Relic gravitational waves produced after preheating

S. Khlebnikov
Department of Physics, Purdue University, West Lafayette, Indiana 47907

I. Tkachev
Department of Physics, The Ohio State University, Columbus, Ohio 43210
and Institute for Nuclear Research of the Academy of Sciences of Russia, Moscow 117312, Russia
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We show that gravitational radiation is produced quite efficiently in interactions of classical waves created by resonant decay of a coherently oscillating field. As an important example we consider simple models of chaotic inflation, where we find that today's ratio of energy density in gravitational waves per octave to the critical density of the Universe can be as large as 10^{-12} at the maximal wavelength of order 10^5 cm. In the pure $\lambda\phi^4/4$ model with inflaton self-coupling $\lambda = 10^{-13}$, the maximal today's wavelength of gravitational waves produced by this mechanism is of order 10^6 cm, close to the upper bound of operational LIGO and TIGA frequencies. The energy density of waves in this model, though, is likely to be well below the sensitivity of LIGO or TIGA at such frequencies. We discuss the possibility that in other models the interaction of classical waves can lead to an even stronger gravitational radiation background. [S0556-2821(97)01914-0]

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I. INTRODUCTION

Recent research in inflationary cosmology has attracted attention to highly nonequilibrium states created in a decay of a coherently oscillating field after the end of inflation. These states could support a number of nonequilibrium phenomena, such as nonthermal symmetry restoration [1] and baryogenesis [1,2] shortly after or during the decay of the oscillating field.

In this paper we want to show that nonequilibrium states produced by the decay of coherent oscillations of a field are a quite efficient source of a stochastic background of gravitational waves.

There are several possible processes in the early Universe capable of producing a stochastic background of relic gravitational waves. One is the parametric amplification of vacuum graviton fluctuations during inflation [4]. This process is efficient on all superhorizon scales. Waves with lowest frequencies cause inhomogeneities of cosmic microwave background [5]. In conventional scenarios, this restricts the amplitude of high-frequency gravitational waves to be far below [6] the experimental limits accessible for direct detection experiments in the near future; this conclusion changes in superstring-motivated cosmologies [7]. Another source of gravitational radiation is classical emission that accompanies collisions of massive bodies. A natural source in this class in the early Universe is a strongly first-order phase transition when gravitational waves are produced in collisions of bubbles of a new phase [8–10], in particular the phase transition that terminates first-order inflation [9]. Gravitational radiation is also emitted during the decay of a cosmic string network [11].

In this paper we discuss a new source of relic gravitational waves. Coherent oscillations of a scalar field can produce large fluctuations of Bose fields via parametric amplification (parametric resonance) [12]. As was shown in Ref. [13] (see also Ref. [14] for a discussion of a quantum-to-classical transition in a different context), when fluctuations amplified in this manner reach sufficiently large values, they become essentially classical. Namely, at that time, the fields and their canonical momenta approximately commute, and the quantum averages can be approximated by classical averages over realizations of the inital data. The classical fluctuations emerging in this way can be viewed either as classical waves traveling through the Universe or, at least qualitatively, as quantum ''particles'' in states with large occupation numbers. The fluctuations interact with the oscillating background and one another. This interaction, which we call rescattering [13], is accompanied by gravitational radiation. That is the effect we want to estimate.

Favorable conditions for an effective parametric resonance in cosmology naturally appear in inflationary models [15,16]. We consider two types of simple inflationary models here. One type of model has two scalar fields with an interaction potential of the form $g^2\phi^2X^2/2$, and the resonance produces mostly fluctuations of a scalar field X other than the field ϕ that oscillates (although subsequent rescattering processes produce large fluctuations of the field ϕ as well). We consider a range of moderate values of the coupling g^2 (see below); in this case fluctuations of X are not suppressed too strongly by nonlinear effects (cf. Refs. [17–19]). In simplest models of this type, ϕ is the inflaton itself. For these models, we find that typically $\sim 10^{-5}$ of the total energy of the Universe goes into gravitational waves, at the time of their production. The minimal today's frequency $f_{\rm min}$ of these waves is typically of order 10^5 Hz, and today's spectral density at this frequency can be as large as 10^{-12} of the critical density.

Another type of model we considered was the pure $\lambda\phi^4$ model of chaotic inflation. In this model, the minimal today's frequency $f_{\rm min}$ is of order 10^4 Hz, close to the upper bound of the operational Laser Interferometer Gravitational Wave Observatory (LIGO) and Truncated Icosahedral Gravitational Wave Antenna (TIGA) frequencies [3]: 10 Hz $\lesssim f_{\rm LIGO} \lesssim 10^4$ Hz. We do not yet have efficient means of extrapolating our numerical results for today's spectral intensity to the minimal frequency of this model, but we do not expect it to be above 10^{-11} of the critical density. That would be well below the sensitivity of LIGO or TIGA at frequencies of order 10^4 Hz.

Nevertheless, as the precise content of Bose fields that could be important in the dynamics in the early Universe is at present unknown, we believe it is premature to rule out the possibility of experimental detection, even already by LIGO or TIGA, of gravity waves produced by the mechanism that we consider here. At the end of this paper, we discuss the possibility of a stronger background of gravitational waves in models with more fields or more complicated potentials.

II. GENERAL FEATURES OF POST-INFLATIONARY DYNAMICS

In scenarios where inflation ends by resonant decay of coherent field oscillations, post-inflationary dynamics has two rapid stages. At the first of these, called preheating [15], fluctuations of Bose fields interacting with the oscillating field grow exponentially fast, as a result of parametric resonance, and achieve large occupation numbers. At the second stage, called semiclassical thermalization [13], rescattering of produced fluctuations smears out the resonance peaks in power spectra and leads to a slowly evolving state, in which the power spectra are smooth [13,17,18]. The system begins to exhibit chaotic behavior characteristic of a classical nonlinear system with many degrees of freedom. In the course of subsequent slow evolution, the power spectra propagate to larger momenta; we expect that eventually this process will lead to a fully thermalized state (which does not admit a semiclassical description).

