''Faster than light'' photons in dilaton black hole spacetimes

H. T. Cho*
Tamkang University, Department of Physics, Tamsui, Taipei, Taiwan, Republic of China
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We investigate the phenomenon of ''faster than light'' photons in a family of dilaton black hole spacetimes. For radially directed photons, we find that their light-cone condition is modified even though the spacetimes are spherically symmetric. They also satisfy the ''horizon theorem'' and the ''polarization sum rule'' of Shore. For orbital photons, the dilatonic effect on the modification of the light-cone condition can become more dominant than the electromagnetic and the gravitational ones as the orbit gets closer to the event horizon in the extremal or near-extremal cases. [S0556-2821(97)08320-3]

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I. INTRODUCTION

In 1980, Drummond and Hathrell [1] discovered that photons may propagate faster than the ''speed of light'' c in curved spacetimes if QED one-loop quantum effects are taken into account. For instance, they found that in a Schwarzschild black hole spacetime, an orbital photon is superluminal in one polarization and subluminal in the other. This in fact gives a gravitational analogue to the phenomenon of electromagnetic birefringence [2]. Because of the one-loop vacuum polarization, the photon exists part of the time as a virtual e^+e^- pair. This introduces a size on the photon of the order of the Compton wavelength of the electron $\lambda_c = 1/m$, where m is the mass of the electron (here we use the notation $\hbar = c = \epsilon_0 = 1$). After acquiring a size, the motion of the photon could thus be altered by the tidal effects of the spacetime curvature. As discussed in Ref. [1] however, this change of speed does not necessarily imply any violation of causality.

In the past few years, the study of this phenomenon has been extended by Daniels and Shore to Reissner-Nordström (RN) [3] and Kerr [4] black hole spacetimes. Two general features, called the ''horizon theorem'' and the ''polarization sum rule,'' emerge from these considerations [5]. First, the horizon theorem states that the velocity of radial photons remains equal to c at the event horizon. In fact it has been found that in spherically symmetric spacetimes like the Schwarzschild and RN ones, the velocity of radial photons does not change at all. Second, the polarization sum rule states that the polarization averaged velocity shift is proportional to the matter energy-momentum tensor. Therefore velocity shifts of the two polarizations are equal and opposite in Ricci flat spacetimes.

In this work we would like to study this phenomenon in dilaton black hole spacetimes. Dilaton gravity arises from low-energy effective string actions. The corresponding black hole solutions [6,7] exhibit quite different causal structures from the usual RN solutions. For example, the inner horizon is a spacelike surface of singularity in the dilaton case versus a regular surface in the usual one. Therefore it would be interesting to see if there is any peculiarity in the propagation of photons in these rather different black hole spacetimes.

In the next section we consider the effective action of dilaton gravity which includes one-loop quantum effects from matter. Using the geometric optics approximation, we derive the equation of photon propagation. In Sec. III, the propagations of photons in different dilaton black hole spacetimes, parametrized by the dilaton coupling constant $\hat{a}$, are studied. The light-cone condition of the specific cases of radial and orbital photons is considered in more details in Secs. IV and V, respectively. Conclusions and discussions are presented in Sec. VI.

II. PHOTON PROPAGATION IN DILATON GRAVITY

We shall be using the action for dilaton gravity,

$$S = \int d^4x \sqrt{-g}[R - 2(\nabla\phi)^2 - e^{-2\hat{a}\phi}F^2], \tag{2.1}$$

where ϕ is the dilaton field and $\hat{a}$ is the dilaton coupling. The one-loop quantum effects from matter are summarized in the effective action [3]

$$\begin{aligned} S_1 = \frac{1}{m^2}\int d^4x \sqrt{-g}[&aRF_{\mu\nu}F^{\mu\nu} + bR_{\mu\nu}F^{\mu\sigma}F^{\nu}{}_{\sigma} \\ &+ cR_{\mu\nu\sigma\tau}F^{\mu\nu}F^{\sigma\tau} + d(\nabla_\mu F^{\mu\nu})(\nabla_\sigma F^{\sigma}{}_{\nu})] \\ &+ \frac{1}{m^4}\int d^4x\sqrt{-g}[z(F_{\mu\nu}F^{\mu\nu})^2 + yF_{\mu\nu}F_{\sigma\tau}F^{\mu\sigma}F^{\nu\tau}], \end{aligned} \tag{2.2}$$

where a, b, c, d, z, and y are constants. For QED corrections,

$$a = \frac{\alpha}{36\pi} \quad b = -\frac{13\alpha}{90\pi} \quad c = \frac{\alpha}{90\pi},$$

$$d = \frac{2\alpha}{15\pi} \quad z = -\frac{\alpha^2}{9} \quad y = \frac{14\alpha^2}{45}, \tag{2.3}$$

where α is the fine structure constant and m is the electron mass. Here we are effectively making a local expansion in

*Electronic address: htcho@mail.tku.edu.tw

powers of R/m^2 and $\alpha F^2/m^4$ where R and F are generic curvature and field strength, and are retaining only the leading order terms.

We derive the equation of motion for the electromagnetic field by taking the variation

$$\frac{\delta(S+S_1)}{\delta A_\nu}=0\Rightarrow\nabla_\mu(e^{-2\hat{a}\phi}F^{\mu\nu})-\frac{1}{m^2}a\nabla_\mu(RF^{\mu\nu})-\frac{1}{2m^2}b\nabla_\sigma(R^{\mu\sigma}F_\mu{}^\nu-R^{\mu\nu}F_\mu{}^\sigma)-\frac{1}{m^2}c\nabla_\mu(R^{\mu\nu}{}_{\sigma\tau}F^{\sigma\tau})$$
$$+\frac{1}{2m^2}d(\Box\nabla_\sigma F^{\sigma\nu}-\nabla^\mu\nabla^\nu\nabla_\sigma F^\sigma{}_\mu)-\frac{2}{m^4}z(F^{\sigma\tau}F_{\sigma\tau}\nabla_\mu F^{\mu\nu}+2F^{\mu\nu}F_{\sigma\tau}\nabla_\mu F^{\sigma\tau})$$
$$-\frac{2}{m^4}y(F^{\nu\tau}F_{\sigma\tau}\nabla_\mu F^{\mu\sigma}+F^{\mu\sigma}F_{\sigma\tau}\nabla_\mu F^{\nu\tau}+F^{\mu\sigma}F^{\nu\tau}\nabla_\mu F_{\sigma\tau})=0. \tag{2.4}$$

