Effect of violation of quantum mechanics on neutrino oscillation

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The effect of quantum mechanics violation due to quantum gravity on neutrino oscillation is investigated. It is found that the mechanism introduced by Ellis, Hagelin, Nanopoulos, and Srednicki through the modification of the Liouville equation can affect neutrino oscillation behavior and may be taken as a new solution of the solar neutrino problem. [S0556-2821(97)03222-0]

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I. INTRODUCTION

More than 20 years ago, Hawking found that quantum mechanics allows black holes to emit particles in a thermal spectrum [1,2]. Because the black hole creates particles in pairs, with one particle always falling into the hole and the other possibly escaping to infinity, part of the information about the state of the system is lost down the black hole and the final situation is represented by a density matrix rather than a pure quantum state [3]. Hawking proposed that if such a decay of a pure quantum state into a mixed state can occur with a macroscopic configuration such as a black hole, it also ought to occur on a microscopic elementary particle level because of quantum fluctuations of the metric which could be interpreted as virtual black holes which appear and disappear again [4]. Furthermore, Hawking introduced a new operator, called the superscattering operator, to describe the process. This operator can map the initial mixed states to final mixed states [3].

The evolution of pure states into mixed states has aroused considerable attention in physics. Page showed that any such dynamics can lead to conflict with *CPT* conservation [5]. Banks, Peskin, and Susskind found that, in such a theory which allows the evolution of pure states into mixed states, there is a serious conflict between energy-momentum conservation and locality [6]. Thereafter, Ellis, Hagelin, Nanopoulos, and Srednicki (EHNS) set up a modified Hamiltonian formalism for the time evolution of density matrices which includes violation of quantum mechanics such as the evolution of pure states into mixed states [7]. Following EHNS, Ellis, Mavromatos, and Nanopoulos reconsidered the analysis of EHNS for the K_0-$\overline{K_0}$ system. They suggested that this new source of *CPT* violation might fully account for the observed CP violation in the K_0-$\overline{K_0}$ system [8,9]. But Huet and Peskin using the classic results of the Carithers *et al.* [10] and CERN-Heidelberg experiments [11] and the results from CPLEAR [12] determined the two of the three new CPT-violation parameters α, β, and γ of EHNS. They argued that the CP violation observed in the K_0-$\overline{K_0}$ system is dominantly quantum mechanical in nature and of CPT-conserving origin [13]. Works along this direction are still going on [14,15].

On the other hand, the oscillation among neutrinos of different flavors such as ν_e-ν_μ is much like the strangeness oscillation phenomenon in the K_0-$\overline{K_0}$ system [16]. Because the neutrino oscillations in vacuum and/or in matter are in connection with the solar neutrino experiments and as a possible solution of the solar neutrino problem, it has caused a great interest in this subject for many years [17–19].

In this work, we use the EHNS mechanism to investigate the effect of the quantum mechanics violation proposed by Hawking on neutrino oscillations. To make this paper self-contained, we will introduce the modified Liouville equation of EHNS following Huet and Peskin [13] and the relative formulas about neutrino oscillation in Sec. II. Then we will list some numerical results in Sec. III. The conclusion and a discussion are given in Sec. IV.

II. FORMALISM

The neutrino weak eigenstates may not coincide with the eigenstates of its mass matrix. If such is the case, due to the different time evolution properties, oscillations will occur [16–19].

We consider the simplest case of two neutrinos. Let $|\nu_1\rangle$ and $|\nu_2\rangle$ be the mass eigenstates with masses m_1 and m_2. Suppose that neutrinos mix through a vacuum mixing angle θ; then, the weak eigenstates are

$$|\nu_e\rangle=\cos\theta|\nu_1\rangle+\sin\theta|\nu_2\rangle,$$
$$|\nu_\mu\rangle=-\sin\theta|\nu_2\rangle+\cos\theta|\nu_2\rangle. \tag{1}$$

