamma_\alpha(1-\gamma_5)\mu\\&+G_S\,\bar{s}\,u\,\overline{\nu}(1+\gamma_5)\mu+G_P\,\bar{s}\,\gamma_5u\,\overline{\nu}(1+\gamma_5)\mu\\&+G_V\,\bar{s}\,\gamma^\alpha u\,\bar{\nu}\,\gamma_\alpha(1-\gamma_5)\mu\\&+G_A\,\bar{s}\,\gamma^\alpha\gamma_5u\,\bar{\nu}\,\gamma_\alpha(1-\gamma_5)\mu+\text{H.c.},\end{aligned} \tag{2}$$

where G_F is the Fermi constant, θ_C is the Cabibbo mixing angle, and G_S, G_P, G_V, and G_A, arising from new physics, denote scalar, pseudoscalar, vector, and axial vector interactions, respectively. From the interactions in Eq. (2), we can write the amplitude of the decay $K^+_{\mu 2\gamma}$ in terms of "inner bremsstrahlung" (IB) and "structure-dependent" (SD) contributions, which can be written as [3,14]

$$M=M_{\rm IB}+M_{\rm SD},$$

with

$$M_{\rm IB}=ie\frac{G_F}{\sqrt{2}}\sin\theta_C f_K m_\mu \epsilon^*_\alpha K^\alpha,$$

$$M_{\rm SD}=-ie\frac{G_F}{\sqrt{2}}\sin\theta_C \epsilon^*_\mu L_\nu H^{\mu\nu}, \tag{3}$$

where

$$K^\alpha=\overline{u(p_\nu)}(1+\gamma_5)\left(\frac{p^\alpha}{p\cdot q}-\frac{2p^\alpha_\mu+\not{q}\gamma^\alpha}{2p_\mu\cdot q}\right)v(p_\mu,s_\mu),$$

$$L_\nu=\overline{u(p_\nu)}\gamma_\nu(1-\gamma_5)v(p_\mu,s_\mu),$$

$$H^{\mu\nu}=\frac{F_A}{m_K}(-g^{\mu\nu}p\cdot q+p^\mu q^\nu)+i\frac{F_V}{m_K}\epsilon^{\mu\nu\alpha\beta}q_\alpha p_\beta, \tag{4}$$

ϵ_α is the photon polarization vector, p, p_ν, p_μ, and q are the four-momenta of K^+, ν, μ^+, and γ, respectively, s_μ is the polarization vector of the muon, and f_K, F_V, and F_A are the form factors given by

$$f_K=f^0_K(1+\Delta_P+\Delta_A),$$

$$F_A=F^0_A(1+\Delta_A),$$

$$F_V=F^0_V(1-\Delta_V), \tag{5}$$

with

$$\Delta_{(P,A,V)}=\frac{\sqrt{2}}{G_F\sin\theta_C}\left(\frac{G_P m_K^2}{(m_s+m_u)m_\mu},G_A,G_V\right). \tag{6}$$

Here f^0_K is the kaon decay constant and $F^0_{V(A)}$ the vector (axial-vector) form factor, defined by

$$\langle 0|\overline{s}\gamma^\mu\gamma_5 u|K^+(p)\rangle=-if^0_K p^\mu,$$

$$\int dx e^{iqx}\langle 0|T(J^\mu_{\rm em}(x)\overline{s}\gamma^\nu\gamma_5 u(0))|K^+(p)\rangle$$

$$=-f^0_K\left(g^{\mu\nu}+\frac{p^\mu(p-q)^\nu}{p\cdot q}\right)+\frac{F^0_A}{m_K}(g^{\mu\nu}p\cdot q-p^\mu q^\nu),$$

$$\int dx e^{iqx}\langle 0|T(J^\mu_{\rm em}(x)\overline{s}\gamma^\nu u(0))|K^+(p)\rangle$$

$$=i\frac{F^0_V}{m_K}\epsilon^{\mu\nu\alpha\beta}q_\alpha p_\beta, \tag{7}$$

and the hadronic matrix elements involving the scalar and pseudoscalar currents in Eq. (5) are given by [15]

$$\langle 0|\overline{s}\gamma_5 u|K^+(p)\rangle=if^0_K\frac{m_K^2}{m_s+m_u},$$

$$\int dx e^{iqx}\langle 0|T(J^\mu_{\rm em}(x)\overline{s}\gamma_5 u(0))|K^+(p)\rangle=f^0_K\frac{m_K^2}{m_s+m_u}\frac{p^\mu}{p\cdot q},$$

$$\int dx e^{iqx}\langle 0|T(J^\mu_{\rm em}(x)\overline{s}u(0))|K^+(p)\rangle=0, \tag{8}$$

where $J^\mu_{\rm em}$ is the electromagnetic current. Numerically, one has $f^0_K=0.16$ GeV from the experiment and $F^0_V=-0.095$ and $F^0_A=-0.043$ found in the chiral perturbation theory at one-loop level [14].

We use the standard techniques to calculate the probability of the process $K^+\to\mu^+\nu\gamma$ as a function of the four-momenta of the particles and the polarization four-vector s_μ of the muon. We write the components of s_μ in terms of $\vec{\xi}$, a unit vector along the muon spin in its rest frame, as

$$s_0=\frac{\vec{p}_\mu\cdot\vec{\xi}}{m_\mu},\quad \vec{s}=\vec{\xi}+\frac{s_0}{E_\mu+m_\mu}\vec{p}_\mu. \tag{9}$$

In the rest frame of K^+, the partial decay width is found to be

$$d\Gamma=\frac{1}{2m_K}|M|^2(2\pi)^4\delta(p-p_\mu-p_\nu-q)$$

$$\times\frac{d\vec{q}}{(2\pi)^3 2E_q}\frac{d\vec{p}_\mu}{(2\pi)^3 2E_\mu}\frac{d\vec{p}_\nu}{(2\pi)^3 2E_\nu}, \tag{10}$$

with

$$|M|^2=\rho_0(x,y)[1+(P_L\vec{e}_L+P_N\vec{e}_N+P_T\vec{e}_T)\cdot\vec{\xi}], \tag{11}$$

where $\vec{e}_i$ $(i=L,N,T)$ are the unit vectors along the longitudinal, normal and transverse components of the muon polarization, defined by

$$\vec{e}_L=\frac{\vec{p}_\mu}{|\vec{p}_\mu|},$$

$$\vec{e}_N=\frac{\vec{p}_\mu\times(\vec{q}\times\vec{p}_\mu)}{|\vec{p}_\mu\times(\vec{q}\times\vec{p}_\mu)|},$$

$$\vec{e}_T=\frac{\vec{q}\times\vec{p}_\mu}{|\vec{q}\times\vec{p}_\mu|}, \tag{12}$$