Next we write

$$F_{\mu\nu}=\overline{F_{\mu\nu}}+\hat{f}_{\mu\nu}, \tag{2.5}$$

where $\overline{F_{\mu\nu}}$ is the background electromagnetic field and $\hat{f}_{\mu\nu}$ is the photon field. To obtain the equation of motion for the photon, we put Eq. (2.5) into Eq. (2.4) and take the part linear in $\hat{f}_{\mu\nu}$. In addition, we assume that typical variations of the background electromagnetic, gravitational, and dilatonic fields, characterized by the scale L, are much smaller than that of the photon field, that is,

$$L\gg\lambda, \tag{2.6}$$

where λ is the photon wavelength. Then derivatives like $\nabla\overline{F}$ and ∇R can be neglected as compared to $\nabla\hat{f}$. Hence, from Eq. (2.4) we have

$$\nabla_\mu\hat{f}^{\mu\nu}-\frac{1}{m^2}ae^{2\hat{a}\phi}R(\nabla_\mu\hat{f}^{\mu\nu})-\frac{1}{2m^2}be^{2\hat{a}\phi}(R^{\mu\sigma}\nabla_\sigma\hat{f}_\mu{}^\nu-R^{\mu\nu}\nabla_\sigma\hat{f}_\mu{}^\sigma)-\frac{1}{m^2}ce^{2\hat{a}\phi}R^{\mu\nu}{}_{\sigma\tau}(\nabla_\mu\hat{f}^{\sigma\tau})$$
$$+\frac{1}{2m^2}de^{2\hat{a}\phi}(\Box\nabla_\sigma\hat{f}^{\sigma\nu}-\nabla^\mu\nabla^\nu\nabla_\sigma\hat{f}^\sigma{}_\mu)-\frac{2}{m^4}ze^{2\hat{a}\phi}(\overline{F^{\sigma\tau}}\,\overline{F_{\sigma\tau}}\nabla_\mu\hat{f}^{\mu\nu}+2\overline{F^{\mu\nu}}\,\overline{F_{\sigma\tau}}\nabla_\mu\hat{f}^{\sigma\tau})$$
$$-\frac{2}{m^4}ye^{2\hat{a}\phi}(\overline{F^{\nu\tau}}\,\overline{F_{\sigma\tau}}\nabla_\mu\hat{f}^{\mu\sigma}+\overline{F^{\mu\sigma}}\,\overline{F_{\sigma\tau}}\nabla_\mu\hat{f}^{\nu\tau}+\overline{F^{\mu\sigma}}\,\overline{F^{\nu\tau}}\nabla_\mu\hat{f}_{\sigma\tau})=0. \tag{2.7}$$

Without quantum corrections, $\nabla_\mu\hat{f}^{\mu\nu}=0$. Thus $\nabla_\mu\hat{f}^{\mu\nu}$ is at least of first order (order of α in QED) in perturbation. Since we are considering only first order effects, we should consistently neglect terms in the above equation with $\nabla_\mu\hat{f}^{\mu\nu}$ which are of second order or higher, giving

$$\nabla_\mu\hat{f}^{\mu\nu}-\frac{1}{2m^2}be^{2\hat{a}\phi}(R^{\mu\sigma}\nabla_\sigma\hat{f}_\mu{}^\nu)-\frac{1}{m^2}ce^{2\hat{a}\phi}R^{\mu\nu}{}_{\sigma\tau}(\nabla_\mu\hat{f}^{\sigma\tau})-\frac{4}{m^4}ze^{2\hat{a}\phi}\overline{F^{\mu\nu}}\,\overline{F_{\sigma\tau}}\nabla_\mu\hat{f}^{\sigma\tau}$$
$$-\frac{2}{m^4}ye^{2\hat{a}\phi}(\overline{F^{\mu\sigma}}\,\overline{F_{\sigma\tau}}\nabla_\mu\hat{f}^{\nu\tau}+\overline{F^{\mu\sigma}}\,\overline{F^{\nu\tau}}\nabla_\mu\hat{f}_{\sigma\tau})=0. \tag{2.8}$$

Actually, to neglect terms containing $\nabla_\mu\hat{f}^{\mu\nu}$, we need to have a bound on this derivative. A sufficient condition is that

$$\lambda\gg\lambda_c, \tag{2.9}$$

where λ_c is the electron Compton wavelength.

To study the propagation of the photon, we use the geometric optics approximation [8], in which one writes

$$\hat{f}_{\mu\nu}=f_{\mu\nu}e^{i\theta}, \tag{2.10}$$

where θ is a rapidly varying phase and $f_{\mu\nu}$ a slowly varying amplitude. The momentum of the photon is given by $k_\mu=\nabla_\mu\theta$ and the amplitude can be written as

$$f_{\mu\nu}=k_\mu a_\nu-k_\nu a_\mu, \tag{2.11}$$

where a_μ is the polarization vector satisfying the condition

$$k_\mu a^\mu = 0. \tag{2.12}$$

In this geometric optics approximation, the equation of motion becomes

$$\begin{aligned}
&k_\mu f^{\mu\nu} - \frac{1}{2m^2} b e^{2\hat{a}\phi}(R^{\mu\sigma}k_\sigma f_\mu{}^\nu) - \frac{1}{m^2} c e^{2\hat{a}\phi} R^{\mu\nu}{}_{\sigma\tau}(k_\mu f^{\sigma\tau}) \\
&- \frac{4}{m^4} z e^{2\hat{a}\phi} \overline{F^{\mu\nu}}\,\overline{F_{\sigma\tau}} k_\mu f^{\sigma\tau} - \frac{2}{m^4} y e^{2\hat{a}\phi}(\overline{F^{\mu\sigma}}\,\overline{F_{\sigma\tau}} k_\mu f^{\nu\tau} + \overline{F^{\mu\sigma}}\,\overline{F^{\nu\tau}} k_\mu f_{\sigma\tau}) = 0 \\
&\Rightarrow k^2 a^\nu - \frac{1}{2m^2} b e^{2\hat{a}\phi} R^{\mu\sigma} k_\sigma (k_\mu a^\nu - k^\nu a_\mu) - \frac{2}{m^2} c e^{2\hat{a}\phi} R^{\mu\nu}{}_{\sigma\tau} k_\mu k^\sigma a^\tau \\
&- \frac{8}{m^4} z e^{2\hat{a}\phi} \overline{F^{\mu\nu}}\,\overline{F_{\sigma\tau}} k_\mu k^\sigma a^\tau - \frac{2}{m^4} y e^{2\hat{a}\phi}[\overline{F^{\mu\sigma}}\,\overline{F_{\sigma\tau}} k_\mu (k^\nu a^\tau - k^\tau a^\nu) - \overline{F^{\mu\sigma}}\,\overline{F^{\nu\tau}} k_\mu k_\tau a_\sigma] = 0.
\end{aligned} \tag{2.13}$$

