Baryogenesis during reheating in natural inflation and comments on spontaneous baryogenesis

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We calculate the baryon asymmetry created by the decay of a pseudo Nambu-Goldstone boson (PNGB) whose interactions violate baryon number conservation. Our results are in disagreement with previous results in the original spontaneous baryogenesis models for the asymmetry produced by the decay of an oscillating scalar field with B-number-violating derivative couplings; we find that the net baryon number density is proportional to θ_i^3, where θ_i is the amplitude of the PNGB field in natural inflation at the onset of reheating. While our calculation of the asymmetry is carried out in the context of natural inflation our approach is generally valid for baryogenesis models using decaying classical fields. We include a complete derivation of the number density of particles produced by the decay of a classical scalar field. [S0556-2821(97)07422-5]

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I. INTRODUCTION

In this paper, we calculate the baryon asymmetry obtained during reheating in natural inflation by using an approach that is generally valid for baryogenesis models with decaying classical fields. Our results are in disagreement with the results presented in the original spontaneous baryogenesis papers.

In natural inflation the role of the inflation is played by a pseudo Nambu-Goldstone boson, hereafter referred to as θ, with a potential of the form [1]

$$V(\theta)=\Lambda^4(1-\cos\theta). \tag{1.1}$$

This model was proposed to ''naturally'' provide the flat potential required for inflation to work [2,3]. Here $\theta=\Phi/f$, where Φ is a complex scalar field and f is the scale at which a global symmetry is spontaneously broken; soft explicit symmetry breaking takes place at a lower scale Λ. From Eq. (1.1) one can see that the height of the potential is $2\Lambda^4$ while the width is f. Since the scales of spontaneous and explicit symmetry breaking can ''naturally''be separated by several orders of magnitude, one can obtain $\Lambda\leqslant 10^{-3}f$ as required for successful inflation [4].

In Ref. [14] the results of an extensive study of the conditions under which the θ field can drive inflation are given. After the period of inflationary expansion, the energy density of the θ field is converted to radiation during reheating through its decay to other forms of matter as it oscillates in its potential. Below we shall assume that θ is coupled only to fermions. We treat θ as a classical scalar field coupled to quantized fermion fields Q and L via an interaction term of the form $\overline{Q}Le^{i\theta}+\overline{L}Qe^{-i\theta}$, where Q carries baryon number but L does not. We show that the decay of θ gives rise to a net baryon number density $(n_b-n_{\bar{b}})$ proportional to θ_i^3, where θ_i is the value of the θ field at the onset of reheating.

Our result disagrees with the calculation in the original spontaneous baryogenesis papers [15] where it was argued that the asymmetry is proportional to θ_i to the first power, independent of the details of the baryon number violating couplings of the θ field. Specifically, in previous work, Cohen and Kaplan [15] considered any theory in which a scalar field is derivatively coupled to the baryon current J^μ with a term in the interaction Lagrangian of the form $\mathcal{L}_{\rm int}\propto\partial_\mu\theta J^\mu$, and derived an expression for the baryon asymmetry produced by the decay of the scalar field as it oscillates about its minimum. The pseudo Nambu-Goldstone boson (PNGB) in natural inflation can serve as an example of such a scalar field. Cohen and Kaplan obtained $|\dot{n}_B|=\Gamma f^2|\dot{\theta}|$, where Γ is the decay rate of the θ field and n_B is the net baryon number density. This gives

$$|\Delta n_B|=\Gamma f^2|\Delta\theta|. \tag{1.2}$$

Below we discuss our concerns with this conclusion and present calculations for the specific case of Eq. (1.1); our results *disagree* with Eq. (1.2).

The framework of this paper is as follows. In Sec. II, we write down the Lagrangian density for the inflation field and

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present the equation of motion for θ as it oscillates during the reheating phase, as derived in Ref. [16]. In Sec. III we discuss our concerns with Eq. (1.2) as obtained in Ref. [15] (these concerns were raised in an earlier paper [16] by Dolgov and Freese). We then proceed to calculate the total baryon number and antibaryon number produced during the decay of the θ field, and find a net baryon number density $(n_b - n_{\bar{b}})$ proportional to θ_i^3. We also show that the energy density of the produced particles is equal to the initial energy density of the θ field as a check on our calculation. In Sec. IV, we discuss how constraints on parameters in natural inflation obtained in Ref. [14] affect the quantitative results for baryogenesis. Finally we summarize our results. In the Appendixes we provide details of the calculations outlined in the main body of the paper. In particular, in Appendixes A and B, we include derivations of the number density of particles produced by the decay of a classical scalar field; the number density of particles produced is proportional to the integral over momenta of the one pair production amplitude.

II. THE MODEL

As in Ref. [16] we consider a simple model involving a complex scalar field Φ and fermion fields Q and L with the Lagrangian density[1]

$$\mathcal{L} = -\partial_\mu \Phi^* \partial^\mu \Phi - V(\Phi^*\Phi) + i\overline{Q}\gamma^\mu \partial_\mu Q + i\overline{L}\gamma^\mu \partial_\mu L - m_Q \overline{Q}Q - m_L \overline{L}L + (g\Phi \overline{Q}L + \text{H.c.}). \tag{2.1}$$

Note that, despite their names, Q and L cannot be actual quarks and leptons, since the interaction term does not conserve color. They could, however, represent heavy fermions with other interactions with the fields of the standard model that fix the assignments of global charges. In particular, we shall assume that the field Q carries baryon number while the field L does not. The U(1) symmetry that corresponds to baryon number is therefore identified as

$$\Phi \to e^{i\alpha}\Phi, \quad Q \to e^{i\alpha}Q, \quad L \to L. \tag{2.2}$$

This assignment for Q could be enforced by effective four-fermion interactions between Q and the usual quarks u and d such as $\mathcal{L}_{\text{int}} \sim \overline{Q}u_R \overline{d^c_R} d_R$. Such an interaction could arise from exchange of heavy scalar or vector fields. Assigning baryon number zero to L could be enforced by a renormalizable interaction such as $\mathcal{L}_{\text{int}} \sim \lambda \overline{L} l_L H^\dagger$, where l_L is the neutrino-electron doublet, and H is the Higgs doublet of the standard model. Of course, these interactions are rather baroque; we will not worry about this, however, since our main focus is on the quantum dynamics rather than the construction of an elegant model.

We assume that the global symmetry of Eq. (2.2) is spontaneously broken at an energy scale f via a potential of the form

$$V(|\Phi|) = \lambda(\Phi^*\Phi - f^2/2)^2. \tag{2.3}$$

[1] We use a metric $(-1,1,1,1)$.