respectively, and

$$\rho_0(x,y)=\frac{1}{2}e^2G_F^2\sin^2\theta_C(1-\lambda)\left\{\frac{4m_\mu^2|f_K|^2}{\lambda x^2}\left[x^2+2(1-r_\mu)\left(1-x-\frac{r_\mu}{\lambda}\right)\right]+m_K^4x^2\left[|F_V+F_A|^2\frac{\lambda^2}{1-\lambda}\left(1-x-\frac{r_\mu}{\lambda}\right)\right.\right.$$
$$\left.\left.+|F_V-F_A|^2(y-\lambda)\right]-4m_Km_\mu^2\left[\mathrm{Re}[f_K(F_V+F_A)^*]\left(1-x-\frac{r_\mu}{\lambda}\right)-\mathrm{Re}[f_K(F_V-F_A)^*]\frac{1-y+\lambda}{\lambda}\right]\right\}, \qquad (13)$$

with $\lambda=(x+y-1-r_\mu)/x$, $r_\mu=m_\mu^2/M_K^2$ and $x=2p\cdot q/p^2=2E_\gamma/m_K$ and $y=2p\cdot p_\mu/p^2=2E_\mu/m_K$ being normalized energies of the photon and muon, respectively. If we define the longitudinal, normal and transverse, muon polarization asymmetries by

$$P_i(x,y)=\frac{d\Gamma(\vec{e}_i)-d\Gamma(-\vec{e}_i)}{d\Gamma(\vec{e}_i)+d\Gamma(-\vec{e}_i)} \quad (i=L,N,T), \qquad (14)$$

we find that

$$P_i(x,y)=\frac{\rho_i(x,y)}{\rho_0(x,y)} \quad (i=L,N,T), \qquad (15)$$

with

$$\rho_L(x,y)=-e^2G_F^2\sin^2\theta_C\frac{(1-\lambda)}{2\lambda\sqrt{y^2-4r_\mu}}\left\{\frac{4m_\mu^2|f_K|}{\lambda x^2}[x(\lambda y-2r_\mu)(x+y-2\lambda)-(y^2-4r_\mu)(\lambda x+2r_\mu-2\lambda)]\right.$$
$$-M_K^4\lambda x^2\left[|V+A|^2\frac{\lambda}{1-\lambda}(\lambda y-2r_\mu)\left(1-x-\frac{r_\mu}{\lambda}\right)+|V-A|^2(y^2-\lambda y-2r_\mu)\right]$$
$$\left.-4M_Km_\mu^2\left[\mathrm{Re}\{f_K(V+A)^*\}\lambda\left(1-x-\frac{r_\mu}{\lambda}\right)(2-2x-y)+\mathrm{Re}\{f_K(V-A)^*\}((1-y)(y-\lambda)+2r_\mu-\lambda)\right]\right\},$$

$$\rho_N(x,y)=e^2G_F^2\sin^2\theta_C\frac{(1-\lambda)\sqrt{\lambda y-\lambda^2-r_\mu}}{M_K\lambda\sqrt{y^2-4r_\mu}}\left\{\frac{4m_\mu^3|f_K|^2}{\lambda x}(x+y-2\lambda)-M_K^4m_\mu\lambda x^2\left[|V+A|^2\frac{\lambda}{1-\lambda}\left(1-x-\frac{r_\mu}{\lambda}\right)+|V-A|^2\right]\right.$$
$$\left.-2M_K^3m_\mu\left[\mathrm{Re}\{f_K(V+A)^*\}\left(\frac{(r_\mu-\lambda)(1-x-r_\mu)}{1-\lambda}+\lambda x(1-x)\right)-\mathrm{Re}\{f_K(V-A)^*\}(y-2r_\mu)\right]\right\},$$

$$\rho_T(x,y)=-2e^2G_F^2\sin^2\theta_Cm_K^2m_\mu\frac{1-\lambda}{\lambda}\sqrt{\lambda y-\lambda^2-r_\mu}\left\{\mathrm{Im}[f_K(F_V+F_A)^*]\frac{\lambda}{1-\lambda}\left(1-x-\frac{r_\mu}{\lambda}\right)+\mathrm{Im}[f_K(F_V-F_A)^*]\right\}. \qquad (16)$$

From Eq. (14), it is easily seen that the asymmetries of P_L and P_N are even quantities under time-reversal transformation, while P_T is an odd one. Since we are interested in CP violation, we will concentrate on the transverse part of the polarization shown in Eq. (14). We rewrite $P_L(x,y)$ as

$$P_T(x,y)=P_T^V(x,y)+P_T^A(x,y), \qquad (17)$$

with

$$P_T^V(x,y)=\sigma_V(x,y)[\mathrm{Im}(\Delta_A+\Delta_V)],$$

$$P_T^A(x,y)=[\sigma_V(x,y)-\sigma_A(x,y)]\mathrm{Im}(\Delta_P), \qquad (18)$$

where

$$\sigma_V(x,y)=2e^2G_F^2\sin^2\theta_Cm_K^2m_\mu f_K^0F_V^0\frac{\sqrt{\lambda y-\lambda^2-r_\mu}}{\rho_0(x,y)}$$
$$\times\left[\frac{-1+\lambda}{\lambda}-\left(1-x-\frac{r_\mu}{\lambda}\right)\right],$$

$$\sigma_A(x,y)=2e^2G_F^2\sin^2\theta_Cm_K^2m_\mu f_K^0F_A^0\frac{\sqrt{\lambda y-\lambda^2-r_\mu}}{\rho_0(x,y)}$$
$$\times\left[\frac{-1+\lambda}{\lambda}+\left(1-x-\frac{r_\mu}{\lambda}\right)\right]. \qquad (19)$$

Clearly, to have CP-violating transverse muon polarization of P_T in Eq. (18), at least one of the couplings G_i $(i=P,A,V)$ in Eq. (2) has to exist and be complex.

In the standard model, from Eqs. (17) and (18), we see that the transverse muon polarization is zero since $f_K=f_K^0$ and $F_{V(A)}=F_{V(A)}^0$ which are both real due to $G_i=0$ and $\Delta_i=0$ $(i=P,V,A)$. However, the longitudinal and normal muon polarizations are nonvanishing. In Figs. 1–3, we show the Dalitz plots for ρ_0, P_L, and P_N, respectively. In the

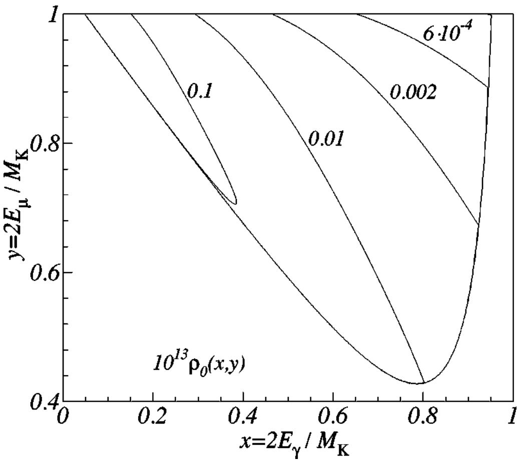

FIG. 1. Dalitz plot of $\rho_0(x,y)$.

nonstandard models, P_T could be nonzero if the new physics couplings G_i $(i=P,A,V)$ have some phases. In Figs. 4 and 5, we display the Dalitz plots of $\sigma_V(x,y)$ and $\sigma_V(x,y)-\sigma_A(x,y)$, respectively. From the figures, we see that they are all in the order of 10^{-1} in most of the allowed parameter space. We shall use σ_V and $\sigma_V-\sigma_A$ as 0.1 in our numerical estimations of the next section. However, it is interesting to note that there is no contribution to the transverse muon polarization if the interaction beyond the standard model contains only left-handed vector current, i.e., $G_V=-G_A$, because of the zero relative phase between the amplitudes of $M_{\rm IB}$ and $M_{\rm SD}$.

