How neutrino oscillations can induce an effective neutrino number of less than three during big bang nucleosynthesis

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Ordinary-sterile neutrino oscillations can generate significant neutrino asymmetry in the early Universe. In this paper we extend this work by computing the evolution of neutrino asymmetries and light element abundances during the big bang nucleosynthesis (BBN) epoch. We show that a significant electron-neutrino asymmetry can be generated in a way that is approximately independent of the oscillation parameters δm^2 and $\sin^2 2\theta$ for a range of parameters in an interesting class of models. The numerical value of the asymmetry leads to the *prediction* that the effective number of neutrino flavors during BBN is either about 2.5 or 3.4, depending on the sign of the asymmetry. Interestingly, one class of primordial deuterium abundance data favors an effective number of neutrino flavors during the epoch of BBN of less than 3. [S0556-2821(97)03822-8]

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I. INTRODUCTION

The possible existence of sterile neutrinos can be motivated by the solar neutrino, atmospheric neutrino, and LSND experiments [1]. There are also interesting theoretical motivations for the existence of light sterile neutrinos. For example, if nature respects an exact unbroken parity symmetry, then three necessarily light mirror neutrinos must exist [2]. In view of this, it is interesting to study the implications of ordinary-sterile neutrino oscillations for both particle physics and cosmology. The effects of ordinary-sterile neutrino oscillations in the early Universe are actually quite remarkable. It turns out that for a wide range of parameters, ordinary-sterile neutrino oscillations generate a large neutrino asymmetry [3] (see also [4]). (A large neutrino asymmetry implies that the Universe has a net nonzero lepton number given that the electron asymmetry is necessarily small due to the charge neutrality requirement.) One important implication of this result is that the bounds on ordinary-sterile oscillation parameters that can be derived mainly from energy density considerations during big bang nucleosynthesis (BBN) are severely affected (see Ref. [5] for a detailed analysis). However, electron lepton number can also affect BBN directly through the modification of nuclear reaction rates. It is this issue that we will study in this paper.

In a previous paper [5], we showed that for a wide range of parameters, the evolution of the lepton number can be approximately described by a relatively simple first-order integro-differential equation. We called the approximation used there the ''static approximation'' because it holds provided that the system is sufficiently smooth. The static approximation is valid in the region where the evolution of the lepton number is dominated by collisions. In particular, for the temperature at which the lepton number is initially produced, this approximation is generally valid for $|\delta m^2| \gtrsim 10^{-2}$ eV^2 [5]. However, it is not expected to be valid for temperatures much less than the temperature at which the lepton number is initially generated. This is because the static approximation discussed in Ref. [5] does not incorporate the Mikheyev-Smirnov-Wolfenstein (MSW) effect [6], which is in fact the dominant process affecting the evolution of the lepton number at low temperatures. For the application considered in Ref. [5], the evolution of the lepton number at low temperatures was not required. However, for the application of the present paper the accurate evolution of the lepton number to temperatures $T \sim 1$ MeV is necessary in order to study its precise effect on BBN reaction rates.

The outline of this paper is as follows. In Sec. II we set the scene with a brief review of the effects of neutrino asymmetry on BBN. In Sec. III we develop a simple formalism describing the evolution of the lepton number at low temperatures where the MSW effect is important. This work can also be viewed as an extension of our previous study [5], where the evolution of the lepton number at higher temperature was studied in detail. In this section we also examine the implications for BBN of direct electron asymmetry generation by ν_e-ν_s oscillations. In Sec. IV we examine a more interesting scenario where electron asymmetry is generated indirectly. In Sec. V we provide a check of our simple formalism (of Sec. III) by numerically solving the exact quantum kinetic equations. Finally, in Sec. VI we conclude.

II. ELECTRON-NEUTRINO ASYMMETRY AND BBN

Standard BBN can give a prediction for the effective number of neutrino flavors, $N_\nu^{\rm eff}$ present during nucleosynthesis. This prediction depends on the baryon to photon number-density ratio η and the primordial helium mass fraction Y_P. A precise determination of the primordial deuterium abundance will provide a quite sensitive measurement of η. Once η is known, the effective number of neutrinos present during nucleosynthesis depends only on Y_P. At present there are two conflicting deuterium observations in different high-redshift low-metallicity quasistellar object absorbers. There is the high-deuterium result of Ref. [7], which suggests that $D/H = (1.9 \pm 0.4) \times 10^{-4}$. On the other hand, in Ref. [8] the low-deuterium result of $D/H = [2.3 \pm 0.3(\text{stat}) \pm 0.3(\text{syst})] \times 10^{-5}$ is obtained. The implications of these results for the prediction of $N_\nu^{\rm eff}$ have been discussed in a number of recent papers [9]. Depending on which of these two values of the deuterium abundance is assumed, different predictions for η are obtained. The high-deuterium result leads to $\eta \sim 2\times 10^{-10}$, while the low deuterium result leads to $\eta \sim 7\times 10^{-10}$ [9]. Each of these predictions for η, together with the inferred primordial abundance of ^{4}He, allows a prediction for $N_\nu^{\rm eff}$ to be made [9]. According to Ref. [10], for example, the high-deuterium case leads to

$$N_\nu^{\rm eff}=2.9\pm 0.3, \tag{1}$$

while the low-deuterium case leads to

$$N_\nu^{\rm eff}=1.9\pm 0.3, \tag{2}$$

where the errors are at 68% C.L. The minimal standard model of course predicts that $N_\nu^{\rm eff}=3$. Thus, if the low-deuterium result were correct, then new physics would presumably be required [11]. Of course, estimating the primordial element abundances is difficult and it is possible that the primordial helium abundance has been underestimated (in other words, even if the low-deuterium measurement is correct $N_\nu^{\rm eff}=3$ is not inconsistent). Fortunately, the situation is continually improving as more observations and analyses are done. In the interim it is useful to identify and study the types of particle physics that can lead to $N_\nu^{\rm eff}<3$.

One possibility is that the electron lepton number is large enough to significantly affect BBN (i.e., $L_{\nu_e}\lesssim 0.01$) [12]. The relationship between an electron-neutrino asymmetry and the effective number of neutrino species arises as follows. A nonzero electron-neutrino asymmetry modifies the nucleon reaction rates ($n+\nu_e \leftrightarrow p+e^-$ and $n+e^+\leftrightarrow p+\overline{\nu_e}$), which keep the neutrons and protons in thermal equilibrium down to temperatures of about 0.7 MeV. A modification of these rates affects the ratio of neutrons to protons and hence changes the prediction of Y_P [12]. A change of Y_P can be equivalently expressed as a change in the effective number of neutrino species $\delta N_\nu^{\rm eff}$ present during nucleosynthesis. These quantities are related by the equation (see, e.g., Ref. [13])

$$\delta Y_P \simeq 0.012\,\delta N_\nu^{\rm eff}. \tag{3}$$

The effect of the electron-neutrino asymmetry on the primordial helium abundance is most important in the temperature region

$$0.4\ \text{MeV} \lesssim T \lesssim 1.5\ \text{MeV}, \tag{4}$$

where the reactions $n+\nu_e\leftrightarrow p+e^-$ and $n+e^+\leftrightarrow p+\overline{\nu_e}$ fix the neutron to proton ratio. For temperatures less than about 0.4 MeV, these reaction rates become so slow that the dominant process affecting the neutron to proton ratio is neutron decay. Note that the helium mass fraction Y_P satisfies the differential equation [14],

$$\frac{dY_P}{dt}=-\lambda(n\to p)Y_P+\lambda(p\to n)(2-Y_P), \tag{5}$$

where $\lambda(n\to p)$ [$\lambda(p\to n)$] is the rate at which neutrons are converted into protons [protons are converted to neutrons]. For temperatures in the range of Eq. (4), $\lambda(n\to p)\simeq \lambda(n+\nu_e\to p+e^-)+\lambda(n+e^+\to p+\overline{\nu_e})$ and $\lambda(p\to n)\simeq \lambda(p+\overline{\nu_e}\to n+e^+)+\lambda(p+e^-\to n+\nu_e)$. The reaction rates (per nucleon) are obtained by integrating the square of the matrix element weighted by the available phase space. For example, the rate for the process $n+\nu_e\to p+e^-$ is given by

$$\lambda(n+\nu_e\to p+e^-) = \int f_\nu(E_\nu)[1-f_e(E_e)]|\mathcal{M}|^2_{n\nu_e\to pe}(2\pi)^{-5} \times \delta^4(n+\nu-p-e)\frac{d^3p_\nu}{2E_\nu}\frac{d^3p_e}{2E_e}\frac{d^3p_p}{2E_p}, \tag{6}$$

where $f_i(E_i)$ is the Fermi-Dirac distribution $f_i(E_i)\equiv[\exp(E_i/T)+1]^{-1}$. These reaction rates are modified in the presence of significant electron-neutrino asymmetry. If the neutrino asymmetry is produced at temperatures above about 1.5 MeV and is constant over the temperature range of Eq. (4), then we only need to add in the appropriate chemical potentials μ_ν and $\mu_{\bar\nu}$ to the distributions f_ν and $f_{\bar\nu}$.

