Instability of cosmological event horizons of nonstatic global cosmic strings

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The stability of the cosmological event horizons found recently by Gregory [Phys. Rev. D **54**, 4955 (1996)] for a class of nonstatic global cosmic strings is studied. It is shown that they are not stable to both test particles and physical perturbations. In particular, the back reaction of the perturbations of null dust fluids will turn them into spacetime singularities. The resulting singularities are strong in the sense that the distortion of test particles diverges logarithmically when these singular hypersurfaces are approaching. [S0556-2821(97)02622-2]

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I. INTRODUCTION

Topological defects which may have been formed in the early Universe have been studied extensively [1] since the pioneering work of Kibble [2]. They may have been formed during phase transitions of the Universe, where the degenerated vacua acquired nonzero expectation values. Depending on the topology of the vacua, the defects could be domain walls, cosmic strings, monopoles, textures, or the hybrids of them. Among these defects, cosmic strings have received particular attention mainly because of their cosmological implications: They might provide the seeds for the formations of galaxies and the large-scale structure of the Universe [1].

Cosmic strings are further classified as local (gauge) and global strings, depending on whether they arise from a local symmetry breaking or a global symmetry breaking. These two kinds of strings have very different properties. In particular, the spacetime of a local (static) string is well behaved and asymptotically approaches a conical spacetime [3], while the spacetime of a global static string is necessarily singular at a finite distance from the symmetric axis, and its deficit angle diverges logarithmically [4]. It is this undesirable property that makes global strings very difficult to use, and most studies of cosmic strings have been restricted only to local strings [1].

However, local strings are tightly constrained by their contribution to the gravitational radiation background [5,6], while global cosmic strings circumvent this constraint and may have similar cosmological implications [7]. Lately, Banerjee *et al.* [8] and Gregory [9] studied nonstatic global strings, and some interesting results were found. In particular, Gregory showed that the spacetime singularities usually appearing in the static case can be replaced by cosmological event horizons (CEH's).[1] This result is very important, as it may make the structure formation scenario of cosmic strings more likely, and may open a new avenue to the study of global strings.

In this paper, we shall study the stability of the CEH's found above by Gregory, and shall show that in general they are not stable against physical perturbations, instead are turned into spacetime singularities. It should be noted that this does not mean that the hope that the spacetimes of nonstatic global cosmic strings might be free of spacetime singularities is already found negative? In fact, Gregory considered a very particular case: the energy-momentum tensor of the string is still time independent. As a result, no gravitational and particle radiation exists. For a more general case, one would expect that CEH's may not be formed at all, or even they are formed but stable. As we know, in the cylindrical case gravitational and particle radiation in general always exists. It is plausible to expect that in some situations the radiation is so strong that the gravitational field is well dilated before any spacetime singularity or horizon is formed. In this respect, it would be very interesting to study nonstatic global cosmic strings in a more general case.

The rest of the paper is organized as follows: In Sec. II we shall briefly review the main properties of the spacetimes studied by Gregory [9], while in Sec. III null dust fluids of test particles are studied, which indicate some singularity behavior of the spacetimes near the CEH's. In Sec. IV we consider ''physical'' perturbations of real particles, and confirm the results obtained in Sec. III. Here ''physical'' is in the sense that the back reaction of the perturbations are taken into account. Finally in Sec. V we derive our main conclusions.

II. SPACETIME FOR NONSTATIC GLOBAL COSMIC STRINGS

For a straight cosmic string, we can always choose a coordinate system that is comoving with the string so that the

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[1] Note that Gregory called the horizons event horizons. However, to be distinguishable with the ones of black holes, following Gibbons and Hawking [10], we call them cosmological event horizons.

spacetime in this system has a cylindrical symmetry. If additionally we require that the string has no rotation, the metric for such a spacetime takes the general form [11]

$$ds^2 = e^{2(\gamma-\psi)}(dt^2 - dR^2) - e^{2\psi}dz^2 - \alpha^2 e^{-2\psi}d\theta^2, \tag{2.1}$$

where γ, ψ, and α are functions of t and R only, and t, R, z, and θ are the usual cylindrical coordinates.

By requiring that the string have fixed proper width and that the spacetime have boost symmetry in the (t,z) plane, Gregory managed to show that the spacetime for a U(1) global string (vortex) is given by [9]

$$\gamma = 2a(R) + b(t), \quad \psi = a(R) + b(t), \quad \alpha = c(R)e^{a(R)+b(t)}, \tag{2.2}$$

where $a(R)$ and $c(R)$ are two arbitrary functions, and $b(t)$ is given by

$$b(t) = \begin{cases} \ln[\cosh(\beta t)], \pm\beta t, & b_0 > 0, \\ b_1 \ln t, & b_0 = 0, \\ \ln[\cos(\beta t)], & b_0 < 0, \end{cases} \tag{2.3}$$

where b_0 and b_1 are arbitrary constants, and $\beta \equiv \sqrt{|b_0|}$. From Eqs. (2.1) and (2.2) we can see that by introducing a new radial coordinate r via the relation