III. TRANSVERSE MUON POLARIZATION IN VARIOUS MODELS

Since $P_T(K^+_{\mu 2\gamma})=0$ at tree level in the standard model, a nonzero value of the transverse muon polarization provides an evidence for new CP-violating source outside the CKM mechanism after taking care of the theoretical background. In such case, P_T may arise from the interference between the tree level amplitude in the standard model and the new CP-violating amplitude. In the following we will study various CP violation models such as the left-right symmetric, two-Higgs-doublets, supersymmetry (SUSY), and discuss the possibilities of having large P_T in these theories.

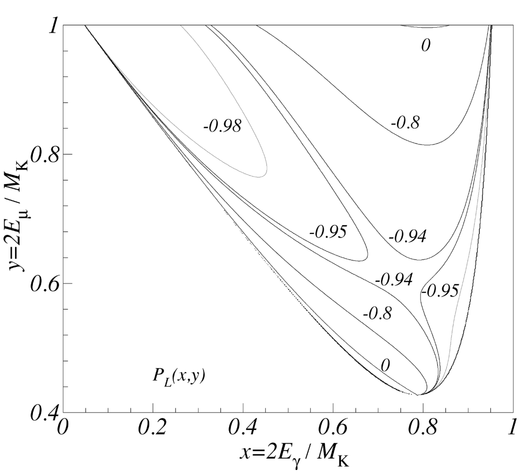

FIG. 2. Dalitz plot of $P_L(x,y)$.

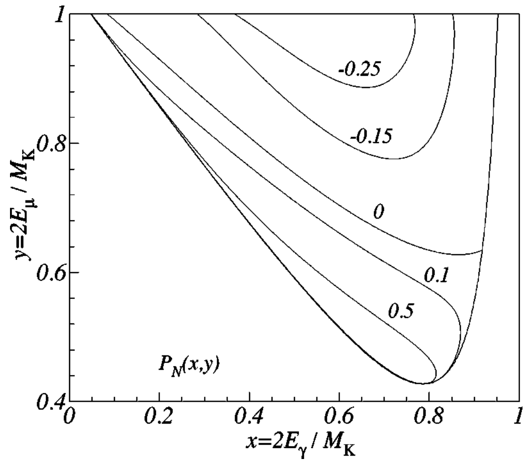

FIG. 3. Dalitz plot of $P_N(x,y)$.

A. Left-right symmetric models

In this subsection we study the prediction of T-violating muon polarization for $K^+_{\mu 2\gamma}$ decay in models with left-right symmetric gauge symmetries ${\rm SU}(2)_L\times{\rm SU}(2)_R\times{\rm U}(1)_{B-L}$ [16]. In these models, the minimal set of Higgs multiplets to break the gauge symmetry down to ${\rm U}(1)_{\rm em}$ is one doublet ϕ and two triplets $\Delta_{L,R}$. The transformations of the Higgs bosons under ${\rm SU}(2)_L\times{\rm SU}(2)_R$ are given by

$$\phi=\begin{pmatrix}\phi_1^{0*} & \phi_2^+\\ -\phi_1^- & \phi_2^+\end{pmatrix}\to U_L\phi U_R^\dagger,$$

$$\Delta_{L,R}=\begin{pmatrix}\delta^+/\sqrt{2} & \delta^{++}\\ \delta^0 & -\delta^+/\sqrt{2}\end{pmatrix}_{L,R}\to U_{L,R}\Delta_{L,R}U_{L,R}^\dagger, \quad (20)$$

with the vacuum expectation values (VEVs) being

$$\langle\phi\rangle=\frac{e^{i\alpha}}{\sqrt{2}}\begin{pmatrix}v_1 & 0\\ 0 & v_2\end{pmatrix},$$

$$\langle\Delta_{L,R}\rangle=\frac{e^{i\theta_{L,R}}}{\sqrt{2}}\begin{pmatrix}0 & 0\\ v_{L,R} & 0\end{pmatrix}, \quad (21)$$

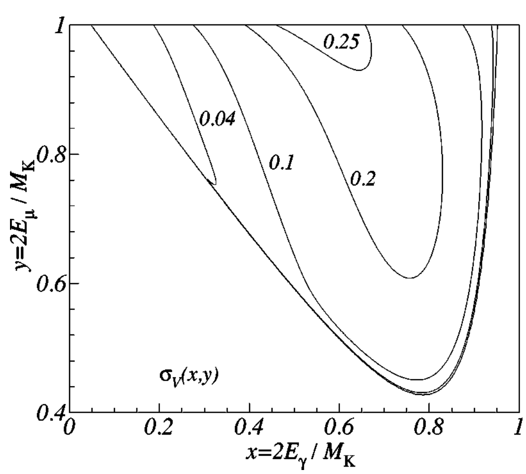

FIG. 4. Dalitz plot of $\sigma_V(x,y)$.

where α and $\theta_{L,R}$ are the CP-violating phases. After the spontaneous symmetry breaking (SSB), the masses of fermions can be generated by the Yukawa coupling terms

$$\mathcal{L}_Y=\overline{Q_L}F\phi Q_R+\overline{Q_L}G\tilde{\phi}Q_R+\text{H.c.}, \tag{22}$$

where the generation indices have been suppressed, $Q_{L,R}$ stand for the left- and right-handed fermions doublets, F and G correspond to the $N\times N$ mass matrices for N generations, and $\tilde{\phi}\equiv\tau_2\phi^*\tau_2$. In addition, the eigenstates of both $W_{L,R}$ bosons related to weak eigenstates can be written as

$$W_1=\cos\xi W_L+\sin\xi W_R,$$
$$W_2=-\sin\xi W_L+\cos\xi W_R, \tag{23}$$

where ξ is the left-right mixing angle. In any event, due to the W_R gauge boson, we have new mixing matrix, called right-handed CKM (RCKM) matrix, coming from the diagonalization of right-handed quarks. The charged current interactions are given by

$$\mathcal{L}_{\rm CC}=\frac{g_L}{\sqrt{2}}W_L^\mu\overline{U}\gamma_\mu K^L P_L D+\frac{g_R}{\sqrt{2}}W_R^\mu\overline{U}\gamma_\mu K^R P_R D+\text{H.c.}, \tag{24}$$

where g_L and g_R are coupling constants for $\text{SU}(2)_L$ and $\text{SU}(2)_R$, $U^T=(u,c,t)$ and $D^T=(d,s,b)$ are the physical states of up-type quarks and down-type quarks, K^L and K^R are the CKM and RCKM matrices, and $P_{L,R}=(1\mp\gamma_5)/2$, respectively.