In many models, we find that during the stage of semiclassical thermalization, or chaotization, fluctuations grow somewhat beyond their values at the end of, the resonance stage. In such cases, the most effective graviton production takes place at the end of, and shortly after, the chaotization stage.

In order not to confine ourselves to any particular type of inflationary scenario, we will not assume that the oscillating field ϕ is the inflaton itself. (It is distinct from the inflaton, for example, in hybrid inflationary models [20].) Let us denote the amplitude of the zero-momentum mode of this field, ϕ_0, at the end of the chaotization stage as $\phi_{\rm ch}$, the frequency of its oscillations as m, and the Hubble parameter at that time as $H_{\rm ch}$. These same parameters at the end of inflation, when the oscillations start, will be denoted as $\phi(0)$, $m(0)$, and $H(0)$. In simplest models of chaotic inflation, in which ϕ is the inflaton, $\phi(0)\sim M_{\rm Pl}$, and $H(0)\sim m(0)$. Although we will use such models for illustrative purposes, our general formulas do not assume these conditions. Oscillations of ϕ cannot start unless $m(0)\gtrsim H(0)$, but on the other hand, they can start at $m(0)\gg H(0)$, if they have to be triggered by some other field. Even if $H(0)\sim m(0)$, as in simple models of chaotic inflation, we still have

$$H_{\rm ch}<H_r\ll m, \tag{1}$$

where H_r is the Hubble parameter at the end of the resonance stage. This is because the frequency of oscillations redshifts slower (if at all) than the Hubble parameter. We will use condition (1) in what follows.

We will consider models, in which oscillating field ϕ interacts with a massless scalar field X. The Lagrangians for these models are of the form

$$L=\tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+\tfrac{1}{2}g^{\mu\nu}\partial_\mu X\partial_\nu X-V(\phi,X), \tag{2}$$

where

$$V(\phi,X)=V_\phi(\phi)+\tfrac{1}{2}g^2\phi^2X^2, \tag{3}$$

and V_ϕ is a potential for the field ϕ. We will consider two types of V_ϕ: $V_\phi=\frac{1}{2}m^2\phi^2$ (massive ϕ) and $V_\phi=\frac{1}{4}\lambda\phi^4$ (massless ϕ). The effect we discuss is not limited to these particular models and exists in a wide variety of inflationary models that have a preheating stage. Note that in the model with massless ϕ, the frequency m of oscillations of ϕ_0 at the end of chaotization is $m\sim\sqrt{\lambda}\phi_{\rm ch}$.

The oscillating ϕ amplifies fluctuations of X via parametric resonance. An important parameter in the problem is the resonance parameter q. Depending on the choice of the potential for ϕ (see above), $q=g^2\phi^2(0)/4m^2$ (massive ϕ) or $q=g^2/4\lambda$ (massless ϕ).

In the case of massive ϕ, it is useful to introduce, in addition to the resonance parameter q, the redshifted resonance parameter at the end of the resonance stage

$$q_r=q\frac{\overline{\phi^2}(t_r)}{\phi^2(0)}, \tag{4}$$

where t_r is the time corresponding to the end of the resonance stage, and $\overline{\phi(t_r)}$ is the amplitude of oscillations of ϕ_0 at that time. In the model with massive ϕ and massless X, parametric resonance can fully develop in an expanding Universe if $q_r\gtrsim 1$ [17]. Similarly, we can introduce

$$q_{\rm ch}=\frac{q\phi_{\rm ch}^2}{\phi^2(0)}. \tag{5}$$

For uniformity of notation, we will sometimes use q_r or $q_{\rm ch}$ instead of q in the case of massless ϕ; there is no difference between the three of these in that case.

Resonant production is most effective for fluctuations of fields that couple to ϕ not too weakly but also not too strongly, those with $q_r\sim 1$. For $q_r\gg 1$, the maximal size of X fluctuations is significantly suppressed by nonlinear effects [17–19]. Because, for instance, superstring models predict a plethora of scalar fields, we expect that some of those will have couplings in the optimal range. So, in what follows we consider moderate values of q_r, $1\lesssim q_r\lesssim 100$.

In the model with massive ϕ, the oscillating zero-momentum mode ϕ_0 drops rapidly at the end of the chaotization stage, and all its energy at that time is transferred to fluctuations [18]. The variance of ϕ, $\langle(\delta\phi)^2\rangle$, at the end of chaotization is thus much larger than $\phi_{\rm ch}^2$. This does not happen in the model with massless ϕ. So, in what follows we consider two cases: $\langle(\delta\phi)^2\rangle\lesssim\phi_{\rm ch}^2$ and $\langle(\delta\phi)^2\rangle\gg\phi_{\rm ch}^2$.

For $q\gtrsim 1$ in the model where a massless ϕ interacts with X, parametric resonance for ϕ itself is insignificant. To study a case different in that respect, we consider also the pure $\lambda\phi^4/4$ model, in which ϕ decays solely due to self-coupling [$g^2=0$ in Eq. (3)].