III. NEUTRINO OSCILLATION GENERATED NEUTRINO ASYMMETRY

We now discuss the effects of neutrino oscillations, assuming that a sterile neutrino exists. Our convention for the neutrino oscillation parameters $\delta m^2_{\alpha s}$ and $\sin^2 2\theta_0$ is as follows. For ν_α-ν_s oscillations (with $\alpha=e,\mu,\tau$), the weak eigenstates ν_α and ν_s are linear combinations of mass eigenstates ν_a and ν_b:

$$\nu_\alpha=\cos\theta_0\nu_a+\sin\theta_0\nu_b,\ \nu_s=-\sin\theta_0\nu_a+\cos\theta_0\nu_b, \tag{7}$$

where the vacuum mixing angle θ_0 is defined so that $\cos 2\theta_0\geqslant 0$. Further, we define the oscillation parameter $\delta m^2_{\alpha s}$ by $\delta m^2_{\alpha s}\equiv m_b^2-m_a^2$. Also, the term ''neutrino'' will sometimes include antineutrino as well. We hope that the correct meaning will be clear from context.

Ordinary-sterile neutrino oscillations can create a significant lepton number provided that $\delta m^2_{\alpha s}<0$ and $|\delta m^2_{\alpha s}|\gtrsim 10^{-4}$ eV^2. For full details, see Refs. [3]–[5]. In the following we consider ν_α-ν_s oscillations in isolation. It is important to note that this is not in general valid because the effective potential (see below) depends on all of the lepton asymmetries. However, it is approximately valid for the ordinary-sterile neutrino oscillations that have the largest $|\delta m^2|$ [5].

The effective potential describing the coherent forward scattering of neutrinos of momentum $p\equiv|\vec{p}|\simeq E$ with the background is [15,6]

$$V_\alpha\equiv V_\alpha(T,p,L^{(\alpha)})=\frac{\delta m^2_{\alpha s}}{2p}(-a+b), \tag{8}$$

where the dimensionless functions a and b are given by

$$a\equiv a(T,p,L^{(\alpha)})=\frac{-4\zeta(3)\sqrt{2}G_F T^3 L^{(\alpha)} p}{\pi^2\delta m^2_{\alpha s}},$$

$$b\equiv b(T,p)=\frac{-4\zeta(3)\sqrt{2}G_F T^4 A_\alpha p^2}{\pi^2\delta m^2_{\alpha s}M_W^2}, \tag{9}$$

and $\zeta(3)\simeq 1.202$ is the Riemann zeta function of 3, G_F is the Fermi constant, M_W is the W-boson mass, $A_e\simeq 17$, and $A_{\mu,\tau}\simeq 4.9$ [16]. The quantity $L^{(\alpha)}$ is given by

$$L^{(\alpha)}=L_{\nu_\alpha}+L_{\nu_e}+L_{\nu_\mu}+L_{\nu_\tau}+\eta, \tag{10}$$

where $L_{\nu_\alpha}\equiv(n_{\nu_\alpha}-n_{\bar{\nu}_\alpha})/n_\gamma$ with n_i being the number density of species i. In kinetic equilibrium n_i and hence $L^{(\alpha)}$ is in general a function of the independent variables μ_i (the chemical potential) and T. For the situation we will be considering, the asymmetry $L^{(\alpha)}$ quickly becomes independent of its initial value (see Ref. [5] for a complete discussion). This effectively means that μ_i is not an independent variable but rather becomes a function of T. The asymmetry $L^{(\alpha)}$ is thus essentially a function of T only and a and b are functions of p and T only. The quantity $\eta\simeq L_N/2$ is a small term ($\sim 10^{-10}$) that arises from the asymmetries of baryons and electrons. The matter mixing angles are expressed in terms of the quantities a and b through [6]

$$\sin^2 2\theta_m\equiv\sin^2 2\theta_m(T,p,L^{(\alpha)})=\frac{\sin^2 2\theta_0}{\sin^2 2\theta_0+(b-a-\cos 2\theta_0)^2},$$

$$\sin^2 2\overline{\theta_m}\equiv\sin^2 2\overline{\theta_m}(T,p,L^{(\alpha)})=\frac{\sin^2 2\theta_0}{\sin^2 2\theta_0+(b+a-\cos 2\theta_0)^2}. \tag{11}$$

Note that the MSW resonance occurs for neutrinos of momentum p when $\theta_m=\pi/4$ and for antineutrinos of momentum p when $\overline{\theta_m}=\pi/4$, which from Eq. (11) implies that $b-a=\cos 2\theta_0$ and $b+a=\cos 2\theta_0$, respectively.

If $\sin^2 2\theta_0\ll 1$, then it can be shown that oscillations with $b<1$ create a lepton number, while the oscillations with $b>1$ destroy a lepton number [5,3]. Since $\langle b\rangle\sim T^6$, it follows that at some point the lepton number creating oscillations dominates over the lepton number destroying oscillations (where the angular brackets denote the thermal momentum average). We will call this temperature T_c. It is given roughly by the temperature where $\langle b\rangle=\cos 2\theta_0\simeq 1$, that is,

$$T_c\simeq 13(16)\left(\frac{|\delta m^2_{\alpha s}|}{eV^2}\right)^{1/6}\ \text{MeV} \tag{12}$$

for ν_e-ν_s ($\nu_{\mu,\tau}$-ν_s) oscillations.

It is important to note that there are two distinct contributions to the rate of change of the lepton number. One contribution is from the oscillations between collisions. The other is from the collisions themselves, which deplete neutrinos and antineutrinos at different rates in a CP asymmetric background. It turns out that for $T\gtrsim T_c$, the lepton number evolution is dominated by collisions for the small vacuum mixing angle case we are considering assuming that $|\delta m^2_{\alpha s}|\gtrsim 10^{-4}$ eV^2 [3–5,17]. Oscillations between collisions, and in particular MSW transitions, can be ignored for $T\gtrsim T_c$ because the interactions are so rapid that the mean distance between collisions $L_{\rm int}$ is much smaller than the matter-oscillation length $L^m_{\rm osc}$ (and consequently the neutrino cannot evolve coherently through the resonance). To see this note that the amplitude of the oscillations at the MSW resonance is given roughly by $\sin^2 L_{\rm int}/2L^m_{\rm osc}$, where $L_{\rm int}\sim 1/G_F^2 p_{\rm res}T^4$ is the interaction length at the resonance and $L^m_{\rm osc}\sim 2p_{\rm res}/\sin 2\theta_0|\delta m^2_{\alpha s}|$ is the oscillation length at the resonance [5]. So, at the resonance,

$$\frac{L_{\rm int}}{L^m_{\rm osc}}\sim\frac{\sin 2\theta_0}{G_F^2 p_{\rm res}T^4}\frac{|\delta m^2_{\alpha s}|}{2p_{\rm res}}\simeq 10^2\sin 2\theta_0\left[\frac{T_c}{T}\right]^6, \tag{13}$$

where we have used the approximation $p_{\rm res}\simeq\langle p\rangle\simeq 3.15T$ for the resonance momentum. Thus $\sin^2 L_{\rm int}/2L^m_{\rm osc}\ll 1$ for $T\gtrsim T_c$, provided that $\sin^2 2\theta_0\lesssim 10^{-4}$, and so the MSW transitions are heavily suppressed. However, $L_{\rm int}/L^m_{\rm osc}\sim 1/T^6$ rapidly increases as T becomes lower. Also, for $T\lesssim T_c$ it turns out that $p_{\rm res}/T\lesssim 0.8$ (see later). Taking these factors into account, $L_{\rm int}/L^m_{\rm osc}\gtrsim 1$ for $T\lesssim T_c/2$, provided that $\sin^2 2\theta_0\gtrsim 3\times 10^{-10}$. In this case the MSW effect will not be suppressed if the oscillations are adiabatic. Furthermore, the oscillations are generally adiabatic for the parameter space of interest in this paper (which turns out to be $|\delta m^2_{\alpha s}|\gtrsim 10^{-1}$ eV^2).

The key task in this paper is to analyze the evolution of the lepton number for $T\lesssim T_c/2$ when MSW transitions become important. (The effect of MSW transitions was noted in Ref. [5], but since the evolution of the lepton number to low temperatures was not required for the application considered there, we did not study the effect of MSW transitions in any detail, except to note that they keep L_{ν_α} growing like $1/T^4$ for low temperatures). Now, through the quantum kinetic equations [18], the evolution of the lepton number can be calculated in a close to exact manner, including the effects of both collisions and oscillations between collisions. However, these complicated coupled equations have two notable drawbacks. First, they do not furnish as much physical insight as one might wish. Second, they are impractically complicated when one wishes to consider a system of more than two neutrino flavors. Since the physics of MSW transitions is the essence of how the lepton number evolves during the BBN epoch and since we will later need to consider a system of four neutrino flavors, we now pursue a very useful approximate approach instead of employing the full quantum kinetic equations. We will, along the way check the veracity of our approximate approach by comparing results with those obtained from the quantum kinetic equations in the two-flavor case (see Sec. V). This will give us confidence in the use of our appproximate formalism in the four-flavor case considered later in this paper.