The resulting scalar field vacuum expectation value (VEV) is $\langle \Phi \rangle = f e^{i\phi/f}/\sqrt{2}$. Below the scale f, we can neglect the radial mode of Φ since it is so massive that it is frozen out; $m_{\text{radial}} = \lambda^{1/2} f$. The remaining light degree of freedom is ϕ, the Goldstone boson of the spontaneously broken U(1). For simplicity of notation we introduce the dimensionless angular field $\theta \equiv \phi/f$. We then obtain an effective Lagrangian density for θ, Q, and L of the form

$$\mathcal{L}_{\text{eff}} = -\frac{f^2}{2}\partial_\mu\theta\partial^\mu\theta + i\overline{Q}\gamma^\mu\partial_\mu Q + i\overline{L}\gamma^\mu\partial_\mu L - m_Q\overline{Q}Q - m_L\overline{L}L + \left(\frac{g}{\sqrt{2}} f\overline{Q}L e^{i\theta} + \text{H.c.}\right). \tag{2.4}$$

The global symmetry is now realized in the Goldstone mode: $\mathcal{L}_{\text{eff}}$ is invariant under

$$Q \to e^{i\alpha}Q, \quad L \to L, \quad \theta \to \theta + \alpha. \tag{2.5}$$

With a rotation of the form in Eq. (2.5) with $\alpha = -\theta$, the Lagrangian can alternatively be written as

$$\mathcal{L}_{\text{eff}} = -\frac{f^2}{2}\partial_\mu\theta\partial^\mu\theta + i\overline{Q}\gamma^\mu\partial_\mu Q + i\overline{L}\gamma^\mu\partial_\mu L - m_Q\overline{Q}Q - m_L\overline{L}L + \left(\frac{g}{\sqrt{2}} f\overline{Q}L + \text{H.c.}\right) + \partial_\mu\theta J^\mu, \tag{2.6}$$

where the fermion current derives from the U(1) symmetry; here, $J^\mu = \overline{Q}\gamma^\mu Q$.

We now assume that the symmetry (2.2) is also subject to a small explicit breaking, which gives rise to a potential as in Eq. (1.1) and which provides a nonzero mass for the field θ. This explicit symmetry breaking could come from Planck scale physics. Alternatively, one can imagine a scenario similar to that involving the QCD axion where, at energy scales of the order of Λ_{QCD}, instanton effects create the fermion condensate $\langle \psi\psi \rangle \sim \Lambda_{\text{QCD}}^3$, giving rise to a mass term for the axion. Note that for the natural inflation model, the required mass scales are much higher than for the QCD axion. The width of the potential must be roughly the Planck mass in order to achieve enough e-foldings of inflation, and the height of the potential must be roughly $\Lambda^4 \sim [10^{16}\ \text{GeV}]^4$ in order for density perturbations appropriate for structure formation to be produced (see the Discussion section at the end of the paper for more detail). Consequently the scale at which the relevant gauge group (*not* QCD) must become strong is roughly the grand unified (GUT) scale. These and other mechanisms such as those found in technicolor and schizon models for generating a potential for pseudo Nambu-Goldstone bosons are discussed in Refs. [4,14].

Initially, as the θ field rolls down towards the minimum of its potential, its potential energy drives inflation. Let θ_i be the value of the θ field at the beginning of the reheating epoch, after inflationary expansion has ended. (We shall ignore spatial variations in the θ field.) During the reheating epoch the θ field oscillates about the minimum of its potential. While θ oscillates it decays to the fields Q and L. The interactions of the fermionic fields create a thermal bath thereby reheating the Universe. Note that we must take $g \ll 1$ so that fermion masses generated for the fermions from the Yukawa coupling, $m_\psi \sim gf$, are small enough that the fermions can in fact be produced by decays of the pseudo Nambu-Goldstone bosons. See Ref. 10 in Dolgov and Freese [16] for further discussion of this point.

The equation of motion for the θ field with the backreaction of the produced fermions was rigorously derived in the one-loop approximation in Ref. [16]. For small deviations of θ from the equilibrium the potential can be approximated as $V(\theta)=\frac{1}{2}m_R^2 f^2\theta^2$ and the equation of motion during the oscillating phase can be effectively written in the well-known form

$$\ddot{\theta}+m_R^2\theta+\Gamma\dot{\theta}=0, \tag{2.7}$$

where m_R is the renormalized θ mass defined as $\lim_{\omega\to\infty} m_R^2[1+(g^2/4\pi^2)\ln(2\omega/m_R)]=m^2$, where m is the bare mass of the θ field, and $\Gamma\equiv g^2 m_R/8\pi$. (Our expressions above differ by a factor of 2 from those in Ref. [16] because a factor of $1/\sqrt{2}$ was dropped from Eq. (2.5) in Ref. [16].) The solution to this equation is

$$\theta(t)=\theta_i e^{-\Gamma t/2}\cos(m_R t), \tag{2.8}$$

where we have assumed that the initial velocity of the θ field is negligible and have therefore set an arbitrary phase in the cosine to zero. The results obtained below can be easily generalized for arbitrary initial conditions. The above solution was derived assuming $m_Q=m_L=0$. However, it can be shown that nonzero values of m_Q and m_L will not change the solution for θ significantly as long as m_Q, $m_L \ll m_R$, which we shall assume below.

III. BARYOGENESIS

Previous calculations and concerns: In previous work, Cohen and Kaplan [15] considered any theory in which a scalar field is derivatively coupled to the baryon current with a term in the interaction Lagrangian of the form $\mathcal{L}_{\rm int} \propto \partial_\mu\theta J^\mu$, and derived an expression for the baryon asymmetry produced by the decay of the scalar field as it oscillates about its minimum. From Eq. (2.6) one can see that our pseudo Nambu-Goldstone boson is an example of such a scalar field as it has the appropriate coupling. Cohen and Kaplan obtained $|\dot{n}_B|=\Gamma f^2|\dot{\theta}|$, where n_B is the net baryon number density. This gives

$$|\Delta n_B|=\Gamma f^2|\Delta\theta|. \tag{3.1}$$

In a previous paper [16] by Dolgov and Freese, several concerns with this interpretation were raised. We will outline two of these concerns again here, and then proceed with a direct calculation of the baryon asymmetry. Our results will *disagree* with Eq. (3.1).

One concern is as follows: in making the identification $|\dot{n}_B|=\Gamma f^2|\dot{\theta}|$, one is comparing an operator equation, namely, the Euler-Lagrange equation $\ddot{\theta}+m^2\theta=\dot{n}_B/f^2$, with an equation of the form of Eq. (2.7), which is obtained after vacuum averaging. In Ref. [16] the average value $\langle\dot{n}_B\rangle$ was found to be not just $-\Gamma f^2\dot{\theta}$ but a more complicated expression [Eq. (3.3) in Ref. [16]].

A second concern is with regard to energy conservation. The initial energy density of the field θ that creates the baryons and antibaryons is $\rho_\theta(t_i)=\frac{1}{2}f^2 m_R^2\theta_i^2$. At the end this energy density has been converted to baryons and antibaryons, with energy density $\rho_{\rm final}>n_B E_B$ where $E_B\sim m_R/2$ is the characteristic energy of the produced fermions (note that n_B refers to the difference between baryon and antibaryon number densities and not to the total number density of produced particles). It must be true that $n_B E_B<\rho_\theta(t_i)$. If we were to use Eq. (3.1) we would see that this requires $\Gamma< m_R\theta_i^2/\Delta\theta$. Using the definition of Γ, we can write this as $g^2/8\pi<\theta_i^2/\Delta\theta$. Clearly there can be particular choices of g, θ_i, and $\Delta\theta$ for which this condition is not satisfied. Since the arguments put forward in Ref. [15] are independent of the value of g, θ_i, or $\Delta\theta$, this counterexample calls into question the validity of Eq. (3.1).

New calculations and results: We now proceed to calculate the net baryon number density of the particles produced during reheating. We perform an explicit calculation and find a different result from Eq. (3.1). The θ field decays to either $Q\bar{L}$ pairs or $\bar{Q}L$ pairs. (The Q and L fields are not the mass eigenstates. Later in this section we consider effects of oscillations between Q and L fields.) As mentioned earlier, we treat the θ field classically, Q and L are quantum fields and Q carries baryon number. For now we ignore any dilution of the baryon number density due to the expansion of the Universe.