In the next section, we shall study the propagation of photons in dilaton black hole spacetimes using these equations.

III. DILATON BLACK HOLE SPACETIMES

One family of electrically charged dilaton black holes [6,7] can be obtained from the action in Eq. (2.1) with the line element

$$ds^2 = -\lambda^2 dt^2 + \lambda^{-2} dr^2 + R^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{3.1}$$

where

$$\lambda = \left(1 - \frac{r_+}{r}\right)^{1/2} \left(1 - \frac{r_-}{r}\right)^{(1-\hat{a}^2)/2(1+\hat{a}^2)}, \tag{3.2}$$

$$R = r\left(1 - \frac{r_-}{r}\right)^{\hat{a}^2/(1+\hat{a}^2)}, \tag{3.3}$$

with $r = r_+$ and $r = r_-$ corresponding to the outer and inner horizons, respectively. r_+ and r_- are related to the mass M and charge Q of the black hole by

$$2GM = r_+ + \left(\frac{1-\hat{a}^2}{1+\hat{a}^2}\right) r_-, \tag{3.4}$$

$$\frac{GQ^2}{4\pi} = \frac{r_+ r_-}{1+\hat{a}^2}, \tag{3.5}$$

where G is the Newton's constant. Moreover, the electric and the dilaton fields are given by

$$\overline{F}_{01} = \frac{Q}{4\pi r^2}, \tag{3.6}$$

$$e^{2\hat{a}\phi} = \left(1 - \frac{r_-}{r}\right)^{2\hat{a}^2/(1+\hat{a}^2)}. \tag{3.7}$$

To introduce a local set of orthonormal frames, we use the vierbein fields $e_\mu{}^a$ defined by

$$g_{\mu\nu} = \eta_{ab} e_\mu{}^a e_\nu{}^b, \tag{3.8}$$

where η_{ab} is the Minkowski metric, and

$$e_\mu{}^a = \mathrm{diag}(\lambda, 1/\lambda, R, R\sin\theta), \tag{3.9}$$

with the inverse

$$e_a{}^\mu = \mathrm{diag}(1/\lambda, \lambda, 1/R, 1/R\sin\theta). \tag{3.10}$$

The vierbein components of the Riemann tensor R_{abcd} are

$$R_{0101} = \frac{d}{dr}\left(\lambda \frac{d\lambda}{dr}\right) \equiv A, \tag{3.11}$$

$$R_{0202} = R_{0303} = \frac{\lambda}{R}\left(\frac{dR}{dr}\right)\left(\frac{d\lambda}{dr}\right) \equiv B, \tag{3.12}$$

$$R_{1212} = R_{1313} = -\frac{\lambda^2}{R}\left(\frac{d^2R}{dr^2}\right) - \frac{\lambda}{R}\left(\frac{dR}{dr}\right)\left(\frac{d\lambda}{dr}\right) \equiv C, \tag{3.13}$$

$$R_{2323} = \frac{1}{R^2} - \frac{\lambda^2}{R^2}\left(\frac{dR}{dr}\right)^2 \equiv D.$$

Using the notation $U^{01}_{ab} \equiv \delta^0_a \delta^1_b - \delta^1_a \delta^0_b$, etc., the Riemann tensor can be expressed more compactly as

$$\begin{aligned}
R_{abcd} = {} & A U^{01}_{ab} U^{01}_{cd} + B(U^{02}_{ab}U^{02}_{cd} + U^{03}_{ab}U^{03}_{cd}) \\
& + C(U^{12}_{ab}U^{12}_{cd} + U^{13}_{ab}U^{13}_{cd}) + D U^{23}_{ab} U^{23}_{cd},
\end{aligned} \tag{3.14}$$

and the vierbein components of the background electric field is

$$\overline{F}_{ab} = \frac{Q}{4\pi r^2} U^{01}_{ab}. \tag{3.15}$$

In this local set of orthonormal frames, the equation of motion for photons in Eq. (2.13) can be written as

$$k^2a^b-\frac{1}{2m^2}be^{2\hat{a}\phi}R^{ac}k_c(k_aa^b-k^ba_a)$$
$$-\frac{2}{m^2}ce^{2\hat{a}\phi}R^{ab}{}_{cd}k_ak^ca^d-\frac{8}{m^4}ze^{2\hat{a}\phi}\overline{F^{ab}}\,\overline{F_{cd}}k_ak^ca^d$$
$$-\frac{2}{m^4}ye^{2\hat{a}\phi}[\overline{F^{ac}}\,\overline{F_{cd}}k_a(k^ba^d-k^da^b)-\overline{F^{ac}}\,\overline{F^{bd}}k_ak_da_c]$$
$$=0. \tag{3.16}$$