The number of physical CP-violating phases in CKM and RCKM matrices with N generations are $N_L=(N-1)(N-2)/2$ and $N_R=N(N+1)/2$, respectively. For example, we have three CP-violating phases from K^R for the case of two generations. Here, we do not assume parity invariant as well as any special relation in the matrices so that, in general, the gauge coupling constant g_L is not equal to g_R and the matrix elements of RCKM are free parameters. Including the left-right mixing parameter ξ we can generate $P_T(K^+_{\mu 2\gamma})$ from the tree diagram as shown in Fig. 6. The quark level four-fermion interaction which contributes to $K^+_{\mu 2\gamma}$ decay is given by

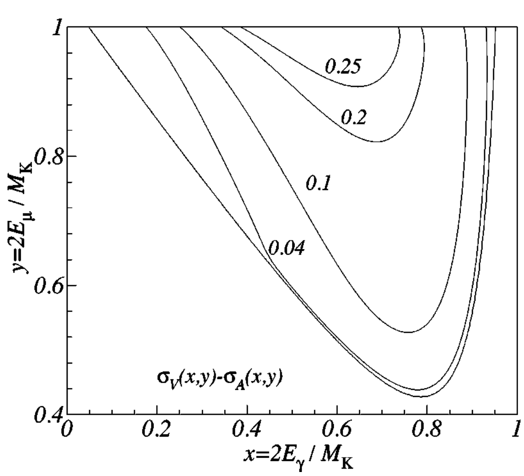

FIG. 5. Dalitz plot of $\sigma_V(x,y)-\sigma_A(x,y)$.

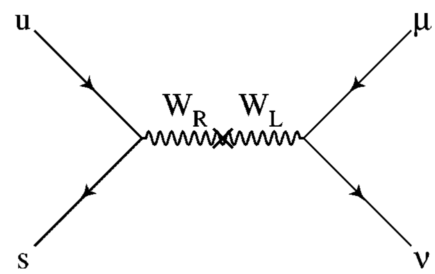

FIG. 6. Tree diagram to $\overline{s}\,\overline{u}\to\mu\overline{\nu}$ induced by the left-right gauge boson mixing.

$$\mathcal{L}_{RL}=-2\sqrt{2}G_F\left(\frac{g_R}{g_L}\right)K_{us}^{R*}\xi\overline{s}\gamma_\mu P_R u\overline{\nu}\gamma^\mu P_L\mu. \tag{25}$$

Comparing Eq. (25) with Eq. (2), we get

$$G_V=G_A=-\frac{G_F}{\sqrt{2}}\frac{g_R}{g_L}K_{us}^{R*}\xi, \tag{26}$$

which leads to

$$\Delta_A=\Delta_V=-\frac{\xi K_{us}^{R*}}{\sin\theta_C}\frac{g_R}{g_L}, \tag{27}$$

in terms of Eq. (5). From the expression of $\delta_{A,V}$ in Eq. (27) and the definition of the muon transverse polarization in Eq. (17), we obtain

$$P_T=2\sigma_V\frac{\xi g_R}{g_L}\text{Im}(K_{us}^{R*}). \tag{28}$$

To illustrate the prediction of the transverse polarization, we consider two-generation case. The RCKM matrix which contains three physical phases can be parametrized as

$$U_R=e^{i\gamma}\begin{bmatrix} e^{-i\delta_2}\cos\theta_C & e^{-i\delta_1}\sin\theta_C \\ -e^{i\delta_1}\sin\theta_C & e^{i\delta_2}\cos\theta_C\end{bmatrix}, \tag{29}$$

$\delta_{1,2}$ and γ are the three physical CP-violating phases, and θ_C is the Cabibbo angle. From the mixing matrix in Eq. (29), we find that

$$\text{Im}(K_{us}^R)=\sin\theta_C\sin(\gamma-\delta_1)\leqslant\sin\theta_C. \tag{30}$$

In the typical left-right symmetric models as shown in Ref. [17], one generally has

$$\xi\frac{g_R}{g_L}<4.0\times10^{-3}, \tag{31}$$

and thus one gets

$$P_T<8.0\times10^{-4} \tag{32}$$

with choosing that $\sigma_V\sim0.1$ and $\text{Im}(K_{us}^{R*})\sim\sin\theta_C$. However, in a class of the specific models studied in Ref. [18], it is found that

$$\xi\frac{g_R}{g_L}<3.3\times10^{-2} \tag{33}$$

for $M_R>549$ GeV. In such models, with the same set of the parameters σ_V and $\mathrm{Im}(K_{us}^{R*})$, one gets

$$P_T<6.6\times10^{-3}, \tag{34}$$

which is within the experimental detecting range. The bounds in Eqs. (32) and (34) can be even larger if one uses a larger value of σ_V. We remark that the muon transverse polarization for the decay of $K^+\to\pi^0\mu^+\nu$, i.e., $P_T(K_{\mu3}^+)$, vanishes in the left-right symmetric models as shown in Ref. [19]. This is because the photon is a vector particle while the pion is a pseudoscalar one.

B. Two-Higgs-doublet models with FCNC

In a two-Higgs-doublet model (THDM) without introducing global symmetry [20], the up and down-type quarks will couple to the both two Higgs doublets. However, the up- and down-type quarks mass matrices cannot be simultaneously diagonalized. Therefore, flavor changing neutral current (FCNC) is induced at tree level. To suppress the effects of the FCNC, one can impose a discrete symmetry. However, the discrete symmetry normally also constrains the scalar potential such that spontaneous CP violation (SCPV) may not occur. In this section, instead of having a discrete symmetry, we assume that the couplings related to the FCNC are small. The various possible theories with naturally small couplings have been explored in Refs. [21,22].