As shown in Appendix A with the Bogolyubov transformation method [17], the average number density n of particle-antiparticle pairs produced by decay of a homogeneous classical scalar field, to lowest order in perturbation theory, is given by

$$n=\frac{1}{V}\sum_{s_1,s_2}\int \widetilde{dp_1}\widetilde{dp_2}|A|^2, \tag{3.2}$$

where A is the one pair production amplitude, subscripts 1 and 2 refer to the final particles produced, and $\widetilde{dp}=d^3p/[(2\pi)^3 2p^0]$. Equation (3.2) can also be obtained using the method presented in Sec. 4-1-1 of Ref. [18], as discussed in Appendix B.

Thus, to lowest order in perturbation theory, the average number density of $Q\bar{L}$ pairs produced during reheating in our model is given by[2]

$$n(Q,\bar{L})=\frac{1}{V}\sum_{s_Q,s_{\bar{L}}}\int \widetilde{dp}\widetilde{dq}|\langle Q(p,s_Q),\overline{L(q,s_L)}|0\rangle|^2. \tag{3.3}$$

We take

$$Q=\sum_s \int \widetilde{dk}[u_k^s b^s e^{+ik\cdot x}+v_k^s d_k^{s\dagger}e^{-ik\cdot x}] \tag{3.4}$$

and a similar expression for L. Here $\{b_k^s,b_{k'}^{s'\dagger}\}=\{d_k^s,d_{k'}^{s'\dagger}\}=(2\pi)^3 2k^0\delta^3(\mathbf{k}-\mathbf{k}')\delta_{ss'}$. Standard algebra gives

[2]Throughout the paper, a state $\langle A(p_1,s_1),\overline{B(p_2,s_2)}|$ corresponds to a final state with an A particle of momentum p_1 and spin s_1 and an anti-B particle with momentum p_2 and spin s_2.

$$n(Q,\overline{L})=\frac{1}{V}\sum_{s_Q,s_{\overline{L}}}\int \widetilde{dp}\widetilde{dq}\left|\langle Q(p,s_Q),\overline{L(q,s_L)}|i\frac{g}{\sqrt{2}}\int d^4x\overline{Q(x)}L(x)e^{i\theta(x)}|0\rangle\right|^2$$

$$=\frac{g^2f^2}{2V}\int \widetilde{dp}\widetilde{dq}\left|(2\pi)^3\delta^3(\mathbf{p}+\mathbf{q})\int_{-\infty}^{\infty}dte^{i2\omega t+i\theta(t)}\right|^2\mathrm{Tr}[(-\not p+m_Q)(-\not q-m_L)], \tag{3.5}$$

where $2\omega=p^0+q^0$. We obtain a similar expression for $n(L,\overline{Q})$ with $\theta(t)$ replaced by $-\theta(t)$. We set the baryon number density n_b to be equal to $n(Q,\overline{L})$ and the antibaryon number density $n_{\overline{b}}$ to be equal to $n(L,\overline{Q})$. Then we have

$$n_{b,\overline{b}}=\frac{g^2f^2}{2\pi^2}\int d\omega\omega^2\left|\int_{-\infty}^{\infty}dt\ e^{2i\omega t}e^{\pm i\theta(t)}\right|^2, \tag{3.6}$$

where the + sign in the exponent refers to baryon number and the − sign to antibaryon number. To carry out the integration over time we expand $e^{i\theta}$ as

$$1+i\theta-\theta^2/2, \tag{3.7}$$

valid for small θ, and use

$$\theta(t)=\begin{cases}\theta_i & \text{for } t\leqslant 0,\\ \theta_i e^{-\Gamma t/2}\cos(m_R t) & \text{for } t\geqslant 0.\end{cases} \tag{3.8}$$

We also use a convergence factor at early times to regularize the integral. We will examine a series of possible terms to find the first nonzero contribution in perturbation theory. The lowest-order term comes from using $e^{i\theta}=1$ from Eq. (3.7) in Eq. (3.6) and gives $\int dte^{2i\omega t}\propto\delta(2\omega)=0$ since we cannot have $\omega=0$ for particle production. The next term in the expansion, the θ term in Eq. (3.7), when squared gives the same contribution to n_b and to $n_{\overline{b}}$. In order to obtain an asymmetry one must consider cross terms. The lowest-order cross term that gives a nonzero contribution to the baryon asymmetry is

$$n_b-n_{\overline{b}}=2\times\frac{g^2f^2}{2\pi^2}\int d\omega\omega^2\left[\frac{\widetilde{\theta}(2\omega)[\widetilde{\theta^2}(2\omega)]^*}{2i}+\text{H.c.}\right], \tag{3.9}$$

where H.c. refers to Hermitian conjugate,

$$\widetilde{\theta}(2\omega)=\int_{-\infty}^{\infty}dt\ e^{2i\omega t}\theta(t) \tag{3.10a}$$

and

$$\widetilde{\theta^2}(2\omega)=\int_{-\infty}^{\infty}dt\ e^{2i\omega t}\theta^2(t). \tag{3.10b}$$

The factor of 2 in Eq. (3.9) arises from the fact that the cross terms in n_b and $n_{\overline{b}}$ terms are the same up to a minus sign. One can see from the form of Eq. (3.9) that we expect the asymmetry to be proportional to θ^3. The details of this calculation are outlined in Appendix C, and the results are presented here.

We obtain

$$n_b=\frac{1}{4}m_Rf^2\theta_i^2+\frac{g^2}{32\pi}m_Rf^2\theta_i^3, \tag{3.11}$$

$$n_{\overline{b}}=\frac{1}{4}m_Rf^2\theta_i^2-\frac{g^2}{32\pi}m_Rf^2\theta_i^3. \tag{3.12}$$

Therefore,

$$n_B\equiv n_b-n_{\overline{b}}=\frac{g^2}{16\pi}m_Rf^2\theta_i^3=\frac{1}{2}\Gamma f^2\theta_i^3. \tag{3.13}$$

We notice that the net baryon number density is proportional to θ_i^3. This disagrees with the calculation in Ref. [15], which gives an asymmetry proportional to θ_i. We also note that the number density of pairs of particles $n_b+n_{\overline{b}}$ is equal to $\frac{1}{2}m_Rf^2\theta_i^2$. Since the energy per pair of particles is m_R, the energy density in the produced particles is $\frac{1}{2}m_R^2f^2\theta_i^2$, which agrees with the initial energy density of the θ field. We have also done the calculation of

$$\rho_{\text{final}}=\frac{1}{V}\sum_{s_Q,s_L}\int \widetilde{dp}\widetilde{dq}(p^0+q^0)[|\langle Q(p,s_Q),\overline{L(q,s_L)}|0\rangle|^2+|\langle L(q,s_L),\overline{Q(p,s_Q)}|0\rangle^2] \tag{3.14}$$

and have verified that we obtain $\frac{1}{2}m_R^2f^2\theta_i^2$.

Mass mixing: In many cases Eq. (3.13) is not yet the complete story because of mass mixing. As we mentioned above the Q and L fields are not mass eigenstates. Therefore a particle that is produced as a Q may later rotate into an L. This effect must be taken into account. Equation (3.13) is completely correct for the case where the fermions Q and L are converted immediately to regular quarks q and leptons l as soon as they are produced (assuming that the temperature is low enough that the q and l cannot convert back into Q and L). In that case, there is no opportunity for mixing to take place, e.g., there is no opportunity for Q to convert to an L. On the other hand, if Q and L do not decay immediately into stable lighter mass particles with appropriate quark quantum numbers, they may have the chance to mix into one another. One can calculate the effects of mixing in either the Q, L basis or in the basis of mass eigenstates; below we will do both.