$$r = \int^r e^{a(R)}dR, \tag{2.4}$$

the metric (2.1) can be written in the form

$$ds^2 = e^{2A(r)}dt^2 - dr^2 - e^{2[A(r)+b(t)]}dz^2 - C^2(r)d\theta^2, \tag{2.5}$$

where $A(r) \equiv a[R(r)]$ and $C(r) \equiv c[R(r)]$. This is exactly the form used by Gregory [9]. As shown by herself, the spacetime inside the core of a string is always singular at a finite distance for the cases $b_0 \leqslant 0$, and has a CEH for $b_0 > 0$. In the following, we shall consider only the case $b_0 > 0$. In this case the metric coefficients have the asymptotic behavior [9]

$$e^{A(r)} \sim \beta(r_0 - r), \quad C(r) \sim C_0 + O(r_0 - r)^2, \tag{2.6}$$

as $r \to r_0^-$, where C_0 is a constant [cf. Eq. (3.14) in Ref. [9]]. For the choice $b(t) = \ln[\cosh(\beta t)]$, the corresponding metric takes the form

$$ds^2 = \beta^2(r_0 - r)^2[dt^2 - \cosh^2(\beta t)dz^2] - dr^2 - C_0^2 d\theta^2, \tag{2.7}$$

in the neighborhood of the hypersurface $r = r_0$. One can show that the singularity appearing at $r = r_0$ in Eq. (2.7) is a coordinate one. This can be seen, for example, by making the following coordinate transformations:

$$X = (r_0 - r)\cosh(\beta t)\cos(\beta z),$$

$$Y = (r_0 - r)\cosh(\beta t)\sin(\beta z),$$

$$T = (r_0 - r)\sinh(\beta t); \tag{2.8}$$

then the metric (2.7) is brought to the form

$$ds^2 = dT^2 - dX^2 - dY^2 - C_0^2 d\theta^2, \tag{2.9}$$

which is locally Minkowski. Thus, the singularity at $r = r_0$ in the coordinates $\{t, r, z, \theta\}$ is indeed a coordinate singularity and represents a conelike CEH in the coordinates $\{T, X, Y, \theta\}$, as one can see from Eq. (2.8) that the hypersurface $r = r_0$ is mapped to

$$X^2 + Y^2 = T^2, \quad (r = r_0). \tag{2.10}$$

For the details, we refer readers to [9].

For $b(t) = \pm\beta t$, the corresponding metric takes the form

$$ds^2 = \beta^2(r_0 - r)^2[dt^2 - e^{\pm\beta t}dz^2] - dr^2 - C_0^2 d\theta^2. \tag{2.11}$$

One can show that the singularities appearing at $r = r_0$ in the above metric also represent CEH's. In fact, if we make the coordinate transformations

$$T = \frac{1}{2}(r_0 - r)[\beta^2 z^2 e^{\beta t} + 2\sinh(\beta t)],$$

$$X = \frac{1}{2}(r_0 - r)[\beta^2 z^2 e^{\beta t} - 2\cosh(\beta t)],$$

$$Y = \beta(r_0 - r)z e^{\beta t}, \tag{2.12}$$

for $b(t) = +\beta t$, the metric (2.11) will be brought to the exact form of Eq. (2.9), while if we make the same transformations as those of Eq. (2.12) but with t being replaced by $-t$ for $b(t) = -\beta t$, the metric (2.11) will be also brought to the same form, Eq. (2.9). Therefore, in the latter two cases the hypersurface $r = r_0$ all represents a CEH. The topology of it is also conical in the Minkowski-like coordinates (T, X, Y, θ), as one can show from Eq. (2.12) that Eq. (2.10) is satisfied, too.

Before proceeding further, we would like to note the following: (a) For the coordinate transformations given by Eq. (2.8), the mapping between (t, r, z) and (T, X, Y) is not one-to-one, while the one given by Eq. (2.12) is. (b) The nature of the CEH's in all three cases is quite similar to that of the extreme Reissner-Nordström black hole [12], in the sense that across $r = r_0$ the coordinate t remains timelike, while r remains spacelike. (c) In the neighborhood $r = r_0$ but with $r > r_0$, Eq. (2.6) should be replaced by

$$e^{A(r)} \sim \beta(r - r_0), \quad C(r) \sim C_0 + O(r - r_0)^2, \quad (r > r_0). \tag{2.13}$$

Substituting Eqs. (2.6) and (2.13) into Eq. (2.4), we find

$$R = \begin{cases} \dfrac{1}{\beta}\ln[\beta(r - r_0)], & r > r_0 \\ -\dfrac{1}{\beta}\ln[\beta(r_0 - r)], & r < r_0 \end{cases} \tag{2.14}$$

which shows that R is a monotonically increasing function of r, except for the point $r = r_0$, at which R diverges.

In the following, we shall consider the stability of these CEH's in two steps: First, in the next section we shall consider test particles near the CEH's along a line suggested by

Helliwell and Konkowski (HK) in the study of the stability of quasiregular singularities [13], at the aim of generalizing the HK conjecture to the case of CEH's. Second, in Sec. IV we shall consider perturbations of null dust fluids. These perturbations are different from the ones studied in Sec. III, in the sense that the back reaction of them to the spacetime backgrounds will be taken into account.