The Yukawa coupling terms in the weak eigenstate can be written by

$$\begin{aligned}\mathcal{L}_Y=&\,\eta_{ij}^D\overline{Q_{Li}}\phi_1D_{Rj}+\eta_{ij}^U\overline{Q_{Li}}\tilde{\phi}_1U_{Rj}+\xi_{ij}^D\overline{Q_{Li}}\phi_2D_{Rj}\\&+\xi_{ij}^U\overline{Q_{Li}}\tilde{\phi}_2U_{Rj}+\eta_{ij}^E\overline{L_i}\phi_1E_{Rj}+\xi_{ij}^E\overline{L_i}\phi_2E_{Rj}+\text{H.c.},\end{aligned} \tag{35}$$

where $\eta_{ij}^{U,D,E}$ and $\xi_{ij}^{U,D,E}$ are dimensionless and real parameters, Q_L and L denote the left-handed quark and lepton doublets, D_R, U_R, and E_R are the right-handed down-type quarks, up-type quarks, and leptons, ϕ_1 and ϕ_2 are the two Higgs doublets with $\tilde{\phi}\equiv i\sigma_2\phi^*$, respectively. The VEVs of the Higgs doublets are given by $\langle\phi_1\rangle=v_1$ and $\langle\phi_2\rangle=\exp(i\theta)v_2$, where θ is the CP-violating phase. From Eq. (35) the Yukawa interactions of quarks and leptons with neutral and charged Higgs bosons can be expressed by

$$\mathcal{L}_{\rm NH}=\frac{1}{\sqrt{2}}\overline{U_L}\tilde{\xi}^UU_R(h^0-iA^0)+\frac{1}{\sqrt{2}}\overline{D_L}\tilde{\xi}^DD_R(h^0+iA^0)+\text{H.c.},$$

$$\mathcal{L}_{\rm CH}=-\overline{U_R}\tilde{\xi}^{U\dagger}K^LD_LH^\dagger+\overline{U_L}K^L\tilde{\xi}^DD_RH^\dagger+\overline{N_L}K^L\tilde{\xi}^EE_RH^\dagger+\text{H.c.}, \tag{36}$$

respectively, with

$$\begin{aligned}\tilde{\xi}^U&\equiv V_L^U(-\eta^U\sin\beta+e^{-i\theta}\xi^U\cos\beta)V_R^{U\dagger},\\ \tilde{\xi}^D&\equiv V_L^D(-\eta^D\sin\beta+e^{i\theta}\xi^D\cos\beta)V_R^{D\dagger},\\ \tilde{\xi}^E&\equiv(-\eta^E\sin\beta+e^{i\theta}\xi^E\cos\beta)V_R^{E\dagger},\end{aligned} \tag{37}$$

where flavor indices are suppressed, $V_{L,R}^{U,D,E}$ are the unitary matrices which transform the fermionic weak eigenstates to the mass eigenstates, and $\tan\beta=v_2/v_1$. However, for simplicity we can reparamterize the diagonal parts of matrix $\tilde{\xi}^E$ to be $(\tilde{\xi}^E)_{ii}\equiv zm_{l_i}$, where z is unknown parameter and m_l is the lepton mass. The parameter z can be bounded by the μ-e universality in tau decay.

From Eq. (36), we find the following four-fermion interaction:

$$\mathcal{L}_{K_{\mu2}^+}=\frac{zm_\mu}{M_H^2}\bar{s}\left(-\sum_j K_{js}^{L*}\tilde{\xi}_{j1}^UP_R+\sum_i\tilde{\xi}_{i2}^{D*}K_{ui}^{L*}P_L\right)u\,\overline{\nu}P_R\mu, \tag{38}$$

where M_H is the mass of charged Higgs particle. From Eq. (36), we clearly see that the off-diagonal elements $\tilde{\xi}_{ij}^{U,D}$ $(i\neq j)$ are related to the FCNC at tree level. To illustrate the polarization effect, we use the ansatz in Ref. [23] by Cheng and Sher, in which the couplings $\tilde{\xi}^{U,D}$ in Eq. (38) are taken to be

$$\tilde{\xi}_{ij}^{U,D}=\lambda_{ij}\frac{\sqrt{m_i^{U,D}m_j^{U,D}}}{v}\quad\text{for }\ i\neq j, \tag{39}$$

where λ_{ij} are undetermined parameters, $v=\sqrt{v_1^2+v_2^2}=(2\sqrt{2}G_F)^{-1/2}$, and $m^{U(D)}$ the masses of up (down)-type quarks. We now simplify Eq. (38) to

$$\mathcal{L}_{K_{\mu2}^+}=\frac{zm_\mu}{M_H^2}\bar{s}K_{us}^{L*}(-\tilde{\xi}_{11}^UP_R+\tilde{\xi}_{22}^{D*}P_L)u\,\overline{\nu}P_R\mu \tag{40}$$

by using Eq. (39). We consider the constraints on $\tilde{\xi}_{11}^U$ and $\tilde{\xi}_{22}^D$ from the $K^0-\overline{K^0}$ mixing, arising from the box diagrams with the u quark as the internal fermion as shown in Fig. 7. From the figures, we find that

$$\begin{aligned}\langle K^0|M_{HH}^{\rm box}|\overline{K^0}\rangle&=-\frac{f_K^2m_K}{6(4\pi)^2M_H^2}(K_{ud}^LK_{us}^L)^2(\tilde{\xi}_{11}^D\tilde{\xi}_{22}^{D*}-\tilde{\xi}_{11}^U\tilde{\xi}_{11}^{U*})^2,\\ \langle K^0|M_{HW}^{\rm box}|\overline{K^0}\rangle&=\frac{4\sqrt{2}G_F}{(4\pi)^2}\frac{f_k^2m_K^3}{3m_s^2}(K_{ud}^LK_{us}^L)^2\times\tilde{\xi}_{11}^D\tilde{\xi}_{22}^{D*}D_{HW}\left(\frac{M_H^2}{M_W^2},\frac{m_u^2}{M_W^2}\right),\end{aligned} \tag{41}$$

with

$$D_{HW}(x,y)=\frac{\ln x}{2(x-1)x^2}+\frac{\ln x}{2x}+\frac{\ln(xy)}{2x^2}. \tag{42}$$

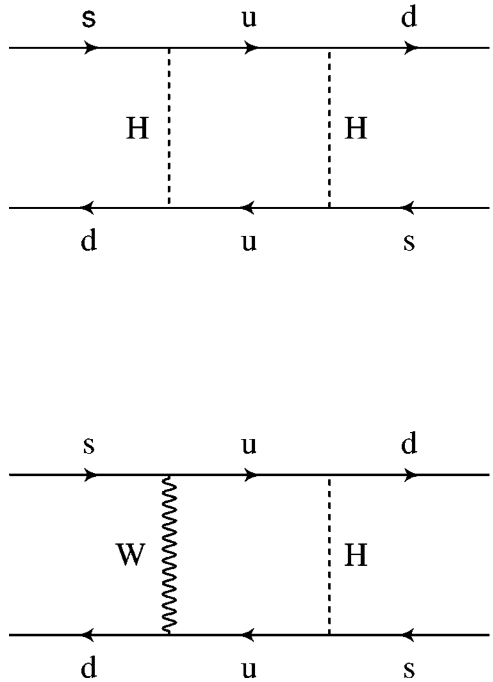

FIG. 7. Box diagrams to $K^0-\overline{K^0}$ mixing with the u quark as the internal fermion.