The mass matrix in the (Q,L) basis is

$$\begin{pmatrix} m_Q & -gf/\sqrt{2} \\ -gf/\sqrt{2} & m_L \end{pmatrix}. \tag{3.15}$$

The mass eigenstates are

$$\psi_1 = \frac{L+\epsilon Q}{\sqrt{1+\epsilon^2}} \quad \text{and} \quad \psi_2 = \frac{Q-\epsilon L}{\sqrt{1+\epsilon^2}} \tag{3.16}$$

with masses $m_Q - gf/(\sqrt{2}\epsilon)$ and $m_L + gf/(\sqrt{2}\epsilon)$, respectively, where $\epsilon = \sqrt{2}gf/(\Delta m + \sqrt{(\Delta m)^2 + 2g^2 f^2})$ and $\Delta m = m_Q - m_L$. Note that $\Delta m = 0$ corresponds to $\epsilon = 1$.

In the ψ_1, ψ_2 basis, one can now calculate the baryon asymmetry as a sum of terms, each of which is a product of a number density of produced particle-antiparticle pairs times the (time-averaged) quark content of the pair:

$$n_B = n(\psi_1, \overline{\psi_2}) |\langle Q|\psi_1\rangle|^2 + n(\psi_2, \overline{\psi_1}) |\langle Q|\psi_2\rangle|^2 - n(\psi_1, \overline{\psi_2}) |\langle \overline{Q|\psi_2}\rangle|^2 - n(\psi_2, \overline{\psi_1}) |\langle \overline{Q|\psi_1}\rangle|^2. \tag{3.17}$$

Here $n(\psi_1, \overline{\psi_2})$ and $n(\psi_2, \overline{\psi_1})$ are the number densities of ψ_1 and $\overline{\psi_2}$ pairs and ψ_2 and $\overline{\psi_1}$ pairs, respectively; and $|\langle Q|\psi_i\rangle|^2$ is the probability that a particle that is produced as a ψ_i (where $i=1,2$) is measured as a Q. Hence, for example, the first term is the product of the number density of $\psi_1 \overline{\psi_2}$ pairs produced times the quark content of ψ_1.

Note that we are here computing a time averaged baryon asymmetry; actually the value of the baryon asymmetry oscillates in time, as discussed in Appendix D. From Eq. (3.16) we see that the probability that $\psi_{1,2}$ is measured as a Q is

$$|\langle Q|\psi_1\rangle|^2 = |\langle \overline{Q|\psi_1}\rangle|^2 = \frac{\epsilon^2}{1+\epsilon^2} \tag{3.18}$$

and

$$|\langle Q|\psi_2\rangle|^2 = |\langle \overline{Q|\psi_2}\rangle|^2 = \frac{1}{1+\epsilon^2}. \tag{3.19}$$

As in Eq. (3.2), the number densities of particle-antiparticle pairs are obtained by squaring the production amplitudes for the pairs,

$$n_{i\bar{j}} = \frac{1}{V} \sum_{s_i, s_{\bar{j}}} \int \widetilde{dk_i} \widetilde{dk_{\bar{j}}} |A_{i\bar{j}}|^2, \tag{3.20}$$

where i and j are either 1 or 2. The amplitude for production of a $\psi_i \overline{\psi_j}$ pair is

$$A_{i\bar{j}} = \langle \psi_i \overline{\psi_j} | i \int d^4x \left(\frac{g}{\sqrt{2}} f e^{i\theta} \overline{Q} L + \text{H.c.} \right) |0\rangle. \tag{3.21}$$

Using Eqs. (3.18), (3.19), and (3.20), we can write Eq. (3.17) as

$$n_B = -\frac{1}{V} \sum_{s_1, s_2} \int \widetilde{dk_1} \widetilde{dk_2} \left(\frac{1-\epsilon^2}{1+\epsilon^2} \right) [|A_{1\bar{2}}|^2 - |A_{2\bar{1}}|^2]. \tag{3.22}$$

Using

$$\overline{Q}L = \left(\frac{1}{1+\epsilon^2} \right) [\overline{\psi_2}\psi_1 - \epsilon \overline{\psi_2}\psi_2 + \epsilon \overline{\psi_1}\psi_1 - \epsilon^2 \overline{\psi_1}\psi_2] \tag{3.23}$$

and its Hermitian conjugate, we calculate the relevant production amplitudes:

$$A_{1\bar{2}} = \left\langle \psi_1, \overline{\psi_2} \right| i \int d^4x \left(\frac{g}{\sqrt{2}} f e^{i\theta} \overline{Q} L + \frac{g}{\sqrt{2}} f e^{-i\theta} \overline{L} Q \right) \left| 0 \right\rangle \tag{3.24a}$$

to find

$$A_{1\bar{2}} = i \frac{g}{\sqrt{2}} f \left(\frac{1}{1+\epsilon^2} \right) \left\langle \psi_1 \overline{\psi_2} \right| \int d^4x (\overline{\psi_1}\psi_2 e^{-i\theta} - \epsilon^2 e^{i\theta} \overline{\psi_1}\psi_2) \left| 0 \right\rangle. \tag{3.24b}$$

Now the two matrix elements in Eq. (3.24b) are similar to the ones we calculated in Eq. (3.5), with $\overline{Q}L$ replaced by $\overline{\psi_1}\psi_2$. Hence, we have

$$A_{1\bar{2}} = \left(\frac{1}{1+\epsilon^2} \right) (A_{L\bar{Q}} - \epsilon^2 A_{Q\bar{L}}). \tag{3.25}$$

Similarly,

$$A_{2\bar{1}} = \text{H.c.}[A_{1\bar{2}}] = \left(\frac{1}{1+\epsilon^2} \right) (-\epsilon^2 A_{L\bar{Q}} + A_{Q\bar{L}}). \tag{3.26}$$

Thus Eq. (3.22) becomes

$$n_B=\left(\frac{1-\epsilon^2}{1+\epsilon^2}\right)^2\left(\sum_{s_Q,s_{\bar{L}}}\int \widetilde{dk_Q}\widetilde{dk_{\bar{L}}}|A_{Q\bar{L}}|^2-\sum_{s_L,s_{\bar{Q}}}\int \widetilde{dk_L}\widetilde{dk_{\bar{Q}}}|A_{L\bar{Q}}|^2\right)=\left(\frac{1-\epsilon^2}{1+\epsilon^2}\right)^2\times(\text{our previous answer}). \quad (3.27)$$

Thus we find that

$$n_B=\frac{1}{2}\,\Gamma f^2\theta_i^3\left(\frac{1-\epsilon^2}{1+\epsilon^2}\right)^2. \quad (3.28)$$

If $m_Q=m_L$, $\epsilon=1$ and the asymmetry vanishes because in this case of the net baryon number of a $(\psi_1,\bar{\psi}_2)$ pair or a $(\psi_2,\bar{\psi}_1)$ pair is 0 and thus no baryon asymmetry is produced.

Another derivation of Eq. (3.28) is given in Appendix D. In the preceding paragraphs we considered particle production and mixing in the mass eigenstate ψ_1,ψ_2 basis. In Appendix D we work in the Q,L basis. We find the oscillations of the baryon asymmetry with time, and obtain the same expression as in Eq. (3.28) for the time-averaged baryon asymmetry.