The independent components of the polarization vector can be projected out using the vectors,

$$l_b=k^aU^{01}_{ab}, \tag{3.17}$$

$$m_b=k^aU^{02}_{ab}, \tag{3.18}$$

$$n_b=k^aU^{03}_{ab}, \tag{3.19}$$

since l^a, m^a, and n^a are independent and orthogonal to k^a. We also introduce the dependent vectors

$$p_b=k^aU^{12}_{ab}=\frac{1}{k^0}(k^1m_b-k^2l_b), \tag{3.20}$$

$$q_b=k^aU^{13}_{ab}=\frac{1}{k^0}(k^1n_b-k^3l_b), \tag{3.21}$$

$$r_b=k^aU^{23}_{ab}=\frac{1}{k^0}(k^2n_b-k^3m_b). \tag{3.22}$$

Contracting the equations of motion in Eq. (3.16) with l^a, m^a, and n^a, respectively, we obtain equations for each independent component of the polarization vector a^a,

$$k^2(a\cdot v)-\frac{1}{2m^2}be^{2\hat{a}\phi}[Al^2+B(m^2+n^2)+C(p^2+q^2)+Dr^2](a\cdot v)$$
$$-\frac{2}{m^2}ce^{2\hat{a}\phi}[A(l\cdot v)(a\cdot l)+B(m\cdot v)(a\cdot m)+B(n\cdot v)(a\cdot n)+C(p\cdot v)(a\cdot p)+C(q\cdot v)(a\cdot q)+D(r\cdot v)(a\cdot r)]$$
$$-\frac{8}{m^4}ze^{2\hat{a}\phi}\left(\frac{Q}{4\pi r^2}\right)^2(l\cdot v)(a\cdot l)-\frac{2}{m^4}ye^{2\hat{a}\phi}\left(\frac{Q}{4\pi r^2}\right)^2[l^2(a\cdot v)+(l\cdot v)(a\cdot l)]=0, \tag{3.23}$$

where $v^a=l^a$, m^a, or n^a. To show the peculiarities of the propagation of photons in these situations we shall concentrate on the cases of radially directed and orbital photons in the next two sections.

IV. RADIAL PHOTONS

For radially directed photons,

$$k^2=k^3=0; \tag{4.1}$$

then,

$$l^a=(k^1,k^0,0,0), \tag{4.2}$$

$$m^a=(0,0,k^0,0), \tag{4.3}$$

$$n^a=(0,0,0,k^0). \tag{4.4}$$

Thus $(a\cdot l)$ corresponds to the unphysical polarization. On the other hand, the equations for the physical polarizations $(a\cdot m)$ and $(a\cdot n)$ in Eq. (3.23) are the same,

$$\Bigg\{k^2-\frac{1}{2m^2}be^{2\hat{a}\phi}[Al^2+B(m^2+n^2)+C(p^2+q^2)]$$
$$-\frac{2}{m^2}ce^{2\hat{a}\phi}\left[Bn^2+C\left(\frac{k^1}{k^0}\right)^2n^2\right]$$
$$-\frac{2}{m^4}ye^{2\hat{a}\phi}\left(\frac{Q}{4\pi r^2}\right)^2l^2\Bigg\}(a\cdot v)=0, \tag{4.5}$$

where $v^a=m^a$ or n^a.

To have nontrivial solutions, we require

$$k^2-\frac{1}{2m^2}be^{2\hat{a}\phi}[Al^2+B(m^2+n^2)+C(p^2+q^2)]$$
$$-\frac{2}{m^2}ce^{2\hat{a}\phi}\left[Bn^2+C\left(\frac{k^1}{k^0}\right)^2n^2\right]$$
$$-\frac{2}{m^4}ye^{2\hat{a}\phi}\left(\frac{Q}{4\pi r^2}\right)^2l^2=0. \tag{4.6}$$

To first order in perturbation, this gives

$$\left|\frac{k^0}{k^1}\right| = 1 - \frac{1}{2m^2}(b+2c)e^{2\hat{a}\phi}(B+C)$$
$$= 1 - \frac{1}{2m^2}(b+2c)\frac{\hat{a}^2}{(1+\hat{a}^2)^2}$$
$$\times\left(\frac{r_-^2}{r^4}\right)\left(1-\frac{r^+}{r}\right)\left(1-\frac{r^-}{r}\right)^{-1}. \tag{4.7}$$

For $\hat{a}^2=0$,

$$\left|\frac{k^0}{k^1}\right| = 1, \tag{4.8}$$

which is the case for Schwarzschild or RN black holes as well as other spherically symmetric spacetimes [1,3], where the light-cone condition for radial photons is not modified. For $\hat{a}^2\neq 0$, the light-cone condition is modified in this dilatonic case even though the spacetime is still spherically symmetric. For example, for QED quantum corrections

$$b+2c = -\frac{13\alpha}{90\pi}+\frac{2\alpha}{90\pi} = -\frac{11\alpha}{90\pi}<0, \tag{4.9}$$

the radial photons in fact propagate superluminally when $r>r_+,r_-$.

At the event horizon $r=r_+$,

$$\left|\frac{k^0}{k^1}\right| = 1, \tag{4.10}$$

which is in accordance with the ‘‘horizon theorem’’ proved by Shore [5], that the light-cone condition for radial photons becomes $k^2=0$ at the event horizon. From Eq. (4.7), we also see that at the inner horizon $r=r_-$ the velocity shift diverges. This reflects the fact that the inner horizon for dilaton black holes is actually a singular surface.

We can also consider the other theorem of Shore [5], the polarization sum rule. For radial photons, to first order in perturbation,

$$R_{ab}k^ak^b = Al^2+B(m^2+n^2)+C(p^2+q^2) = 2(B+C)(k^1)^2. \tag{4.11}$$

Using Eqs. (4.11) and (4.7), we have, for the two polarizations,

$$\sum_{\text{pol}} k^2 = \sum_{\text{pol}}\,[-(k^0)^2+(k^1)^2]$$
$$= \frac{1}{m^2}(b+2c)e^{2\hat{a}\phi}[2(B+C)(k^1)^2)]$$
$$= \frac{1}{m^2}(b+2c)e^{2\hat{a}\phi}R_{ab}k^ak^b, \tag{4.12}$$

which is just the ‘‘polarization sum rule’’ considered by Shore, modified to the dilaton case.