The two states evolve differently; thus,

$$|\nu_e(t)\rangle=\cos\theta e^{-iE_1t}|\nu_1\rangle+\sin\theta e^{-iE_2t}|\nu_2\rangle,$$
$$|\nu_\mu(t)\rangle=-\sin\theta e^{-iE_1t}|\nu_2\rangle+\cos\theta e^{-iE_2t}|\nu_2\rangle. \tag{2}$$

As a result, a state originally $|\nu_e\rangle$ may oscillate into $|\nu_\mu\rangle$ with the probability

$$p[\nu_e\to\nu_\mu(t)]=\sin^2(2\theta)\sin^2[\tfrac{1}{2}(E_2-E_1)t] \tag{3}$$

and the probability for it to remain as itself is

$$p[\nu_e\to\nu_e(t)]=1-\sin^2(2\theta)\sin^2[\tfrac{1}{2}(E_2-E_1)t]. \tag{4}$$

Because of the smallness of neutrino masses, their energy and momentum are very close; hence, we can rewrite the probability as [20,21]

$$p[\nu_e \to \nu_\mu(t)] = \sin^2(2\theta)\sin^2\left(\frac{\delta m^2 L}{4E}\right),$$

$$p[\nu_e \to \nu_e(t)] = 1 - \sin^2(2\theta)\sin^2\left(\frac{\delta m^2 L}{4E}\right), \tag{5}$$

where

$$\delta m^2 = m_{\nu_\mu}^2 - m_{\nu_e}^2. \tag{6}$$

Numerically, $\delta m^2 L/(4E) = (1.266\,932\ldots)\,\delta m^2 L/E$, where δm^2 is measured in eV^2 and L in m, while E in MeV (or L in km while E in GeV).

Now, let us return to the EHNS mechanism [7]. Our description follows that of Huet and Peskin [13].

In conventional quantum mechanics, the density matrix obeys the evolution equation

$$i\,\frac{d}{dt}\,\rho = [H,\rho]. \tag{7}$$

For a two-state system, the density matrix can be expanded by using the Pauli matrix:

$$\rho = \rho^0 1 + \rho^i \sigma^i, \tag{8}$$

where $i=1,2,3$. When expanding the Hamiltonian in the same way, the equation of motion can be written as

$$\frac{d}{dt}\,\rho = 2\epsilon^{ijk} H^i \rho^j \sigma^k. \tag{9}$$

For including the quantum gravity effects which allows pure states to evolve into mixed states, Hawking proposed that the quantum mechanics evolution should be modified. EHNS added the most general linear term

$$-h^{0j}\rho^j 1 - h^{j0}\sigma^j - h^{ij}\sigma^i\rho^j$$

to Eq. (9) [7]. But two restrictions on these terms are evident. First, the probability must be conservation. Second, the entropy of the density matrix should not decrease. These requirements set $h^{0j}=0$ and $h^{j0}=0$, respectively. In the meantime,

$$(\rho^0)^2 \geqslant \sum_{i=1}^{3} (\rho^i)^2$$

and the submatrix h^{ij} should be positive definite [7]. This leads to the equation

$$\frac{d}{dt}\,\rho = 2\epsilon^{ijk} H^i \rho^j \sigma^k - h^{ij}\sigma^i \rho^j. \tag{10}$$

Because the antisymmetric part of h^{ij} can be absorbed into H^i, we may assume that h^{ij} is symmetric.

EHNS simplify this formalism by imposing one further assumption. For the K_0-$\overline{K_0}$ system, the new term does not change strangeness. This requirement becomes

$$h^{1j} = 0. \tag{11}$$

Drawing an analogy between the ν_e-ν_μ system and the K_0-$\overline{K_0}$ system, we suppose that the ν_e-ν_μ system has the same constraints. So we get

$$h = 2\begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \gamma \end{pmatrix}, \tag{12}$$

where α,β,γ are the EHNS parameters and [7,13]

$$\alpha,\gamma > 0, \quad \alpha\gamma > \beta^2.$$

Finally, we get the evolution equations for the components of the density matrix in the matrix form

$$\frac{d}{dt}\begin{pmatrix} \rho^0 \\ \rho^1 \\ \rho^2 \\ \rho^3 \end{pmatrix} = 2\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -H^3 & H^2 \\ 0 & H^3 & -\alpha & -H^1-\beta \\ 0 & -H^2 & H^1-\beta & -\gamma \end{pmatrix}\begin{pmatrix} \rho^0 \\ \rho^1 \\ \rho^2 \\ \rho^3 \end{pmatrix}. \tag{13}$$

