Global monopoles in Brans-Dicke theory of gravity

A. Barros
Departamento de Física, Universidade Federal de Roraima, 69310-270, Boa Vista, RR, Brazil

C. Romero*
Departamento de Física, Universidade Federal da Paraíba, Caixa Postal 5008, 58059-970, João Pessoa, PB, Brazil

(Received 3 July 1997)

The gravitational field of a global monopole in the context of Brans-Dicke theory of gravity is investigated. The space-time and the scalar field generated by the monopole are obtained by solving the field equations in the weak field approximation. A comparison is made with the corresponding results predicted by general relativity. [S0556-2821(97)03122-6]

PACS number(s): 04.50.+h

Monopoles resulting from the breaking of global O(3) symmetry lie among those strange and exotic objects such as cosmic strings and domain walls [1], generally referred to as topological defects of space-time, which may have existed due to phase transitions in the early universe. Likewise for cosmic strings, the most studied of these structures, the gravitational field of a monopole exhibits some interesting properties, particularly those concerning the appearance of nontrivial space-time topologies.

The solutions corresponding to the metrics generated by strings [2], domain walls [2], and global monopoles [3] in the context of general relativity were all first obtained using the weak field approximation.

In a similar approach, the gravitational fields of cosmic strings and domain walls have been obtained regarding Brans-Dicke theory of gravity and more general scalar-tensor theories of gravity [4,5].

In this paper we consider the global monopole and investigate its gravitational field by working out Brans-Dicke equations using once more the weak field approximation, essentially in the same way as in the previous works mentioned above.

Let us consider Brans-Dicke field equations in the form

$$R_{\mu\nu}=\frac{8\pi}{\phi}\left[T_{\mu\nu}-\frac{g_{\mu\nu}}{2}\left(\frac{2\omega+2}{2\omega+3}\right)T\right]+\frac{\omega}{\phi^2}\phi_{,\mu}\phi_{,\nu}+\frac{1}{\phi}\phi_{;\mu;\nu}, \tag{1}$$

$$\Box\phi=\frac{8\pi T}{2\omega+3}, \tag{2}$$

where ϕ is the scalar field, ω is a dimensionless coupling constant, and T denotes the trace of T^{μ}_{ν}— the energy-momentum tensor of the matter fields.

The energy-momentum tensor of a static global monopole can be approximated (outside the core) as [3]

$$T^{\mu}_{\nu}=\text{diag}\left(\frac{\eta^2}{r^2},\frac{\eta^2}{r^2},0,0\right), \tag{3}$$

where η is the energy scale of the symmetry breaking.

As a result of spherical symmetry, we consider $\phi=\phi(r)$ and the line element

$$ds^2=B(r)dt^2-A(r)dr^2-r^2(d\theta^2+\sin^2\theta d\varphi^2). \tag{4}$$

Substituting this into Eq. (1) and Eq. (2), and taking in account Eq. (3) we obtain the following set of equations:

$$\frac{B''}{2A}-\frac{B'}{4A}\left(\frac{A'}{A}+\frac{B'}{B}\right)+\frac{1}{r}\frac{B'}{A}=\frac{8\pi}{\phi}\left[\frac{\eta^2B}{r^2(2\omega+3)}\right]-\frac{B'\phi'}{2A\phi}, \tag{5}$$

$$-\frac{B''}{2B}+\frac{B'}{4B}\left(\frac{A'}{A}+\frac{B'}{B}\right)+\frac{1}{r}\frac{A'}{A}=-\frac{8\pi}{\phi}\left[\frac{\eta^2A}{r^2(2\omega+3)}\right]+\frac{\omega\phi'^2}{\phi^2}+\frac{1}{\phi}\left[\phi''-\frac{A'}{2A}\phi'\right], \tag{6}$$

$$\phi''+\frac{1}{2}\phi'\left[\frac{B'}{B}-\frac{A'}{A}+\frac{4}{r}\right]=-\frac{16\pi}{(2\omega+3)}\left(\frac{\eta^2}{r^2}\right)A, \tag{7}$$

$$1-\frac{r}{2A}\left(\frac{B'}{B}-\frac{A'}{A}\right)-\frac{1}{A}=\frac{8\pi}{\phi}\left[\eta^2\left(\frac{2\omega+2}{2\omega+3}\right)\right]+\frac{r\phi'}{A\phi}, \tag{8}$$

where a prime denotes differentiation with respect to r.

