Disorder parameter for dual superconductivity in gauge theories

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A detailed discussion is given of the disorder parameter for dual superconductivity of lattice gauge theories, introduced in a previous paper, and of its relation to other approaches. New lattice data are reported. Among other results, we find that the dual superconductivity of compact U(1) is type II. [S0556-2821(97)02823-3]

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I. INTRODUCTION

Dual superconductivity of the vacuum is an important phenomenon in gauge theories. It produces confinement of electric charges via dual Meissner effect in the Abelian case. There exist indications that it could be the mechanism of color confinement in QCD [1,2]. It plays a key role in the structure of supersymmetric gauge theories [3]. The simplest case is the compact U(1) gauge theory.

With the Wilson action this theory shows a phase transition at $\beta_c \simeq 1.01$, probably weak first order, from a phase at low β where electric charge is confined, to a phase of free photons [4]. Confinement is detected by measuring the string tension from the vacuum expectation value (VEV) of Wilson loops. The penetration depth of the electric field is finite for $\beta < \beta_c$, indicating the dual Meissner effect, and goes large at the deconfining transition β_c [4].

Monopoles are detected by their Dirac strings as units of 2π magnetic flux through the plaquettes. Their number density is not a disorder parameter for dual superconductivity, in the same way as the number of electric charges is not for ordinary superconductivity. However, empirically, the number density of monopoles is larger in the confined phase and drops to zero above β_c [4].

A legitimate disorder parameter should vanish for symmetry reasons in the deconfined phase and be different from zero in the confining phase. Since dual superconductivity is nothing but the spontaneous breaking of the U(1) symmetry related to the magnetic charge conservation, the VEV of any operator carrying a nonzero magnetic charge can be a disorder parameter. A nonzero VEV of such an operator would indeed indicate that vacuum has not a definite magnetic charge, i.e., that monopoles condense in it in the same way as Cooper pairs do in the ground state of ordinary superconductors. The concept of the disorder parameter has been known for a long time in the community of field theory and statistical mechanics [5,6]. In the pionering numerical simulations in lattice gauge theories, however, emphasis was given to the density of monopoles as indicators of dual superconductivity [4,7].

A rigorous proof was given in Ref. [8] that monopoles condense at low β's in lattice U(1) theory with a Villain action [9]. The proof makes use of the specific form of the action and so did numerical attempts to extract a disorder parameter [10]. However, probably because of the mathematical language of the forms, which is not so familiar to physicists, nobody tried for a long time to export the construction to the generic form of the action or to non-Abelian gauge theories. Indeed, after Abelian projection [11], monopole condensation in non-Abelian gauge theories like QCD always reduces to an effective U(1) with Dirac monopoles [11,12]. Of course, the U(1) effective action is unknown, and therefore a construction of the disorder parameter is needed, which can work with any variant action.

Such a construction was given in Ref. [13] and immediately afterwards was used to demonstrate dual superconductivity of non-Abelian theories [14].

This result prompted the exportation of the construction of Refs. [8,10] from a Villain to a generic action.

In this paper we want to discuss in detail and improve the construction of Refs. [13] (Sec. II), compare it to that of Ref. [8] (Sec. III), showing that they are equivalent, and present a number of numerical results for lattice U(1) with Wilson's action (Sec. IV). We will then compare our [13] way of detecting superconductivity by the quantity $\rho = (d/d\beta)\ln\langle\mu\rangle$, $\langle\mu\rangle$ being the disorder parameter, to direct determination of $\langle\mu\rangle$ or of its effective potential (Sec. IV).

Besides confirming dual superconductivity of U(1) gauge theory in the confined phase we show that it is of the second kind.

The present discussion is also a useful basis to the treatment of the analogous problem in non-Abelian theories, which will be presented elsewhere.

II. DISORDER PARAMETER

The construction of Ref. [13] of the creation operator of a monopole or antimonopole is inspired by Refs. [5,6] and is based on the following simple idea.

In the Schrödinger representation where the field $\vec{A}(x)$ is diagonal, a monopole of charge $2\pi q/e$ sitting in $\vec{y}$ is created by adding the corresponding vector potential $(1/e)\vec{b}(\vec{x}-\vec{y})$ to $\vec{A}(x)$.

This is nothing but a translation of $\vec{A}(x)$, which is generated by the conjugate momentum $\vec{\pi}(x) = \vec{E}(x)$, the electric field operator. In the same way as

$$e^{ipa}|x\rangle = |x+a\rangle, \tag{1}$$

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we have

$$\left|\vec{A}(\vec{x},t)+\frac{1}{e}\vec{b}(\vec{x}-\vec{y})\right\rangle=\mu|\vec{A}(\vec{x},t)\rangle, \tag{2}$$

with

$$\mu(\vec{y},t)=\exp\left[i\frac{1}{e}\int d^3x\ \vec{E}(\vec{x},t)\vec{b}(\vec{x}-\vec{y})\right]. \tag{3}$$