III. TEST NULL DUST FIELDS NEAR THE CEH'S

In a series of papers [13], HK studied the stability of quasiregular singularities by using test fields. In particular, they conjectured that *if one introduces a test field whose energy-momentum tensor (EMT) calculated in a free falling frame mimics the behavior of the Riemann tensor components that indicate a particular type of singularity (quasiregular, nonscalar curvature, or scalar curvature), then a complete nonlinear back-reaction calculation would show that this type of singularity actually occurs*. Recently, this conjecture was further generalized to the stability of Cauchy horizons [14]. Clearly, if this conjecture is true, the stability analysis of spacetime singularities would be considerably simplified. In this section, using HK's ideas we shall study test null dust fields near the CEH's.

For a null dust fluid moving along the outgoing null geodesics defined by n^{μ} in the region $r \leqslant r_0$ (cf. the Appendix), the EMT takes the form

$$T^{-\text{out}}_{\mu\nu} = \rho^{\text{out}}_{-} n_{\mu} n_{\nu}, \tag{3.1}$$

where n_{μ} is a null vector defined as that in Eq. (A9). Then, from the conservation equations $T^{-\text{out}}_{\mu\nu;\lambda} g^{\nu\lambda}=0$, we find

$$e^{A}\left(\frac{\rho^{\text{out}}_{-},_{r}}{\rho^{\text{out}}}+3A'(r)\right)+\left(\frac{\rho^{\text{out}}_{-},_{t}}{\rho^{\text{out}}_{-}}+b'(t)\right)=0, \tag{3.2}$$

which has the solution

$$\rho^{\text{out}}_{-} = \frac{\rho^{\text{out}(0)}_{-} e^{a_1 t - b(t)}}{(r_0 - r)^{3 - a_1/\beta}}, \tag{3.3}$$

where $(),_{x} = \partial/\partial x$, a prime denotes the ordinary derivative with respect to the indicated argument, and a_1 and $\rho^{\text{out}(0)}_{-}$ are two constants, while $A(r)$ and $b(t)$ are given, respectively, by Eqs. (2.6) and (2.3). Projecting $T^{-\text{out}}_{\mu\nu}$ onto the PPON frame defined by Eqs. (A3) and (A4), we find that the nonvanishing components are given by

$$T^{-\text{out}}_{(0)(0)} = T^{-\text{out}}_{(1)(1)} = T^{-\text{out}}_{(0)(1)} = \frac{\rho^{\text{out}(0)}_{-} e^{a_1 t - b(t)}}{(r_0 - r)^{3 - a_1/\beta}} \left\{ \frac{E^2}{\beta^2 (r_0 - r)^2} - \frac{E[E^2 - \beta^2 (r_0 - r)^2]^{1/2}}{[\beta(r_0 - r)]^2} - \frac{1}{2} \right\}. \tag{3.4}$$

Clearly, for $b(t) = \ln[\cosh \beta t], \beta t$, we have to choose $a_1 = \beta$ in order to have the perturbations be finite initially $(t = -\infty)$, while for $b(t) = -\beta t$, we have to choose $a_1 = -\beta$, namely,

$$a_1 = \begin{cases} \beta, & b(t) = \ln[\cosh \beta t], \beta t, \\ -\beta, & b(t) = -\beta t. \end{cases} \tag{3.5}$$

Equations (3.4) and (3.5) show that all the components diverge as $r \to r_0^-$ for all three different choices of $b(t)$, which indicates that if we take the back reaction of the null dust fluid into account, the CEH's appearing on the hypersurface $r = r_0$ in the solutions (2.7) and (2.11) will be turned into spacetime singularities, provided that the HK conjecture still holds here. Since all the corresponding scalars made of $T^{-\text{out}}_{\mu\nu}$ are zero in the present case, the resulted singularities would be expected to be nonscalar curvature singularities, when the back reaction of the null dust fluid is taken into account. The back reaction will be considered in the next section, and it will be shown that it is indeed the case.

In addition to the outgoing null dust fluid, if there also exists an ingoing null fluid moving along the null geodesics defined by l^{μ}, i.e.,

$$T^{-\text{in}}_{\mu\nu} = \rho^{\text{in}}_{-} l_{\mu} l_{\nu}, \tag{3.6}$$

where l_{μ} is defined by Eq. (A9), then, from the conservation equations $T^{-\text{in}}_{\mu\nu;\lambda} g^{\nu\lambda}=0$, one can show that ρ^{in}_{-} is given by

$$\rho^{\text{in}}_{-} = \frac{\rho^{\text{in}(0)}_{-} e^{a_0 t - b(t)}}{(r_0 - r)^{3 + a_0/\beta}}, \tag{3.7}$$

where a_0 and $\rho^{\text{in}(0)}_{-}$ are other two integration constants. Then, the nonvanishing tetrad components of $T^{-\text{in}}_{\mu\nu}$ are given by

$$T^{-\text{in}}_{(0)(0)} = T^{-\text{in}}_{(1)(1)} = -T^{-\text{in}}_{(0)(1)} = \frac{\rho^{\text{in}(0)}_{-} e^{a_0 t - b(t)}}{(r_0 - r)^{3 + a_0/\beta}} \left\{ \frac{E^2}{\beta^2 (r_0 - r)^2} + \frac{E[E^2 - \beta^2 (r_0 - r)^2]^{1/2}}{[\beta(r_0 - r)]^2} - \frac{1}{2} \right\}. \tag{3.8}$$