Now, dividing Eqs. (5) and (6) by B and A, respectively, and adding we get

$$\frac{\alpha}{r}=\frac{\omega\phi'^2}{\phi^2}+\frac{\phi''}{\phi}-\frac{\phi'}{2\phi}\alpha, \tag{9}$$

where we have put

$$\alpha=\frac{A'}{A}+\frac{B'}{B}. \tag{10}$$

Then, Eqs. (7) and (8) read

$$\phi''+\frac{\phi'}{2}\left[\alpha-\frac{2A'}{A}+\frac{4}{r}\right]=-\frac{16\pi}{2\omega+3}\left(\frac{\eta^2}{r^2}\right)A, \tag{11}$$

*Electronic address: cromero@dfjp.ufpb.br

$$1-\frac{r}{2A}\left(\alpha-\frac{2A'}{A}\right)-\frac{1}{A}=\frac{8\pi}{\phi}\left[\eta^2\left(\frac{2\omega+2}{2\omega+3}\right)\right]+\frac{r}{A}\frac{\phi'}{\phi}. \tag{12}$$

At this stage, let us consider the weak field approximation and assume that $A(r)=1+f(r)$, $B(r)=1+g(r)$, and $\phi(r)=\phi_0+\epsilon(r)$, where ϕ_0 is a constant which may be identified with G^{-1} when $\omega\rightarrow\infty$ (G being the Newtonian gravitational constant), and the functions f, g, and ϵ/ϕ_0 should be computed to first order in η^2/ϕ_0, with $|f(r)|$, $|g(r)|$, and $|\epsilon(r)/\phi_0|\ll 1$.

In this approximation it is easy to see that

$$\frac{\phi'}{\phi}=\frac{\epsilon'}{\phi_0[1+\epsilon/\phi_0]}=\frac{\epsilon'}{\phi_0},\quad \frac{\phi''}{\phi}=\frac{\epsilon''}{\phi_0[1+\epsilon/\phi_0]}=\frac{\epsilon''}{\phi_0},$$

$$\frac{B'}{B}=\frac{g'}{1+g}=g',\quad \frac{A'}{A}=\frac{f'}{1+f}=f',$$

and so on.

From Eq. (9) it follows that

$$\frac{\alpha}{r}=\frac{\epsilon''}{\phi_0}. \tag{13}$$

And from Eq. (11) we have

$$\epsilon''+\frac{2\epsilon'}{r}=-\frac{16\pi}{(2\omega+3)}\frac{\eta^2}{r^2}, \tag{14}$$

the solution of which is given by

$$\epsilon=-\frac{16\pi}{2\omega+3}\eta^2\ln\frac{r}{r_0}-\frac{\kappa}{r}, \tag{15}$$

r_0 and κ being integration constants.

On the other hand, considering Eqs. (13) and (15), Eq.(12) becomes

$$f'+\frac{f}{r}=\frac{16\pi\eta^2}{\phi_0(2\omega+3)r}\left(\omega+\frac{1}{2}\right), \tag{16}$$

which yields the solution

$$f=\frac{8\pi\eta^2(2\omega+1)}{\phi_0(2\omega+3)}+\frac{l}{r}, \tag{17}$$

where l is an arbitrary constant.

Therefore,

$$A=1+f=1+\frac{8\pi\eta^2(2\omega+1)}{\phi_0(2\omega+3)}+\frac{l}{r} \tag{18}$$

and

$$A^{-1}=1-\frac{8\pi\eta^2(2\omega+1)}{\phi_0(2\omega+3)}-\frac{l}{r}. \tag{19}$$

It is currently known that solutions of Brans-Dicke field equations do not always go over general relativity solutions when $\omega\rightarrow\infty$ [6]. However, as the term $\omega\phi_{,\mu}\phi_{,\nu}/\phi^2$ in Eq. (1) is neglected in the weak field approximation we expect that in the limit $\omega\rightarrow\infty$ our solution reduces to Barriola-Vilenkin space-time, which is given by [3]

$$ds^2=\left(1-8\pi G\eta^2-\frac{2GM}{r}\right)dt^2-\left(1-8\pi G\eta^2-\frac{2GM}{r}\right)^{-1}dr^2-r^2(d\theta^2+\sin^2\theta d\varphi^2). \tag{20}$$