The magnetic charge operator being

$$Q=\int d^3x\vec{\nabla}(\vec{\nabla}\wedge\vec{A}(\vec{x},t)), \tag{4}$$

the commutator $[Q,\mu]$ can be evaluated by use of the canonical commutation relations

$$[E_i(\vec{x},t),A_j(\vec{z},t)]=-i\delta_{ij}\delta^3(\vec{x}-\vec{z}), \tag{5}$$

giving

$$\begin{aligned}[Q(t),\mu(\vec{y},t)]&=\frac{1}{e}\int d^3x\vec{\nabla}(\vec{\nabla}\wedge\vec{b}(\vec{x}-\vec{y}))\cdot\mu(\vec{y},t)\\&=\frac{q}{2e}\mu(\vec{y},t)\int d^3x\vec{\nabla}\left(\frac{\vec{r}}{r^3}\right)=2\pi\frac{q}{e}\mu(\vec{y},t).\end{aligned} \tag{6}$$

In deriving Eq. (6) the Dirac string has been removed.

A choice for $\vec{b}(\vec{x}-\vec{y})$ can be

$$\vec{b}(\vec{x}-\vec{y})=\frac{q}{2}\frac{\vec{r}\wedge\vec{n}_3}{r(r-\vec{r}\cdot\vec{n}_3)}. \tag{7}$$

Alternative choices differ by a gauge transformation $\vec{b}\to\vec{b}+\vec{\nabla}\Phi$, which leaves the operator invariant if the Gauss law $\vec{\nabla}\cdot\vec{E}=0$ is satisfied.

On the lattice the building block of the theory is the link $\mathrm{U}_\mu(n)$, which is an element of the gauge group. For U(1), $\mathrm{U}_\mu(n)=e^{i\theta_\mu(n)}$ and the plaquette $\Pi_{\mu\nu}$, which is the parallel transport along the elementary square in the plane $\mu\nu$ at the site n, is

$$\Pi_{\mu\nu}(n)=\exp[i\theta_{\mu\nu}(n)], \tag{8}$$

with

$$\theta_{\mu\nu}=\Delta_\mu\theta_\nu-\Delta_\nu\theta_\mu\underset{a\to 0}{\simeq}a^2eF_{\mu\nu}. \tag{9}$$

The lattice version of the electric field is then

$$a^2E_i\simeq\frac{1}{e}\mathrm{Im}\ \Pi^{0i}+O(a^4), \tag{10}$$

and a definition of the operator μ on the lattice [13] can be

$$\begin{aligned}\mu(\vec{y},n_0)&=\exp\left[-\beta\sum_{n} b^i(\vec{n}-\vec{y})\mathrm{Im}\Pi^{0i}(\vec{n},n_0)\right]\\&=\exp\left[-\beta\sum_{\vec{n}} b^i(\vec{n}-\vec{y})\sin[\theta_{0i}(\vec{n},n_0)]\right],\end{aligned} \tag{11}$$

with $\beta=1/e^2$. Here $b^i(\vec{n})$ is the discretized version of the monopole field, Eq. (7). The factor β in front of the exponent comes from the factor $1/e$ in the monopole charge times the normalization factor in Eq. (10). Usual Wick rotation to the Euclidean region has been performed.

The form (11) was successfully used in Ref. [13].

A better definition of μ can be given, which coincides with Eq. (11) in practice, but automatically respects the compactness of the theory, in that it shifts the exponent of the links, and not the links themselves. In formulas,

$$\begin{aligned}\mu(\vec{y},m_0)=\exp\Big\{\beta\sum_{\vec{n}}\ &\{\cos[\theta^{0i}(\vec{n},m_0)+b^i(\vec{y}-\vec{n})]\\&-\cos[\theta^{0i}(\vec{n},m_0)]\}\Big\}.\end{aligned} \tag{12}$$

For small b^i the definition (12) coincides with Eq. (11).

More generally, if $\Sigma_{\mu\nu n}S(\theta_{\mu\nu}(n))$ is the action, μ will be defined as

$$\begin{aligned}\mu(\vec{y},m_0)=\exp\Big\{\beta\sum_{\vec{n}}\ &[S(\theta^{0i}(\vec{n},m_0)+b_i(\vec{n}-\vec{y}))\\&-S(\theta^{0i}(\vec{n},m_0))]\Big\}\end{aligned} \tag{13}$$

and will tend to the expression (11) as the lattice spacing a goes to zero, when the action tends to the continuum action.

The prescription of excluding Dirac string on a lattice being either to locate the monopole at $\vec{y}$ between two neighboring sites or to eliminate in the sum the arrow of sites where $\vec{b}$ is singular, it is easy to verify that the definitions (12) and (11) give the same results from the practical point of view.