Similar to the outgoing case, to have the perturbations be finite initially $(t = -\infty)$, we have to choose $a_0 = a_1$, where a_1 is given by Eq. (3.5). Then, from Eq. (3.8) we can see that these components also diverge. Since now we have

$$T^{-\mu\nu} T^{-}_{\mu\nu} = 2\rho^{\text{out}}_{-} \rho^{\text{in}}_{-} = 2\rho^{\text{out}(0)}_{-} \rho^{\text{in}(0)}_{-} \frac{e^{2[a_0 t - b(t)]}}{(r_0 - r)^6}, \tag{3.9}$$

which always diverges as $r \to r_0^-$, we can see that the resulting singularities should be scalar curvature ones, when the back reaction of the two null dust fluids are taken into account, where

$$T^{-}_{\mu\nu} \equiv T^{-\text{out}}_{\mu\nu} + T^{-\text{in}}_{\mu\nu}. \tag{3.10}$$

Similarly, we can consider test null dust fields in the region $r \geqslant r_0$, and will obtain the same conclusions. Thus, the above considerations suggest that all the CEH's appearing in the solutions (2.7) and (2.11) are not stable against perturbations for all the three different choices of $b(t)$ with $b_0 > 0$.

IV. PERTURBATIONS NEAR THE CEH'S

In this section, let us consider perturbations of null dust fluids near the CEH's. For the sake of convenience, we shall work with the coordinates t and R, in terms of which the metrics (2.7) and (2.11) can be cast in the form

$$ds^2=e^{-\Omega_{(0)}}(dt^2-dR^2)-e^{-h_{(0)}}[e^{\Phi_{(0)}}dz^2+e^{-\Phi_{(0)}}d\theta^2], \tag{4.1}$$

where

$$\Omega_{(0)}=2\beta R,\quad h_{(0)}=\beta R-b(t)-\ln C_0,$$
$$\Phi_{(0)}=-\beta R+b(t)-\ln C_0, \tag{4.2}$$

for $r\leqslant r_0$, and

$$\Omega_{(0)}=-2\beta R,\quad h_{(0)}=-[\beta R+b(t)+\ln C_0],$$
$$\Phi_{(0)}=\beta R+b(t)-\ln C_0, \tag{4.3}$$

for $r\geqslant r_0$, and the function $b(t)$ is given by Eq. (2.3).

As shown in [15], the null dust fluids given by

$$T_{\mu\nu}=\rho^{\text{in}}l_\mu l_\nu+\rho^{\text{out}}n_\mu n_\nu, \tag{4.4}$$

have contributions only to the metric coefficients g_{tt} and g_{RR}. Specifically, if we set

$$\{\Omega,h,\Phi\}=\{\Omega_{(0)}+f(u)+g(v),h_{(0)},\Phi_{(0)}\}, \tag{4.5}$$

the metric

$$ds^2=e^{-\Omega}(dt^2-dR^2)-e^{-h}(e^{\Phi}dz^2+e^{-\Phi}d\theta^2), \tag{4.6}$$

will satisfy the Einstein field equations $R_{\mu\nu}-g_{\mu\nu}R/2=T_{\mu\nu}$, with ρ^{out} and ρ^{in} being given, respectively, by

$$\rho^{\text{out}}=g'(v)h_{,v},\quad \rho^{\text{in}}=f'(u)h_{,u}, \tag{4.7}$$

and now

$$l_\mu=e^{-\Omega/2}(\delta^t_\mu+\delta^R_\mu),\quad n_\mu=e^{-\Omega/2}(\delta^t_\mu-\delta^R_\mu),$$
$$u\equiv\frac{t+R}{\sqrt{2}},\quad v\equiv\frac{t-R}{\sqrt{2}}, \tag{4.8}$$

and $f(u)$ and $g(v)$ are arbitrary functions of their indicated arguments. Note that although $T_{\mu\nu}$ now takes the same form as that considered in the last section, it has a fundamental difference: now it acts as a source of the spacetime. As a result, the back reaction of it is automatically fully taken into account. When $f(u)$, $g(v)$, and their first derivatives are very small, the two dust fluids can be considered as perturbations of the spacetime given by Eqs. (4.1)–(4.3). In the following, let us consider the three cases $b(t)=\ln[\cosh(\beta t)]$, $+\beta t$, $-\beta t$, separately.