Thermalization: After the θ field has decayed into ψ_1 and ψ_2 particles, thermal equilibrium can be established if these particles have other interactions with each other and with other particles. As long as one introduces interactions such as $\chi\bar{\psi}_1\psi_1$ and $\bar{\chi}\bar{\psi}_2\psi_2$ as a part of a realistic model, the number of ψ_1-$\bar{\psi}_1$ particles and of ψ_2-$\bar{\psi}_2$ particles does not change, thereby preserving the baryon asymmetry. (Interactions such as $\chi\bar{\psi}_1\psi_2+\text{H.c.}$ would, however, destroy the baryon asymmetry.) The fields ψ_1 and ψ_2 will annihilate or decay to lighter particles that will thermalize. If these interactions preserve the net baryon number, then the asymmetry will survive.

Quantitative results: So far we have not included the effects of the expansion of the Universe. For baryon number created when $H\leqslant\Gamma$, we may neglect the expansion and directly use the results obtained above in Eq. (3.28) with θ_i replaced with the value of θ at $H=\Gamma$. Since the θ field dominates the cosmic energy density, the condition $H=\Gamma$ fixes the amplitude of θ at that moment to be

$$\theta_1=\sqrt{3/4\pi}(\Gamma m_{\text{Pl}}/fm_R)\approx 0.02g^2 m_{\text{Pl}}/f\ll 1. \quad (3.29)$$

In the early stages of reheating with $\theta>\theta_1$, expansion of the Universe must be taken into account.

The decay of the θ field produces relativistic $\psi_{1,2}$ and $\bar{\psi}_{1,2}$ with energies $\omega\approx m_R/2$. This state is far from thermal equilibrium [the temperature of the thermalized plasma in Eq. (3.30) below may be smaller than the ψ masses]. The rate of thermalization depends upon the interaction strength of the fermions created in the θ decay. It is typically higher than the decay rate because $g\ll 1$ to ensure reasonable fermion masses. Thermalization could occur either through annihilation of ψ_1 and $\bar{\psi}_1$ or ψ_2 and $\bar{\psi}_2$ into light particles or through their decays and subsequent elastic scattering. Assuming that these processes are fast we can roughly estimate the reheat temperature in the instantaneous decay approximation, $\rho_{\text{rad}}=\rho_\theta(t=\Gamma^{-1})$, as

$$T_{\text{reh}}=(90/8\pi^3 g_*)^{1/4}\sqrt{\Gamma m_{\text{Pl}}}\approx 0.15gg_*^{-1/4}\Lambda\sqrt{m_{\text{Pl}}/f}, \quad (3.30)$$

where we have taken $m_R=\Lambda^2/f$.

The entropy density after thermalization is given by $s=4\pi^2 g_* T_{\text{reh}}^3/90$. It is conserved in the comoving volume if the expansion of the Universe is adiabatic, in particular in the absence of first-order phase transitions as the Universe cools. Baryonic charge density is also assumed to be conserved inside a comoving volume during and after thermalization and so the baryon-to-entropy ratio n_B/s remains constant in the course of expansion.

First we find the baryon asymmetry produced after $H\leqslant\Gamma$ so that expansion may be neglected (subscript 1 refers to this case). Using Eqs. (3.28), (3.29), and (3.30) we find

$$\left(\frac{n_B}{s}\right)_1\approx 10^{-4}\,\frac{g^5}{g_*^{1/4}}\left(\frac{m_{\text{Pl}}}{f}\right)^{3/2}\frac{f}{\Lambda}\left(\frac{1-\epsilon^2}{1+\epsilon^2}\right)^2. \quad (3.31)$$

In the models studied in Ref. [14] $(f/m_{\text{Pl}})\sim 1$ and $f/\Lambda=10^6-10^3$, so to get a reasonable baryon asymmetry we need a rather large coupling, $g>10^{-2}$ (for $\epsilon\ll 1$).

In fact the asymmetry should be noticeably larger than that given by Eq. (3.31). The result that we got above refers to the case when $H<\Gamma$ but the process of particle production starts much earlier when $H\approx m_R$ and the inflation field begins to oscillate around the bottom of the potential. The net baryon number density produced while $H>\Gamma$ is again proportional to θ^3, as it is associated with the interference between the θ and the θ^2 terms in $|\int dt\, e^{2i\omega t}(1+i\theta-\theta^2/2)|^2$ in Eq. (3.6). The generation of the asymmetry is more efficient at early times $(H>\Gamma)$ since the amplitude of the θ field, which goes down with the scale factor as $R^{-3/2}$, is larger. However, when $H>\Gamma$ one must include the effects of the expansion of the Universe on the production of the baryon asymmetry. This makes the exact calculations considerably more complicated. Still we can roughly estimate the asymmetry in the following way. The difference between the production of particles and antiparticles is most profound at early times, $\Delta t_a\sim 1/m_R$, when θ is larger. The total number of particles produced in time Δt_a is proportional to $\Gamma\Delta t_a n_\theta$ and, as we mention above, the baryon number asymmetry must vary as θ^3. Therefore, a reasonable estimate of the net baryon number density created while $H>\Gamma$ is $n_B\sim\Gamma f^2\theta_i^3$. Between the time of peak production of baryon asymmetry at $t_a\sim 1/m_R$ and the peak entropy production at $t_b\sim 1/\Gamma$ we will take the baryon asymmetry to be diluted by a factor of $(R_a/R_b)^3\sim(t_a/t_b)^2\sim(\Gamma/m_R)^2$ due to the expansion of the Universe, where we have taken the Universe to behave as matter dominated with $R\propto t^{2/3}$ in the usual fashion during reheating. Thus the baryon-to-entropy ratio at time t_b and afterwards is $(n_B/s)_2\sim\Gamma f^2\theta_i^3(\Gamma/m_R)^2/s$. The calculation of the entropy density is exactly the same as described above Eq. (3.31), while the baryonic charge density is larger than the $H<\Gamma$ case by a factor of $(\theta_i/\theta_1)^3(\Gamma/m_R)^2=\theta_i/\theta_1=m_R/\Gamma=8\pi/g^2\gg 1$. Consequently, we get that the total baryon asymmetry of the Universe is approximately equal to

$$\left(\frac{n_B}{s}\right)_2=\frac{\theta_i}{\theta_1}\left(\frac{n_B}{s}\right)_1\approx 3\times 10^{-3}\,\frac{g^3}{g_*^{1/4}}\left(\frac{m_{\rm Pl}}{f}\right)^{3/2}\frac{f}{\Lambda}\left(\frac{1-\epsilon^2}{1+\epsilon^2}\right)^2. \tag{3.32}$$

Here subscript 2 refers to the case where expansion has been included. Henceforth we use Eq. (3.32) as our estimate of the baryon asymmetry produced.

IV. DISCUSSION

In Ref. [14], the authors obtain constraints on the parameters Λ and f. The stipulation that a large fraction of the Universe after inflation have inflated by at least 60 e-foldings gives $f\geqslant 0.06M_{\rm Pl}$. A stronger constraint can be obtained by requiring the formation of galaxies to take place early enough in the history of the universe; in this way one obtains $f\geqslant 0.3M_{\rm Pl}$. A constraint on Λ is derived by using Cosmic Background Explorer (COBE) data on the density fluctuation amplitude and is plotted in Fig. 1 of Ref. [14]; the upper bound on Λ thus obtained ranges from 10^{13} to 10^{16} GeV for f between $0.3M_{\rm Pl}$ and $1.2M_{\rm Pl}$. If one desires the density fluctuations from inflation to be responsible for the large-scale structure of our Universe and hence for the COBE anisotropy, then Λ must be equal to the above values rather than simply being bounded by these numbers.