III. CALCULATIONAL PROCEDURES

Because the time scale of processes that give rise to gravitational radiation is much smaller than the time scale of the expansion of the Universe $H_{\rm ch}^{-1}$, the energy of gravitational waves can be approximately computed starting from the well-known formulas for flat space-time. Total energy of gravitational waves radiated in direction $\mathbf{n}=\mathbf{k}/\omega$ in flat space-time is [21]

$$\frac{dE}{d\Omega}=2G\Lambda_{ij,lm}(\mathbf{n})\int_0^\infty \omega^2 T^{ij*}(\mathbf{k},\omega)T^{lm}(\mathbf{k},\omega)d\omega, \quad (6)$$

where $T^{ij}(\mathbf{k},\omega)$ are Fourier components of the stress tensor, and $\Lambda_{ij,lm}$ is a projection tensor made of the components of $\mathbf{n}$ and Kronecker's δ's:

$$\Lambda_{ij,lm}(\mathbf{n})=\delta_{il}\delta_{jm}-2n_jn_m\delta_{il}+\tfrac{1}{2}n_in_jn_ln_m-\tfrac{1}{2}\delta_{ij}\delta_{lm} +\tfrac{1}{2}\delta_{ij}n_ln_m+\tfrac{1}{2}\delta_{lm}n_in_j. \quad (7)$$

For models with two fields (ϕ and X) with potentials (3) and moderate q_r, we will present both analytical estimates and numerical calculations based on Eq. (6). For analytical estimates, we choose a specific process—gravitational bremsstrahlung that accompanies creation and annihilation of fluctuations of the field ϕ. This is not the only process that can produce a significant amount of gravitational radiation. For example, because the equations of motion for fluctuations contain oscillating terms, due to the interaction with ϕ_0, the collection of X and ϕ fluctuations works (until ϕ_0 decays completely) as a gravitational antenna. The reason why we concentrate on bremsstrahlung from ϕ is that it is a significant source of gravity waves with *small frequencies*. Indeed, we will see that both the frequency dependence and the overall magnitude of the effect are reasonably close to those of the *full* intensity for small frequencies that we obtain numerically. So, bremsstrahlung from ϕ appears to be at least one of the main sources of gravitational radiation with small frequencies in the states we consider here.

Numerical calculations were done for simple models of chaotic inflation, in which ϕ is the inflaton itself. The calculations were done in conformal time, in which the evolution of the system is Hamiltonian (for more detail, see Ref. [17]), and then rescaled back to the physical time. We computed directly Fourier transforms $T^{ij}(\mathbf{k},\omega)$ over successive intervals of conformal time, the duration of which was taken small compared to the time scale of the expansion yet large enough to accommodate the minimal frequency of gravity waves we had in these calculations. Intensities from different intervals were summed up with the weight that takes into account expansion of the Universe (see more on that below). Being a finite size effect, the minimal frequency of our numerical calculations was much larger than the actual minimal frequency of gravity waves, $\omega_{\rm min}\sim H_{\rm ch}$. In the models with two fields (ϕ and X), the spectrum obtained numerically is approximated reasonably well by the bremsstrahlung spectrum at small frequencies. This allows us to extrapolate the numerical results to $\omega\sim\omega_{\rm min}$ using the frequency dependence of bremsstrahlung.

For the pure $\lambda\phi^4/4$ model, the analytical method that we use to estimate the intensity of radiation does not apply at frequencies for which we have numerical data. So, we do not have efficient means of extrapolating our numerical results to $\omega_{\rm min}$. For this model, we contented ourselves with numerical simulations.

IV. ANALYTICAL ESTIMATES

In general, X and ϕ fluctuations produced by parametric resonance scatter off the homogeneous oscillating background (condensate) of ϕ, knocking ϕ out of the condensate and into modes with nonzero momenta. In cases when resonance amplifies mostly fluctuations of X, the scattering process can be viewed as a decay of X, $X_{\mathbf{k}}\to X_{\mathbf{k}'}+\phi_{\mathbf{p}}$, in the time-dependent background field of the condensate. It can also be thought of as evaporation of the inflaton condensate. There is also the inverse process (condensation).

We will assume that bremsstrahlung from ϕ is at least one of the main sources of gravity waves with small frequencies (we will specify the notion of ''small'' frequency below). For estimation purposes, we will neglect its interference with possible small-frequency radiation from other sources.

As we noted in the Introduction, one can describe the states we are considering either as collections of interacting classical waves, or as collections of ''particles'' in modes with large occupation numbers. For estimating the intensity of bremsstrahlung that accompanies creation and annihilation of fluctuations of the field ϕ, the description of these fluctuations in terms of particles is more convenient.

For the notion of particle, i.e., an entity moving freely after it has been created (or before it is destroyed), to have anything but purely qualitative meaning, the energy of a freely moving ''particle'' should not be modulated too strongly by its interactions with the background. In the models (2) with $q_r\gtrsim 1$, fluctuations of ϕ are reasonably well described as particles, even in the case of massless ϕ where they are coupled to the oscillating ϕ_0. Indeed, typical momenta p of ϕ fluctuations at the end of the chaotization stage are of order m, the frequency of oscillations of ϕ_0 at that time. The frequency of oscillations of a mode with momentum $p\sim m$ (the would-be energy of a particle) is only moderately modulated by its coupling to the oscillating ϕ_0. Note also that for $q_r\gtrsim 1$, the Hartree correction to the frequency squared of ϕ, g^2X^2, does not exceed m^2 itself [18], so interaction with X also does not modulate frequencies of fluctuations of ϕ too much.

So, let us consider fluctuations of ϕ as particles, neglecting modulations of their energy, and estimate the intensity of gravitational bremsstrahlung emitted by these particles in the scattering processes described above.

As we will see shortly, to describe radiation with small enough frequencies, we can regard the particles—quanta of ϕ—as having both position and momentum reasonably well defined and thus behaving in that respect as classical particles. Consider the stress tensor of a single classical particle:

$$T_1^{ij}(\mathbf{x},t)=f(t)\,\frac{p^i p^j}{p^0}\,\delta^3(\mathbf{x}-\mathbf{r}(t)), \tag{8}$$

where p^μ and $\mathbf{r}$ are the particle's time-dependent four-momentum and position. The function $f(t)$ describes how the particle was created or destroyed; if the creation and destruction processes can be regarded as instantaneous, $f(t)=\theta(t-t_1)$ for a particle created at time t_1, and $f(t)=\theta(t_2-t)$ for a particle destroyed at time t_2. Using identity transformations, we write the Fourier transform of the stress tensor (8) as

$$\omega T_1^{ij}(\mathbf{k},\omega)=\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{p^i p^j}{p^0(1-\mathbf{nv})}f(t)\frac{\partial}{\partial t}e^{i\omega t-i\mathbf{kr}}dt, \tag{9}$$

where $\mathbf{v}\equiv\dot{\mathbf{r}}$ is the particle's velocity. The uncertainty that we can tolerate in the position of a particle, to be able to use this formula, is of order $1/k$. Uncertainty we can tolerate in the momentum is of order of the momentum spread among the particles Δp. Thus, both the position and the momentum can be sufficiently well defined when $\omega=k\ll\Delta p$. In the models with two fields, at the end of chaotization, $\Delta p\sim m$, so Eq. (9) applies for $\omega\ll m$.