After we solve Eq. (13), we can get the density matrix

$$\rho = \begin{pmatrix} \rho^0+\rho^3 & \rho^1 - i\rho^2 \\ \rho^1 + i\rho^2 & \rho^0 - \rho^3 \end{pmatrix} \tag{14}$$

and the value of the observable o can be given in terms of it:

$$\langle o \rangle = \mathrm{Tr}[\rho\ o]. \tag{15}$$

III. RESULTS

For the neutrino, its Hamiltonian is diagonalized on the basis of $|\nu_1\rangle$, $|\nu_2\rangle$,

$$H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}, \tag{16}$$

and so we have

$$H^1 = H^2 = 0,$$

$$H^3 = (E_1 - E_2)/2 \approx -\delta m^2/(4E). \tag{17}$$

Now, Eq. (13) takes the very simplified form

$$\frac{d}{dt}\begin{pmatrix} \rho^0 \\ \rho^1 \\ \rho^2 \\ \rho^3 \end{pmatrix} = 2\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \delta m^2/(4E) & 0 \\ 0 & -\delta m^2/(4E) & -\alpha & -\beta \\ 0 & 0 & -\beta & -\gamma \end{pmatrix} \times \begin{pmatrix} \rho^0 \\ \rho^1 \\ \rho^2 \\ \rho^3 \end{pmatrix}. \tag{18}$$

In the following, we will consider the case in which the neutrino is originally in the state $|\nu_e\rangle$, from Eq. (1):

$$\rho_{\nu_e} = \begin{pmatrix} \cos^2\theta & \cos\theta\,\sin\theta \\ \cos\theta\,\sin\theta & \sin^2\theta \end{pmatrix} \tag{19}$$

and

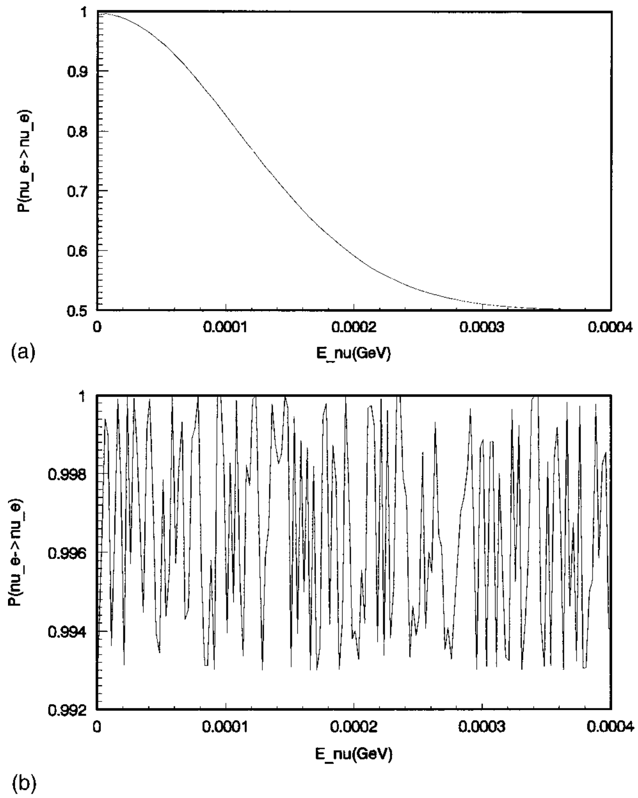

FIG. 1. $P(\nu_e \to \nu_e)$ as a function of E_ν for the small mixing: (a) with quantum mechanics violation and (b) without quantum mechanics violation, where $L=1$ AU$=1.496\times 10^{11}$ m.