If the action is the Wilson's action [15]

$$S=\sum_{n,(\mu\nu)}\beta(\cos(\theta_{\mu\nu})-1), \tag{14}$$

then the vacuum expectation value of μ is given by

$$\langle\mu\rangle=\frac{1}{Z}\int\left[\prod_{\mu,n}d\theta_\mu(n)\right]\exp(S)\ \mu \tag{15}$$

or, making use of Eq. (12),

$$\langle\mu(\vec{y},m_0)\rangle=\frac{1}{Z}\int\left[\prod_{\mu,n}d\theta_\mu(n)\right]\exp(S+S'), \tag{16}$$

where S' is the exponent of Eq. (12).

Adding S' simply amounts to modify the (0,i) plaquettes on the time slice n_0, by addition of b_i to θ_{0i},

$$S+S'=\sum_n \sum_{(i,j)=1}^{3} \beta(\cos[\theta_{ij}(n)-1])$$
$$+\sum_{n,n_0\neq m_0} \beta(\cos[\theta_{0i}(n)-1])$$
$$+\sum_{\vec{n}} \{\cos[\theta_{0i}(\vec{n},m_0)+b^i(\vec{m}-\vec{n})]-1\}. \quad (17)$$

If a number of monopoles and antimonopoles are created at time n_0, b_i should be the sum of the corresponding vector potentials. The generic correlation function $\langle\mu(x_1)\cdots\mu(x_n)\rangle$ is defined as $\langle\mu\rangle$ in Eq. (16), with the change from S to $S+S'$ extended to all the time slices where monopoles or antimonopoles are created.

So, for example, the correlation function where a monopole is created in $\vec{y}=0$ at $t=0$ and destroyed at $t=m_0$ is given by

$$\langle\mu(\vec{y},0)\ \overline{\mu(\vec{y},m_0)}\rangle=\frac{1}{Z}\int \exp(S+S'_{\mu\bar{\mu}}).$$

$S+S'$ differs from S by the replacement

$$\theta_{0i}(\vec{n},0)\rightarrow\theta_{0i}(\vec{n},0)+b_i(\vec{n}-\vec{y}) \quad \text{at } t=0,$$
$$\theta_{0i}(\vec{n},m_0)\rightarrow\theta_{0i}(\vec{n},m_0)-b_i(\vec{n}-\vec{y}) \quad \text{at } t=m_0. \quad (18)$$

Monopole condensation can be detected from the asymptotic value of $\langle\mu(\vec{y},0)\ \overline{\mu(\vec{y},m_0)}\rangle$. Indeed, as m_0 grows large, by the cluster property

$$\langle\mu(\vec{y},0)\ \overline{\mu(\vec{y},m_0)}\rangle\simeq C\ \exp(-m_0M)+\langle\mu\rangle^2. \quad (19)$$

Notice that $\langle\mu\rangle=\langle\overline{\mu}\rangle$ by C invariance, and the position $\vec{y}$ is irrelevant by translation invariance. M is the mass of the lowest state with monopole charge q in units of inverse lattice spacing.

To visualize that μ really creates a monopole at $t=0$ consider again the change it produces according to Eq. (18). Since

$$\theta_{0i}(\vec{n},0)=\theta_i(\vec{n},1)-\theta_0(\vec{n},0)-\theta_0(\vec{n}+\hat{\imath},0)+\theta_0(\vec{n},0), \quad (20)$$

the change (18) of θ_{0i} can be considered as a shift:

$$\theta_i(\vec{n},1)\rightarrow\theta_i(\vec{n},1)-b_i(\vec{n}-\vec{y}). \quad (21)$$

A change of variables,

$$\theta_i'=\theta_i(\vec{n},1)-b_i(\vec{n}-\vec{y}), \quad (22)$$

in the Feynman integral (16), which leaves the measure invariant, brings back the plaquette θ_{0i} to its unperturbed form. However, the change of variables (22) changes the (i,j) plaquette at $n_0=1$ as follows:

$$\theta_{ij}(\vec{n},1)\rightarrow\theta_{ij}(\vec{n},1)+\Delta_i b_j(\vec{n}-\vec{y})-\Delta_j b_i(\vec{n}-\vec{y}). \quad (23)$$

This means that at $n_0=1$ the magnetic field of a monopole located at $\vec{n}=\vec{y}$ is added to the original configuration. The change of variables (21) also affects the plaquette $\theta_{0i}(\vec{n},2)$, and amounts to the shift

$$\theta_{0i}(\vec{n},2)\rightarrow\theta_{0i}(\vec{n},2)-b_i(\vec{n}-\vec{y}). \quad (24)$$

Again a change of variables $\theta_i(\vec{n},2)\rightarrow\theta_i(\vec{n},2)-b_i(\vec{n}-\vec{y})$ restores $\theta_{0i}(\vec{n},2)$ to the initial form at the price of adding a monopole at time $t=2$ and of producing a shift in the form (24) on $\theta_{0i}(\vec{n},3)$. This procedure can be iterated. At $t=m_0$ this procedure ends, because b_i cancels with the shift of opposite sign corresponding to the creation of the antimonopole.

Thus the correlator $\langle\mu(\vec{y},0)\ \overline{\mu(\vec{y},m_0)}\rangle$ simply consists in having a monopole propagating in time, from 0 to m_0.