If the baryon asymmetry produced above is accompanied by an equal lepton asymmetry, so that $B-L=0$, it will be wiped out by baryon number violating sphaleron processes unless the reheat temperature is below 100 GeV. The low reheat temperature condition may be a desirable feature of our model as many inflation models have difficulty creating a high reheat temperature. Furthermore, we shall require that $T_{\rm reh}>10$ MeV so that we reproduce standard nucleosynthesis. If, in addition, one requires the density fluctuations from inflation to serve as the explanation for the COBE data rather than merely being bounded by it, then Λ is determined as a function of f as described in the previous paragraph; then the combination of these constraints implies that $10^{-14}<g<10^{-10}$ for Λ and f equal to 10^{13} GeV and $0.3M_{\rm Pl}$, respectively, and the asymmetry generated by the mechanism considered above is by far below the necessary observed value. However, if Λ is merely bounded by COBE measurements (density fluctuations must then be generated some other way than by the inflation), then g can be much larger as can the baryon asymmetry. Alternatively if a nonzero $(B-L)$ is generated, for example, if the L fields carry no lepton number, then it is not destroyed by the electroweak processes and the coupling constant g need not be so small.

In our perturbative calculations of the number of pairs of particles produced we have assumed that the masses of the fermions are smaller than the mass m_R of the theta field and that $gf<m_{Q,L}$; otherwise the perturbative approach is not applicable. This implies that $gf<m_R=\Lambda^2/f$ or $g<(\Lambda/f)^2$. In this case, the baryon asymmetry is rather small as $(n_B/s)_2<10^{-3}(\Lambda/f)^5(m_{\rm Pl}/f)^{1.5}<10^{-18}$ (in obtaining this limit we have included the simultaneous constraint on Λ and f from density fluctuation constraints in Ref. [14]). If, however, θ is not the inflation field, as in the original version of the spontaneous baryogenesis scenario [15], then the parameters Λ and f do not necessarily satisfy the above bounds and the asymmetry may be quite large, especially if $f\ll m_{\rm Pl}$. In such a case, one would have to redo the calculation of the entropy if θ does not dominate the energy density of the Universe when it decays. A period of inflation prior to the decay of the PNGB would also be required so that θ and, consequently, the baryon asymmetry have the same sign within present-day domains of sizes 100 Mpc or greater (constraints on the minimum size scale of domains of matter and antimatter in a matter-antimatter symmetric universe are discussed in the following references [19]).

An interesting possibility is that the mass of fermions is not below m_R and the perturbative approach is not applicable. The nonperturbative calculations in this case are more complicated and will be presented elsewhere.

In conclusion, we have calculated the baryon asymmetry created by a pseudo Nambu-Goldstone boson with baryon-number-violating couplings in the context of natural inflation. We have obtained a general result for the baryon asymmetry created by the decay of an oscillating scalar field with baryon-number-violating couplings and demonstrated explicitly that the asymmetry is not proportional to θ_i to the first power as claimed in earlier work.

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APPENDIX A: NUMBER DENSITY OF PRODUCED PARTICLES IN TERMS OF ONE PAIR PRODUCTION AMPLITUDE

Here we use the Bogolyubov transformation method to obtain Eq. (3.2). We show that in the lowest order of perturbation theory, the average number density of particle-antiparticle pairs produced by decay of the initial scalar field is given by

$$n=\frac{1}{V}\sum_{s_1,s_2}\int \widetilde{dp_1}\widetilde{dp_2}|A|^2,$$

where A is the one pair production amplitude and subscripts 1 and 2 refer to the final particle and antiparticle produced. For simplicity we will work with scalar fields here; the generalization to production of fermions is similar and has been performed in Ref. [20].

We begin with a classical scalar field $\phi(t)$ coupled to a quantum complex scalar χ:

$$L_{\rm int}=g\phi(t)\chi^*\chi. \tag{A1}$$

At early times $t\rightarrow-\infty$, we take $L_{\rm int}=0$ so that χ is expanded in terms of creation and annihilation operators:

$$\chi=\int \widetilde{dk}[a_{\mathbf{k}}\exp(-i\omega t+i\mathbf{k}\cdot\mathbf{x})+b_{\mathbf{k}}^{\dagger}\exp(i\omega t-i\mathbf{k}\cdot\mathbf{x})], \quad (A2)$$

where $\omega=\sqrt{\mathbf{k}^2+m^2}$. Here the commutators are $[a_{\mathbf{k}_1},a_{\mathbf{k}_2}^{\dagger}]=(2\pi)^3 2k_1^0\delta^{(3)}(\mathbf{k}_1-\mathbf{k}_2)$ and a similar relation holds for the antiparticle creation and annihilation operators $b_{\mathbf{k}}$. Then, at later times, $\phi(t)\neq 0$ and Eq. (A2) is replaced by

$$\chi=\int \widetilde{dk}[a_{\mathbf{k}}f_k(t)\exp(i\mathbf{k}\cdot\mathbf{x})+b_{\mathbf{k}}^{\dagger}f_k^*(t)\exp(-i\mathbf{k}\cdot\mathbf{x})], \quad (A3)$$

with equation of motion

$$[\partial_t^2+\mathbf{k}^2+m^2-g\phi(t)]f_k(t)=0. \quad (A4)$$

The subscript on f_k, and on α_k and β_k below, refers to $|\mathbf{k}|$ and not to the momentum four vector. For continuity at early times $f_k(t\rightarrow-\infty)=\exp(-i\omega t)$. We also assume that $\phi(t)\rightarrow 0$ for $t\rightarrow\infty$. Then we have

$$f_k(t\rightarrow+\infty)\rightarrow\alpha_k e^{-i\omega t}+\beta_k e^{i\omega t}, \quad (A5)$$

so that $\chi(t)$ evolves as

$$\chi(t\rightarrow+\infty)=\int \widetilde{dk}[\exp(-i\omega t+i\mathbf{k}\cdot\mathbf{x})(\alpha_k a_{\mathbf{k}}+\beta_k^* b_{-\mathbf{k}}^{\dagger}) + \exp(i\omega t-i\mathbf{k}\cdot\mathbf{x})(\alpha_k^* b_{\mathbf{k}}^{\dagger}+\beta_k a_{-\mathbf{k}})]. \quad (A6)$$

One can define new creation and annihilation operators for particles,

$$\tilde{a}_{\mathbf{k}}=\alpha_k a_{\mathbf{k}}+\beta_k^* b_{-\mathbf{k}}^{\dagger}, \quad (A7a)$$

and for antiparticles,

$$\tilde{b}_{\mathbf{k}}=\alpha_k b_{\mathbf{k}}+\beta_k^* a_{-\mathbf{k}}^{\dagger}. \quad (A7b)$$

Then the operator of final particle number is given by $\tilde{N}_{\mathbf{k}}=\tilde{a}_{\mathbf{k}}^{\dagger}\tilde{a}_{\mathbf{k}}/[2k^0V]$.