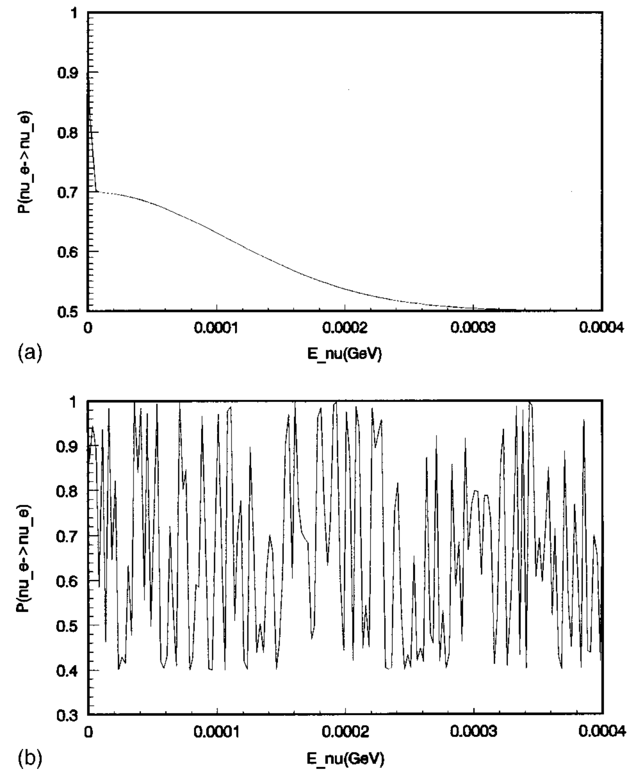

FIG. 2. $P(\nu_e \to \nu_e)$ as a function of E_ν for the large mixing: (a) with quantum mechanics violation and (b) without quantum mechanics violation, where $L=1$ AU$=1.496\times 10^{11}$ m.

$$\rho_{\nu_\mu}=\begin{pmatrix} \sin^2\theta & -\cos\theta\,\sin\theta \\ -\cos\theta\,\sin\theta & \cos^2\theta \end{pmatrix}. \tag{20}$$

Then the initial condition is

$$\rho(t=0)=\rho_{\nu_e}, \tag{21}$$

i.e.,

$$\rho^0=\tfrac{1}{2}, \quad \rho^1=\tfrac{1}{2}\sin(2\theta), \quad \rho^2=0, \quad \rho^3=\tfrac{1}{2}\cos(2\theta). \tag{22}$$

The most important observables are

$$p(\nu_e\to\nu_e)=\mathrm{Tr}[\rho(t)\quad\rho_{\nu_e}] \tag{23}$$

and

$$p(\nu_e\to\nu_\mu)=\mathrm{Tr}[\rho(t)\quad\rho_{\nu_\mu}]. \tag{24}$$

We will investigate the effects of the quantum mechanics violation due to quantum gravity on these two observables, and so we should compare the two cases, i.e., the vacuum oscillation with the EHNS modification and the usual one without the EHNS modification. Notice that in all cases $p(\nu_e\to\nu_e)+p(\nu_e\to\nu_\mu)=1$; i.e., the probability is conserved. So we only need to write down one of them.

Solving Eq. (18) analytically is very tedious, and so we solve it numerically. We should give the EHNS parameters and the relative oscillation parameters of the neutrino first. According to the recent experiments and analysis [8,10,14,22], we take the values as [14]

$$\alpha\leqslant 4\times 10^{-17}\ \mathrm{GeV},$$

$$|\beta|\leqslant 3\times 10^{-19}\ \mathrm{GeV},$$

$$\gamma\leqslant 7\times 10^{-21}\ \mathrm{GeV}.$$

In this paper, we will use the upper limits of them. But it should be pointed out that these parameters are obtained in the neutral kaon system and they are conjectured to be of order $0(m_K^2/M_{\mathrm{Pl}})\approx 2\times 10^{-20}$ GeV [7,13], where M_{Pl} is the Planck mass. For the neutrino, we deal with the ultrarelativity case, as a generalization from the K_0-$\overline{K_0}$ system to the neutrino system; we suppose that these parameters are of order E_ν^2/M_{Pl}. So we multiply them by the factor E_ν^2/m_K^2. Here we take