Using the experimental value on the $K^0-\overline{K^0}$ mixing, we obtain

$$|\tilde{\xi}^D_{11}\tilde{\xi}^{D*}_{22}-\tilde{\xi}^U_{11}\tilde{\xi}^{U*}_{11}|<7.3\times10^{-5}M_H\ \ \mathrm{GeV}^{-1},$$

$$|\tilde{\xi}^D_{11}\tilde{\xi}^{D*}_{22}|<1.3\times10^{-2}m_s\frac{\mathrm{GeV}^{-1}}{\sqrt{D_{HW}\left(\dfrac{M_H^2}{M_W^2},\dfrac{m_u^2}{M_W^2}\right)}}. \quad (43)$$

For $M_H\sim200$ GeV, we find that $|\tilde{\xi}^D_{11}\tilde{\xi}^{D*}_{22}|<5.6\times10^{-3}$ and thus,

$$|\tilde{\xi}^U_{11}|<8.6\times10^{-3}\sqrt{M_H}. \quad (44)$$

The strict constraint on z, as pointed out by Grossman [24], is from the μ-e universality in τ decay as well as the perturbativity bound, and is given by

$$|z|<\min(1.1\times10^{-2}M_H\ \ \mathrm{GeV}^{-2},2.0\ \ \mathrm{GeV}^{-1}). \quad (45)$$

One notes that if $M_H>175$ GeV, the bound on z mainly comes from the perturbativity as shown in Eq. (45). From Eqs. (2), (6), and (40), we get

$$G_P=-\frac{zm_\mu}{4M_H^2}K^{L*}_{us}(\tilde{\xi}^U_{11}+\tilde{\xi}^{D*}_{22}), \quad (46)$$

and

$$\Delta_P=-\frac{m_K^2}{m_s+m_u}\frac{\sqrt{2}z}{4G_FM_H^2}(\tilde{\xi}^U_{11}+\tilde{\xi}^{D*}_{22}), \quad (47)$$

which leads to the muon transverse polarization as

$$P_T=(\sigma_V-\sigma_A)\frac{m_K^2}{m_s+m_u}\frac{\sqrt{2}z}{4G_FM_H^2}\mathrm{Im}(\tilde{\xi}^U_{11}+\tilde{\xi}^{D*}_{22}). \quad (48)$$

Using the constraints in Eqs. (44) and (45), we obtain

$$P_T\leqslant0.048 \quad (49)$$

for $|\sigma_V-\sigma_A|\sim0.1$ and $M_H\sim100$ GeV.

Using the interaction in Eq. (40) by neglecting the contribution of $\tilde{\xi}^{D*}_{22}$, the similar estimation for the transverse muon polarization in $K^+\to\pi^0\mu^+\nu$ can be done. By taking the same values of parameters as in the decay of $K^+_{\mu2\gamma}$, we find that $P_T(K^+_{\mu3})$ could be as large as the current experimental limit, i.e., 1×10^{-2}. Therefore, P_T in both decays of $K^+_{\mu2\gamma}$ and $K^+_{\mu3}$ can be very large in the Higgs models with FCNC. We note that the muon polarization effects of the two modes in the three-Higgs-doublet models with NFC could be also large as studied in Refs. [15,19,25].

C. Supersymmetric models

In this subsection we consider the effects on P_T in theories with SUSY. It is known that, in general, SUSY theories would contain couplings with the violation of baryon or/and lepton numbers, that could induce the rapid proton decay. To avoid such couplings, one usually assigns R parity, defined by $R\equiv(-1)^{3B+L+2S}$ to each field [26], where $B(L)$ and S denote the baryon (lepton) number and the spin, respectively. Thus, the R parity can be used to distinguish the particle $(R=+1)$ from its superpartner $(R=-1)$. Recently, Ng and Wu [27] have investigated the T-violating P_T for $K^+_{\mu2\gamma}$ in SUSY models with R parity and they find that when the squark family mixings are taken into account, large enhancement effects would appear due to the heavy quark masses and large $\tan\beta=v_2/v_1$. In this paper, we will concentrate on the SUSY models without R parity. To evade the stringent constraint from proton decay, we can simply require that B-violating couplings do not coexist with the L-violating ones. In the following we consider the theories with the violation of the R parity and the lepton number. In such cases, the superpotential is given by

$$W_{\not L}=\frac{1}{2}\lambda_{ijk}L_iL_jE^c_k+\lambda'_{ijk}L_iQ_jD^c_k, \quad (50)$$

where the subscript ijk are the generation indices, L and E^c denote the chiral superfields of lepton doublets and singlets, and Q and D^c are the chiral superfields of quark doublets and down-type quark singlets, respectively. We note that the first two generation indices of λ_{ijk} are antisymmetry, i.e., $\lambda_{ijk}=-\lambda_{jik}$. The corresponding Lagrangian is

$$\begin{aligned}\mathcal{L}_{\not L}=&\frac{1}{2}\lambda_{ijk}[\overline{\nu^c_{Li}}e_{Lj}\tilde{e}^*_{Rk}+\overline{e}_{Rk}\nu_{Li}\tilde{e}_{Lj}+\overline{e}_{Rk}e_{Lj}\tilde{\nu}_{Li}-(i\leftrightarrow j)]\\&+\lambda'_{ijk}[\overline{\nu^c_{Li}}d_{Lj}\tilde{d}^*_{Rk}+\overline{d}_{Rk}\nu_{Li}\tilde{d}_{Rk}+\overline{d}_{Rk}d_{Lj}\tilde{\nu}_{Li}\\&-\overline{e^c_{Ri}}u_{Lj}\tilde{d}^*_{Rk}-\overline{d}_{Rk}e_{Li}\tilde{u}_{Lj}-\overline{d}_{Rk}u_{Lj}\tilde{e}_{Li}]+\mathrm{H.c.}\end{aligned} \quad (51)$$

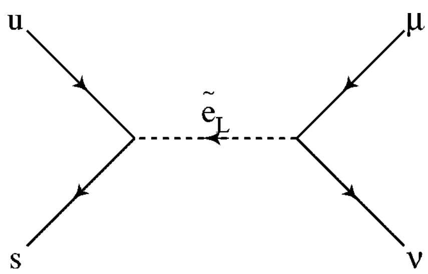

FIG. 8. Tree diagram to $\overline{s}u\to\mu\overline{\nu}$ with the slepton as the intermediate state.

From Eq. (51), we find that the four-fermion interaction of $\overline{s}u\to\mu\overline{\nu}$ with the slepton as the intermediate state shown in Fig. 8 can be written as

$$\mathcal{L}_{RV}=-\frac{\lambda^*_{2i2}\lambda'_{i12}}{M^2_{\tilde{e}_{Li}}}\overline{s}P_L u\,\overline{\nu}P_R\mu, \tag{52}$$

where $M^2_{\tilde{e}_{Li}}$ is the slepton mass.