For definiteness we will assume that the lepton number created at the temperature $T=T_c$ is positive in sign [19]. In this case note that $a,b>0$ given also that $\delta m^2_{\alpha s}<0$. The momentum of the antineutrino oscillation resonance (obtained from the condition $b+a=\cos 2\theta_0\simeq 1$) typically moves to quite low values $p_{\rm res}/T\lesssim 0.8$ (for $T\lesssim T_c$). In contrast, the neutrino oscillation resonance momentum obtained from $-a+b\simeq 1$ moves to a very high value $p_{\rm res}/T\gg 1$ (see Fig. 2 in Sec. V for an illustration of this). As $b\simeq\langle b\rangle\sim T^6$ becomes smaller, the neutrino momentum resonance $p_{\rm res}/T$ very quickly becomes so high that its effects can be neglected because the resonance occurs in the tail of the neutrino momentum distribution. Thus, for $T<T_c$, we can, to a good

approximation ignore the neutrinos and simply study the effects of the MSW transitions on the antineutrinos. In this case all the antineutrinos that pass through the resonance are converted into sterile neutrinos and vice versa (the MSW effect). The rate of change of the lepton number is thus related to the number of antineutrinos minus the number of sterile antineutrinos that pass through the resonance. Note that this rate is independent of the precise value of $\sin^2 2\theta_0$ provided that $\sin^2 2\theta_0 \ll 1$. Under this assumption

$$\frac{dL_{\nu_\alpha}}{dT} = -\left(\frac{N_{\bar{\nu}_\alpha} - N_{\bar{\nu}_s}}{n_\gamma}\right) T \left|\frac{d}{dT}\left(\frac{p_{\rm res}}{T}\right)\right|, \qquad (14)$$

where N_i describes the momentum distribution of species i, so that $n_i = \int_0^\infty N_i dp$. In thermal equilibrium,

$$N_{\bar{\nu}_\alpha} = \frac{1}{2\pi^2} \frac{p^2}{1+\exp\left(\frac{p+\mu_{\bar{\nu}}}{T}\right)}. \qquad (15)$$

In Eq. (14) the factor $Td(p_{\rm res}/T)/dT \simeq dp_{\rm res}/dT - p/T = dp_{\rm res}/dT - dp/dT$ is the rate at which $p_{\rm res}$ changes relative to the neutrino momentum (for neutrinos with momentum $p \sim p_{\rm res}$). Note that $N_{\nu_s} \simeq 0$ if the number density of sterile neutrinos is negligible. The functions N_i in Eq. (14) are evaluated at the resonance momentum $p_{\rm res}$ obtained as a function of $L_{\nu_\alpha}(T)$ and T from the resonance condition $a \simeq \cos 2\theta_0 \simeq 1$,

$$\frac{p_{\rm res}}{T} \equiv \frac{p_{\rm res}[T, L_{\nu_\alpha}(T)]}{T} = \frac{-\pi^2 \delta m^2_{\alpha s}}{8\zeta(3)\sqrt{2} G_F T^4 L_{\nu_\alpha}}, \qquad (16)$$

where we have considered the case η, $L_{\nu_\beta} \ll L_{\nu_\alpha}$ for $\beta \neq \alpha$. Note that this expression is only valid for $T \lesssim T_c/2$ where the b term can be neglected. Using

$$\frac{d(p_{\rm res}/T)}{dT} = \frac{\partial(p_{\rm res}/T)}{\partial T} + \frac{\partial(p_{\rm res}/T)}{\partial L_{\nu_\alpha}} \frac{dL_{\nu_\alpha}}{dT} = -4\frac{p_{\rm res}}{T^2} - \frac{p_{\rm res}}{TL_{\nu_\alpha}} \frac{dL_{\nu_\alpha}}{dT}, \qquad (17)$$

Eq. (14) yields

$$\frac{dL_{\nu_\alpha}}{dT} = \frac{-4Xp_{\rm res}/T}{T + Xp_{\rm res}/L_{\nu_\alpha}}, \qquad (18)$$

where we have assumed $d(p_{\rm res}/T)/dT < 0$. The useful dimensionless quantity X is given by

$$X \equiv X[T, p, \mu_{\bar{\nu}_\alpha}(T), N_{\bar{\nu}_s}(T)] = \frac{T}{n_\gamma}(N_{\bar{\nu}_\alpha} - N_{\bar{\nu}_s}) \qquad (19)$$

and it is evaluated at $p = p_{\rm res}$.

Equation (18) is a nonlinear equation in L_{ν_α}. The right-hand side of this equation depends on L_{ν_α} through $p_{\rm res}$ directly, through the dependence of X on $p_{\rm res}$ and through the number densities. In order to solve this equation, we need to write the chemical potentials in terms of L_{ν_α}. Now, for each temperature T, the neutrino asymmetry is created at the neutrino momentum $p_{\rm res}$. However, for temperatures greater than about 1 MeV [20] the effect of the weak interactions is to thermalize quickly the neutrino momentum distributions. This means that the neutrino asymmetry is approximately distributed throughout the neutrino momentum spectrum via chemical potentials for the neutrinos and antineutrinos. In general,

$$L_{\nu_\alpha} = \frac{1}{4\zeta(3)}\int_0^\infty \frac{x^2 dx}{1+e^{x+\tilde{\mu}_\nu}} - \frac{1}{4\zeta(3)}\int_0^\infty \frac{x^2 dx}{1+e^{x+\tilde{\mu}_{\bar{\nu}}}}, \qquad (20)$$

where $\tilde{\mu}_i \equiv \mu_i/T$ and $i = \nu, \bar{\nu}$. Expanding Eq. (20), we find that

$$L_{\nu_\alpha} \simeq -\frac{1}{24\zeta(3)}[\pi^2(\tilde{\mu}_\nu - \tilde{\mu}_{\bar{\nu}}) - 6(\tilde{\mu}_\nu^2 - \tilde{\mu}_{\bar{\nu}}^2)\ln 2 + (\tilde{\mu}_\nu^3 - \tilde{\mu}_{\bar{\nu}}^3)], \qquad (21)$$

which is an exact equation for $\tilde{\mu}_\nu = -\tilde{\mu}_{\bar{\nu}}$; otherwise it holds to a good approximation provided that $\tilde{\mu}_{\nu,\bar{\nu}} \lesssim 1$. For $T \gtrsim T^\alpha_{\rm dec}$ (where $T^e_{\rm dec} \simeq 3$ MeV and $T^{\mu,\tau}_{\rm dec} \simeq 5$ MeV are the chemical decoupling temperatures) $\mu_{\nu_\alpha} \simeq -\mu_{\bar{\nu}_\alpha}$ because processes such as $\nu_\alpha + \bar{\nu}_\alpha \leftrightarrow e^+ + e^-$ are rapid enough to make $\tilde{\mu}_\nu + \tilde{\mu}_{\bar{\nu}} \simeq \tilde{\mu}_{e^+} + \tilde{\mu}_{e^-} \simeq 0$. However, for 1 MeV$\lesssim T \lesssim T^\alpha_{\rm dec}$, weak interactions are rapid enough to thermalize approximately the neutrino momentum distributions, but not rapid enough to keep the neutrinos in chemical equilibrium. In this case, the value of $\tilde{\mu}_\nu$ is approximately frozen at $T \simeq T^\alpha_{\rm dec}$, while the antineutrino chemical potential $\tilde{\mu}_{\bar{\nu}}$ continues increasing until $T \simeq 1$ MeV.

We also need to specify the initial condition in order to solve Eq. (18). To do so we need to know the value of L_{ν_α} at some temperature $T_i < T_c$ at which MSW transitions are already dominant. This L_{ν_α} value can be obtained by solving the exact quantum kinetic equations, based on the density matrix, which incorporate both collision and oscillation effects. Fortunately, it turns out that the subsequent evolution of the lepton number is reasonably insensitive to what temperature T_i is chosen as the initial temperature for Eq. (18), provided that T_i is chosen during the epoch after T_c for which $L_{\nu_\alpha} \ll 1$ [21]. When the asymmetry $L_{\nu_\alpha} \ll 1$, Eq. (18) can be simplified to $dL_{\nu_\alpha}/dT \simeq -4L_{\nu_\alpha}/T$, which means that $L_{\nu_\alpha}T^4$ is approximately constant. This means that $p_{\rm res}/T$ is also approximately constant given that L_{ν_α} is related to the resonance momentum $p_{\rm res}$ by Eq. (16). As we will discuss in Sec. V, a numerical solution of the quantum kinetic equations shows that $p_{\rm res}/T$ is generally in the range

$$0.2 \lesssim p_{\rm res}/T \lesssim 0.8 \qquad (22)$$

for T values around $T_i = T_c/2$ when the oscillation parameters have been chosen to lie in the parameter space of interest, which turns out to be $|\delta m^2_{\alpha s}| \gtrsim 10^{-1}$ eV^2 and $\sin^2 2\theta_0 \gtrsim 5\times 10^{-10}$ (eV$^2/|\delta m^2_{\alpha s}|)^{1/6}$. [This lower bound for the mixing angle ensures that a suitably large asymmetry is created at T_c [5]. The result of Eq. (22) can also be gleaned from the static approximation based results of Ref. [5].] We will from now on use a value of about $T_c/2$ for T_i.

Before presenting the results of a numerical solution of Eq. (18) for the final asymmetry L_{ν_α}, it is interesting to note that an approximately correct answer is easily obtained from the following argument. As T falls below $T_c/2$, the asymmetry keeps increasing. This eventually forces the rate of change of L_{ν_α} to decrease substantially. Recall that dL_{ν_α}/dT is proportional to the how quickly the resonance momentum $p_{\rm res}$ moves as per Eq. (14). When L_{ν_α} is large, $p_{\rm res}/T$ must move to large values in order to create a lepton number. Eventually, $p_{\rm res}/T\to\infty$ and all of the antineutrinos that have passed through the resonance have been converted into sterile neutrinos. Thus, assuming that the initial number of sterile neutrinos is negligible and also neglecting the modification of the distribution due to the chemical potential, we expect that the final value of the lepton number $L^f_{\nu_\alpha}$ is given roughly by

$$\frac{L^f_{\nu_\alpha}}{h}\simeq\frac{1}{4\zeta(3)}\int_{p_{\rm in}/T}^{\infty}\frac{x^2dx}{1+e^x}\simeq\frac{3}{8},\tag{23}$$

where $h=T^3_{\nu_\alpha}/T^3_\gamma$ (note that $h\simeq1$ for $T\gtrsim m_e\simeq0.5$ MeV) and $p_{\rm in}/T$ is the value of $p_{\rm res}/T$ [and is in the range of Eq. (22)] at $T\simeq T_c/2$. It is interesting that the final asymmetry is approximately independent of $p_{\rm in}/T$ and hence also of $\delta m^2_{\alpha s}$. This is because $p_{\rm in}/T$ from Eq. (22) is always small.