V. ORBITAL PHOTONS

For orbital photons, take

$$k^1=k^2=0 \quad\text{and}\quad \theta=\frac{\pi}{2}; \tag{5.1}$$

then,

$$l^a=(0,k^0,0,0), \tag{5.2}$$

$$m^a=(0,0,k^0,0), \tag{5.3}$$

$$n^a=(k^3,0,0,k^0). \tag{5.4}$$

Thus $(a\cdot n)$ is the unphysical polarization. From Eq. (3.23), the r polarization $(a\cdot l)$ and the θ polarization $(a\cdot m)$ satisfy the equations

$$\Bigg\{k^2 - \frac{1}{2m^2}be^{2\hat{a}\phi}[Al^2+B(m^2+n^2)+Cq^2+Dr^2]$$
$$-\frac{2}{m^2}ce^{2\hat{a}\phi}\left[A+C\left(\frac{k^3}{k^0}\right)^2\right]l^2 - \frac{8}{m^4}ze^{2\hat{a}\phi}\left(\frac{Q}{4\pi r^2}\right)^2 l^2$$
$$-\frac{4}{m^4}ye^{2\hat{a}\phi}\left(\frac{Q}{4\pi r^2}\right)^2 l^2\Bigg\}(a\cdot l)=0 \tag{5.5}$$

and

$$\Bigg\{k^2 - \frac{1}{2m^2}be^{2\hat{a}\phi}[Al^2+B(m^2+n^2)+Cq^2+Dr^2]$$
$$-\frac{2}{m^2}ce^{2\hat{a}\phi}\left[B+D\left(\frac{k^3}{k^0}\right)^2\right]m^2 - \frac{2}{m^4}ye^{2\hat{a}\phi}\left(\frac{Q}{4\pi r^2}\right)^2 l^2\Bigg\}$$
$$\times(a\cdot m)=0. \tag{5.6}$$

For the polarizations $(a\cdot l)$ or $(a\cdot m)$ to be nonzero, we obtain the photon velocities for these two polarizations:

$$\left|\frac{k^0}{k^3}\right|_{r\ \text{pol}} = 1-\frac{1}{4m^2}be^{2\hat{a}\phi}(A+B+C+D)-\frac{1}{m^2}ce^{2\hat{a}\phi}(A+C)$$
$$-\frac{4}{m^4}ze^{2\hat{a}\phi}\left(\frac{Q}{4\pi r^2}\right)^2-\frac{2}{m^4}ye^{2\hat{a}\phi}\left(\frac{Q}{4\pi r^2}\right)^2, \tag{5.7}$$

$$\left|\frac{k^0}{k^3}\right|_{\theta\ \text{pol}} = 1-\frac{1}{4m^2}be^{2\hat{a}\phi}(A+B+C+D)-\frac{1}{m^2}ce^{2\hat{a}\phi}(B+D)$$
$$-\frac{1}{m^4}ye^{2\hat{a}\phi}\left(\frac{Q}{4\pi r^2}\right)^2. \tag{5.8}$$

Therefore, the light-cone condition is also modified for orbital photons, and the velocities of the photons for the two polarizations are different, a phenomenon of gravitational birefringence [1].

To see how the light-cone condition is modified, we shall examine more closely the RN case with $\hat{a}^2=0$ and the stringy case with $\hat{a}^2=1$. For $\hat{a}^2=0$,

$$\lambda=\left(1-\frac{r_+}{r}\right)^{1/2}\left(1-\frac{r_-}{r}\right)^{1/2}, \tag{5.9}$$

$$R=r, \tag{5.10}$$

with

$$2GM=r_+ + r_-, \tag{5.11}$$

$$\frac{GQ^2}{4\pi}=r_+ r_-. \tag{5.12}$$

Then

$$A+B+C+D=\frac{GQ^2}{2\pi r^4}, \tag{5.13}$$

$$A+C=-\frac{3GM}{r^3}+\frac{GQ^2}{\pi r^4}, \tag{5.14}$$

$$B+D=\frac{3GM}{r^3}-\frac{GQ^2}{2\pi r^4}, \tag{5.15}$$

and

$$\left|\frac{k^0}{k^3}\right|_{r\ \mathrm{pol}}=1-\frac{b}{4m^2}\left(\frac{GQ^2}{2\pi r^4}\right)-\frac{c}{m^2}\left(-\frac{3GM}{r^3}+\frac{GQ^2}{\pi r^4}\right)-\frac{4z}{m^4}\left(\frac{Q}{4\pi r^2}\right)^2-\frac{2y}{m^4}\left(\frac{Q}{4\pi r^2}\right)^2, \tag{5.16}$$

$$\left|\frac{k^0}{k^3}\right|_{\theta\ \mathrm{pol}}=1-\frac{b}{4m^2}\left(\frac{GQ^2}{2\pi r^4}\right)-\frac{c}{m^2}\left(\frac{3GM}{r^3}-\frac{GQ^2}{2\pi r^4}\right)-\frac{1}{m^4}y\left(\frac{Q}{4\pi r^2}\right)^2, \tag{5.17}$$

which are the same as the results in [3]. For QED corrections, Eqs. (5.16) and (5.17) become

$$\left|\frac{k^0}{k^3}\right|_{r\ \mathrm{pol}}=1+\frac{1}{30}\left(\frac{\alpha}{\pi}\right)\left(\frac{1}{m^2}\right)\left(\frac{GM}{r^3}\right)+\frac{1}{36}\left(\frac{\alpha}{\pi}\right)\left(\frac{1}{m^2}\right)\left(\frac{GQ^2}{4\pi r^4}\right)-\frac{8}{45}\left(\frac{\alpha^2}{m^4}\right)\left(\frac{Q}{4\pi r^2}\right)^2, \tag{5.18}$$

$$\left|\frac{k^0}{k^3}\right|_{\theta\ \mathrm{pol}}=1-\frac{1}{30}\left(\frac{\alpha}{\pi}\right)\left(\frac{1}{m^2}\right)\left(\frac{GM}{r^3}\right)+\frac{17}{180}\left(\frac{\alpha}{\pi}\right)\left(\frac{1}{m^2}\right)\times\left(\frac{GQ^2}{4\pi r^4}\right)-\frac{14}{45}\left(\frac{\alpha^2}{m^4}\right)\left(\frac{Q}{4\pi r^2}\right)^2. \tag{5.19}$$