The construction above simply generalizes to more complicated forms of the action, where Wilson loops other than plaquettes enter.

III. COMPARISON WITH OTHER APPROACHES

In this section we want to discuss the relation of our approach to that of Ref. [8].

In the language of Ref. [8] $\theta_\mu(n)$ is a one-form associated with the links and $d\theta$ is the two-form associated with the plaquettes or the field strength tensor.

In this language the partition function is

$$Z=\int \mathcal{D}[\theta]\ \Phi_\beta(d\theta). \quad (25)$$

For Wilson's action,

$$\Phi_\beta=\exp\left(\beta\sum_{\text{plaq}}[\cos(d\theta)-1]\right). \quad (26)$$

For Villain's action,

$$\Phi_\beta=\sum_n \exp\left\{-\frac{\beta}{2}\sum_{\text{plaq}}\|d\theta+2\pi n\|^2\right\}. \quad (27)$$

To define a disorder operator $\langle\mu\rangle$ the action is modified by adding a two-form X to $d\theta$. We define

$$Z(X)=\int \mathcal{D}[\theta]\ \Phi_\beta(d\theta+X) \quad (28)$$

and

$$\langle\mu\rangle=\frac{Z(X)}{Z(0)}. \quad (29)$$

Any change of X of the form $X\rightarrow X+d\Lambda$ leaves $\langle\mu\rangle$ invariant, in that $d\Lambda$ corresponds to a shift of θ to $\theta+\Lambda$ which is reabsorbed by a change of the (periodic) integration variables.

Since a generic X can be written as (Hodge decomposition)

$$X=d\alpha+\delta\frac{1}{\Delta}dX, \tag{30}$$

the above invariance implies that $\langle\mu\rangle$ only depends on dX.

dX is a three-form $[dX]_{\mu\nu\alpha}$ and its dual $*dX$ is a one-form, which is a magnetic current, since X is a field strength. Explicitly

$$dX_{\mu\nu\alpha}=-(\partial_\alpha X_{\mu\nu}+\partial_\mu X_{\nu\alpha}+\partial_\nu X_{\alpha\mu}) \tag{31}$$

and

$$J^M_\rho=\frac{1}{6}\varepsilon_{\rho\mu\nu\alpha}\,dX_{\mu\nu\alpha}. \tag{32}$$

The magnetic current (32) is identically conserved. In the language of forms

$$\delta J^M=0. \tag{33}$$

The magnetic charge density which describes the creation of a monopole of charge $2\pi q$ in the site $\vec{y}$ at time y^0, and its destruction at time y'^0 is

$$J^M_0(\vec{x},x^0)=2\pi q\,\delta^3(\vec{x}-\vec{y})[\theta(x^0-y^0)-\theta(x^0-y'^0)]. \tag{34}$$

Since the current is conserved,

$$\begin{aligned}\vec{\nabla}\vec{J}^M&=-\Delta_0 J^M_0\\&=-2\pi q\,\delta^3(\vec{x}-\vec{y})[\delta(x^0-y^0)-\delta(x^0-y'^0)].\end{aligned} \tag{35}$$

A solution of Eq. (35) is

$$\vec{J}^M(\vec{x},x^0)=2\pi q\frac{1}{4\pi}\frac{\vec{x}-\vec{y}}{|\vec{x}-\vec{y}\,|^3}[\delta(x^0-y^0)-\delta(x^0-y'^0)]. \tag{36}$$

The corresponding X is then

$$\overline{X}=\delta\frac{1}{\Delta}J^M. \tag{37}$$

The correlation function of a monopole antimonopole will then be

$$\langle\mu(\vec{y},y_0)\ \overline{\mu}(\vec{y},y'^0)\rangle=\frac{Z(\overline{X})}{Z(0)}. \tag{38}$$

This is the construction of Ref. [8].

Notice that $Z(X)$ is periodic in X (with period 2π) since the action is compact. In fact Z only depends on dX and is periodic also in dX with the same period. This can be rigorously proved by going to a Fourier transform:

$$Z(dX+2\pi n)=Z(dX). \tag{39}$$

Consider now a one-form Ω on the dual lattice, with support on a line. If $\delta\Omega=0$, the support must be a closed line. If Ω is integer valued in units of 2π, the change

$$dX=*J^M\to dX=*J^M+\Omega$$

leaves Z invariant.