The number of particles in the final state of momentum $\mathbf{k}$ is given by

$$N_{\mathbf{k}}=\langle 0|\tilde{N}_{\mathbf{k}}|0\rangle=|\beta_k|^2. \quad (A8)$$

Thus the total number density of produced particles is

$$n=\frac{1}{V}\frac{V}{(2\pi)^3}\int d^3k\, N_{\mathbf{k}}=\int \frac{d^3k}{(2\pi)^3}|\beta_k|^2. \quad (A9)$$

This result, obtained by the method of Bogolyubov coefficients, can be found in Refs. [17,21].

Now we shall calculate β_k in perturbation theory. Expanding $f=f_0+f_1$, we have $f_0=\exp(-i\omega t)$ and the equation of motion (A4) becomes

$$(\partial_t^2+\mathbf{k}^2+m^2)f_1=g\phi(t)\exp(-i\omega t). \quad (A10)$$

Using the Green's function method we find

$$f_1(t)=-g\int \frac{d\omega'}{2\pi}\frac{\tilde{\phi}(\omega'-\omega)}{\omega'^2-\mathbf{k}^2-m^2}e^{-i\omega' t}. \quad (A11)$$

Taking the residue at the pole $\omega'=-\sqrt{\mathbf{k}^2+m^2}=-\omega$, we find the coefficient of $\exp(+i\omega t)$ to be

$$\beta_k=ig[\tilde{\phi}(2\omega)]^*/2\omega. \quad (A12)$$

Now, for comparison, let us calculate the field theory amplitude with the interaction Lagrangian given by Eq. (A1):

$$A=\left\langle k_1,\bar{k}_2\right| i\int d^4x g\phi(t)\chi^*\chi\left|0\right\rangle. \quad (A13)$$

Perturbatively the matrix element is easy to calculate using Eq. (A2), and we find

$$A=ig(2\pi)^3\delta^3(\mathbf{k}_1+\mathbf{k}_2)\int dt\,\phi(t)\exp[i(\omega_1+\omega_2)t], \quad (A14)$$

so that

$$|A^2|=g^2V(2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2)|\tilde{\phi}(\omega_1+\omega_2)|^2. \quad (A15)$$

Now if we integrate over $\widetilde{dk_1}\widetilde{dk_2}$, we find that

$$n=\frac{1}{V}\int \widetilde{dk_1}\widetilde{dk_2}|A|^2=\int \frac{d^3k}{(2\pi)^3}g^2\frac{|\tilde{\phi}(2\pi)|^2}{4\omega^2}. \quad (A16)$$

This is exactly Eq. (A9) with β_k given by Eq. (A12). Thus we have shown that the number density of produced particles is given by the integral of the one pair production amplitude squared.

APPENDIX B: SECOND DERIVATION OF NUMBER DENSITY OF PRODUCED PARTICLES IN TERMS OF ONE PAIR PRODUCTION AMPLITUDE

Equation (3.2) can also be obtained using the method presented in Sec. 4-1-1 of Ref. [18]. [We have ignored the higher-order vacuum graphs that give the exponential factor $\exp(-\bar{n})$ in Eqs. (4-23), of Ref. [18].] We have verified that we obtain the Poisson distribution for the number of $(Q,\bar{L})$ pairs and $(\bar{Q},L)$ pairs as in Ref. [18]. Indeed the derivation of the Poisson distribution can be done exactly along the same lines as in Ref. [18]. The only difference is that in the example considered in this book the matrix element describes the production of a single photon by an external current while in our case it gives the amplitude for production of a *pair* of particles. For the multiparticle production amplitude this gives rise to a different normalization, namely, in the case of the production of n photons the amplitude contains the factor $1/\sqrt{n!}$ connected with identical photons while for the case of production of n pairs of $Q\bar{L}$ (or charge conjugate) the amplitude contains $1/n!$. In the case of photons the multiparticle amplitude squared contains the following n-dependent factors: $|A_n^\gamma|^2\sim|(n!)(1/n!)(1/\sqrt{n!})|^2\sim 1/n!$. The first factor of $n!$ comes from $n!$ combinations that appear when the photon production operator acts on the multiphoton state $\langle k_1,k_2,\ldots,k_n\|(a_k^+)^n$. The factor of $1/n!$ comes from the expansion of the action $S=\exp(i\int d^4xA^\mu J_\mu)$, and the factor of $1/\sqrt{n!}$ comes from the normalization of the n-photon state. So the net result is proportional to $1/n!$, which is exactly what is needed to get the Poisson distribution $p_n=\exp(-\bar{n})\bar{n}^n/n!$. In the case of the production of n pairs, we have the same $1/n!$ from the expansion of the action, but now we get $1/n!$ coming from the normalization and not $1/\sqrt{n!}$ as before. However, the action of the product of the creation operators of Q and $\bar{L}$, which can be symbolically written as $(a_Q^+ b_L^+)^n$, gives now an overall factor of $n!$ from the action of, say, $(a_Q^+)^n$, as above, and also the sum of $n!$ equal but not interfering terms, each of them being proportional to a different delta function of the momenta, $\delta(p_{Q_j}+p_{L_k})$. Thus in the matrix element squared we will get the same overall factor $1/n!$, which is necessary for the Poisson distribution.

APPENDIX C: CALCULATION OF BARYON ASYMMETRY

Here we calculate the lowest-order nonzero contribution to the baryon asymmetry; we derive Eq. (3.13) from Eqs. (3.9) and (3.10). As our starting point, we have

$$n_b-n_{\bar{b}}=\frac{g^2f^2}{\pi^2}\int d\omega\,\omega^2\left[\frac{\tilde{\theta}(2\omega)[\widetilde{\theta^2}(2\omega)]^*}{2i}+\text{H.c.}\right], \tag{C1}$$

where

$$\tilde{\theta}(2\omega)=\int_{-\infty}^{\infty}dt\,e^{2i\omega t}\theta(t) \tag{C2}$$

and

$$\widetilde{\theta^2}(2\omega)=\int_{-\infty}^{\infty}dt\,e^{2i\omega t}\theta^2(t). \tag{C3}$$

Using Eq. (3.8), we find that

$$\tilde{\theta}(2\omega)=\frac{\theta_i}{4i\omega}\left[\frac{(-\Gamma/2+im_R)}{(-\Gamma/2+im_R+2i\omega)}-\frac{(\Gamma/2+im_R)}{(-\Gamma/2-im_R+2i\omega)}\right] \tag{C4}$$

and

$$\widetilde{\theta^2}(2\omega)^*=-\frac{\theta_i^2}{4i\omega}\left[\frac{(im_R+\Gamma/2)}{2im_R+2i\omega+\Gamma}+\frac{(-im_R+\Gamma/2)}{2i\omega-2im_R+\Gamma}+\frac{\Gamma}{2i\omega+\Gamma}\right]. \tag{C5}$$

Thus

$$\begin{aligned}\tilde{\theta}\widetilde{\theta}^{2*}=\frac{\theta_i^3}{16\omega^2}\Bigg[&\frac{(-m_R^2-\Gamma^2/4)}{(2im_R+2i\omega+\Gamma)(2i\omega+im_R-\Gamma/2)}+\frac{(m_R^2-\Gamma^2/4+\Gamma im_R)}{(2i\omega-2im_R+\Gamma)(2i\omega+im_R-\Gamma/2)}+\frac{\Gamma(im_R-\Gamma/2)}{(2i\omega+\Gamma)(2i\omega+im_R-\Gamma/2)}\\&-\frac{-m_R^2+i\Gamma m_R+\Gamma^2/4}{(2im_R+2i\omega+\Gamma)(2i\omega-im_R-\Gamma/2)}-\frac{m_R^2+\Gamma^2/4}{(2i\omega-2im_R+\Gamma)(2i\omega-im_R-\Gamma/2)}-\frac{\Gamma(im_R+\Gamma/2)}{(2i\omega+\Gamma)(2i\omega-im_R-\Gamma/2)}\Bigg].\end{aligned} \tag{C6}$$