To compare the magnitudes of various terms, we define, following Daniels and Shore [3], the accretion limit charge

$$Q_0\equiv\sqrt{\frac{4\pi}{\alpha}}GMm. \tag{5.20}$$

In terms of Q_0,

$$\left|\frac{k^0}{k^3}\right|_{r\ \mathrm{pol}}=1+\frac{1}{240}\left(\frac{\alpha}{\pi}\right)\frac{1}{(GMm)^2}\left(\frac{2GM}{r}\right)^3\left[1+\frac{5}{12}\left(\frac{Gm^2}{\alpha}\right)\times\left(\frac{Q}{Q_0}\right)^2\left(\frac{2GM}{r}\right)-\frac{2}{3}\left(\frac{Q}{Q_0}\right)^2\left(\frac{2GM}{r}\right)\right], \tag{5.21}$$

$$\left|\frac{k^0}{k^3}\right|_{\theta\ \mathrm{pol}}=1+\frac{1}{240}\left(\frac{\alpha}{\pi}\right)\frac{1}{(GMm)^2}\left(\frac{2GM}{r}\right)^3\left[-1+\frac{17}{12}\left(\frac{Gm^2}{\alpha}\right)\times\left(\frac{Q}{Q_0}\right)^2\left(\frac{2GM}{r}\right)-\frac{7}{6}\left(\frac{Q}{Q_0}\right)^2\left(\frac{2GM}{r}\right)\right]. \tag{5.22}$$

In the square brackets, the three terms represent, respectively, the gravitational effect identical to the Schwarzschild case, the indirect effect of the charge due to its modification of the gravitational field, and the contribution of the electromagnetic field itself. The second term includes a factor

$$\frac{Gm^2}{\alpha}\simeq 10^{-43}, \tag{5.23}$$

so that it is much smaller than the third term. On the other hand, for the gravitational contribution to be comparable to the electromagnetic one, one must require

$$Q\simeq Q_0 \tag{5.24}$$

for $r\simeq 2GM$. Then the first and the third terms are of the same magnitude:

$$\left|\frac{k^0}{k^3}\right|_{r\ \mathrm{pol}}=1+\frac{1}{240}\left(\frac{\alpha}{\pi}\right)\frac{1}{(GMm)^2}\left(\frac{2GM}{r}\right)^3\times\left[1-\frac{2}{3}\left(\frac{Q}{Q_0}\right)^2\left(\frac{2GM}{r}\right)\right], \tag{5.25}$$

$$\left|\frac{k^0}{k^3}\right|_{\theta\ \mathrm{pol}}=1-\frac{1}{240}\left(\frac{\alpha}{\pi}\right)\frac{1}{(GMm)^2}\left(\frac{2GM}{r}\right)^3\times\left[1+\frac{7}{6}\left(\frac{Q}{Q_0}\right)^2\left(\frac{2GM}{r}\right)\right]. \tag{5.26}$$

For θ polarization, the velocity is always smaller than c. For r polarization, the velocity could be larger than c if

$$1-\frac{2}{3}\left(\frac{Q}{Q_0}\right)^2\frac{2GM}{r}>0\Rightarrow\frac{r}{2GM}>\frac{2}{3}\left(\frac{Q}{Q_0}\right)^2, \tag{5.27}$$

which is again the same conclusion as in [3].

For comparison, we note that the extremal value Q_{ext} for a RN black hole is given by

$$Q_{\rm ext}=\sqrt{4\pi GM^2} \tag{5.28}$$

and

$$\frac{Q_0}{Q_{\rm ext}}=\sqrt{\frac{Gm^2}{\alpha}}. \tag{5.29}$$

Hence, for the gravitational effect to be of significance, the charge of the black hole must be much smaller than the extremal value, that is,

$$Q\simeq Q_0\ll Q_{\rm ext}. \tag{5.30}$$

However, if $Q\simeq Q_{\rm ext}\gg Q_0$, the electromagnetic term dominates. Then Eqs. (5.21) and (5.22) become

$$\left|\frac{k^0}{k^3}\right|_{r\ \rm pol}=1-\frac{1}{360}\left(\frac{\alpha}{\pi}\right)\frac{1}{(GMm)^2}\left(\frac{2GM}{r}\right)^4\left(\frac{Q}{Q_0}\right)^2, \tag{5.31}$$

$$\left|\frac{k^0}{k^3}\right|_{\theta\ \rm pol}=1-\frac{7}{1440}\left(\frac{\alpha}{\pi}\right)\frac{1}{(GMm)^2}\left(\frac{2GM}{r}\right)^4\left(\frac{Q}{Q_0}\right)^2, \tag{5.32}$$

for $r\simeq 2GM$. The velocities for both polarizations are subluminal but different from each other, and this is just the phenomenon of electromagnetic birefringence [2].

Next we turn to the stringy case $\hat{a}^2=1$, which we take as a typical example for nonzero $\hat{a}^2$. With $\hat{a}^2=1$,

$$r_+=2GM, \tag{5.33}$$

$$r_-=Q^2/4\pi M; \tag{5.34}$$

then,

$$e^{2\phi}(A+B+C+D)=\frac{GQ^2}{2\pi r^4}, \tag{5.35}$$

$$e^{2\phi}(A+C)=\left(GM-\frac{GQ^2}{4\pi r}\right)^{-1}\left(\frac{1}{4r^3}\right)\times\left[-12(GM)^2+22(GM)\left(\frac{GQ^2}{4\pi r}\right)-12\left(\frac{GQ^2}{4\pi r}\right)^2+\left(\frac{r}{GM}\right)\left(\frac{GQ^2}{4\pi r}\right)^2\right], \tag{5.36}$$

$$e^{2\phi}(B+D)=\left(GM-\frac{GQ^2}{4\pi r}\right)^{-1}\left(\frac{1}{4r^3}\right)\left[12(GM)^2-14(GM)\left(\frac{GQ^2}{4\pi r}\right)+4\left(\frac{GQ^2}{4\pi r}\right)^2-\left(\frac{r}{GM}\right)\left(\frac{GQ^2}{4\pi r}\right)^2\right], \tag{5.37}$$

$$e^{2\phi}=\frac{1}{GM}\left(GM-\frac{GQ^2}{4\pi r}\right). \tag{5.38}$$

First we assume that the charge

$$Q\simeq Q_0\ll Q_{\rm ext}, \tag{5.39}$$

where the extremal condition $r_+=r_-$ now gives

$$Q_{\rm ext}=\sqrt{8\pi GM^2}. \tag{5.40}$$

Hence, $Q\ll Q_{\rm ext}$ implies that

$$GM\gg\frac{GQ^2}{4\pi r} \tag{5.41}$$

for $r\simeq 2GM$. In this approximation, we can expand the expressions in Eqs. (5.36) and (5.37),