On the other hand, in the pure $\lambda\phi^4/4$ model, fluctuations do not grow during chaotization, so for the most efficient graviton production we should consider the end of the resonance stage. The power spectrum of fluctuations at the end of resonance, and long into chaotization, is concentrated in rather narrow peaks, so that $\Delta p\ll m$. Then, Eq. (9) applies only at quite low frequencies, much lower than those we had in numerical simulations. As a result, for this model, we could not check our analytical estimates against numerical simulations.

In the rest of this section we consider the models with two fields. For $\omega\ll m$, not only we can use the semiclassical expression for the stress tensor of ϕ, but in addition each act of evaporation (or condensation) of a ϕ particle can be regarded as instantaneous. So, we can use the small-frequency approximation, familiar from similar problems in electrodynamics [22]. It amounts to integrating by parts in Eq. (9) and then replacing the exponential with unity. After that, the integral is trivially taken and depends only on the particle's final (or initial) momentum.

Substituting the small-frequency limit of Eq. (9) in Eq. (6), we obtain the energy of gravitational waves radiated in a single act of evaporation (or condensation):

$$\frac{d^2E}{d\Omega d\omega}=\frac{G}{(2\pi)^2}\,\frac{p^4\sin^4\theta}{p_0^2(1-v\cos\theta)^2}, \tag{10}$$

where p is the magnitude of the final (or initial) momentum $\mathbf{p}$ of the ϕ particle, $v=p/p_0$, and θ is the angle between $\mathbf{p}$ and the radiation wave vector $\mathbf{k}$.

To obtain the total power $\mathcal{P}$ radiated by a unit volume, we multiply Eq. (10) by the rate at which ϕ fluctuations are created (or destroyed), add the contributions from the creation and annihilation processes together, and integrate the result over momenta with $n_\phi(p)/(2\pi)^3$, where $n_\phi(p)$ are the occupation numbers of ϕ fluctuations, and also over the solid angle Ω. The rates of the creation and annihilation processes are almost equal (see below). We obtain

$$\frac{d\mathcal{P}}{d\omega}\approx 4G\int_0^\infty n_\phi(p)\mathcal{R}(p)F(v)\,\frac{p^4dp}{(2\pi)^3}, \tag{11}$$

where $\mathcal{R}(p)$ is the evaporation (condensation) rate and the function

$$F(v)=\frac{4}{v^2}\left(2-\frac{4}{3}v^2-\frac{1-v^2}{v}\ln\frac{1+v}{1-v}\right) \tag{12}$$

arises after integration over direction of particle's momentum; we have assumed that both the occupation numbers and the rate depend only on the absolute value of momentum. Notice that the spectrum (11) is ω independent, as characteristic of small-frequency bremsstrahlung.

To estimate the rate $\mathcal{R}$, we will consider, qualitatively, X fluctuations also as ''particles'' characterized by their own occupation numbers $n_X(k)$. This is not a strictly defensible view, as X fluctuations are strongly coupled to the oscillating background, but it should do for estimation purposes.

Consider first the case $\langle(\delta\phi)^2\rangle\lesssim\phi_{\rm ch}^2$. The rate $\mathcal{R}(p)$ can then be estimated as

$$\mathcal{R}(p)\sim\frac{g^4\phi_{\rm ch}^2}{m}\int\frac{d^3k}{(2\pi)^3}\,\frac{n_X(\mathbf{k})n_X(\mathbf{k}-\mathbf{p})}{8\omega_{\mathbf{k}}\omega_{\mathbf{k}-\mathbf{p}}p^0}, \tag{13}$$

where a factor of $1/m$ appears instead of the usual energy δ function because energy of the ''particles'' participating in the process is not conserved, due to time dependence of ϕ_0. This factor estimates the time scale at which energy nonconservation sets in; in our case, it is the period of the oscillations of ϕ_0.

Notice that the rate (13) is larger than the net kinetic rate, which would enter a kinetic equation for ϕ, by a factor of order of a typical occupation number of X. The net kinetic rate is the difference between two (large) numbers, the rate at which collisions supply particles to a given mode and the rate at which they remove them. But each collision is accompanied by bremsstrahlung, so $\mathcal{R}$ is not the net kinetic rate but a rate of the ''in'' and ''out'' processes separately.

We can use the following estimates [18]: $p\sim k\sim m,\omega_{\mathbf{k}}^2\sim g^2\phi_{\rm ch}^2$, $n_X\sim 1/g^2$. Using these estimates, we obtain $\mathcal{R}(p)\sim m/g^2$, for $p\sim m$. Such a large value of the rate $[\mathcal{R}(p)\gg m]$ means that a typical collision produces not a single ϕ particle but a ''complex'' of $O(1/g^2)$ particles with almost equal values of momentum. At the end of chaotization, $n_\phi\sim q_{{\rm ch}/g^2}$ (so that $\langle\delta\phi^2\rangle\sim\phi_{\rm ch}^2$ [18]). Using that we obtain for the power

$$\frac{d\mathcal{P}}{d\omega}\sim\frac{m^4\phi_{\rm ch}^2}{g^2M_{\rm Pl}^2}. \tag{14}$$

An estimate for the total energy density of gravitational waves is obtained by multiplying the power $\mathcal{P}$ by the time Δt during which the radiation was substantial. For the ratio of $\rho_{\rm GW}$ per octave to the total energy density of the Universe after chaotization, we obtain

$$\left(\frac{1}{\rho_{\rm tot}}\frac{d\rho_{\rm GW}}{d\ln\omega}\right)_{\rm ch}\sim\frac{m^4\phi_{\rm ch}^2}{g^2M_{\rm Pl}^2\rho_{\rm tot}}\omega\Delta t\sim\frac{m^2}{g^2M_{\rm Pl}^2}\omega\Delta t, \qquad (15)$$

where in the last relation we used $\rho_{\rm tot}\sim m^2\phi_{\rm ch}^2$. The time during which the radiation was substantial is determined by the redshift, $\Delta t\sim H_{\rm ch}^{-1}$. For the model with massless ϕ recall that $m^2\sim\lambda\phi_{\rm ch}^2$.