Actually, the final value of the lepton number is somewhat less than $3/8=0.375$ if it is created when $T\gtrsim1$ MeV. This is because the number density of antineutrinos is continually reduced as the lepton number is thermally distributed via the chemical potential. Thus $L^f_{\nu_\alpha}$ depends on the temperature at which L_{ν_α} becomes large (10^{-2} roughly) and thus on $|\delta m^2_{\alpha s}|$. Numerically solving Eq. (18), assuming that the initial number of sterile neutrinos is negligible, we find that the final value of the lepton number is [22]

$$L^f_{\nu_\alpha}/h\simeq0.35\quad\text{for } |\delta m^2_{\alpha s}|/\text{eV}^2\lesssim3,$$

$$L^f_{\nu_\alpha}/h\simeq0.23\quad\text{for } 3\lesssim|\delta m^2_{\alpha s}|/\text{eV}^2\lesssim3000,$$

$$L^f_{\nu_\alpha}/h\simeq0.29\quad\text{for } |\delta m^2_{\alpha s}|/\text{eV}^2\gtrsim3000.\tag{24}$$

In numerically solving Eq. (18) we start the evolution at $T\simeq T_c/2$ with $p_{\rm in}/T$ in the range of Eq. (22) and with a corresponding L_{ν_α} obtained through Eq. (16). We find that $L^f_{\nu_\alpha}$ is approximately independent of the initial value of L_{ν_α} for $p_{\rm in}/T$ in this range.

The temperature where the final neutrino asymmetry is reached is approximately,

$$T^f_\nu\simeq0.5(|\delta m^2|/\text{eV}^2)^{1/4}\ \text{MeV}.\tag{25}$$

This result can be obtained analytically by using the resonance relation (16) with $L_{\nu_\alpha}\simeq L^f_{\nu_\alpha}$ and $p_{\rm res}/T\sim6$ (since $L^f_{\nu_\alpha}$ is not reached until $p_{\rm res}/T\gg1$ and we take $p_{\rm res}/T\sim6$ for definiteness).

Equation (18) is an approximation based on the neglect of collisions and the assumption of complete MSW conversion. By numerically integrating the exact quantum kinetic equations [18], we have checked that Eq. (18) does indeed accurately describe the evolution of the neutrino asymmetry in the range 1 MeV $\lesssim T\lesssim T_c/2$. We will discuss this and provide an illustrative example in Sec. V.

As preparation for the application of the above formalism to BBN, we need to discuss how an asymmetry in ν_e can be generated in the context of an overall neutrino mixing scenario. There are two generic ways of producing a nonzero L_{ν_e}. First, ν_e-ν_s oscillations can generate L_{ν_e} directly. Alternatively, ν_τ-ν_s (and/or ν_μ-ν_s) oscillations can generate a large L_{ν_τ} (and/or L_{ν_μ}), some of which is then transferred to L_{ν_e} by ν_τ-ν_e (and/or ν_μ-ν_e) oscillations.

The direct way of generating L_{ν_e} is only possible in special circumstances. Either $|\delta m^2_{es}|\gg|\delta m^2_{\tau s}|$, $|\delta m^2_{\mu s}|$ or ν_s has significant mixing with ν_e only. Only in these circumstances can we consider ν_e-ν_s oscillations in isolation. For this case we have estimated the effects of the neutrino asymmetry on BBN by writing a nucleosynthesis code. We find that $-1.8\lesssim\delta N^{\rm eff}_\nu\lesssim-0.1$ requires a $|\delta m^2_{es}|$ in the range 0.5$-$7 eV^2. For $|\delta m^2_{es}|\lesssim0.5$ eV^2 the lepton number is created too late to significantly affect BBN, while for $|\delta m^2_{es}|\gtrsim7$ eV^2 the lepton number is created so early that it leads to $\delta N_\nu\lesssim-1.8$ and thus appears to be too great a modification of BBN to be consistent with the observations. Note, however, that for $\sin^22\theta_0$ large enough, the sterile neutrino can be excited at temperatures before a significant lepton number is generated (which for $|\delta m^2_{es}|\sim1$ eV is $T\gtrsim13$ MeV). This can lead to an increase in the energy density, which can (partially) compensate for a large positive electron lepton number.

While the above direct way of generating L_{ν_e} is a possibility, we believe that a more interesting possibility is that L_{ν_e} is generated indirectly. As we will show, this mechanism gives $\delta N_\nu\sim-0.5$ (assuming $L_{\nu_e}>0$) for a wide range of parameters. This mechanism is also the only possibility if $|\delta m^2_{\tau s}|\gg|\delta m^2_{es}|$ (or $|\delta m^2_{\mu s}|\gg|\delta m^2_{es}|$), assuming that ν_s mixes with all three ordinary neutrinos.

IV. AN EXAMPLE WITH FOUR NEUTRINOS

Consider the system comprising $\nu_\tau,\nu_\mu,\nu_e,\nu_s$. An experimental motivation for the sterile neutrino comes from the current neutrino anomalies. There are several ways in which the sterile neutrino can help solve these problems. For example, the solar neutrino problem can be solved if $\delta m^2_{es}/\text{eV}^2\simeq10^{-6}$ and $\sin^22\theta_0\simeq10^{-2}$ (small angle MSW solution) [6] or if $10^{-2}\lesssim|\delta m^2_{es}|/\text{eV}^2\lesssim10^{-10}$ and $\sin^22\theta_0\simeq1$ (maximal oscillation solution) [23]. Alternatively, ν_μ-ν_s oscillations can solve the atmospheric neutrino problem if $|\delta m^2_{\mu s}|\simeq10^{-2}\text{eV}^2$ and $\sin^22\theta_0\simeq1$ [24].

We will assume that $m_{\nu_\tau}\gg m_{\nu_\mu},m_{\nu_e},m_{\nu_s}$, which means that

$$|\delta M^2|\equiv|\delta m^2_{\tau e}|\simeq|\delta m^2_{\tau s}|\simeq|\delta m^2_{\tau\mu}|\gg|\delta m^2_{es}|,|\delta m^2_{\mu s}|,|\delta m^2_{\mu e}|.\tag{26}$$

With the above assumption, ν_τ-ν_s oscillations initially create significant L_{ν_τ} at the temperature $T=T_c\simeq 16(|\delta M^2|/\mathrm{eV}^2)^{1/6}$. As before, we will assume that the sign of L_{ν_τ} is positive [19]. The effect of ν_τ-ν_e and ν_τ-ν_μ oscillations is to generate L_{ν_e} and L_{ν_μ} in such a way that $L^{(e)}-L^{(\tau)}=L_{\nu_e}-L_{\nu_\tau}\to 0$ and $L^{(\mu)}-L^{(\tau)}=L_{\nu_\mu}-L_{\nu_\tau}\to 0$, respectively [4,5,25]. [Note that if $L_{\nu_\tau}>0$, then the MSW resonances for ν_τ-ν_e and ν_τ-ν_μ oscillations occur for antineutrinos (given also our assumption that $m_{\nu_\tau}>m_{\nu_{e,\mu}}$) and so the signs of L_{ν_e} and L_{ν_μ} are also positive.] However, the rate of change of the lepton number due to collisions, the dominant process at higher T, is typically too small to generate L_{ν_e} from L_{ν_τ} efficiently [4,5]. However, as L_{ν_τ} becomes large at lower T, the lepton number can be efficiently transferred by MSW transitions. (When $L_{\nu_\tau}\ll 1$, MSW transitions cannot efficiently create L_{ν_e} because $N_{\bar{\nu}_\tau}-N_{\bar{\nu}_e}\simeq 0$ and MSW transitions only interchange $\bar{\nu}_\tau$ with $\bar{\nu}_e$ without changing their overall number density.) The rate of change of the lepton number due to $\bar{\nu}_\alpha$-$\bar{\nu}_\beta$ oscillations is simply given by the difference in rates for which $\bar{\nu}_\alpha$ antineutrinos and $\bar{\nu}_\beta$ antineutrinos pass through the resonance (assuming that $\sin^2 2\theta_0\ll 1$). We need to consider the three resonances $\bar{\nu}_\tau$-$\bar{\nu}_s$, $\bar{\nu}_\tau$-$\bar{\nu}_e$, and $\bar{\nu}_\tau$-$\bar{\nu}_\mu$ for our system. We denote the resonance momenta of these resonances by p_1, p_2, and p_3, respectively. The rate of change of the lepton numbers due to MSW transitions is governed approximately by the differential equations

$$\frac{dL_{\nu_\tau}}{dT}=-X_1\left|\frac{d(p_1/T)}{dT}\right|-X_2\left|\frac{d(p_2/T)}{dT}\right|-X_3\left|\frac{d(p_3/T)}{dT}\right|,$$