In the notation of Ref. [8], $\vec{J}^M$ is denoted by $2\pi qB$ and J^M_0 by $-2\pi q\omega$ and

$$d\overline{X}=2\pi q(B-\omega). \tag{40}$$

Any X with the same dX will give the same correlation function (38). The construction presented in Sec. II corresponds to the choice

$$\begin{aligned}\overline{X'_{0i}}&=b_i(\vec{x})[\delta(x^0-y^0)-\delta(x^0-y'^0)],\\ \overline{X'_{ij}}&=0,\end{aligned} \tag{41}$$

or, in the dual language,

$$\begin{aligned}(*\overline{X'})_{0i}&=0,\\ (*\overline{X'})_{ij}&=\varepsilon_{ijk}b_k(\vec{x})[\delta(x^0-y^0)-\delta(x^0-y'^0)],\end{aligned} \tag{42}$$

and

$$*d\overline{X'_\mu}=\delta(*\overline{X'})_\mu=-\sum_\rho\ \Delta_\rho(*X)_{\rho\mu}. \tag{43}$$

Explicitly

$$\delta(*\overline{X'})_0=0,\quad \delta(*\overline{X'})_i=-\sum_k\ \Delta_k(*X)_{ki},$$

and, by Eqs. (41) and (7),

$$\begin{aligned}\delta(*\overline{X'})_i={}&2\pi q\frac{1}{4\pi}\frac{x_j-y_j}{|\vec{x}-\vec{y}|^3}[\delta(x_0-y_0)-\delta(x_0-y'_0)]\\&-2\pi q\,\delta(x_1-y_1)\,\delta(x_2-y_2)\,\theta(x_3-y_3)[\delta(x_0-y_0)\\&-\delta(x_0-y'_0)].\end{aligned} \tag{44}$$

Our $*d\overline{X'}$ differs from $*d\overline{X}$ (40) of Ref. [8] by a one-form integer valued in units of 2π, with support on a closed line. Therefore our correlator coincides with that of Ref. [8], not only for the Villain action, but for the generic form of the action.

This section is a cultivated way of presenting the argument already given at the end of the last section.

IV. NUMERICAL RESULTS FOR THE DISORDER PARAMETER

As discussed in Sec. II, we measure the correlation function

$$\mathcal{D}(x^0)=\langle\mu(\vec{x},x^0),\overline{\mu}(\vec{x},0)\rangle\simeq Ae^{-Mx^0}+\langle\mu\rangle^2. \tag{45}$$

The aim is to extract $\langle\mu\rangle^2$, which will signal dual superconductivity, and M, which is the lowest mass in the sector of magnetically charged excitations.

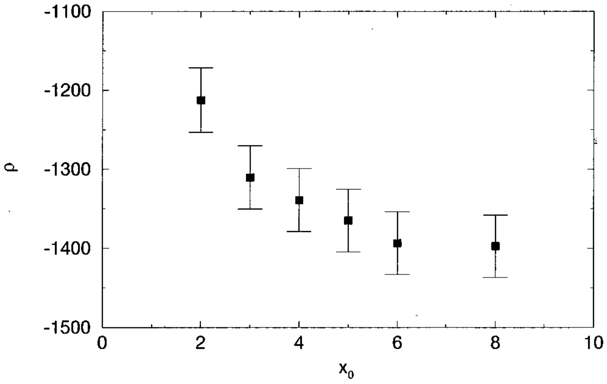

FIG. 1. Monopole antimonopole correlation in time (lattice $8^3\times 16$).

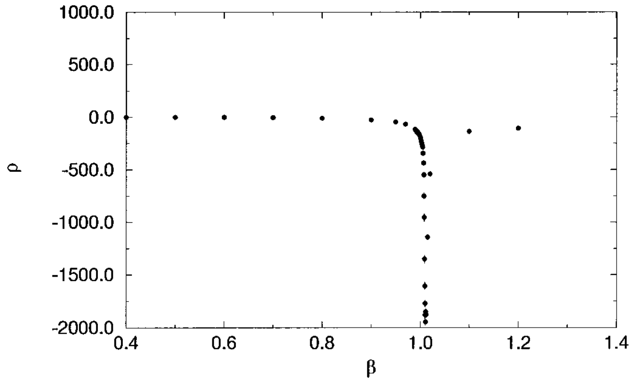

FIG. 2. ρ_∞ as a function of β. The negative peak signals the phase transition (lattice $8^3\times 16$).

A direct determination of $\mathcal{D}$ can be done, as we will discuss below, but is rather noisy from the numerical point of view. The reason for this is that $\mathcal{D}$,

$$\mathcal{D}=\frac{1}{Z}\int \mathcal{D}\theta\ \exp(S+S'), \tag{46}$$

is the average of $\exp(S')$, S' being the modification of the action on the time slices $t=0$ and $t=x^0$, and S' fluctuates roughly like the square root of the spatial volume.

A way to go around this difficulty is to measure instead of $\mathcal{D}$ the quantity [12]

$$\rho(\vec{x},x^0,\vec{x},0)=\frac{d}{d\beta}\ln\mathcal{D}. \tag{47}$$

At a large distance ($x^0\to\infty$),

$$\rho_\infty\simeq 2\frac{d}{d\beta}\ln\langle\mu\rangle, \tag{48}$$

and since $\rho(\beta=0)=1$, $\langle\mu\rangle$ can be reconstructed as

$$\langle\mu\rangle=\exp\left(\frac{1}{2}\int \rho(\beta')d\beta'\right). \tag{49}$$

From Eq. (46),

$$\rho_\infty=\langle S\rangle_S-\langle S+S'\rangle_{S+S'}. \tag{50}$$

The definition of ρ is analogous to the definition of the internal energy in terms of the partition function in statistical mechanics. ρ is now a well-defined quantity and easy to measure and, as we shall see, can give all the information needed to detect dual superconductivity.