For the opposite case, $\langle(\delta\phi)^2\rangle\gg\phi_{\rm ch}^2$, we need to replace $\phi_{\rm ch}^2$ in estimates (13)–(15) by $\langle(\delta\phi)^2\rangle$, and to use $\rho_{\rm tot}\sim m^2\langle(\delta\phi)^2\rangle$. We see that the final estimate in Eq. (15) remains unchanged.

These estimates apply at the end of the chaotization stage and extend into the infrared up to frequencies of the order of the horizon scale at that time, $\omega\sim H_{\rm ch}$. (We will present estimates for today's $\rho_{\rm GW}$ below.)

As an example, consider the model with massive ϕ in which ϕ is the inflaton itself, and $m=10^{-6}M_{\rm Pl}$. Take $g^2=10^{-6}$, which in this case corresponds to $q\approx2\times10^4$ [17]. We obtain $\rho_{\rm tot}^{-1}d\rho_{\rm GW}/d\ln\omega\sim10^{-6}$ at $\omega\sim H_{\rm ch}$. (This number will be three orders of magnitude larger at $w\sim m$).

Our analytical estimates are rather crude, as we were somewhat cavalier with numerical factors. They do bring out, however, not only the frequency dependence of the effect but also its parametric dependence. Notice, for example, that if the last estimate in Eq. (15) contained a different power of q^2, that would change the estimate by many orders of magnitude, in drastic disagreement with our numerical results.

V. BACKGROUND GRAVITATIONAL RADIATION TODAY

Let us now translate the above estimates into estimates for gravitational background radiation today. It is important that gravitational radiation produced after preheating has not interacted with matter since then [23].

First, let us estimate the physical wavelength today, $l=2\pi/k_0$, that corresponds to a wave vector $k_{\rm ch}$ at the end of the chaotization stage. We have $k_0=k_{\rm ch}a_{\rm ch}/a_0$, where a_0 is the scale factor today. Thermal equilibrium was established at some temperature T_*, and the Universe was radiation dominated at that time. This happened when relevant reaction rates became equal to the expansion rate. We can find the Hubble parameter at that time as $H_*^2=8\pi G\rho_*/3$, where $\rho_*=g_*\pi^2T_*^4/30$, and $g_*\equiv g(T_*)$; $g(T)$ stands for the effective number of ultrarelativistic degrees of freedom at temperature T. The expansion factor from $T=T_*$ down to $T=T_0\approx3$ K is given by $a_*/a_0=(g_0/g_*)^{1/3}(T_0/T_*)=(g_0^{1/3}/g_*^{1/12})(8\pi^3/90)^{1/4}(T_0/\sqrt{H_*M_{\rm Pl}})$, where $g_0\equiv g(T_0)$. Depending upon model parameters, the Universe could expand from $T=T_{\rm ch}$ to $T=T_*$ as matter or radiation dominated. We can write $H_*=H_{\rm ch}(a_{\rm ch}/a_*)^\alpha$, where $\alpha=2$ for the radiation-dominated case and $\alpha=3/2$ for a matter-dominated Universe. We obtain $k_0=k_{\rm ch}a_{\rm ch}/a_0=k_{\rm ch}(a_{\rm ch}/a_*)(a_*/a_0)\approx1.2k_{\rm ch}(H_{\rm ch}M_{\rm Pl})^{-1/2}(a_{\rm ch}/a_*)^{1-\alpha/2}T_0$. The model-dependent factor $(a_{\rm ch}/a_*)$ does not enter this relation in the radiation-dominated case and enters in power 1/4 for a matter-dominated Universe. In the simple models that we consider below, this factor gives a not very important correction (at most an order of magnitude). For today's wavelength, corresponding to wave vector $k_{\rm ch}$ at the end of chaotization, we find $l=2\pi/k_0\approx0.5(M_{\rm Pl}H_{\rm ch})^{1/2}k_{\rm ch}^{-1}(a_*/a_{\rm ch})^{1-\alpha/2}$ cm.

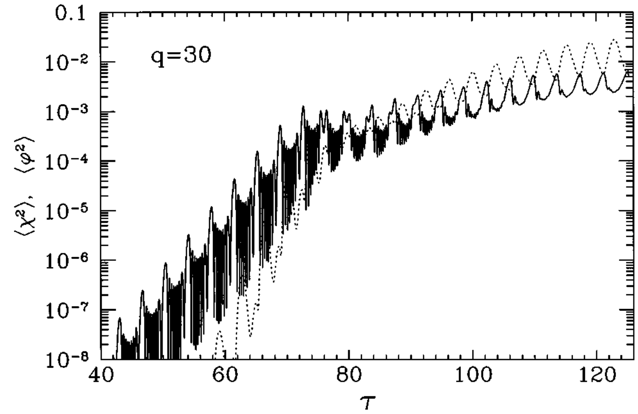

FIG. 1. Variances of fields X (solid curve) and ϕ (dotted curve) as functions of conformal time in the model with massless inflaton for $\lambda=10^{-13}$ and $q=30$.

The smallest wave vector of radiation that could be produced at the end of chaotization is of order $H_{\rm ch}$. The corresponding maximal today's wavelength $l_{\rm max}$ will fall into the range of the LIGO detector, i.e., $l_{\rm max}>3\times10^6$ cm, when $H_{\rm ch}<10^5$–10^6 GeV.