$$\frac{dL_{\nu_\mu}}{dT}=+X_3\left|\frac{d(p_3/T)}{dT}\right|,\quad \frac{dL_{\nu_e}}{dT}=+X_2\left|\frac{d(p_2/T)}{dT}\right|, \tag{27}$$

where

$$X_1\equiv\frac{T}{n_\gamma}(N_{\bar{\nu}_\tau}-N_{\bar{\nu}_s}),\quad X_2=\frac{T}{n_\gamma}(N_{\bar{\nu}_\tau}-N_{\bar{\nu}_e}),$$

$$X_3=\frac{T}{n_\gamma}(N_{\bar{\nu}_\tau}-N_{\bar{\nu}_\mu}), \tag{28}$$

and the X_i are evaluated at $p=p_i$ $(i=1,2,3)$. Note that X_i depends on T through the ratio p_i/T and through the dependence of the various chemical potentials on T. Observe that

$$\frac{d(p_i/T)}{dT}=\frac{\partial(p_i/T)}{\partial T}+\frac{\partial(p_i/T)}{\partial L_{\nu_e}}\frac{dL_{\nu_e}}{dT}+\frac{\partial(p_i/T)}{\partial L_{\nu_\mu}}\frac{dL_{\nu_\mu}}{dT}+\frac{\partial(p_i/T)}{\partial L_{\nu_\tau}}\frac{dL_{\nu_\tau}}{dT}, \tag{29}$$

with

$$\frac{\partial(p_1/T)}{\partial L_{\nu_\tau}}=2\frac{\partial(p_1/T)}{\partial L_{\nu_\mu}}=2\frac{\partial(p_1/T)}{\partial L_{\nu_e}}=\frac{-2(p_1/T)}{L^{(\tau)}},$$

$$\frac{\partial(p_2/T)}{\partial L_{\nu_\tau}}=-\frac{\partial(p_2/T)}{\partial L_{\nu_e}}=\frac{-(p_2/T)}{L^{(\tau)}-L^{(e)}},\quad \frac{\partial(p_2/T)}{\partial L_{\nu_\mu}}=0,$$

$$\frac{\partial(p_3/T)}{\partial L_{\nu_\tau}}=-\frac{\partial(p_3/T)}{\partial L_{\nu_\mu}}=\frac{-(p_3/T)}{L^{(\tau)}-L^{(\mu)}},\quad \frac{\partial(p_3/T)}{\partial L_{\nu_e}}=0,$$

$$\frac{\partial(p_i/T)}{\partial T}=\frac{-4p_i}{T^2}. \tag{30}$$

By the symmetry of the problem, $L_{\nu_\mu}=L_{\nu_e}$, $p_2=p_3$, and $dL_{\nu_\mu}/dT=dL_{\nu_e}/dT$ [26]. Using this simplification, we find that

$$\frac{dL_{\nu_e}}{dT}=\frac{dL_{\nu_\mu}}{dT}=\frac{A}{B},\quad \frac{dL_{\nu_\tau}}{dT}=\frac{\alpha}{y_1}+\frac{\beta}{y_1}\frac{dL_{\nu_e}}{dT}, \tag{31}$$

where $A=\gamma y_1+\alpha\delta$ and $B=y_1y_2-\beta\delta$, with

$$\alpha=-4X_1\left[\frac{p_1}{T^2}\right]-8X_2\left[\frac{p_2}{T^2}\right],$$

$$\beta=-2X_1\left[\frac{p_1}{TL^{(\tau)}}\right]+2X_2\left[\frac{p_2}{T(L^{(\tau)}-L^{(e)})}\right],$$

$$\gamma=4X_2\left[\frac{p_2}{T^2}\right],\quad \delta=+X_2\left[\frac{p_2}{T(L^{(\tau)}-L^{(e)})}\right],$$

$$y_1=1+2X_1\left[\frac{p_1}{TL^{(\tau)}}\right]+2X_2\left[\frac{p_2}{T(L^{(\tau)}-L^{(e)})}\right],$$

$$y_2=1+X_2\left[\frac{p_2}{T(L^{(\tau)}-L^{(e)})}\right]. \tag{32}$$

In deriving this equation we have assumed that $d(p_1/T)/dT<0$ and $d(p_2/T)/dT<0$. Observe that $X_2d(p_2/T)/dT=-A/B$ and thus for self-consistency Eq. (31) is only valid provided that $A/B<0$ (given that $X_2<0$). If $d(p_2/T)/dT>0$, then Eq. (31) becomes

$$\frac{dL_{\nu_e}}{dT}=\frac{dL_{\nu_\mu}}{dT}=\frac{\tilde{A}}{\tilde{B}},\quad \frac{dL_{\nu_\tau}}{dT}=\frac{\tilde{\alpha}}{\tilde{y}_1}+\frac{\tilde{\beta}}{\tilde{y}_1}\frac{dL_{\nu_e}}{dT}, \tag{33}$$

where $\tilde{A},\tilde{B},\tilde{\alpha},\tilde{\beta},\tilde{y}_1$ have the same form as A,B,α,β,y_1 except that $X_2\to -X_2$. In this case, $X_2d(p_2/T)/dT=\tilde{A}/\tilde{B}$. It follows that Eq. (33) is only self-consistent provided that $\tilde{A}/\tilde{B}<0$. Observe that $d(p_2/T)/dT$ must be continuous, which means that $d(p_2/T)/dT$ changes sign only when A changes sign and Eq. (31) maps onto Eq. (33) continuously because $\tilde{A}=-A$ (and thus $\tilde{A}=A$ at the point where $A=0$). If $d(p_1/T)/dT$ changes sign at some point $p_1/T=q$ then we must make the replacement $X_1\simeq 0$ for $p_1/T<q$ [assuming that initially $d(p_1/T)/dT<0$] since the previous MSW transitions have populated ν_s for $p_1/T<q$.

In solving Eq. (31) we will assume that the initial number of sterile neutrinos can be neglected (this will be valid for a wide range of parameters as will be discussed later). We start the evolution of Eq. (31) when $T \simeq T_c/2$ [with T_c given by Eq. (12) for ν_τ-ν_s oscillations]. There is a range of values of L_{ν_τ} at this point that is related to the range of $p_{\rm res}/T$ [Eq. (22)] through Eq. (16). Performing the numerical integration, we find that the final electron neutrino asymmetry is [27]

$$L^f_{\nu_e}/h \simeq 2.0 \times 10^{-2} \quad \text{for } 10 \lesssim |\delta M^2|/\text{eV}^2 \lesssim 3000,$$

$$L^f_{\nu_e}/h \simeq 1.7 \times 10^{-2} \quad \text{for } |\delta M^2|/\text{eV}^2 \gtrsim 3000 \tag{34}$$

and recall that $h \equiv T^3_{\nu_\alpha}/T^3_\gamma$. We found that $L^f_{\nu_e}$ is approximately independent of $p_{\rm res}/T$ for $p_{\rm res}/T$ in the range given by Eq. (22). We also found numerically that $L^f_{\nu_e}$ is approximately independent of the initial value of L_{ν_e} (at $T \simeq T_c/2$) so long as $L_{\nu_e} \lesssim L_{\nu_\tau}$ at this temperature (which should be valid since efficient generation of L_{ν_e} does not occur until much lower temperatures where L_{ν_τ} has become very large). In addition, we found that $L^f_{\nu_e}$ is independent of the precise value of the initial temperature (so long as the initial temperature is less than T_c and $L_{\nu_\tau} \ll 1$ at this temperature). The reason for this independence is simply due to the fact that significant generation of L_{ν_e} cannot occur until L_{ν_τ} becomes large ($\gtrsim 10^{-2}$). The final asymmetry $L^f_{\nu_e}$ is also independent of $\sin^2 2\theta_0$ so long as $\sin^2 2\theta_0 \ll 1$ for aforementioned reasons. Finally, and perhaps of most interest, we find that $L^f_{\nu_e}$ is almost independent of $|\delta M^2|$ so long as $|\delta M^2| \gtrsim 3$ eV^2. For $|\delta M^2| \lesssim 3$ eV^2, L_{ν_τ} does not become large until $T \lesssim 1$ MeV. For temperatures in this range, the effect of L_{ν_τ} cannot be described in terms of chemical potentials because the weak interactions are too weak to thermalize the neutrino distribution. For this reason, $L^f_{\nu_e}$ should be much smaller since the $\bar{\nu}_\tau$-$\bar{\nu}_e$ resonance (which occurs at a momentum that is always greater than the $\bar{\nu}_\tau$-$\bar{\nu}_s$ resonance) simply interchanges almost equal numbers of $\bar{\nu}_\tau$'s and $\bar{\nu}_e$'s.

We now apply the above analysis to BBN. Recall that the neutrino oscillations affect $N^{\rm eff}_\nu$ in two ways. First, the creation of $L^f_{\nu_e}$ and the related modification of the neutrino momentum distributions directly affects the nuclear reaction rates that determine the neutron to proton ratio. Second, the oscillations can modify the energy density of the Universe by the excitation of the sterile neutrino and the modification of the neutrino momentum distributions due to chemical potentials. We first discuss the energy density question.