We have made simulations on a $6^3\times 12$, $8^3\times 16$, and $10^3\times 20$ lattices putting the time axis along the long edge of the lattice. The algorithm used was the heat bath. The typical number of configurations was 1.3×10^5 for $6^3\times 12$, and 6×10^4 for $8^3\times 16$ and $10^3\times 20$ lattices. The computer used was APE-QUADRIX.

A typical behavior of ρ versus x^0 is shown in Fig. 1, for a $8^3\times 16$ lattice, showing that an asymptotic value is reached by ρ as a function of x^0. The mass M of the exponential in Eq. (45) can be estimated and is typically $\sim(2-3)/a$. We will come back again to this point in the following.

The quantity ρ_∞ as a function of β is plotted in Fig. 2. For all of our lattices sizes ρ_∞ is negative and sharply decreases approaching β_c. This corresponds, by Eq. (49) to a behavior of $\langle\mu\rangle$ which slowly decreases from the value $\langle\mu\rangle=1$ at $\beta=0$, and has a sharp drop at β_c.

To better analyze this behavior we compare it for the three lattice sizes under study. For $\beta<\beta_c$ below the negative peak, ρ increases with L, showing that as $L\to\infty$, $\langle\mu\rangle$ reaches a finite, nonzero value. Magnetic U(1) is therefore spontaneously broken, and for $\beta<\beta_c$ the system is a dual superconductor (Fig. 3).

For $\beta\simeq\beta_c$ we know that the typical correlation length of the system becomes large. There is evidence that the transition is weak first order [16], with some controversy [17].

The correlation length ξ becomes large as β approaches β_c in a range of β's and eventually stops growing before reaching it. This means that, in the neighborhood of β_c,

$$\mu=\mu\left(\frac{\xi}{L},\frac{a}{\xi}\right)\simeq\mu\left(\frac{\xi}{L}\right). \tag{51}$$

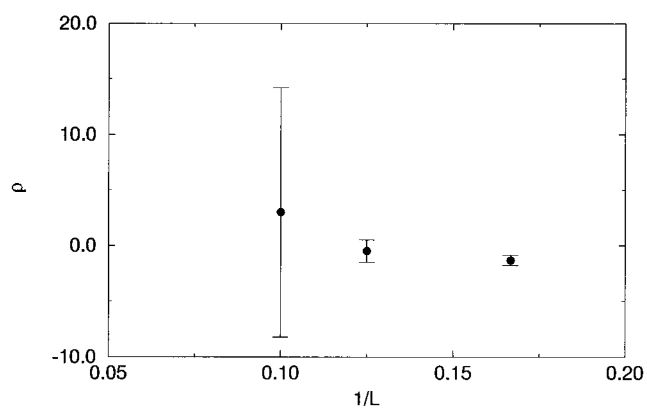

FIG. 3. ρ_∞ versus $1/L$ for $\beta=1.009$.

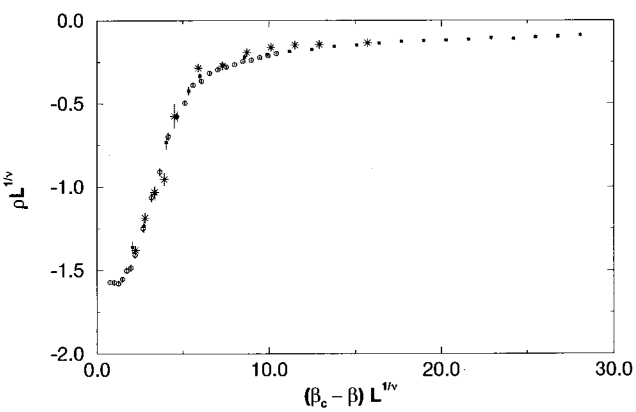

FIG. 4. Finite size scaling. $\rho L^{1/\nu}$ is plotted versus $(\beta_c-\beta)L^{1/\nu}$.

If the transition were second order, a critical index ν would exist such that

$$\xi \underset{\beta\to\beta_c^-}{\simeq} (\beta_c-\beta)^{-\nu}. \tag{52}$$

In our case some effective index ν could anyhow exist, describing a behavior of ξ of the form (52) in the above-mentioned range of β's. Then ξ/L can be traded with $L^{1/\nu}(\beta_c-\beta)$ and a finite size scaling behavior results,

$$\mu=\mu[L^{1/\nu}(\beta_c-\beta)], \tag{53}$$

implying for $\rho=(d/d\beta)\ln\langle\mu\rangle$ a scaling behavior

$$\frac{\rho}{L^{1/\nu}}=f(L^{1/\nu}(\beta_c-\beta)). \tag{54}$$

Equation (54) allows a determination of ν and β_c, together with a determination of the exponent δ by which $\langle\mu\rangle$ tends to zero at β_c in the infinite volume limit.