For illustration, let us consider simple models of chaotic inflation with potentials (3), where the oscillating field ϕ is the inflaton itself. The Hubble parameter at the end of the chaotization stage in these models can be extracted from numerical integrations of Refs. [13,17,18]. For example, for massless inflaton we get $H_{\rm ch}/M_{\rm Pl}=(3\lambda/2\pi)^{1/2}\tau_{\rm ch}^{-2}$, where $\tau_{\rm ch}$ is conformal time at the end of the chaotization stage. For $\lambda=10^{-13}$ and, say, $q=30$, we get (see Fig. 1) $\tau_{\rm ch}\approx125$. This gives $H_{\rm ch}/M_{\rm Pl}\approx1.4\times10^{-11}$ and $l_{\rm max}\approx1.3\times10^5$ cm. Another example is the case of massive inflaton with $m=10^{-6}M_{\rm Pl}$ and $q=10^4$. Here we find $H_{\rm ch}/M_{\rm Pl}\approx2\times10^{-9}$. This gives $l_{\rm max}$ in the range 10^4–10^5 cm.

An interesting case is that of the pure $\lambda\phi^4/4$ model [$g^2=0$ in Eq. (3)]. In this case, fluctuations do not grow during chaotization (they actually decrease due to redshift), so the most efficient production of gravity waves takes place after the end of the resonance (preheating) stage. So, instead of $H_{\rm ch}$ in the above formulas we use H_r, the Hubble parameter at the end of resonance. We have $H_r/M_{\rm Pl}=(3\lambda/2\pi)^{1/2}\tau_r^{-2}$, where τ_r is conformal time at the end of resonance. In this model, resonance develops slower than that in models with $q_r\gtrsim1$. As a result, H_r is smaller, and today's maximal wavelength is larger. For $\lambda=10^{-13}$, we get $\tau_r\approx500$ [13], and $H_r/M_{\rm Pl}\approx9\times10^{-13}$, which gives, for radiation produced at that time, $l_{\rm max}\approx5\times10^5$ cm. Note, that by the time $\tau\approx1000$ the inflaton still has not decayed in this model, fluctuations are still highly nonequilibrium, and gravity waves generation will continue even at later time. This will increase $l_{\rm max}$ by at least another factor of 2 bringing it close to the upper boundary of operational LIGO and TIGA frequencies.

Now, let us estimate today's intensity of radiation. The today's ratio of energy in gravity waves to that in radiation is related to $(\rho_{\rm GW}/\rho_{\rm tot})_{\rm ch}$ via

$$\left(\frac{\rho_{\rm GW}}{\rho_{\rm rad}}\right)_0=\left(\frac{\rho_{\rm GW}}{\rho_{\rm tot}}\right)_{\rm ch}\left(\frac{a_{\rm ch}}{a_*}\right)^{4-2\alpha}\left(\frac{g_0}{g_*}\right)^{1/3}. \qquad (16)$$

For the often-used parameter $\Omega_g(\omega)\equiv(\rho_{\rm crit}^{-1}d\rho_{\rm GW}/d\ln\omega)_0$, where $\rho_{\rm crit}$ is the critical energy density, we obtain, using Eq. (15),

$$\Omega_g(\omega)h^2\sim\Omega_{\rm rad}h^2\frac{m^2}{g^2M_{\rm Pl}^2}\frac{\omega}{H_{\rm ch}}\left(\frac{a_{\rm ch}}{a_*}\right)^{4-2\alpha}\left(\frac{g_0}{g_*}\right)^{1/3}, \qquad (17)$$

where $\Omega_{\rm rad}$ is today's value of the ratio $\rho_{\rm rad}/\rho_{\rm crit}$: $\Omega_{\rm rad}h^2=4.31\times10^{-5}$ [24]. When the oscillating zero-momentum mode decays completely during the stage of chaotization, $\phi_{\rm ch}^2$ in Eq. (17) is replaced by $\langle(\delta\phi)^2\rangle_{\rm ch}$.

For example, in the case of massless inflaton with $\lambda=10^{-13}$ and $q=30$, we have $\phi_{\rm ch}^2\sim10^{-5}M_{\rm Pl}^2$, and with $g_*/g_0\sim100$ the estimate (17) gives $\Omega_gh^2\sim10^{-12}$ at $l_{\rm max}\sim10^5$ cm.

VI. NUMERICAL RESULTS

Using the fully nonlinear method of Ref. [13], we have studied numerically gravitational radiation in the model where ϕ is the massless inflaton with $V_\phi=\lambda\phi^4/4$. We have considered two distinct cases: the pure $\lambda\phi^4$ model and the model where the inflaton interacts with a massless field X according to Eq. (3). In either case, the model is classically conformally invariant, and, in conformal variables, the equations of motion reduce to those in flat space-time. The rescaled conformal time τ is related to time t by $\sqrt{\lambda}\phi(0)dt=a(\tau)d\tau/a(0)$. The rescaled conformal fields χ and φ are related to the original fields by $X=\chi\phi(0)a(0)/a(\tau)$ and $\phi=\varphi\phi(0)a(0)/a(\tau)$. In this model, $\phi(0)\approx0.35M_{\rm Pl}$ and $a(\tau)/a(0)\approx0.51\tau+1$ [17].

In the conformal variables, the equations of motion become

$$\ddot\varphi-\nabla^2\varphi+\varphi^3+4q\chi^2\varphi=0,$$

$$\ddot\chi-\nabla^2\chi+4q\varphi^2\chi=0. \qquad (18)$$

We recall that $q\equiv g^2/4\lambda$. We have solved these equations of motion directly in the coordinate space, on a 128^3 spatial lattice, in a box with periodic boundary conditions. The initial conditions for fluctuations correspond to conformal vacuum at the time when the oscillations of φ_0 start [13,18]. The initial conditions for the coherently oscillating inflaton's zero-momentum mode are $\varphi_0(0)=1$, $\dot\varphi_0(0)=0$.