For $T > T_c$ the ν_τ-ν_s oscillations can excite the sterile neutrino (and antineutrino). In Ref. [5], a detailed study was done that found that $\rho_{\nu_s}/\rho_\nu \lesssim 0.6$ provided that

$$\sin^2 2\theta_0 \lesssim 4 \times 10^{-5} \left[\frac{\text{eV}^2}{|\delta M^2|} \right]^{1/2}. \tag{35}$$

Furthermore, $\rho_{\nu_s}/\rho_\nu \to 0$ very quickly as $\sin^2 2\theta_0 \to 0$. In particular, we found that $\rho_s/\rho_\nu \lesssim 0.1$ for

$$\sin^2 2\theta_0 \lesssim 5 \times 10^{-6} \left[\frac{\text{eV}^2}{|\delta M^2|} \right]^{1/2}. \tag{36}$$

Note that after the lepton number is created, the oscillations no longer excite significant numbers of sterile neutrinos until $L_{\nu_\tau} \gtrsim 10^{-2}$. At this point the $\bar{\nu}_\tau$-$\bar{\nu}_s$ oscillations (recall that we are assuming that $L_{\nu_\tau} > 0$) transfer $\bar{\nu}_\tau \to \bar{\nu}_s$. The effect of these oscillations on the overall energy density depends on the temperature where $L_{\nu_\tau} \gtrsim 10^{-2}$ occurs, which in turn depends on $|\delta M^2|$. There are essentially three regions to consider: $10 \lesssim |\delta M^2|/\text{eV}^2 \lesssim 3000$, $|\delta M^2|/\text{eV}^2 \lesssim 10$, and $|\delta M^2|/\text{eV}^2 \gtrsim 3000$.

For $10 \lesssim |\delta M^2|/\text{eV}^2 \lesssim 3000$, we have numerically calculated the final number and mean energies of $\bar{\nu}_\tau, \bar{\nu}_s, \bar{\nu}_e, \bar{\nu}_\mu$ (the number and energy densities of the neutrinos are approximately unchanged in this region). Normalizing the number density to the number of neutrinos when $\mu_\nu = 0$, $n_0 \equiv \frac{3}{4}\zeta(3)T^3/\pi^2$, we find

$$\frac{n_{\bar{\nu}_e}}{n_0} = \frac{n_{\bar{\nu}_\mu}}{n_0} \simeq 0.95, \quad \frac{n_{\bar{\nu}_\tau}}{n_0} \simeq 0.44, \quad \frac{n_{\bar{\nu}_s}}{n_0} \simeq 0.66. \tag{37}$$

Note that the total number is approximately unchanged (i.e., $0.95 \times 2 + 0.44 + 0.66 \simeq 3$). We find the final mean energy for the $\bar{\nu}_s$, $\langle E_s \rangle$, to be slightly less than the mean energy for a Fermi-Dirac distribution with $\mu_\nu = 0$, $\langle E_{\rm FD} \rangle \simeq 3.15T$ [$\langle E_s \rangle / \langle E_{\rm FD} \rangle \simeq 0.88$]. For this reason there is a small overall change in energy density, equivalent to about $\delta N^{\rm eff}_\nu \simeq -0.05$. For $|\delta M^2| \gtrsim 3000$ eV^2, $L^f_{\nu_\tau}$ is reached for $T \gtrsim T^\tau_{\rm dec}$ and so $\mu_{\nu_\tau} \simeq -\mu_{\bar{\nu}_\tau}$. In this case, there is an additional contribution to the energy density coming from the ν_τ neutrinos due to the negative chemical potential μ_{ν_τ}. In this case we find that the overall change in the energy density is considerably larger and equivalent to $\delta N^{\rm eff}_\nu \simeq 0.4$. Finally, for $|\delta M^2| \lesssim 10$ eV^2, the change in the energy density quickly becomes completely negligible because the weak interactions are unable to thermalize the neutrino distributions. The oscillations simply transfer $\bar{\nu}_\tau$ to $\bar{\nu}_s$ and the total number and energy density remain unchanged.

We now turn to the effect of $L^f_{\nu_e}$ and the corresponding modification of the momentum distributions on $N^{\rm eff}_\nu$ through nuclear reaction rates. For $10 \lesssim |\delta M^2|/\text{eV}^2 \lesssim 3000$, the distribution of $L^f_{\nu_e}$ can be approximately described by chemical potentials $\tilde{\mu}_{\bar{\nu}} \simeq 0.06$ and $\tilde{\mu}_\nu \simeq 0$. For $|\delta M^2| \gtrsim 3000$ eV^2, the lepton number is created above the chemical decoupling temperature. In this case the distribution of $L^f_{\nu_e}$ can be approximately described by chemical potentials $\tilde{\mu}_{\bar{\nu}} \simeq 0.025$ and $\tilde{\mu}_\nu \simeq -0.025$. We find that for $|\delta M^2| \gtrsim 10$ eV^2, $L = L^f_{\nu_e}$ is reached for $T \gtrsim 1.5$ MeV. Thus, to a good approximation, the chemical potentials $\tilde{\mu}_{\nu_e}, \tilde{\mu}_{\bar{\nu}_e}$ are approximately constant during the nucleosynthesis era. Using our BBN code, we find that the modification of Y_P due to the chemical potentials is $\delta Y_P \simeq -0.005$ for $\tilde{\mu}_{\bar{\nu}} \simeq 0.06$, $\tilde{\mu}_\nu \simeq 0$ and $\delta Y_P \simeq -0.006$ for $\tilde{\mu}_{\bar{\nu}} \simeq 0.025$, $\tilde{\mu}_\nu \simeq -0.025$. From Eq. (3), this translates into a reduction of the effective number of neutrino degrees of freedom during nucleosynthesis. Including the effects of the change in energy density discussed earlier, we find that

$$\delta N_\nu^{\rm eff} \simeq -0.5 \quad \text{for } 10 \lesssim |\delta M^2|/\text{eV}^2 \lesssim 3000,$$

$$\delta N_\nu^{\rm eff} \simeq -0.1 \quad \text{for } |\delta M^2|/\text{eV}^2 \gtrsim 3000. \tag{38}$$

For this result, we have considered the case of negligible excitation of sterile neutrinos for temperatures above T_c, that is, Eq. (36) has been assumed. Note that if we had assumed that L_{ν_τ} was negative instead of positive, then the sign of L_{ν_e} is also negative and the change in Y_P due to the asymmetry is opposite in sign as well. This leads to $\delta N_\nu^{\rm eff} \simeq +0.4(0.9)$ for $10 \lesssim |\delta M^2|/\text{eV}^2 \lesssim 3000$ ($|\delta M^2|/\text{eV}^2 \gtrsim 3000$).

In our analysis we have neglected the effects of the ν_μ-ν_s, ν_μ-ν_e, and ν_e-ν_s oscillations. It is usually possible to neglect these oscillations if $|\delta m^2| \ll |\delta M^2|$ because the lepton number created by ν_τ-ν_s oscillations is large enough to suppress the oscillations that have much smaller δm^2. Of course, in some circumstances these oscillations cannot be neglected. For example, in Ref. [5], we showed that the effects of maximal ν_μ-ν_s oscillations with $|\delta m^2_{\mu s}| \simeq 10^{-2}$ eV^2 (as suggested by the atmospheric neutrino anomaly [24]) can only be neglected if $|\delta m^2_{\tau s}| \gtrsim 30$ eV^2. Interestingly, this parameter space overlaps considerably with the parameter space where $\delta N_\nu^{\rm eff} \simeq -0.5$, according to Eq. (38). Note that this parameter space is also suggested if the τ neutrino is a significant component of dark matter.

V. EVOLUTION OF THE LEPTON NUMBER FROM THE EXACT QUANTUM KINETIC EQUATIONS

In this section we study the evolution of the neutrino asymmetry by numerically integrating the exact quantum kinetic equations [18]. This formalism allows a nearly exact calculation to be performed that is valid at both high and low temperatures. As we have discussed, for high temperatures $T \gtrsim T_c$ the evolution of the lepton number is dominated by collisions (assuming $|\delta m^2| \gtrsim 10^{-4}$ eV^2), while at lower temperatures the evolution of lepton number is dominated by oscillations between collisions (MSW effect).

The system of an active neutrino oscillating with a sterile neutrino can be described by a density matrix [18,28]. Below we very briefly outline this formalism. The density matrices describing an ordinary neutrino of momentum p oscillating with a sterile neutrino are given by

$$\rho_\nu(p) = \frac{1}{2} P_0(p)[1 + \mathbf{P}(p)\cdot\sigma],$$

$$\rho_{\bar\nu}(p) = \frac{1}{2}\overline{P_0}(p)[1 + \overline{\mathbf{P}}(p)\cdot\sigma], \tag{39}$$

where $\mathbf{P}(p) = P_x(p)\hat{x} + P_y(p)\hat{y} + P_z(p)\hat{z}$. (It will be understood throughout this section that the density matrices and the quantities P_i also depend on time t or, equivalently, temperature T.) The number distributions of ν_α and ν_s are given by

$$N_{\nu_\alpha} = \frac{1}{2} P_0(p)[1 + P_z(p)] N^{eq}_{\nu_\alpha},$$

$$N_{\nu_s} = \frac{1}{2} P_0(p)[1 - P_z(p)] N^{\rm eq}_{\nu_\alpha}, \tag{40}$$

where

$$N^{\rm eq}_{\nu_\alpha} = \frac{1}{2\pi^2} \frac{p^2}{1 + \exp\left(\frac{p + \mu_\nu}{T}\right)} \tag{41}$$

is the equilibrium number distribution. Note that there are analogous equations for the antineutrinos [with $\mathbf{P}(p) \to \overline{\mathbf{P}}(p)$ and $P_0 \to \overline{P_0}$]. The evolution of $P_0(p)$ and $\mathbf{P}(p)$ are governed by the equations [18]