Now we must integrate each of the terms in Eq. (C6) as indicated in Eq. (C1). The lower limit of the integral is $m_Q+m_L\ll m_R$ and we use $\Gamma\ll m_Q+m_L$. We find that the first term cancels with its Hermitian conjugate, the third and sixth terms are 0, the second and fourth terms cancel each other, and the fifth term plus its Hermitian conjugate is responsible for the final result given in Eq. (3.13),

$$n_B\equiv n_b-n_{\bar{b}}=\frac{g^2}{16\pi}m_Rf^2\theta_i^3=\frac{1}{2}\Gamma f^2\theta_i^3. \tag{C7}$$

APPENDIX D: THE EFFECTS OF MIXING IN THE Q,L BASIS

We will consider the decay of θ to a $Q\bar{L}$ pair (superscript 1 for this decay channel), and the decay of θ to a $\bar{Q}L$ pair (superscript 2 for this decay channel). For the first decay channel, from Eq. (3.16) we see that a Q produced at the time $t=0$ is given by

$$\psi(0)=Q=s\psi_1+c\psi_2, \tag{D1a}$$

where

$$c=\frac{1}{\sqrt{1+\epsilon^2}}\quad\text{and}\quad s=\frac{\epsilon}{\sqrt{1+\epsilon^2}}. \tag{D1b}$$

Similarly,

$$\overline{\chi(0)}=\bar{L}=c\bar{\psi}_1-s\bar{\psi}_2. \tag{D2}$$

We will let the fields ψ and χ evolve in time, mixing their Q and L components as they travel. The time evolution of $\psi(t)$ can be modeled as follows:

$$\psi(t)=(se^{-i\Delta\omega t}\psi_1+c\psi_2)\exp(-i\omega_2 t), \tag{D3}$$

where $\Delta\omega=\omega_1-\omega_2$. We now wish to ask the question: what is the Q content at some time t of the field ψ, which was initially pure Q? Using Eq. (3.16), we can write Eq. (D3) as

$$\psi(t)=[(c^2+s^2e^{-i\Delta\omega t})Q-sc(1-e^{-i\Delta\omega t})L]\exp(-i\omega_2 t). \tag{D4}$$

The quark content is given by the magnitude squared of the coefficient of the first term, so that

$$n_Q^{(1)}(t)=[c^4+s^4+2c^2s^2\cos\Delta\omega t]\,\frac{1}{V}\times\sum_{s_Q,s_{\bar{L}}}\int \widetilde{dk_Q}\widetilde{dk_{\bar{L}}}|A_{Q\bar{L}}|^2. \tag{D5}$$

Similarly, from the same decay process $\theta\to Q+\bar{L}$, the $\bar{L}$ that is produced can convert to a $\bar{Q}$ so that we have

$$n_{\bar{Q}}^{(1)}(t)=\frac{1}{V}\sum_{s_Q,s_{\bar{L}}}2s^2c^2(1-\cos\Delta\omega t)\int \widetilde{dk_Q}\widetilde{dk_{\bar{L}}}|A_{Q\bar{L}}|^2. \tag{D6}$$

From $\theta\to L\bar{Q}$, one can obtain $\bar{Q}$ at a later time from oscillations of either the L or the $\bar{Q}$ and find contributions:

$$n_{\bar{Q}}^{(2)}(t)=[c^4+s^4+2c^2s^2\cos\Delta\omega t]\,\frac{1}{V}\times\sum_{s_L,s_{\bar{Q}}}\int \widetilde{dk_L}\widetilde{dk_{\bar{Q}}}|A_{L\bar{Q}}|^2 \tag{D7}$$

and

$$n_Q^{(2)}(t)=\frac{1}{V}\sum_{s_L,s_{\bar{Q}}}2s^2c^2(1-\cos\Delta\omega t)\int \widetilde{dk_L}\widetilde{dk_{\bar{Q}}}|A_{L\bar{Q}}|^2. \tag{D8}$$

Thus the baryon asymmetry at any time t is

$$\begin{aligned} n_B(t)&=n_Q^{(1)}(t)+n_Q^{(2)}(t)-n_{\bar{Q}}^{(1)}(t)-n_{\bar{Q}}^{(2)}(t)=[(c^2-s^2)^2+4s^2c^2\cos\Delta\omega t]\sum_{s_L,s_Q}(|A_{Q\bar{L}}|^2-|A_{L\bar{Q}}|^2)\\ &=\left[\left(\frac{1-\epsilon^2}{1+\epsilon^2}\right)^2+4s^2c^2\cos\Delta\omega t\right]\sum_{s_L,s_Q}(|A_{Q\bar{L}}|^2-|A_{L\bar{Q}}|^2). \end{aligned} \tag{D9}$$

One can see that the baryon asymmetry oscillates in time as a cosine about the average value. When one takes a time average, the cosine term averages to zero, and one reproduces the result in Eq. (3.28),

$$n_B=\frac{1}{2}\,\Gamma f^2\theta_i^3\left(\frac{1-\epsilon^2}{1+\epsilon^2}\right)^2. \tag{D10}$$

Our derivation above assumes in Eqs. (D5)–(D8) that all $Q\bar{L}$ pairs and all $L\bar{Q}$ pairs were produced at the same time. If one considers that all pairs are not produced at the same time then an average over all pairs would also cancel the $\cos\Delta\omega t$ term in Eqs. (D5)–(D8).

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[4] Natural inflation can be realized in many realistic particle physics models in which a Nambu-Goldstone boson acquires a potential of the form in Eq. (1.1). In a class of Z_2 symmetric models the combination of terms like $m_1\bar{\psi}\psi$ and $m_0(\bar{\psi}\psi e^{i\theta}$ $+$H.c.) can give rise to potential as in Eq. (1.1) for θ with $\Lambda^2\sim m_0m_1$. These ''schizon'' models are further described in Refs. [5,6,7]. In superstring models, nonperturbative effects in the hidden sector can give rise to fermion condensates and, consequently, a potential for the model independent axion (the imaginary part of the dilaton field). In Refs. [8,9], the hidden E_8' sector in $E_8\times E_8'$ heterotic string theory becomes strongly interacting, generating gaugino condensates that lead to supersymmetry breaking and a potential for the model independent axion. Problems with the stability of the dilaton potential in such models have prompted others to consider multiple gaugino condensation models that give a suitable potential for the axion [10,11,12]. Fermion condensates in technicolor theories can also give rise to potentials of the form above for fields coupled to the fermions. Also, a theory with an antisymmetric tensor field $B_{\mu\nu}$ (which arises, for example, in string theory) with a field strength

$$H^{\mu\nu\lambda}=\partial^\mu B^{\nu\lambda}+\partial^\lambda B^{\mu\nu}+\partial^\nu B^{\lambda\mu}$$

has an effective action that can be expressed in terms of a scalar field with a potential of the form in Eq. (1.1) [13]. In a variant of these models, the tensor field can be coupled to a fundamental real scalar field u with the symmetry-breaking potential of the form $V(u)=(\lambda'/4!)(u^2-6m^2/\lambda')^2$. This also leads to a potential as in Eq. (1.1) for the scalar θ.

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