(A) $b(t)=\ln[\cosh(\beta t)]$. In this case, Eqs. (4.2)–(4.5) yield

$$\Omega=f^-(u)+g^-(v)+2\beta R,$$
$$h=\beta R-\ln[\cosh(\beta t)]-\ln C_0,$$
$$\Phi=-\beta R+\ln[\cosh(\beta t)]-\ln C_0, \tag{4.9}$$

for $r\leqslant r_0$, and

$$\Omega=f^+(u)+g^+(v)-2\beta R,$$
$$h=-\{\beta R+\ln[\cosh(\beta t)]+\ln C_0\},$$
$$\Phi=\beta R+\ln[\cosh(\beta t)]-\ln C_0, \tag{4.10}$$

for $r\geqslant r_0$. Substituting Eqs. (4.9) and (4.10) into Eq. (4.7), we find

$$\rho^{\text{out}}_-=-\frac{\beta e^{\beta t}g^{-\prime}(v)}{\sqrt{2}\cosh\beta t},\quad \rho^{\text{in}}_-=\frac{\beta e^{-\beta t}f^{-\prime}(u)}{\sqrt{2}\cosh\beta t}\quad (r\leqslant r_0),$$
$$\rho^{\text{out}}_+=\frac{\beta e^{-\beta t}g^{+\prime}(v)}{\sqrt{2}\cosh\beta t},\quad \rho^{\text{in}}_+=-\frac{\beta e^{\beta t}f^{+\prime}(u)}{\sqrt{2}\cosh\beta t}\quad (r\geqslant r_0). \tag{4.11}$$

Note that it is not necessary to take $f^-(u)$ and $g^-(v)$ the same forms as $f^+(u)$ and $g^+(v)$, since now we consider the perturbations in both sides of the hypersurface $r=r_0$ independently. However, to have physically reasonable perturbations, we require

$$g^{-\prime}(v)<0,\quad g^{+\prime}(v)>0,$$
$$f^{-\prime}(u)>0,\quad f^{+\prime}(u)<0, \tag{4.12}$$

so that $\rho^{\text{out}}_\pm$ and $\rho^{\text{in}}_\pm$ are all not negative.

When $f^\pm(u)$, $g^\pm(v)$, and their first derivatives are very small, the radial timelike geodesics given by Eq. (A3) would be a very good approximation of the corresponding ones of the metric (4.6). Consequently, the tetrad frames given by Eqs. (A3) and (A4) would serve well as the corresponding PPON of the solutions given by Eqs. (4.9) and (4.10). Projecting the EMT onto this frame, we find that the nonvanishing tetrad components of it are given by

$$T^\pm_{(0)(0)}=T^\pm_{(1)(1)}=\frac{1}{2}\{D_+(A)\rho^{\text{in}}_\pm+D_-(A)\rho^{\text{out}}_\pm\},$$
$$T^\pm_{(0)(1)}=T^\pm_{(1)(0)}=\frac{1}{2}\{D_+(A)\rho^{\text{in}}_\pm-D_-(A)\rho^{\text{out}}_\pm\}, \tag{4.13}$$

where $\rho^{\text{in}}_\pm$, $\rho^{\text{out}}_\pm$ are given by Eqs. (4.11), and

$$D_\pm(A)=\left[\frac{E}{e^A}\pm\epsilon\left(\frac{E^2}{e^{2A}}-1\right)^{1/2}\right]^2, \tag{4.14}$$

with A being given by Eqs. (2.6) and (2.13). Clearly, as $r\to r_0$, these components all diverge. Note that in writing the above expressions we have used the fact that $f^\pm(u)$ and $g^\pm(v)$ are very small to set $\exp\{f^\pm(u)+g^\pm(v)\}=1$. This will be also the case for two other cases to be considered below. Combining Eqs. (4.11) with Eqs. (4.13) and (4.14), we can see that the perturbations are finite at the initial $t=-\infty$, but all will focus into a spacetime singularity when they arrive at $r=r_0$. That is, the perturbations turn the CEH's into spacetime curvature singularities. The nature of the singularity is a scalar one. This can be seen, for example, from the Kretschmann scalar,

$$\mathcal{R}=R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma}=\begin{cases}-\dfrac{4\beta^2e^{2[2\beta R+f^-(u)+g^-(v)]}}{\cosh^2\beta t}f^{-\prime}(u)g^{-\prime}(v), & r\leqslant r_0,\\ \dfrac{4\beta^2e^{-2[2\beta R-f^+(u)-g^+(v)]}}{\cosh^2\beta t}f^{+\prime}(u)g^{+\prime}(v), & r\geqslant r_0,\end{cases} \tag{4.15}$$

which diverges like $(\tau_0-\tau)^{-1}$, as $r\to r_0$, as long as $f^{\pm\prime}(u)g^{\pm\prime}(v)\neq 0$, as one can show from Eqs. (A6) and (A7). When only the outgoing null dust fluid exists, the singularity degenerates to a nonscalar curvature singularity, as it can be shown that now all 14 scalars made out of the Riemann tensor are zero. However, in any case the singularity is strong in the sense that the distortion, which is equal to the twice integral of the tetrad components of the Riemann tensor, becomes unbounded, for example,

$$\int\int R^{\pm}_{(0)(2)(0)(2)}d\tau d\tau$$
$$=\pm\frac{\sqrt{2}\beta}{4}\int\int\frac{1}{\cosh\beta t}[D_+(A)e^{\pm\beta(t-2R)}f^{\pm\prime}(u)-D_-(A)e^{\mp\beta(t+2R)}g^{\pm\prime}(v)]d\tau d\tau\sim g^{\pm\prime}(v)\ln(\tau_0-\tau), \tag{4.16}$$