Using fast Fourier transform (FFT), we monitored the power spectra, $P_\phi(k)$ and $P_\chi(k)$, of the fields φ and χ. These power spectra are proportional to $\varphi_{\mathbf k}^*\varphi_{\mathbf k}$ and $\chi_{\mathbf k}^*\chi_{\mathbf k}$ averaged over the direction of $\mathbf{k}$, where $\varphi_{\mathbf k}$ and $\chi_{\mathbf k}$ are the Fourier transforms of the fields. We normalized the power spectra in such a way that Parseval's theorem reads $\int d^3kP_\varphi(k)=L^{-3}\int d^3x[\varphi(x)-\varphi_0]^2\equiv{\rm Var}(\varphi)$, and similarly for the field χ. Since the system is on average homogeneous, the volume average in the last equation is equivalent to an average over realizations of the initial data (the ensemble average): ${\rm Var}(\varphi)=\langle[\varphi(x)-\langle\varphi\rangle]^2\rangle$.

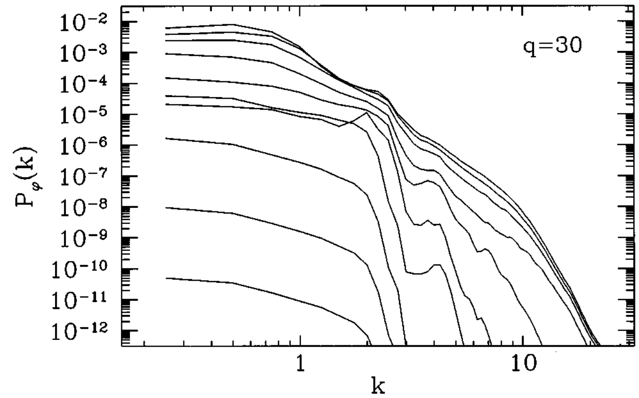

FIG. 2. Power spectrum of the field ϕ for the same model as in Fig. 1, output every period at the maxima of $\varphi_0(\tau)$; k is rescaled comoving momentum (see text).

The time evolution of the variances $\langle\chi^2\rangle$ and $\langle(\delta\varphi)^2\rangle$ for resonance parameter $q=30$ (taken as an exmaple) is shown in Fig. 1. We see that in this case, the parametric resonance ends at $\tau\approx73$. The resonance stage is followed by a plateau (cf. Refs. [17,18]). At the plateau, the variances of fluctuations do not grow, but an important restructuring of the power spectrum of χ takes place. The power spectrum of χ changes from being dominated by a resonance peak at some nonzero momentum to being dominated by a peak near zero. (For some q, though, the strongest peak is close to zero already during the resonance.) When the peak near zero becomes strong enough, the growth of variances resumes (in Fig. 1, that happens at $\tau\approx84$), and the system enters the chaotization stage.

The power spectrum of ϕ during the time interval of Fig. 1 is shown in Fig. 2. The rescaled comoving momentum k is related to the physical momentum $k_{\rm phys}$ as $k_{\rm phys}=\sqrt{\lambda}\phi(0)a(0)k/a(\tau)$. Note that in the range $1\lesssim k\lesssim10$, the power spectrum is approximately a power law, as characteristic of Kolmogorov spectra [25].

Interaction of the fields with gravity is not conformal, and the flat-space-time formula (6) for the energy of gravity waves can be used only after we make an approximation. The approximation replaces the actual expanding Universe with a sequence of static Universes. Specifically, conformal time was divided into steps of $\Delta\tau=L$, where L is the size of the integration box, and at each step the physical variables (fields, frequency, and momenta) were obtained from the conformal ones, using for $a(\tau)$ the actual scale factor taken at the middle point of the step. The energy of gravitational waves was computed at each step using Eq. (6), with the corresponding physical variables. Namely, we calculated the stress tensor $T_{ij}(\mathbf{x},t)$ of the physical fields on solutions to Eq. (18) and then used FFT to find its Fourier transform $T_{ij}(\mathbf{k},\omega)$ in the box $L^3\times\Delta\tau$ for each time step. Note, that the potential term of T_{ij} does not contribute to Eq. (6), $\Lambda_{ij,lm}g^{ij}g^{lm}=0$, so we need to calculate only the "kinetic" part of the stress tensor, $T_{ij}^{\rm kin}\equiv\partial_i\phi\partial_j\phi+\partial_iX\partial_jX$ (in the case with two fields). Also, since the radiation is isotropic, we calculated the right-hand side of Eq. (6) for one direction only, $\mathbf{n}=(1,0,0)$. Finally, the energies of gravitational waves produced at all the steps were summed up.

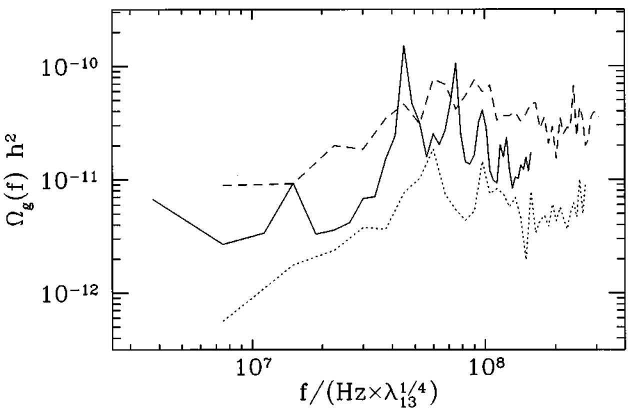

FIG. 3. Today's spectral density of gravitational waves in the pure $\lambda\phi^4$ model (solid line) and in the model where interaction $g^2\phi^2X^2$ with a massless scalar field X is added; the dashed line corresponds to $q=30$, and the dotted line corresponds to $q=105$, where $q=g^2/4\lambda$, and $\lambda_{13}\equiv\lambda/10^{-13}$. We used $g_*/g_0=100$.