$$\frac{\partial}{\partial t}\mathbf{P}(p) = \mathbf{V}(p) \times \mathbf{P}(p) + [1 - P_z(p)]\left[\frac{\partial}{\partial t}\ln P_0(p)\right]\hat{\mathbf{z}} - \left[D(p) + \frac{d}{dt}\ln P_0(p)\right][P_x(p)\hat{\mathbf{x}} + P_y(p)\hat{\mathbf{y}}],$$

$$\frac{\partial}{\partial t} P_0(p) \simeq R(p). \tag{42}$$

The quantity $\mathbf{V}(p)$ is given by

$$\mathbf{V}(p) = \beta(p)\hat{\mathbf{x}} + \lambda(p)\hat{\mathbf{z}}, \tag{43}$$

where $\beta(p)$ and $\lambda(p)$ are defined by

$$\beta(p) = \frac{\delta m^2}{2p}\sin 2\theta_0,$$

$$\lambda(p) = -\frac{\delta m^2}{2p}[\cos 2\theta_0 - b(p) \pm a(p)], \tag{44}$$

in which the plus (minus) sign corresponds to neutrino (antineutrino) oscillations. The dimensionless variables $a(p)$ and $b(p)$ contain the matter effects and are given in Eq. (9). The quantity $D(p)$ is the quantum damping parameter resulting from the collisions of the neutrino with the background. According to Ref. [29], the damping parameter is half of the total collision frequency, i.e., $D(p) = \Gamma_{\nu_\alpha}(p)/2$. Finally, note that in Eq. (42) the function $R(p)$ is related to $\Gamma_{\nu_\alpha}(p)$ and its specific definition is given in Ref. [18]. For temperatures above 1 MeV, we can make the useful approximation of setting $N_{\nu_\alpha} = N^{\rm eq}_{\nu_\alpha}$ and $N_{\bar\nu_\alpha} = N^{\rm eq}_{\nu_\alpha}$. This means that $P_0(p) = 2/[1 + P_z(p)]$, $\overline{P_0}(p) = 2/[1 + \overline{P_z}(p)]$, and consequently

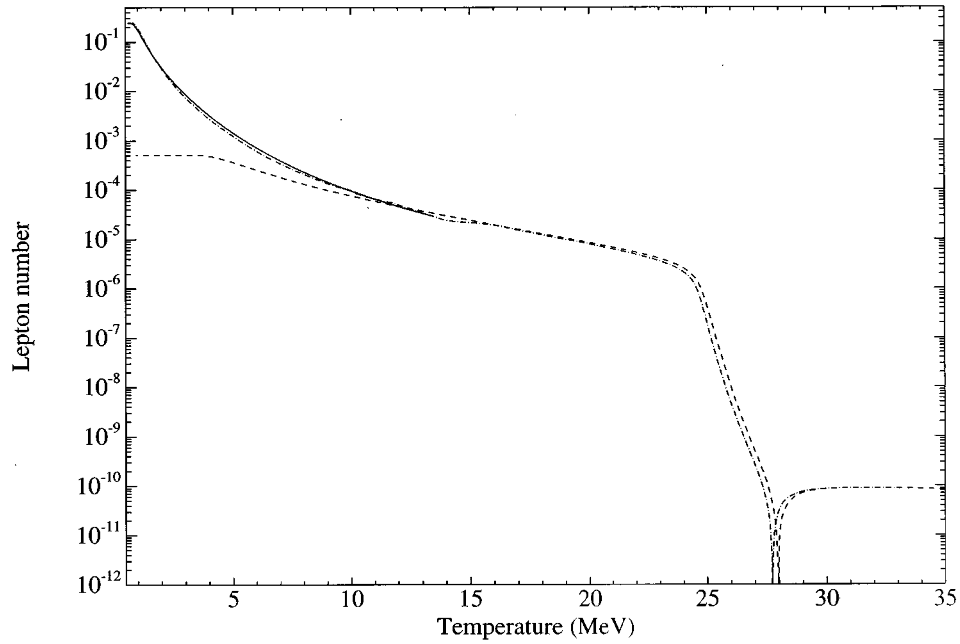

FIG. 1. Evolution of the ν_τ-ν_s oscillation generated lepton number asymmetry L_{ν_τ}. We have taken by way of example the parameter choice $\delta m^2 = -10\ \mathrm{eV}^2$ and $\sin^2 2\theta_0 = 10^{-9}$. The dash-dotted line is the result of the numerical integration of the quantum kinetic equations [Eqs. (49) and (42)]. The solid line is the result from the numerical integration of Eq. (18), while the dashed line is the static approximation developed in Ref. [5].

$$\frac{\partial P_0(p)}{\partial t} = \frac{-2}{[1+P_z(p)]^2}\frac{\partial P_z(p)}{\partial t},$$

$$\frac{\partial \overline{P_0}(p)}{\partial t} = \frac{-2}{[1+\overline{P_z}(p)]^2}\frac{\partial \overline{P_z}(p)}{\partial t}. \tag{45}$$

For the numerical work, the continuous variable p/T is replaced by a finite set of momenta $x_n \equiv p_n/T$ (with $n=1,\ldots,N$). The variables $\mathbf{P}(p)$ and $P_0(p)$ are replaced by the set of N variables $\mathbf{P}(x_n)$ and $P_0(x_n)$. The evolution of each of these variables is governed by Eqs. (42), where for each value of n the variables $\mathbf{V}(p)$ and $D(p)$ are replaced by $\mathbf{V}(x_n)$ and $D(x_n)$. Thus the oscillations of the neutrinos and antineutrinos can be described by $8N$ simultaneous differential equations.

The rate of change of the lepton number is given by

$$\frac{dL_{\nu_\alpha}}{dt} = \frac{d}{dt}\left[\frac{(n_{\nu_\alpha} - n_{\overline{\nu_\alpha}})}{n_\gamma}\right] = -\frac{d}{dt}\left[\frac{(n_{\nu_s} - n_{\overline{\nu_s}})}{n_\gamma}\right]. \tag{46}$$

Thus, using Eq. (40),

$$\frac{dL_{\nu_\alpha}}{dt} = \frac{1}{2}\frac{d}{dt}\left[\frac{1}{n_\gamma}\int [\overline{P_0}(1-\overline{P_z})N^{\rm eq}_{\overline{\nu_\alpha}} - P_0(1-P_z)N^{\rm eq}_{\nu_\alpha}]dp\right]. \tag{47}$$

Taking the time differentiation inside the integral we find that

$$\frac{dL_{\nu_\alpha}}{dt} \simeq \frac{1}{2}\int\left(\frac{\partial[\overline{P_0}(1-\overline{P_z})]}{\partial t}\frac{N^{\rm eq}_{\overline{\nu_\alpha}}}{n_\gamma} - \frac{\partial[P_0(1-P_z)]}{\partial t}\frac{N^{\rm eq}_{\nu_\alpha}}{n_\gamma}\right)dp, \tag{48}$$

where we have used the result that $N^{\rm eq}_{\nu_\alpha}dp/n_\gamma$ is approximately independent of t. Expanding this equation using Eq. (45), we find

$$\frac{dL_{\nu_\alpha}}{dt} = \frac{1}{n_\gamma}\int\left(N^{\rm eq}_{\nu}\frac{2}{[1+P_z]^2}\frac{\partial P_z}{\partial t} - N^{\rm eq}_{\overline{\nu}}\frac{2}{[1+\overline{P_z}]^2}\frac{\partial \overline{P_z}}{\partial t}\right)dp. \tag{49}$$

Equations (42) and (49) can be numerically integrated to obtain the evolution of L_{ν_α} [30,31]. We illustrate this with an example. For definiteness we will consider the ν_τ, ν_s system. In Fig. 1 we take $\delta m^2_{\tau s} = -10$ and $\sin^2 2\theta_0 = 10^{-9}$ (we set $\eta = 4\times 10^{-10}$ and took $L_{\nu_\alpha}=0$ initially [32]). The result of numerically integrating Eqs. (42) and (49) is shown in the figure by the dash-dotted line. Also shown in Fig. 1 (dashed line) is the ''static approximation'' [Eqs. (94) and (93) of Ref. [5]]. As discussed in Ref. [5], the static approximation assumes that the system is sufficiently smooth and that the dominant contribution to the rate of change of the lepton number is collisions. As shown in Fig. 1, the static approximation is a good approximation at high temperatures. How-