as $r\to r_0$. It is interesting to note that, although the tetrad components of $T_{\mu\nu}$ are singular as $r\to r_0$, the scalar $T^{\mu\nu}T_{\mu\nu}$ is not. In fact, from Eq. (4.13) it can be shown that

$$T^{\pm}_{\mu\nu}T^{\pm\mu\nu}=T^{\pm}_{(a)(b)}T^{\pm(a)(b)}=2\rho^{\text{in}}_{\pm}\rho^{\text{out}}_{\pm}=-\frac{\beta^2f^{\pm\prime}(u)g^{\pm\prime}(v)}{2\cosh^2\beta t}$$
$$\sim(\tau_0-\tau), \tag{4.17}$$

as $r\to r_0$. Thus, the formation of the spacetime singularity is mainly due to the focus of the corresponding gravitational fields. This is different from what we can get from Eq. (3.9) for the test particles.

(B) $b(t)=+\beta t$. When $b(t)=+\beta t$, from Eqs. (4.2)–(4.5) we find that

$$\Omega=f^-(u)+g^-(v)+2\beta R,$$
$$h=-\beta(t-R)-\ln C_0,$$
$$\Phi=\beta(t-R)-\ln C_0 \tag{4.18}$$

for $r\leqslant r_0$, and

$$\Omega=f^+(u)+g^+(v)-2\beta R,$$
$$h=-\beta(t+R)-\ln C_0,$$
$$\Phi=\beta(t+R)-\ln C_0 \tag{4.19}$$

for $r\geqslant r_0$. Substituting Eqs. (4.18) and (4.19) into Eq. (4.7), we find that

$$\rho^{\text{out}}_{-}=-\sqrt{2}\beta g^{-\prime}(v),\quad \rho^{\text{in}}_{-}=0\quad (r\leqslant r_0),$$
$$\rho^{\text{out}}_{+}=0,\quad \rho^{\text{in}}_{+}=-\sqrt{2}\beta f^{+\prime}(u)\quad (r\geqslant r_0). \tag{4.20}$$

The above expressions show that in the region $r\leqslant r_0$ now there exists only an outgoing dust cloud, while in the region $r\geqslant r_0$ only ingoing. With the same arguments as those given in the last subsection, we take the tetrad frames given by Eqs. (A3) and (A4) as a good approximation to the corresponding PPON of the solutions given by Eqs. (4.18) and (4.19), when $f^{\pm}(u)$, $g^{\pm}(v)$, and their first derivatives are very small. Then, projecting the EMT onto this frame, we find that the nonvanishing tetrad components of it are given by

$$T^{-}_{(0)(0)}=T^{-}_{(1)(1)}=-T^{-}_{(0)(1)}=-\frac{\beta}{\sqrt{2}}D_-(A)g^{-\prime}(v)\quad (r\leqslant r_0),$$

$$T^{+}_{(0)(0)}=T^{+}_{(1)(1)}=T^{+}_{(0)(1)}=-\frac{\beta}{\sqrt{2}}D_+(A)f^{+\prime}(u)\quad (r\geqslant r_0), \tag{4.21}$$

with A being given by Eqs. (2.6) and (2.13), and $D_\pm$ are defined by Eq. (4.14). Clearly, as $r\to r_0$, these components all diverge, although at the initial $t=-\infty$, $r\neq r_0$ they are finite. That is, the perturbations, similar to the last subcase, turn the CEH's into spacetime curvature singularities. The nature of the singularity is a nonscalar one, as one can show that now all 14 scalars built from the Riemann tensor are zero. However, the singularity is strong in the sense that the distortion diverges like $\ln(\tau_0-\tau)$ as $r\to r_0$, as we can see from the following integrations:

$$\int\int R^{-}_{(0)(2)(0)(2)}d\tau d\tau=\frac{1}{\sqrt{2}\beta}\int\int\frac{g^{-\prime}(v)}{(r_0-r)^2}D_-(A)d\tau d\tau$$
$$\sim\ln(\tau_0-\tau),$$

$$\int\int R^{+}_{(0)(2)(0)(2)}d\tau d\tau=\frac{1}{\sqrt{2}\beta}\int\int\frac{f^{+\prime}(u)}{(r-r_0)^2}D_+(A)d\tau d\tau$$
$$\sim\ln(\tau_0-\tau). \tag{4.22}$$

(C) $b(t)=-\beta t$. When $b(t)=-\beta t$, Eqs. (4.2)–(4.5) yield

$$\Omega = f^-(u)+g^-(v)+2\beta R,$$

$$h=\beta(t+R)-\ln C_0,$$

$$\Phi=-\beta(t+R)-\ln C_0, \tag{4.23}$$

for $r\leqslant r_0$, and

$$\Omega = f^+(u)+g^+(v)-2\beta R,$$

$$h=\beta(t-R)-\ln C_0,$$

$$\Phi=-\beta(t-R)-\ln C_0, \tag{4.24}$$

for $r\geqslant r_0$. Substituting Eqs. (4.23) and (4.24) into Eq. (4.7), we find

$$\rho_-^{\rm out}=0, \quad \rho_-^{\rm in}=\sqrt{2}\beta f^{-\prime}(u) \quad (r\leqslant r_0),$$

$$\rho_+^{\rm out}=\sqrt{2}\beta g^{+\prime}(v), \quad \rho_+^{\rm in}=0 \quad (r\geqslant r_0). \tag{4.25}$$