Then, we should have

$$\lim_{\omega\rightarrow\infty} l=2GM, \tag{21}$$

where M is the mass of the monopole core. Indeed, if we take $\eta=0$ in a region outside the monopole core, then a simple comparison of the r-dependent term in Eq. (18) with the corresponding term of Brans-Dicke solution for a spherically symmetric matter distribution in the weak field approximation [7], which may be written as

$$ds^2=\left[1-\frac{2M}{r\phi_0}\left(1+\frac{1}{2\omega+3}\right)\right]dt^2-\left[1+\frac{2M}{r\phi_0}\times\left(1-\frac{1}{2\omega+3}\right)\right]dr^2-r^2(d\theta^2+\sin^2\theta d\varphi^2), \tag{22}$$

gives

$$l=\frac{2M}{\phi_0}\left[1-\frac{1}{2\omega+3}\right].$$

The same argument concerning the scalar field leads us to $\kappa=-2M/(2\omega+3)$. Thus, we have

$$A=1+\frac{8\pi\eta^2}{\phi_0}\left(\frac{2\omega+1}{2\omega+3}\right)+\frac{2M}{r\phi_0}\left(1-\frac{1}{2\omega+3}\right), \tag{23}$$

$$\phi=\phi_0-\frac{16\pi\eta^2}{2\omega+3}\ln\frac{r}{r_0}+\frac{2M}{(2\omega+3)r}. \tag{24}$$

From Eq. (10) it is straightforward to verify that

$$B=\frac{a}{A}\left[1-\frac{4M}{r\phi_0(2\omega+3)}+\frac{16\pi\eta^2}{\phi_0(2\omega+3)}\ln\frac{r}{r_0}\right], \tag{25}$$

where a is an integration constant. For convenience let us rescale the time by putting $a=1-16\pi\eta^2/\phi_0(2\omega+3)$. Then,

$$B=\frac{1}{A}\left[1-\frac{4M}{r\phi_0(2\omega+3)}+\frac{16\pi\eta^2}{\phi_0(2\omega+3)}\ln\frac{r}{r_0}\right]\times\left[1-\frac{16\pi\eta^2}{\phi_0(2\omega+3)}\right]. \tag{26}$$

Taking into account Eq. (23) we obtain

$$B=1-\frac{8\pi\eta^2}{\phi_0}+\frac{16\pi\eta^2}{\phi_0(2\omega+3)}\ln\frac{r}{r_0}-\frac{2M}{r\phi_0}\left[1+\frac{1}{2\omega+3}\right]. \tag{27}$$

Following Barriola-Vilenkin's reasoning we drop the mass term in Eqs. (23), (24), and (27) as it is negligible on the astrophysical scale. Thus, we have finally

$$A(r)=1+\frac{8\pi\eta^2(2\omega+1)}{\phi_0(2\omega+3)}, \tag{28}$$

$$B(r)=1-\frac{8\pi\eta^2}{\phi_0}+\frac{16\pi\eta^2}{\phi_0(2\omega+3)}\ln\frac{r}{r_0}, \tag{29}$$

$$\phi(r)=\phi_0-\frac{16\pi\eta^2}{2\omega+3}\ln\frac{r}{r_0}. \tag{30}$$

It is not difficult to show that the line element defined by the functions $A(r)$ and $B(r)$ above is conformally related to the Barriola-Vilenkin monopole solution. To do so, let us consider the coordinate transformation given by the equations

$$B(r)=h(r^*)\left(1-\frac{8\pi\eta^2}{\phi_0}\right), \tag{31}$$

$$A(r)dr^2=h(r^*)\left(1+\frac{8\pi\eta^2}{\phi_0}\right)dr^{*2}, \tag{32}$$

$$r=h^{1/2}(r^*)r^*, \tag{33}$$

where $h(r^*)$ is to be calculated and $h(r^*)=1+q(r^*)$, with $|q(r^*)|\ll 1$.

Differentiating Eq. (33) we obtain

$$dr^2=(1+\dot{q}r^*+q)dr^{*2}, \tag{34}$$

where the overdot stands for a derivative with respect to r^*.