The quality of the scaling is shown in Fig. 4. Points corresponding to different lattice sizes follow the same universal curve only for the appropriate values of β_c and ν, Eq. (55). If β_c or ν are changed by one standard deviation from the values of Eq. (55), points from different lattices start splitting apart from each other. A best square fit gives

$$\beta_c=1.011\,60(5), \tag{55}$$

$$\nu=0.29(2).$$

The value (55) of β_c is consistent with a determination based on completely different methods [16]. As for ν it is consistent within two standard deviations with the value expected for a first order phase transition (0.25). It is also compatible with the determination of [17]. If $\mu\to(\beta_c-\beta)^{\delta}$,

$$\frac{\rho}{L^{1/\nu}}\simeq-\frac{\delta}{L^{1/\nu}(\beta_c-\beta)}. \tag{56}$$

An estimate for δ from the behavior in Fig. 4 is

$$\delta=1.1\pm0.2. \tag{57}$$

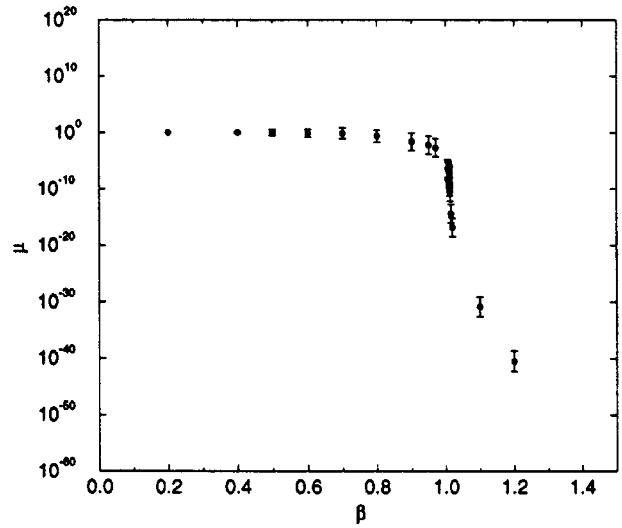

FIG. 5. $\langle\mu\rangle$ determined from measured ρ (lattice $10^3\times20$).

It is obtained by a best fit of Eq. (56) to the universal curve of Fig. 4 in a range of values of $3\leqslant(\beta_c-\beta)L^{1/\nu}\leqslant10$ where scaling is expected to hold, and $\xi\leqslant L$.

In the region $\beta\to\infty$ ρ can be computed in the weak coupling approximation [13]. The result is, for lattices $L^3\times2L$,

$$\rho=-5.05L+4.771+O(1/L), \tag{58}$$

giving $\rho\to-\infty$ or $\langle\mu\rangle=0$ in the infinite volume limit, in agreement with general arguments [5]: Only as $V\to\infty$ does the disorder parameter vanish in the disordered phase, if boundary conditions are not free. The leading term of Eq. (56) accounts for numerical data already at rather low values of β. At $\beta=1.2$ and $L=6$ it gives $\rho=-51.06$, to be compared with the numerical value -52.85 ± 0.43, and at $\beta=1.8$ and $L=8$ it gives $\rho=-71.26$, to be compared with the numerical value -77.22 ± 0.80. The plot of $\langle\mu\rangle$ computed from the measured ρ by use of the Eq. (49) is shown in Fig. 5. The vertical scale is logarithmic and errors are quite large. The mass of the monopole in Eq. (45) should scale properly in the limit $\beta\to\beta_c$ but we have large errors and this behavior is not clearly visible (Fig. 6).

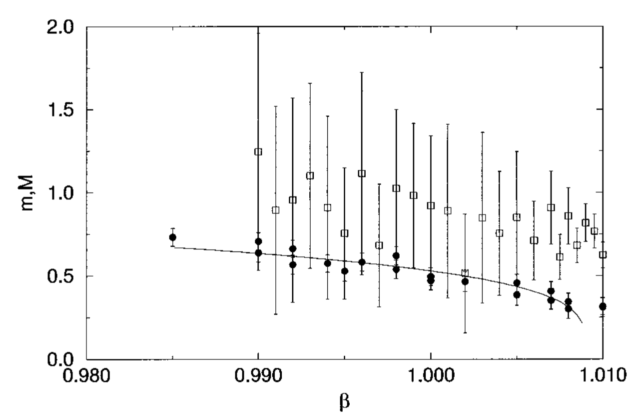

FIG. 6. Mass of the monopole M (squares) and mass of the dual photon m (circles) versus β.

In order to determine if the superconductor is of the first kind or second kind we have also measured the penetration depth $1/m_A$ of the electric field on the lines of Ref. [4].[1] A constant electric field parallel to the space boundary of the lattice is put on a face of the space lattice and its value is determined inside the bulk as a function of the distance from the boundary. An exponential behavior is found, with a penetration depth which properly scales by approaching the critical point, consistently with the effective critical index.