$$e^{2\phi}(A+C)=-\frac{3GM}{r^3}+\frac{5GQ^2}{8\pi r^4}+\cdots, \tag{5.42}$$

$$e^{2\phi}(B+D)=\frac{3GM}{r^3}-\frac{GQ^2}{8\pi r^4}+\cdots. \tag{5.43}$$

Keeping only leading terms in the expressions above, Eqs. (5.7) and (5.8) become

$$\left|\frac{k^0}{k^3}\right|_{r\ \rm pol}=\text{RN result}+\frac{c}{m^2}\left(\frac{3GQ^2}{8\pi r^4}\right)+\frac{4z}{m^4}\left(\frac{GQ^2}{4\pi rGM}\right)\times\left(\frac{Q}{4\pi r^2}\right)^2+\frac{2y}{m^4}\left(\frac{GQ^2}{4\pi rGM}\right)\left(\frac{Q}{4\pi r^2}\right)^2, \tag{5.44}$$

$$\left|\frac{k^0}{k^3}\right|_{\theta\ \rm pol}=\text{RN result}-\frac{c}{m^2}\left(\frac{3GQ^2}{8\pi r^4}\right)+\frac{y}{m^4}\left(\frac{GQ^2}{4\pi rGM}\right)\left(\frac{Q}{4\pi r^2}\right)^2. \tag{5.45}$$

For QED corrections,

$$\left|\frac{k^0}{k^3}\right|_{r\ \rm pol}=\text{RN result}+\frac{1}{240}\left(\frac{\alpha}{\pi}\right)\frac{1}{(GMm)^2}\left(\frac{2GM}{r}\right)^3\times\left[\frac{1}{4}\left(\frac{Gm^2}{\alpha}\right)\left(\frac{Q}{Q_0}\right)^2\left(\frac{2GM}{r}\right)+\frac{1}{3}\left(\frac{Gm^2}{\alpha}\right)\left(\frac{Q}{Q_0}\right)^4\left(\frac{2GM}{r}\right)^2\right], \tag{5.46}$$

$$\left|\frac{k^0}{k^3}\right|_{\theta\ \rm pol}=\text{RN result}+\frac{1}{240}\left(\frac{\alpha}{\pi}\right)\frac{1}{(GMm)^2}\left(\frac{2GM}{r}\right)^3\times\left[-\frac{1}{4}\left(\frac{Gm^2}{\alpha}\right)\left(\frac{Q}{Q_0}\right)^2\left(\frac{2GM}{r}\right)+\frac{7}{12}\left(\frac{Gm^2}{\alpha}\right)\left(\frac{Q}{Q_0}\right)^4\left(\frac{2GM}{r}\right)^2\right]. \tag{5.47}$$

The extra terms are subleading with respect to the RN results because they are proportional to $Gm^2/\alpha\simeq 10^{-43}$. Therefore, conclusions from the RN case will not be altered. The smallness of the dilatonic effects in this case can be understood from the fact that for the dilaton black holes that we are considering, the dilaton charge is given by [7]

$$D=\frac{1}{4\pi}\oint_S d^2S^\mu\ \ \nabla_\mu\phi=\frac{Q^2}{8\pi GM}. \tag{5.48}$$

Hence, for $Q\ll Q_{\rm ext}=\sqrt{8\pi GM^2}$,

$$D\ll\frac{Q_{\rm ext}^2}{8\pi GM}=M. \tag{5.49}$$

The dilatonic effect is therefore much smaller than the gravitational effect.

On the other hand, if $Q\simeq Q_{\rm ext}$ or $D\simeq M$, the dilatonic effect should be of importance. Thus we shall now look at this extremal case, with

$$Q=Q_{\rm ext}=\sqrt{8\pi GM^2}. \tag{5.50}$$

Remember that in the RN case, the electromagnetic effect dominates over the gravitational one for $Q\simeq Q_{\rm ext}\gg Q_0$ and $r\simeq 2GM$, as in Eqs. (5.31) and (5.32).

With Eq. (5.50), Eqs. (5.35) – (5.38) simplify to

$$e^{2\phi}(A+B+C+D)=\frac{1}{r^2}\left(\frac{2GM}{r}\right)^2, \tag{5.51}$$

$$e^{2\phi}(A+C)=-\frac{3}{2r^2}\left(\frac{2GM}{r}\right)\left(1-\frac{2GM}{r}\right), \tag{5.52}$$

$$e^{2\phi}(B+D)=\frac{1}{2r^2}\left(\frac{2GM}{r}\right)\left(3-\frac{2GM}{r}\right), \tag{5.53}$$

$$e^{2\phi}=1-\frac{2GM}{r}. \tag{5.54}$$

Then,

$$\begin{aligned}\left|\frac{k^0}{k^3}\right|_{r\ \rm pol}=1&-\frac{b}{4m^2}\left(\frac{1}{r^2}\right)\left(\frac{2GM}{r}\right)^2\\&+\frac{c}{m^2}\left(\frac{3}{2r^2}\right)\left(\frac{2GM}{r}\right)\left(1-\frac{2GM}{r}\right)\\&-\frac{2}{m^4}(2z+y)\left(\frac{1}{8\pi Gr^2}\right)\left(\frac{2GM}{r}\right)^2\left(1-\frac{2GM}{r}\right),\end{aligned} \tag{5.55}$$

$$\begin{aligned}\left|\frac{k^0}{k^3}\right|_{\theta\ \rm pol}=1&-\frac{b}{4m^2}\left(\frac{1}{r^2}\right)\left(\frac{2GM}{r}\right)^2\\&-\frac{c}{m^2}\left(\frac{1}{2r^2}\right)\left(\frac{2GM}{r}\right)\left(3-\frac{2GM}{r}\right)\\&-\frac{y}{m^4}\left(\frac{1}{8\pi Gr^2}\right)\left(\frac{2GM}{r}\right)^2\left(1-\frac{2GM}{r}\right).\end{aligned} \tag{5.56}$$