From the interaction in Eq. (52), we get

$$G_P=\frac{\lambda^*_{2i2}\lambda'_{i12}}{4M^2_{\tilde{e}_{Li}}}, \tag{53}$$

which leads to

$$\Delta_P=\frac{\sqrt{2}}{4G_F\sin\theta_C}\frac{m_K^2}{(m_s+m_u)m_\mu}\frac{\lambda^*_{2i2}\lambda'_{i12}}{M^2_{\tilde{e}_{Li}}} \tag{54}$$

for $i=1,3$. We therefore obtain the transverse muon polarization of $K^+_{\mu 2\gamma}$ as

$$P_T=-(\sigma_V-\sigma_A)\frac{\sqrt{2}}{4G_F\sin\theta_C}\frac{m_K^2}{(m_s+m_u)m_\mu}\frac{\mathrm{Im}(\lambda^*_{2i2}\lambda'_{i12})}{M^2_{\tilde{e}_{Li}}}. \tag{55}$$

In order to give the bounds on the R-parity violating couplings, we need to examine the various processes induced by the FCNC. We first study the decay of $\mu\to e\gamma$. Based on the analysis in Ref. [28], the total branching ratio for $\mu\to e\gamma$ decay can be expressed by

$$B(\mu\to e\gamma)=\frac{12\pi^2}{G_F}(|A_{LR}|^2+|A_{RL}|^2), \tag{56}$$

where

$$A_i=A_i^{\lambda}+A_i^{\lambda'}+A_i^{\Delta m}, \tag{57}$$

with $i=LR$ and RL. In Eq. (57), the amplitudes $A_i^{\Delta m}$ stand for the neutralino- and slepton-mediated contributions and Δm is the soft breaking mass. For convenience, we only consider A_i^{λ} contributions. Using the results in Ref. [28], we get

$$A^{\lambda}_{LR}=\frac{e}{96\pi^2}\sum_{i,k=1}^{3}\lambda_{i1k}\lambda_{i2k}\left(\frac{1}{M^2_{\tilde{\nu}_i}}-\frac{1}{2}\frac{1}{M^2_{\tilde{e}_{Rk}}}\right),$$

$$A^{\lambda}_{RL}=\frac{e}{96\pi^2}\sum_{i,j=1}^{3}\lambda_{ij1}\lambda_{ij2}\left(\frac{1}{M^2_{\tilde{\nu}_i}}-\frac{1}{2}\frac{1}{M^2_{\tilde{e}_{Lj}}}\right). \tag{58}$$

Therefore, the bounds on R-parity violating couplings λ are given by

$$\frac{|\lambda_{31k}\lambda_{32k}|}{M^2}<4.6\times10^{-8}\frac{1}{\mathrm{GeV}^2},\quad \text{for } k=1,2; \tag{59}$$

$$\frac{|\lambda_{ij1}\lambda_{ij2}|}{M^2}<2.3\times10^{-8}\frac{1}{\mathrm{GeV}^2},\quad \text{for } i,j=1,2,$$

where we have assumed that $M_{\tilde{\nu}_\tau}\simeq M_{\tilde{e}_R}\simeq M_{\tilde{e}_L}\simeq M$. For simplicity, we take $|\lambda_{31k}|\sim|\lambda_{32k}|$ and $|\lambda_{211}|\sim|\lambda_{212}|$, and we have

$$\frac{|\lambda_{322}|}{M}<2.1\times10^{-4}\frac{1}{\mathrm{GeV}},$$

$$\frac{|\lambda_{212}|}{M}<1.5\times10^{-4}\frac{1}{\mathrm{GeV}}. \tag{60}$$

The bounds on λ' can be extracted from the experimental limit on $K^+\to\pi\nu\overline{\nu}$ as shown in Ref. [29]. One finds that

$$\frac{|\lambda'_{ijk}|}{M_{\tilde{d}_{Rk}}}<1.2\times10^{-4}\frac{1}{\mathrm{GeV}},\quad \text{for } j=1,2, \tag{61}$$

where $M_{\tilde{d}_{Rk}}\sim M$ is the sdown-quark mass.

From Eq. (55) and the bounds in Eqs. (60) and (61), we find

$$P_T(K^+_{\mu 2\gamma})\leqslant 0.01 \tag{62}$$

for $|\sigma_V-\sigma_A|\sim0.1$.

The interaction in Eq. (52) could also yield transverse muon polarization in $K^+\to\pi^0\mu^+\nu$. We find [19] that $P_T(K^+_{\mu3})<10^{-2}$ by using the same parameter values as in the case of $K^+_{\mu2\gamma}$. Thus, in R-parity violation SUSY models with R-parity violation, one can also get a large prediction of $P_T(K^+_{\mu3})$.

D. Leptoquark model

There exist three scalar leptoquark models contributing to the decay $K^+_{\mu2\gamma}$ through the tree diagrams which is similar to the cases for $K^+_{\mu3}$ shown in Ref. [19]. The quantum numbers of the leptoquarks under the standard group $\mathrm{SU}(3)_C\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ are [19,30,31]

$$\phi_1=\left(3,2,\frac{7}{3}\right)\quad (\text{model I}),$$

$$\phi_2=\left(3,1,-\frac{2}{3}\right) \quad (\text{model II}),$$

$$\phi_3=\left(3,3,-\frac{2}{3}\right) \quad (\text{model III}), \tag{63}$$

respectively. The general couplings involving these leptoquarks are given by [31]

$$\mathcal{L}^{\mathrm{I}}=(\lambda_1\overline{Q_L}e_R+\lambda_1'\overline{u_R}L_L)\phi_1+\text{H.c.},$$

$$\mathcal{L}^{\mathrm{II}}=(\lambda_2\overline{Q_L}L_L^c+\lambda_2'\overline{u_R}e_R^c)\phi_2+\text{H.c.},$$

$$\mathcal{L}^{\mathrm{III}}=\lambda_3\overline{Q_L}L_L^c\phi_3+\text{H.c.}, \tag{64}$$

where $Q=\binom{u}{d}$ and $L=\binom{\nu}{e}$. Here the coupling constants λ_k ($k=1,\ldots,3$) are complex and thus CP violation could arise from the Yukawa interactions in Eq. (64). We assume that CP violation in $K\to\pi\pi$ decays can be accounted for by the nonvanishing KM phase, and investigate the effect on the muon polarization of adding another CP violation mechanism in Eq. (64).