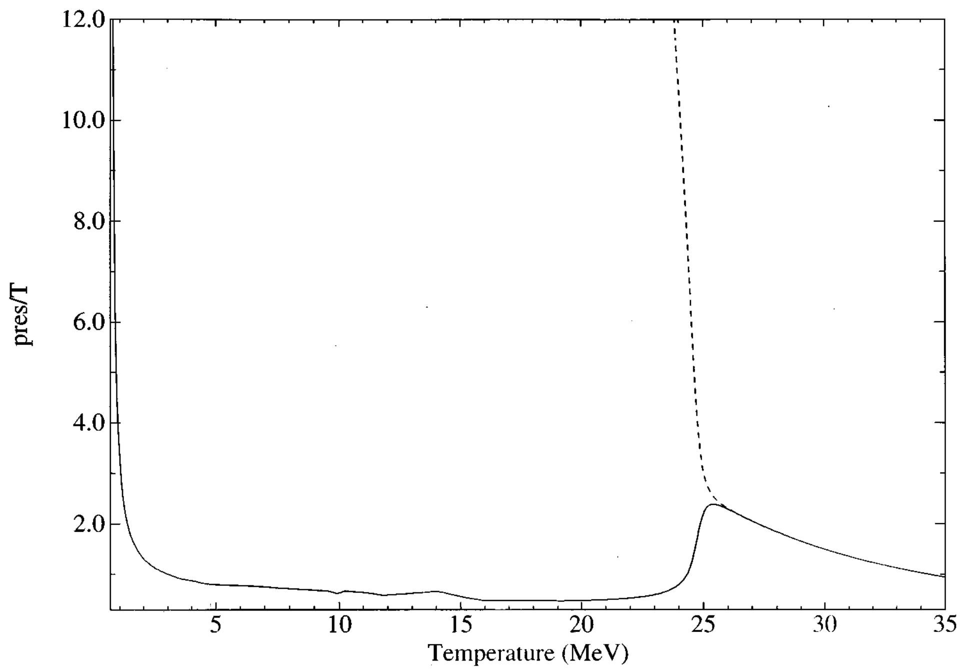

FIG. 2. Evolution of the neutrino (dashed line) and antineutrino (solid line) resonance momenta for the example of Fig. 1.

ever, as discussed in Ref. [5], the static approximation does not include the MSW effect, which is the dominant physical process at low temperatures. As expected, the MSW effect keeps L_{ν_τ} growing like $L_{\nu_\tau} \sim 1/T^4$ for much lower temperatures. We have also checked our simplified equation (18) for the evolution of lepton number due to MSW transitions. We started the evolution of this equation at $T = T_c/2 \simeq 13.5$ MeV with the value of L_{ν_τ} at this point obtained from the quantum kinetic equations of $L_{\nu_\tau} \simeq 2.92 \times 10^{-5}$. The subsequent evolution of L_{ν_τ} obtained from numerically integrating Eq. (18) is given in Fig. 1 by the solid line. As the figure shows, Eq. (18) is a very good approximation for the evolution of the neutrino asymmetry at low temperatures. This provides a useful check of the validity of the approximate approach used in Sec. IV for the $\nu_e, \nu_\mu, \nu_\tau, \nu_s$ four-flavor system.

It is instructive to examine the evolution of the neutrino and antineutrino resonance momenta. Recall that the resonance for neutrinos occurs when $b - a = \cos 2\theta_0$, while the resonance for antineutrinos occurs when $b + a = \cos 2\theta_0$. Let us write

$$b = \lambda_1 p^2, \quad a = \lambda_2 p, \tag{50}$$

where λ_1 and λ_2 are independent of p and can be obtained from Eq. (9). Note that $\lambda_1, \lambda_2 > 0$ given that $\delta m^2_{\alpha s} < 0$ and assuming $L^{(\alpha)} > 0$. Solving the resonance conditions $b \pm a = \cos 2\theta_0$, we find that the resonance momenta satisfy

$$p_{\rm res} = \frac{\lambda_2 + \sqrt{\lambda_2^2 + 4\lambda_1 \cos 2\theta_0}}{2\lambda_1} \quad \text{for neutrinos,} \tag{51}$$

$$p_{\rm res} = \frac{-\lambda_2 + \sqrt{\lambda_2^2 + 4\lambda_1 \cos 2\theta_0}}{2\lambda_1} \quad \text{for antineutrinos.}$$

In Fig. 2 we have plotted the evolution of the resonance momenta for the neutrinos and antineutrinos. As this example illustrates, the neutrino resonance momentum moves to very high values as $T \lesssim T_c$, while the antineutrino resonance momentum moves to very low values (which in this example is $p_{\rm res}/T \simeq 0.6$ for $T \simeq T_c/2$). We have found that this behavior is quite general, with the antineutrino resonance $p_{\rm res}/T$ in the range (22) at $T = T_c/2$ as $\sin^2 2\theta_0$ and δm^2 are varied.

VI. CONCLUSION

In summary, we have extended previous work on the neutrino oscillation generated lepton number in the early Universe by studying the evolution of the lepton number at low temperatures where the MSW effect is important. We applied this work to examine the implications of the neutrino asymmetry for BBN in two illustrative models. In the first model, electron-neutrino asymmetry was created directly by ν_e-ν_s oscillations, while in the second model the electron-neutrino asymmetry was created indirectly by the reprocessing of a τ neutrino asymmetry.

One result of this study is that the naive conclusion that sterile neutrinos only increase the effective number of neutrino species ($N_\nu^{\rm eff}$) during the nucleosynthesis era is actually wrong. Neutrino asymmetries generated by neutrino oscillations can naturally lead to a decrease in $N_\nu^{\rm eff}$. Furthermore in the case where the electron-neutrino asymmetry is transferred from the τ or μ neutrino asymmetries, the electron-neutrino asymmetry is approximately independent of $|\delta m^2|$ and $\sin^2 2\theta_0$ for a wide range of parameters. This leads to a prediction of $\delta N_\nu^{\rm eff} \simeq -0.5$ if the asymmetry is positive for an interesting class of models. Remarkably, this prediction is supported by some recent observations that actually suggest $N_\nu^{\rm eff} < 3$.

ACKNOWLEDGMENTS

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[17] In the case where $|\delta m^2_{\alpha s}| \lesssim 10^{-5}$ eV^2, the lepton number is created so late that the evolution of the lepton number is dominated by oscillations between collisions. This case was examined by K. Kainulainen, K. Enqvist, and J. Maalampi, Nucl. Phys. **B349**, 754 (1991), who showed that the neutrino asymmetry is too small to directly affect BBN through the modification of nuclear reaction rates. Recently, D. P. Kirilova and M. V. Chizhov, Phys. Lett. B **393**, 375 (1997), have shown that the asymmetry can affect BBN indirectly through its effect on the oscillations that can distort the electron-neutrino distribution. This mechanism requires very small $|\delta m^2_{es}| \sim 10^{-7}$ eV^2 and assumes that the effects of the μ and τ neutrinos can be neglected.

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[19] The sign of the asymmetry L_{ν_α} generated by ν_α-ν_s oscillations depends on the sign of the initial asymmetries η and L_{ν_β} (with $\beta \neq \alpha$) [5]. Assuming that $m_{\nu_\tau} \gg m_{\nu_\mu}, m_{\nu_e}, m_{\nu_s}$, then $\mathrm{sgn}(L_{\nu_\alpha}) = \mathrm{sgn}(\tilde{\eta})$, where $\tilde{\eta} \equiv \eta + L_{\nu_e} + L_{\nu_\mu} + L_{\nu_\tau} - L_{\nu_\alpha}$ [5]. Thus, for example, L_{ν_α} should be positive if the initial $L_{\nu_e}, L_{\nu_\mu}, L_{\nu_\tau} > 0$.

[20] The rate at which weak interactions thermalize the neutrino distributions is approximately just the total collision rate. The thermal average of the total collision rate is $\Gamma_{\nu_\alpha} \simeq h_\alpha G_F^2 T^5$, with $h_e \simeq 4.0$ and $h_{\mu,\tau} \simeq 2.9$ [see, e.g., K. Enqvist, K. Kainulainen, and M. Thomson, Nucl. Phys. **B373**, 498 (1992)]. The demand that this collision rate be greater than the expansion rate, that is, $\Gamma_{\nu_\alpha} \gtrsim H \simeq 5.5 T^2/M_P$, implies that $T \gtrsim 1$ MeV.

[21] This is because collisions also keep $p_{\rm res}/T$ approximately constant and in the range (22) (for $T < T_c$), except at very low temperatures where the collision rate is very low.

[22] Note that in the case $\alpha = e$, the laboratory bound $m_{\nu_e} \lesssim 3$ eV (assuming a Majorana mass) implies that $|\delta m^2_{es}| \lesssim 9$ eV^2. Also note that the cosmology energy bound suggests that $m_{\nu_\mu}, m_{\nu_\tau} \lesssim 40$ eV, which implies $|\delta m^2_{\mu s}|, |\delta m^2_{\tau s}| \lesssim 1600$ eV^2.

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[26] Actually, if p_2 is exactly equal to p_3, then the two-component formalism will not be strictly valid. However, p_2 will be only approximately p_3 because, for example, $\delta m^2_{\tau\mu}$ will be slightly different to $\delta m^2_{\tau e}$ due to the m_{ν_μ},m_{ν_e} mass difference.

[27] Note that we found numerically that for $|\delta M^2|\lesssim 350\ \mathrm{eV}^2$, the evolution of A and B is such that $A<0$ and $B<0$ [with $d(p_2/T)/dT<0$] occur at some temperature during the evolution. At this point there is no solution for dL_{ν_e}/dT since Eq. (31) cannot be applied self-consistently. For points in this region, L_{ν_e} is increasing so rapidly that our simplified equations (27) are not valid. For points in this region, we obtained dL_{ν_e}/dT from Eq. (27) but with $d(p_2/T)/dT$ evaluated at the previous integration step.

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[30] In Fig. 1 we have plotted $|L_{\nu_\tau}|$. We found that L_{ν_τ} changes sign at the temperature $T=T_c$. As we discussed in Ref. [5], this behavior is expected.

[31] In integrating Eqs. (42) and (49) we have only evaluated these equations for momenta in the region around the neutrino and antineutrino resonances.

[32] Note that the evolution of L_{ν_α} is essentially independent of the initial conditions so long as $L_{\nu_\alpha}\lesssim 10^{-5}$ [5].