For QED corrections,

$$\begin{aligned}\left|\frac{k^0}{k^3}\right|_{r\ \rm pol}=1&+\left(\frac{\alpha}{\pi}\right)\left(\frac{1}{m^2r^2}\right)\left(\frac{2GM}{r}\right)\\&\times\left[\frac{13}{360}\left(\frac{2GM}{r}\right)+\frac{1}{60}\left(1-\frac{2GM}{r}\right)\right.\\&\left.-\frac{1}{45}\left(\frac{\alpha}{Gm^2}\right)\left(\frac{2GM}{r}\right)\left(1-\frac{2GM}{r}\right)\right],\end{aligned} \tag{5.57}$$

$$\begin{aligned}\left|\frac{k^0}{k^3}\right|_{\theta\ \rm pol}=1&+\left(\frac{\alpha}{\pi}\right)\left(\frac{1}{m^2r^2}\right)\left(\frac{2GM}{r}\right)\\&\times\left[\frac{13}{360}\left(\frac{2GM}{r}\right)-\frac{1}{180}\left(3-\frac{2GM}{r}\right)\right.\\&\left.-\frac{7}{180}\left(\frac{\alpha}{Gm^2}\right)\left(\frac{2GM}{r}\right)\left(1-\frac{2GM}{r}\right)\right].\end{aligned} \tag{5.58}$$

In most cases the last terms in the square brackets in Eqs. (5.57) and (5.58), which come from the electromagnetic part, still dominate because of the factor of α/Gm^2. However, when $r=2GM$, these terms vanish, and

$$\left|\frac{k^0}{k^3}\right|_{r\ \rm pol}=1+\frac{13}{1440}\left(\frac{\alpha}{\pi}\right)\frac{1}{(GMm)^2}, \tag{5.59}$$

$$\left|\frac{k^0}{k^3}\right|_{\theta\ \rm pol}=1+\frac{1}{160}\left(\frac{\alpha}{\pi}\right)\frac{1}{(GMm)^2}. \tag{5.60}$$

This indicates that the dilatonic effect in fact becomes the most dominant one at the event horizon, and the velocities of the photons there in both polarizations are superluminal. There is a caveat here though. For $\hat{a}^2>0$, the inner horizon is a singular surface, and in the extremal case the inner and event horizons merge. As one approaches the event horizon, which is now a singular surface, the curvature becomes very large. This would contradict our assumption that $L\gg\lambda$. Thus the conclusion above can at best be taken only as an indication that the dilatonic effect would become more and more important for determining the light-cone condition of orbital photons as one gets nearer to the event horizon in the extremal or near-extremal cases.

VI. CONCLUSIONS AND DISCUSSIONS

We have investigated the phenomenon of ''faster than light'' photons, which occurs due to the matter quantum corrections to the photon propagator, in a family of dilaton black holes parametrized by $\hat{a}^2$. For $\hat{a}^2=0,\ 1$, and 3, we have the usual RN, the stringy [7], and the Kaluza-Klein black holes [6], respectively. Dilaton black holes are particular examples of black holes with scalar hairs, and their singularity structures are also quite different. It is therefore interesting to see if peculiar behaviors are present when this phenomenon is considered in this case.

Indeed we find that for radially directed photons, the light-cone condition is modified despite the fact that the spacetimes are spherically symmetric. This result is different from the cases of Schwarzschild and RN black holes in which the light-cone condition for radial photons is unchanged. In this dilatonic case, the light-cone condition of the radial photons also satisfies the ''horizon theorem'' and the ''polarization sum rule'' of Shore [5].

For orbital photons, we concentrate on the stringy case, $\hat{a}^2=1$, as a typical example for nonzero $\hat{a}^2$. With the charge $Q\simeq Q_0 \ll Q_{\rm ext}$, where Q_0 is the accretion limit charge and $Q_{\rm ext}$ the extremal charge, the pure gravitational effect (Schwarzschild part) is comparable to the electromagnetic one. The dilaton charge is much smaller than the mass, Eq. (5.49), and so the dilatonic effect is negligible. On the other hand, if $Q\simeq Q_{\rm ext}$, the electromagnetic effect will still dominate over the gravitational one, as in the RN case, so that the velocities of the photons in both polarizations are subluminal. The only exception is that as one approaches the event horizon, the dilatonic effect becomes more and more important, even more so than the electromagnetic one, and the velocities of the photons change to being superluminal. This indicates that the dilatonic effect is crucial in determining this ''faster than light'' phenomenon when the photon is near the event horizon in the extremal or near-extremal cases.

In arriving at the above results, we have taken the approximations

$$L \gg \lambda \gg \lambda_c, \tag{6.1}$$

where L represents the scale of the typical variation of the background fields, λ the photon wavelength, and λ_c the Compton wavelength. For $\hat{a}^2>0$, the inner horizon becomes singular. In addition, the inner and the event horizons merge in the extremal case. When the orbital photon is very near to the event horizon as mentioned above, the curvature becomes so large that our approximation may no longer be valid. Hence, we have taken this result for the orbital photons only as an indication of the growing importance of the dilatonic effect as one gets near to the event horizon.

It is therefore of interest to extend our investigation beyond the assumed range above if, for example, one wants to consider the extremal black holes more carefully. To relax the condition $\lambda \gg \lambda_c$ requires the summation of terms like $\Sigma_i (R/m^2)(\nabla^i/m^i)F^2$ in the effective action. As discussed in Ref. [3], this may be achieved by the new summation technique of Barvinsky and Vilkovisky [9]. Whereas to go beyond the other constraint $L \gg \lambda$, we need to push the formalism to the strong field regime and one must then use nonperturbative approximations. For instance, one may try to consider the light-cone condition of a massless fermion in the Gross-Neveu model [10] in curved space by the $1/N$ approximation to extract nonperturbative effects.

In Sec. V, we have concentrated on the dilaton black holes with $\hat{a}^2=1$ because this corresponds to the stringy case, which is most interesting to us. Moreover, this case should be typical enough that similar results are expected for other nonzero values of $\hat{a}^2$. On the other hand, the low-energy effective action of string theories has introduced a whole new set of black holes [11]. The ones that we have considered here are the simplest. We can therefore extend our consideration to, for instance, dyonic dilaton black holes [6,12], black holes with axions [13], rotating dilaton black holes [14], and even black holes with nontrivial dilaton potentials [15,16].

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