Thus, in the present case in the region $r\leqslant r_0$ there exists only an ingoing dust cloud, while in the region $r\geqslant r_0$ only outgoing. Therefore, now the dust clouds cannot be considered as perturbations, but rather as emission of null fluids from the CEH's. To study the stability of the CEH's in this case we have to consider other solutions. However, the following considerations indicate that they may not be stable, too. Projecting the EMT onto the frame given by Eqs. (A3) and (A4), we find that

$$T^-_{(0)(0)}=T^-_{(1)(1)}=T^-_{(0)(1)}=\frac{\beta}{\sqrt{2}}D_+(A)f^{-\prime}(u) \quad (r\leqslant r_0),$$

$$T^+_{(0)(0)}=T^+_{(1)(1)}=-T^+_{(0)(1)}=\frac{\beta}{\sqrt{2}}D_+(A)g^{+\prime}(v) \quad (r\geqslant r_0). \tag{4.26}$$

From Eq. (4.14) and the above expressions we can see that the back reaction of the emission also turns the CEH's into spacetime singularities. Similar to the last subcase, the nature of the singularity is a nonscalar one but strong, as the distortion also diverges as $r\to r_0$,

$$\int\int R^-_{(0)(2)(0)(2)}d\tau d\tau=-\frac{1}{\sqrt{2}\beta}\int\int\frac{f^{-\prime}(u)}{(r_0-r)^2}D_+(A)d\tau d\tau\sim\ln(\tau_0-\tau),$$

$$\int\int R^+_{(0)(2)(0)(2)}d\tau d\tau=-\frac{1}{\sqrt{2}\beta}\int\int\frac{g^{+\prime}(v)}{(r-r_0)^2}D_-(A)d\tau d\tau\sim\ln(\tau_0-\tau). \tag{4.27}$$

Thus, for the perturbations that have nonvanishing components along the ingoing null geodesics defined by l_μ in the region $r\leqslant r_0$, or for the perturbations that have nonvanishing components along the outgoing null geodesics defined by n_μ in the region $r\geqslant r_0$, we would expect that the CEH's will be turned into spacetime singularities.

V. CONCLUDING REMARKS

In this paper, we have considered the stability of the CEH's for a class of nonstatic global cosmic strings found recently by Gregory [9], and found that they are not stable against perturbations. In particular, the back reaction of null dust fluids will turn them into spacetime singularities. Thus resulted singularities are strong in the sense that the distortion of test particles diverges when these singular hypersurfaces are approaching.

Recently, we have shown that the CEH's of topological domain walls are also not stable against massless scalar field [16] and null dust fluids [17]. Thus, a natural question is are all the cosmological event horizons not stable? If some are but others not, what are the criteria for them? It was exactly this consideration that motivated us to study the test particles in Sec. III, at the aim of generalizing the HK conjecture to the study of the stability of the CEH's. Comparing the results obtained in Sec. III with the ones obtained in Sec. IV, it is really remarkable to see that the HK conjecture can be used directly to the study of the stability of CEH's (as far as the examples considered in this paper are concerned), except for the case where $b(t)=\ln[\cosh(\beta t)]$. In the latter case, although the study of the test particles gives a correct prediction for the nature of the resulted singularities, but the quantity $T_{\mu\nu}T^{\mu\nu}$ for the test particle diverges, while for the real perturbations it does not. As a matter of fact, the divergence of the Kretschmann scalar is due to the nonlinear interaction of gravitational fields. Thus, to properly extend the HK conjecture to the case of CEH's, we need also to take the gravitational interaction among gravitational waves into account, a case which is now under our investigation.

Finally, we would like to note that although the CEH's found by Gregory are not stable, and after the back reaction of perturbations is taken into account, they will be turned into spacetime singularities, the hope that the time dependence of the spacetimes for global cosmic strings may be free of spacetime singularities has not been negated completely. As we mentioned in the Introduction, the class of spacetimes considered by Gregory is not the most general spacetimes for nonstatic global cosmic strings. For some other cases, one may expect that the gravitational and particle radiation is so strong that the gravitational field of a global string may be well dilated before any spacetime singularity or event horizon is formed. Spacetimes with cylindrical symmetry are quite different from those with spherical symmetry. In the former case gravitational radiation in general always exists.

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APPENDIX

In this appendix, we shall briefly review some main properties of the spacetimes given by

$$ds^2=e^{2A(r)}dt^2-dr^2-e^{2[A(r)+b(t)]}dz^2-C^2(r)d\theta^2, \tag{A1}$$

where A, b, and C are arbitrary functions of their indicated arguments.