Substituting Eq. (34) into Eq. (32) one gets

$$q(r^*)=\frac{16\pi\eta^2}{\phi_0(2\omega+3)}\ln\frac{r^*}{r_0}, \tag{35}$$

whence

$$h(r^*)=1+\frac{16\pi\eta^2}{\phi_0(2\omega+3)}\ln\frac{r^*}{r_0}. \tag{36}$$

In order to verify the consistency of this result with Eq. (29) let us calculate $B(r)$ directly from Eqs. (31) and (36). Keeping only linear terms in η^2/ϕ_0 and using Eq. (33), we have, then,

$$\begin{aligned}B(r)&=\left(1+\frac{16\pi\eta^2}{\phi_0(2\omega+3)}\ln\frac{r^*}{r_0}\right)\left(1-\frac{8\pi\eta^2}{\phi_0}\right)\\&=1-\frac{8\pi\eta^2}{\phi_0}+\frac{16\pi\eta^2}{\phi_0(2\omega+3)}\ln\frac{r}{r_0}.\end{aligned} \tag{37}$$

Therefore, the line element (4) which represents the space-time generated by the monopole may be written in terms of the new coordinate r^* as

$$\begin{aligned}ds^2=&\left(1+\frac{16\pi\eta^2}{\phi_0(2\omega+3)}\ln\frac{r^*}{r_0}\right)\left[\left(1-\frac{8\pi\eta^2}{\phi_0}\right)dt^2\right.\\&\left.-\left(1+\frac{8\pi\eta^2}{\phi_0}\right)dr^{*2}-r^{*2}(d\theta^2+\sin^2\theta d\varphi^2)\right].\end{aligned} \tag{38}$$

Rescaling the time and defining a new radial coordinate $r=(1+4\pi\eta^2/\phi_0)r^*$ we end up with

$$\begin{aligned}ds^2=&\left(1+\frac{16\pi\eta^2}{\phi_0(2\omega+3)}\ln\frac{r}{r_0}\right)\left[dt^2-dr^2-\left(1-\frac{8\pi\eta^2}{\phi_0}\right)\right.\\&\left.\times r^2(d\theta^2+\sin^2\theta d\varphi^2)\right].\end{aligned} \tag{39}$$

In order to obtain the correct Newtonian limit from Brans-Dicke field equations the constant ϕ_0 must be given by [7]

$$\phi_0=\left(\frac{2\omega+4}{2\omega+3}\right)\frac{1}{G}.$$

Then, the final form of Eq. (39) reads

$$\begin{aligned}ds^2=&\left(1+\frac{16\pi\eta^2 G}{(2\omega+4)}\ln\frac{r}{r_0}\right)\left\{dt^2-dr^2\right.\\&\left.-\left[1-8\pi\eta^2 G\left(\frac{2\omega+3}{2\omega+4}\right)\right]r^2(d\theta^2+\sin^2\theta d\varphi^2)\right\}.\end{aligned} \tag{40}$$

Thus, we have shown that in the weak field approximation equation (40) represents the space-time generated by a global monopole in Brans-Dicke theory of gravity. Analogously to the general relativity case this curved space-time also presents a deficit solid angle in the hypersurfaces $t=\text{const}$. The area of a sphere of radius r in these spaces would be given by

$$4\pi r^2\left[1-8\pi\eta^2 G\left(\frac{2\omega+3}{2\omega+4}\right)+\frac{16\pi\eta^2 G}{(2\omega+4)}\ln\frac{r}{r_0}\right],$$

rather than $4\pi r^2$.

Also, a simple comparison of Eq. (40) with the Barriola-Vilenkin solution shows that for large values of ω both space-times are related by a conformal transformation. In this case the motion of light rays is the same in the two space-times. For finite values of ω, null geodesics in the space-time of Brans-Dicke global monopole are still closely related to their counterpart in general relativity. Indeed, the only change predicted by Brans-Dicke theory reduces, in this case, to the replacement of the Newtonian gravitational constant G by the ω-dependent "effective" gravitational constant

$$G_0=\left(\frac{2\omega+3}{2\omega+4}\right)G.$$

For a value of ω consistent with solar system observations, say, $\omega\sim 500$ [8], it would mean that massless particles traveling in the space-time described by Eq. (40) would experience a gravitational strength $G_0 \sim 0.999G$.