The corresponding mass is shown in Fig. 6 together with the mass extracted from the correlation length, Eq. (45). It appears clearly that $M \geqslant 2m_A$, indicating that the superconductor is of the second kind. This same problem has been approached by looking at the Abrikosov flux tubes generated by propagating charges. The idea is to compare the dependence of the electric field inside the tube on the transverse distance $x_\perp$ from the center of the tube, with what is expected from London equations. Their result is that the system seems to be at the border between first and second kinds [18]. The method is ingenuous. However, derivatives are approximated by finite differences, the penetration depth being a few lattice spacings (2–3), and this can produce systematic errors. Our method would give a more precise determination if we were able to determine better the mass M of Eq. (45). The question deserves further study.

Finally we want to comment on the possibility of determining numerically $\langle\mu\rangle$ directly and not through the measurement of ρ. As we have seen this is not strictly necessary, since ρ gives complete information about the phase transition. However, the problem has some interest by itself.

The definition of $\langle\mu\rangle$ is $\langle e^{\beta S'}\rangle$, the average being performed with the weight $\mathcal{D}\theta e^{\beta S}/Z$. S' is itself a random variable in this ensemble which has some average value $\langle S'\rangle$ with a width $\sigma=\sqrt{\langle S'^2\rangle-\langle S'\rangle^2}$.

A general theorem of probability theory states that if a random variable is distributed with a probability law $p(x)$, with $\int p(x)dx=1$, then its average $x_n=(1/n)\Sigma_k x_k$ is distributed as a Gaussian for large n if and only if [19]

$$\lim_{X\to\infty}\frac{X^2\int_{|x|>X}p(x)dx}{\int_{|x|<X}x^2p(x)dx}=0. \tag{59}$$

If Eq. (59) holds, then

$$\langle x_n\rangle \underset{n\to\infty}{\longrightarrow} \langle x\rangle=\int x\ p(x)dx \tag{60}$$

and the width of the distribution is, in this limit,

$$\sigma_n=\frac{\sigma}{\sqrt{n}},$$

with

$$\sigma^2=\langle x^2\rangle-\langle x\rangle^2.$$

[1]In Ref. [4] the field was called magnetic.

If we denote by y the variable $\beta S'-\langle\beta S'\rangle$ and by $\pi(y)$ its probability distribution, then the variable μ,

$$\mu=\exp(\beta S')=\overline{\mu}\exp(y)\quad[\overline{\mu}=\exp(\langle\beta S'\rangle)],$$

will be distributed as

$$p(\mu)=\pi\left[\ln\left(\frac{\mu}{\overline{\mu}}\right)\right]d\ln\left(\frac{\mu}{\overline{\mu}}\right). \tag{61}$$

If π decreases as $\exp(-y^2/2\sigma_y^2)$ as $y\to\infty$, then the probability distribution (61) obeys the hypothesis (59) of the theorem of central limit. In fact a much slower decrease would be enough.

If, for the sake of the argument, we assume that $\pi(y)$ is Gaussian, then we easily compute, by use of Eq. (61),

$$\langle\mu\rangle=\overline{\mu}\exp\left(\frac{\sigma_y^2}{2}\right),$$

$$\sigma_\mu=\overline{\mu}\exp(\sigma_y^2). \tag{62}$$

Equations (62) show why a direct determination of $\langle\mu\rangle$ is affected by wild fluctuations: The width is indeed bigger than the value of $\langle\mu\rangle$ itself. The exponential dependence on S' strongly distorts the distribution when going from S' to μ.

The histogram of the values of μ is related to the constrained potential by the relation [10,20]

$$\exp[-V(\Phi)]=\int[\mathcal{D}\theta]\exp(\beta S)\delta(\mu-\Phi).$$

$V(\Phi)$ has a minimum at $\langle\beta S'\rangle+\sigma_y^2/2$. If instead we construct the histogram of $\beta S'$ itself, the minimum will appear at $\langle\beta S'\rangle$ which is displaced by $\sigma_y^2/2$ with respect to the real minimum.

The problem is that the histogram in μ is exponentially large to fill adequately, since μ fluctuates on an exponential scale (typical values of μ on a configuration for a reasonable lattice size range from 10^{150} to 0). A histogram of $\ln\mu$, i.e., of $\beta S'$, is easier to compute.

However, to go back to the distribution in μ, i.e., to compute $\langle\mu\rangle$ and σ_μ, we must know the distribution $\pi(y)$ with great precision. In the Gaussian approximation the solution is given by Eq. (62). A cluster expansion can be attempted, to evaluate non-Gaussian effects, but the problem is only shifted. Higher cumulants of $\pi(y)$ are more and more noisy to determine numerically, and the computer time needed becomes comparable to the one needed for the direct determination of $\langle\mu\rangle$. Finally a finite size scaling analysis would be needed, analogous to what we did in Sec. IV.

This is to justify why we used ρ to extract information on the phase transition, instead of $\langle\mu\rangle$ itself or of its effective potential. The problem is currently under further study.

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