It can be shown that the corresponding radial timelike geodesics have the first integral

$$\dot t=\frac{E}{e^{2A(r)}},\quad \dot r=\epsilon\left(\frac{E^2}{e^{2A(r)}}-1\right)^{1/2}, \tag{A2}$$

where $\epsilon=+1$ corresponds to the outgoing geodesics, while $\epsilon=-1$ to the ingoing geodesics.

Denoting the tangent vector to the geodesics by $\lambda^\mu_{(0)}$,

$$\lambda^\mu_{(0)}\equiv\frac{dx^\mu}{d\tau}=\dot t\,\delta^\mu_t+\dot r\,\delta^\mu_r=\dot t\,\delta^\mu_t+e^{-a(R)}\dot r\,\delta^\mu_R, \tag{A3}$$

we can construct other three orthogonal spacelike vectors

$$\lambda^\mu_{(1)}=e^{-A(r)}\dot r\,\delta^\mu_t+e^{A(r)}\dot t\,\delta^\mu_r=e^{-a(R)}\dot r\,\delta^\mu_t+\dot t\,\delta^\mu_R,$$

$$\lambda^\mu_{(2)}=e^{-[A(r)+b(t)]}\delta^\mu_z,\quad \lambda^\mu_{(3)}=C^{-1}(r)\,\delta^\mu_\theta. \tag{A4}$$

Then, it can be shown that

$$\lambda^\mu_{(i)}\lambda_{(j)\mu}=\eta_{ij},\quad \lambda^\mu_{(i);\nu}\lambda^\nu_{(0)}=0\quad (i,j=0,1,2,3), \tag{A5}$$

where η_{ij} is the Minkowski metric. The above equations show that the four unit vectors $\lambda^\mu_{(i)}$ form a free falling frame or parallel-propagated orthogonal frame (PPON) along the timelike geodesics.

For the particular solutions of $A(r)$ given by Eqs. (2.6) and (2.13), Eq. (A2) has the following integration:

$$e^{-2\beta R}=\beta^2(r_0-r)^2=\beta^2(\tau_0^2-\tau^2),$$

$$e^{2\beta t}=\frac{\tau_0+\tau}{\tau_0-\tau}\quad (r\leqslant r_0) \tag{A6}$$

for $r\leqslant r_0$, and

$$e^{2\beta R}=\beta^2(r-r_0)^2=\beta^2(\tau_0^2-\tau^2),$$

$$e^{2\beta t}=\frac{\tau_0+\tau}{\tau_0-\tau}\quad (r\geqslant r_0) \tag{A7}$$

for $r\geqslant r_0$, where τ_0 is chosen such that when $r\to r_0$, we have $\tau\to\tau_0$.

On the other hand, the corresponding Kretschmann scalar to the metric (A1) is given by

$$\mathcal{R}\equiv R_{\alpha\beta\gamma\lambda}R^{\alpha\beta\gamma\lambda}=4\left\{\left(\frac{C''}{C}\right)^2+2\left(\frac{A'C'}{C}\right)^2\right\}+4\{2(A''+A'^2)^2+[A'^2-(b''+b'^2)e^{-2A}]^2\}. \tag{A8}$$

Choosing a null tetrad frame, on the other hand, as

$$l_\mu=\frac{1}{\sqrt2}(e^{A(r)}\delta^t_\mu+\delta^r_\mu)=\frac{e^{a(R)}}{\sqrt2}(\delta^t_\mu+\delta^R_\mu),$$

$$n_\mu=\frac{1}{\sqrt2}(e^{A(r)}\delta^t_\mu-\delta^r_\mu)=\frac{e^{a(R)}}{\sqrt2}(\delta^t_\mu-\delta^R_\mu),$$

$$m_\mu=\frac{1}{\sqrt2}[e^{A(r)+b(t)}\delta^z_\mu+iC(r)\,\delta^\theta_\mu],$$

$$\overline{m}_\mu=\frac{1}{\sqrt2}[e^{A(r)+b(t)}\delta^z_\mu-iC(r)\,\delta^\theta_\mu], \tag{A9}$$

we find that the nonvanishing Weyl scalars are given by

$$\Psi_2=-\frac12 C_{\mu\nu\lambda\delta}[l^\mu n^\nu l^\lambda n^\delta-l^\mu n^\nu m^\lambda\overline{m^\delta}]=\frac{1}{12}\left\{-\frac{C''}{C}+A''+\frac{C'}{C}A'+(b''+b'^2)e^{-2A}\right\},$$

$$\Psi_0=-C_{\mu\nu\lambda\delta}l^\mu m^\nu l^\lambda m^\delta=-3\Psi_2,$$

$$\Psi_4=-C_{\mu\nu\lambda\delta}n^\mu\overline{m^\nu}n^\lambda\overline{m^\delta}=-3\Psi_2. \tag{A10}$$

Thus, we have

$$\Psi_0\Psi_4=9\Psi_2^2. \tag{A11}$$

Then, according to the theorem given in [18], we find that the metric (A1) is always Petrov type D, except for the degenerate case where $\Psi_2=0$, which is Petrov type O.

Finally, we would like to note that $l_\mu(n_\mu)$ defines an ingoing (outgoing) radial null geodesic congruence [15].

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