In terms of each charge components of the leptoquarks we rewrite Eq. (64) as

$$\mathcal{L}^{\mathrm{I}}=\sum_{i,j}\left\{\left[\lambda_1^{ij}\overline{u}_i\frac{1}{2}(1+\gamma_5)e_j+\lambda'^{ij}_1\overline{u}_i\frac{1}{2}(1-\gamma_5)e_j\right]\phi_1^{(5/3)}+\left[\lambda_1^{ij}\overline{d}_i\frac{1}{2}(1+\gamma_5)e_j+\lambda'^{ij}_1\overline{u}_i\frac{1}{2}(1-\gamma_5)\nu_j\right]\phi_1^{(2/3)}\right\}+\text{H.c.},$$

$$\mathcal{L}^{\mathrm{II}}=\sum_{i,j}\left\{\lambda_2^{ij}\left[-\overline{u}_i\frac{1}{2}(1+\gamma_5)e_j^c+\overline{d}_i\frac{1}{2}(1+\gamma_5)\nu_j^c\right]+\lambda'^{ij}_2\overline{u}_i\frac{1}{2}(1-\gamma_5)e_j^c\right\}\phi_2^{(-1/3)}+\text{H.c.},$$

$$\mathcal{L}^{\mathrm{III}}=\sum_{i,j}\lambda_3^{ij}\left\{\overline{u}_i\frac{1}{2}(1+\gamma_5)\nu_j^c\phi_3^{(2/3)}+\left[\overline{u}_i\frac{1}{2}(1+\gamma_5)e_j^c+\overline{d}_i\frac{1}{2}(1+\gamma_5)\nu_j^c\right]\phi_3^{(-1/3)}+\overline{d}_i\frac{1}{2}(1+\gamma_5)e_j^c\phi_3^{(-4/3)}\right\}+\text{H.c.}, \tag{65}$$

where i,j are family indices and Q_e in $\phi_k^{(Q_e)}$ are the electric charges. From Eq. (65), we see that the relevant terms for the process $K^+_{\mu2\gamma}$ are the ones involving $\phi_1^{(2/3)}$, $\phi_2^{(-1/3)}$, and $\phi_3^{(-1/3)}$ couplings, respectively. We will concentrate on these terms in our discussions. The effective interactions from these leptoquark exchanges are

$$\mathcal{L}_{\mathrm{eff}}^{\mathrm{I}}=\frac{\lambda_1^{22}(\lambda'^{1i}_1)^*}{4M_{\phi_1}^2}\overline{s}(1+\gamma_5)\mu\,\overline{\nu_i}(1+\gamma_5)u+\text{H.c.},$$

$$\mathcal{L}_{\mathrm{eff}}^{\mathrm{II}}=\frac{1}{4M_{\phi_2}^2}\left[-\lambda_2^{2i}(\lambda_2^{12})^*\overline{s}(1+\gamma_5)\nu_i^c\overline{\mu^c}(1-\gamma_5)u+\lambda_2^{2i}(\lambda'^{12}_2)^*\overline{s}(1+\gamma_5)\nu_i^c\overline{\mu^c}(1+\gamma_5)u\right]+\text{H.c.}, \tag{66}$$

$$\mathcal{L}_{\mathrm{eff}}^{\mathrm{III}}=\frac{\lambda_3^{2i}(\lambda_3^{12})^*}{4M_{\phi_3}^2}\overline{s}(1+\gamma_5)\nu_i^c\overline{\mu^c}(1-\gamma_5)u+\text{H.c.}, \tag{67}$$

where M_{ϕ_k} ($k=1,\ldots,3$) are the masses of $\phi_1^{(2/3)}$, $\phi_2^{(-1/3)}$, and $\phi_3^{(-1/3)}$, respectively. Using the Fierz transformations and Eq. (6), we have

$$\Delta_P^{\mathrm{I}}=-\frac{\sqrt{2}m_K^2}{G_F\sin\theta_C(m_s+m_u)m_\mu}\frac{\lambda_1^{22}(\lambda'^{1i}_1)^*}{8M_{\phi_1}^2},$$

$$\Delta_P^{\mathrm{II}}=-\frac{\sqrt{2}m_K^2}{G_F\sin\theta_C(m_s+m_u)m_\mu}\frac{\lambda_2^{2i}(\lambda'^{12}_2)^*}{8M_{\phi_2}^2}, \tag{68}$$

which give rise to

$$[P_T^{\mathrm{I}},P_T^{\mathrm{II}}]=(\sigma_V-\sigma_A)\frac{\sqrt{2}}{G_F\sin\theta_C}\frac{m_k^2}{(m_s+m_u)m_\mu}\times\left[\frac{\lambda_1^{22}(\lambda'^{1i}_1)^*}{M_{\phi_2}^2},\frac{\lambda_2^{2i}(\lambda'^{12}_2)^*}{8M_{\phi_2}^2}\right], \tag{69}$$

respectively, where we have neglected the tensor interactions and the contribution of $\mathcal{L}^{\mathrm{III}}$ because it contains only left-handed vector current interactions which have no relative phases between M_{IB} and M_{SD}.

For model I, since the leptoquarks model can give a large contribution to the moun transverse polarization in $K^+_{\mu3}$ [19], we may use the present experimental bound on $P_T(K^+_{\mu3})$ to constrain the CP-violating parameters in Eq. (67). Explicitly, we find

TABLE I. Summary of the upper values of (1) $P_T(K^+_{\mu3})$ and (2) $P_T(K^+_{\mu2\gamma})$ for (a) current experimental limits, (b) sensitivities in KEK-PS E246, (c) theoretical background (FSI), (d) standard CKM model, (e) left-right symmetric models, (f) multi-Higgs models with NFC, (g) multi-Higgs models without NFC, (h) SUSY with R parity, (i) SUSY without R parity, and (j) leptoquark models.

	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
(1)	10^{-2}	5×10^{-4}	10^{-6}	0	0	10^{-2}	10^{-2}	10^{-2}	10^{-2}	10^{-2}
(2)		10^{-3}	10^{-3}	0	7×10^{-3}	5×10^{-3}	5×10^{-2}	2×10^{-2}	1×10^{-2}	5×10^{-3}

$$\frac{|\lambda_1^{22}(\lambda'^{1i}_1)^*|}{M^2_{\phi_1}} < 4.4\times 10^{-8}, \tag{70}$$

and therefore we get

$$|P_T^I(K^+_{\mu 2\gamma})| < 4.8\times 10^{-3} \tag{71}$$

for $|\sigma_V - \sigma_A| \sim 0.1$. Similar results can be obtained in model II.

IV. CONCLUSIONS

We have studied the transverse muon polarization in the decay of $K^+ \to \mu^+ \nu\gamma$ in various CP violation theories. We have explicitly demonstrated that the polarization effect can be large in models with the left-right symmetry, multi-Higgs bosons, SUSY, and leptoquarks, repectively. These results as well as that from other CP violation models are summarized in Table I. The estimations for the transverse muon polarization in $K^+_{\mu 3}$ are also presented in Table I. It is interesting to see that a large $P_T(K^+_{\mu 2\gamma})$ corresponds to a large $P_T(K^+_{\mu 3})$ in most of the CP violation models shown in the table except the left-right symmetric theories, in which $P_T(K^+_{\mu 3}) = 0$. Therefore, the decay of $K^+_{\mu 2\gamma}$ has comparable or even more sensitivity with that of $K^+_{\mu 3}$ to the new CP violation mechanism.

In conclusion, the transverse muon polarization of $K^+ \to \mu^+ \nu\gamma$ could be at the level of 10^{-2} in the nonstandard CP violation theories, which may be detectable at the ongoing KEK expeiment of E246 as well as the proposed BNL experiment. The measurement of such effect is a clean signature of CP violation beyond the standard model since that from the theoretical background, i.e., FSI, is $\leqslant 10^{-3}$.

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