$$m_K\sim 500\ \mathrm{MeV};$$

then, the EHNS parameters for neutrino that will be used are

$$\alpha=4\times 10^{-17}\times(E_\nu/500)^2\ \mathrm{GeV},$$

$$\beta=3\times 10^{-19}\times(E_\nu/500)^2\ \mathrm{GeV},$$

$$\gamma=7\times 10^{-21}\times(E_\nu/500)^2\ \mathrm{GeV}, \tag{25}$$

where E_ν is measured in MeV.

The neutrino oscillation parameters we take as [21]

$$\delta m^2\sim 6\times 10^{-6}\ \mathrm{eV}^2, \quad \sin^2(2\theta)\sim 7\times 10^{-3}, \tag{26}$$

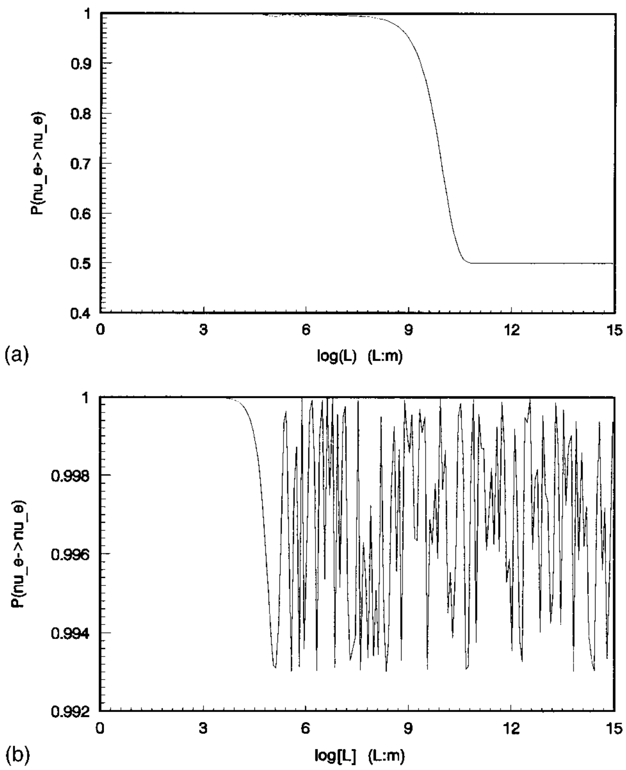

FIG. 3. $P(\nu_e \rightarrow \nu_e)$ as a function of $\log[L]$ for the small mixing: (a) with quantum mechanics violation and (b) without quantum mechanics violation, where $E_\nu = 0.6$ MeV.