Today's Ω_g, in physical units, that had been accumulated by conformal time $\tau=200$ is shown in Fig. 3 for two values of q: $q=30$ (dashed line) and $q=105$ (dotted line). The solid line corresponds to the pure $\lambda\phi^4$ model (no interaction with X field) and includes contributions from times up to $\tau=1600$. In the latter case, the peaks in Ω_g at frequencies $f\gtrsim 4\times10^7$ Hz, seen in the figure, are correlated with peaks in the power spectrum of ϕ at early stages of chaotization, cf. Ref. [13]. Since we included contributions to Ω_g from times long after the time when the peaks in the power spectrum disappeared, and yet the peaks in Ω_g had not been washed out, we suggest that in this model the peaks in Ω_g are a feature potentially observable at present. Observation of such peaks could select a particular model of inflation.

We see also that in the model with two interacting fields (ϕ and X), the linear ω dependence at small ω, characteristic of bremsstrahlung, is overall well borne out, both for $q=30$ and $q=105$. The minimal today's frequency in this model is $f_{\mathrm{min}}\sim10^5$ Hz. Extrapolating the results of Fig. 3 to that minimal frequency using the linear law, we obtain the magnitude of $\Omega_g h^2$ of order 10^{-12} for $q=30$ and of order 10^{-13} for $q=105$. The analytical estimate (17) (using $\phi_{\mathrm{ch}}^2\sim10^{-5}M_{\mathrm{Pl}}^2$) also gives $\Omega_g h^2$ of order 10^{-12} and 10^{-13} for these q, at the minimal frequency.

In the pure $\lambda\phi^4/4$ model, we could not discern any simple pattern of frequency dependence for Ω_g at small frequencies. This is consistent with our discussion of this model in Sec. IV: if the linear frequency dependence sets in at all in this model, that happens only at frequencies much smaller than those in Fig. 3. This makes it difficult for us to extrapolate our numerical results for this model to the maximal today's wavelength, $l_{\mathrm{max}}\sim10^6$ cm, or, equivalently, minimal frequency f_{min} of order 10^4 Hz. We have no reason to suppose, however, that $\Omega_g h^2$ can actually increase to small frequencies, beyond its value of order 10^{-11} at $f\sim10^6$ Hz.

To confirm that Fig. 3 is not a numerical artifact, we have made runs in which the interaction was switched off, i.e., g^2 was set to zero, at $\tau=100$, for the case $q=30$. To exclude also the effect of self-interaction, the term $\lambda\phi^4$ was replaced at that time by $m^2\phi^2$. The system then becomes a collection of free particles (plus the oscillating zero-momentum background) and, if solved exactly in infinite space, should not radiate. In our simulations, the intensity of radiation during the interval from $\tau=100$ to $\tau=125$ was three orders of magnitude smaller than it was during the same time interval with the interaction present.

VII. CONCLUSION

We have shown that gravitational radiation is produced quite efficiently in interactions of classical waves created by resonant decay of a coherently oscillating field. For simple models of chaotic inflation in which the inflaton interacts with another scalar field, we find that today's ratio of energy density in gravitational waves per octave to the critical density of the Universe can be as large as 10^{-12} at the maximal wavelength of order 10^5 cm. In the pure $\lambda\phi^4$ model, the maximal today's wavelength of gravitational waves produced by this mechanism is of order 10^6 cm, close to the upper bound of operational LIGO and TIGA frequencies. The energy density of the waves in this model is likely to be well below the sensitivity of LIGO or TIGA at such frequencies.

In other types of inflationary models (or even with other parameters), the effect can be much stronger. We do not exclude that among these there are cases in which it can be observable already by LIGO or TIGA. The relevant situations are as follows:

(1) At some values of the coupling constant g^2 (or the resonance parameter q), the most resonant momenta are close to $k=0$. In the model with massless inflaton this happens, for example, for $q=100$ (and does not happen for $q=30$ or $q=105$, which we discussed so far; in these cases, the most resonant momenta are at $k\sim1$). The lowest frequency that we had in the box in our numerical simulations for $q=100$ was $f\approx8\times10^6$ Hz. At that frequency, Ω_g for $q=100$ was almost two orders of magnitude larger than that for $q=105$. In addition, the entire spectrum of gravitational waves appears to be shifted to the left with respect to the spectra shown in Fig. 3. Cases when resonance is "tuned" to be close to $k=0$ deserve further study. The question remains, how large, in such cases, the intensity of gravitational waves can be at the horizon scale, H_{ch}.

(2) It is important to consider in detail those models of inflation (for example, hybrid inflation [20]), in which the oscillating field need not be the inflaton itself, and so the frequency of the oscillations may be unrelated to the inflaton parameters. Of interest are also models in which the restriction on the inflaton mass, imposed by the normalization of density fluctuations on large scales, is relaxed. In the extreme case, the oscillating field is unrelated to generation of primordial density fluctuations (or inflation itself), and the frequency of its oscillations may be lowered, with the corresponding increase in today's wavelength of gravitational radiation. For the intensity of the radiation to be of a potentially observable magnitude, the initial amplitude of the field has to be close to the Planck scale.

(3) In models where large fluctuations produced at preheating cause nonthermal phase transitions, as suggested in Ref. [1], domains or strings can form. A large amount of gravitational radiation can be produced in collisions of do-

main walls, in a way somewhat similar to how it happens [9,10] in models of first-order inflation, or in decays of a string network, cf. Ref. [11]. In particular, in cases when domains are formed, intensity of gravitational radiation at the horizon scale, at the moment when the domain structure disappears, is expected to be much larger than in cases without domains.

It should be noted that certain alternative methods of detection of gravitational waves can be particularly well suited for the new source of gravitational radiation that we discuss here. For example, in Ref. [26] the possibility of electromagnetic detection was considered for the gravitational waves with frequency $f\approx 10^8$ Hz and with amplitude comparable to our expectations.

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