In conclusion we see that in going from general relativity to Brans-Dicke theory both space-time curvature and topology are affected by the presence of the scalar field. In particular the deficit solid angle becomes ω dependent. As a consequence, following Barriola and Vilenkin's argument concerning light propagation in the gravitational field of a global monopole one can easily show that a light signal propagating from a source S to an observer O when S,O and the monopole are perfectly aligned produces an image with the form of a ring of angular diameter given by

$$\delta\Omega = 8\pi^2\eta^2\left(\frac{2\omega+3}{2\omega+4}\right)G\,\frac{l}{l+d},$$

where d and l are the distances from the monopole to the observer and to the source, respectively.

Another interesting physical property in connection with Brans and Dicke's global monopole involves the appearance of gravitational forces exerted by the monopole on the matter around it. This effect is absent in the case of general relativity's monopole as was shown in Ref. [3]. To see how this gravitational effect comes about one has to work out the Newtonian potential associated with Eq. (40). As is well known in Galilean coordinates the motion of a nonrelativistic test particle in a weak gravitational field is given by the equation [9]

$$\ddot{x}^i = -\frac{1}{2}\frac{\partial h_{00}}{\partial x^i}, \tag{41}$$

where $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and $\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ is the Minkowski metric tensor. In order to express Eq. (40) in Galilean coordinates let us consider the transformation

$$t = \left[1 - 4\pi\eta^2 G\left(\frac{2\omega+3}{2\omega+4}\right)\right]T,$$

$$r = \left[1 + 4\pi\eta^2 G\left(\frac{2\omega+3}{2\omega+4}\right) - 4\pi\eta^2 G\left(\frac{2\omega+3}{2\omega+4}\right)\ln\frac{R}{R_0}\right]R,$$

with

$$R_0 = \left[1 - 4\pi\eta^2 G\left(\frac{2\omega+3}{2\omega+4}\right)\right]r_0 .$$

Then, we have

$$ds^2 = \left[1 - 8\pi\eta^2 G\left(\frac{2\omega+3}{2\omega+4}\right) + \frac{16\pi\eta^2 G}{2\omega+4}\ln\frac{R}{R_0}\right]dT^2 - \left[1 - 8\pi\eta^2 G\left(\frac{2\omega+1}{2\omega+4}\right)\ln\frac{R}{R_0}\right](dx^2+dy^2+dz^2), \tag{42}$$

with $R = [x^2+y^2+z^2]^{1/2}$. Thus, Eq. (41) becomes, finally

$$\ddot{x}^i = -\frac{4\pi\eta^2 G}{(\omega+2)}\frac{x^i}{R^2}, \tag{43}$$

which shows explicitly that particles around the monopole are subject to an attractive force exerted by it.

Naturally, if it turns out to be that global monopoles possess any kind of physical reality, then a number of other effects such as quantum particle creation [10], vacuum polarization [11], and gravitational scattering [12], among others, which would be in principle amenable to observation may be investigated with the help of Eq. (40), thereby providing alternative ways for testing the predictable power of both general relativity and Brans-Dicke theory.

C.R. was partially supported by CNPq (Brazil).

[1] A. Vilenkin and E. P. Shellard, *Cosmic Strings and Other Topological Defects* (Cambridge University Press, Cambridge, England, 1994).

[2] A. Vilenkin, Phys. Rev. D **23**, 852 (1981).

[3] M. Barriola and A. Vilenkin, Phys. Rev. Lett. **63**, 341 (1989).

[4] A. Barros and C. Romero, J. Math. Phys. **36**, 5800 (1995).

[5] M. E. X. Guimarães, Class. Quantum Grav. **14**, 435 (1997).

[6] C. Romero and A. Barros, Phys. Lett. A **173**, 243 (1993); M. A. Scheel, S. L. Shapiro, and S. A. Teukolsky, Phys. Rev. D **51**, 4236 (1995).

[7] C. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961).

[8] C. M. Will, *Theory and Experiment in Gravitational Physics*, revised ed. (Cambridge University Press, Cambridge, England, 1993).

[9] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Pergamon, Oxford, 1971).

[10] W. A. Hiscock, Phys. Lett. B **234**, 469 (1990).

[11] F. Mazzitelli and C. Lousto, Phys. Rev. D **43**, 468 (1991).

[12] P. O. Mazur and J. Papavassiliou, Phys. Rev. D **44**, 1317 (1991).