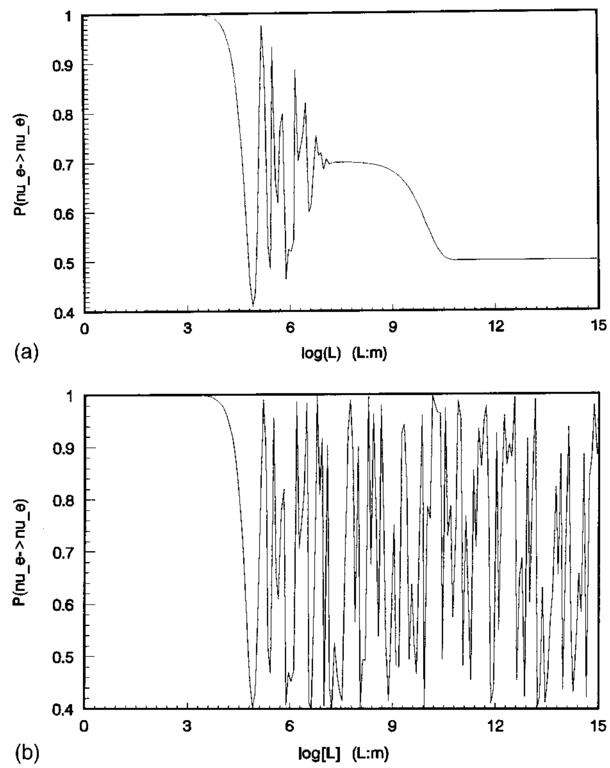

FIG. 4. $P(\nu_e \rightarrow \nu_e)$ as a function of $\log[L]$ for the large mixing: (a) with quantum mechanics violation and (b) without quantum mechanics violation, where $E_\nu = 0.6$ MeV.

for the small-mixing solution, and

$$\delta m^2 \sim 9 \times 10^{-6} \ \text{eV}^2, \quad \sin^2(2\theta) \sim 0.6, \tag{27}$$

for the large-mixing solution.

Substituting these parameters into Eqs. (18)–(24), we can solve $P(\nu_e \rightarrow \nu_e)$ numerically.

For a given distance $L = 1$ AU$= 1.496 \times 10^{11}$ m, the results of the dependence of the neutrino survival probability on the neutrino energy E_ν for the small- and the large-mixing solutions are shown in Figs. 1 and 2, respectively.

For a given energy such as the average of solar neutrino energy $E_\nu \sim 0.6$ MeV [21], the dependence of the survival probability on distance L is given by

$$p(\nu_e \rightarrow \nu_e) \approx 0.5 + 0.0035 e^{-2.92 \times 10^{-7} L} \cos(2.5 \times 10^{-5} L) + 0.4965 e^{-1.022 \times 10^{-10} L}, \tag{28}$$

for the small mixing, and

$$p(\nu_e \rightarrow \nu_e) \approx 0.5 + 0.3 e^{-2.92 \times 10^{-7} L} \cos(3.8 \times 10^{-5} L) + 0.2 e^{-1.022 \times 10^{-10} L}, \tag{29}$$

for the large mixing. The probabilities for the small mixing and the large mixing with no quantum mechanics violation are

$$p(\nu_e \rightarrow \nu_e) \approx 0.9965 + 0.0035 \cos(2.5 \times 10^{-5} L) \tag{30}$$

and

$$p(\nu_e \rightarrow \nu_e) \approx 0.7 + 0.3 \cos(3.8 \times 10^{-5} L), \tag{31}$$

respectively, where L is measured in m. The results are illustrated in Figs. 3 and 4.

IV. CONCLUSION AND DISCUSSION

We have investigated the effect of the quantum mechanics violation on neutrino oscillation. We find that the EHNS mechanism can affect the neutrino oscillation behaviors and hence may be taken as a new solution of the neutrino problem.

It is remarkable that the solutions are exponentially decaying or exponentially increasing [23] and as t or $L \rightarrow \infty$, $P(\nu_e \rightarrow \nu_e)$ decay to $\frac{1}{2}$, while $P(\nu_e \rightarrow \nu_\mu)$ increase to $\frac{1}{2}$. This is compatible with the conclusion of EHNS [7]. Here we have discussed the two-generation case. For the three-generation case, $P(\nu_e \rightarrow \nu_e)$ will decay to $\frac{1}{3}$; this value occurs in the domain 0.25–0.35, which experiments suggest.

For the solar neutrino problem, the matter effect should be considered, but here it does not change the long distance or long time asymptotic characteristic of the neutrino oscillating behavior. Details of the matter effect and generalization to the three-generation case will be reported in the future.

However, because of the indefiniteness of the EHNS parameters α, β, γ and the supposed generalization to the relativity case, our results supply only a qualitative illumination of the effects of quantum mechanics violation on neutrino oscillation.

Recently, Reznik proposed another modified motion equation for the density matrix [23]. It also affects the neutrino oscillation, but differently from EHNS, its solution is oscillated. Further work on this subject is being done.

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