Resonant spin-flavor precession of neutrinos and pulsar velocities

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Young pulsars are known to exhibit large space velocities, up to 10^3 km/s. We propose a new mechanism for the generation of these large velocities based on an asymmetric emission of neutrinos during the supernova explosion. The mechanism involves the resonant spin-flavor precession of neutrinos with a transition magnetic moment in the magnetic field of the supernova. The asymmetric emission of neutrinos is due to the distortion of the resonance surface by matter polarization effects in the supernova magnetic field. The requisite values of the field strengths and neutrino parameters are estimated for various neutrino conversions caused by their Dirac or Majorana-type transition magnetic moments. [S0556-2821(97)03722-3]

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I. INTRODUCTION

In this paper we propose a new mechanism for generating large birth velocities of pulsars which is related to the possible existence of transition magnetic moments of neutrinos. Observations imply that pulsars have rapid proper motions with a mean space velocity of 450 ± 90 km/s [1]. In particular, observations in young supernova remnants have identified pulsars with velocities up to 900 km/s [2]. Such high velocities of young pulsars are most probably associated with the supernova event in which the pulsar is born. Many different models have been formulated to explain the origin of these velocities: asymmetric collapse [3], asymmetry due to misalignment of dipole magnetic field and rotation axis [4], a ''runaway star'' produced by a supernova explosion in a close binary star [5], and recoil momentum from the asymmetric production of neutrinos [6] in the supernova's magnetic field. The latter possibility is especially interesting since neutrinos carry away more than 99% of the supernova's gravitational binding energy and so even a $\sim1\%$ asymmetry in the neutrino emission could lead to the observed recoil velocities of pulsars. However, this mechanism requires a magnetic field $B\gtrsim10^{16}$ G which may be somewhat too strong for supernovae. It is also possible that the asymmetry in the neutrino momenta will be washed out by multiple scattering, absorption, and reemission of the neutrinos in the core of the supernova [7].

Recently, a very interesting new mechanism for the generation of the pulsar velocities has been proposed by Kusenko and Segrè (KS) [8]. The idea is that the neutrino emission from a cooling protoneutron star can be asymmetric due to resonant neutrino oscillations [Mikheyev-Smirnov-Wolfenstein (MSW) effect [9]] in the supernova's magnetic field. The anisotropy of the neutrino emission is driven by the polarization of the medium in a strong magnetic field, which distorts the resonance surface. According to KS, this mechanism requires magnetic fields $B\gtrsim3\times10^{14}$ G. However, this estimate was recently criticized by Qian who showed that in fact magnetic fields $B\gtrsim10^{16}$ G are necessary [10]. We will come back to this question later on. An advantage of the KS scenario is that it is free from the above-mentioned shortcoming of the asymmetric production mechanism: the neutrinos carrying the momentum asymmetry are free streaming from the supernova and therefore the asymmetry is hardly affected by the interaction of neutrinos with matter. This mechanism is very attractive since it predicts a well-defined correlation between the strength of the magnetic field and the observed space velocites of pulsars [11]. As a followup, in [12] the same authors have considered the effects of sterile-to-active neutrino oscillations, where now neutral-current effects are important. Thus, as was stressed in [8,11,12], pulsar motions may be a valuable source of information on neutrino properties. One should therefore study all possible mechanisms of asymmetric neutrino conversions in supernovae that might be the cause of the observed velocities of pulsars.

The mechanism that we propose here is similar to the KS one—it is based on the observation that matter polarization in the strong magnetic field of a supernova distorts the resonance surface of the neutrino conversion. The difference from the KS scenario is in the nature of the neutrino transition involved: in our case it is the resonant spin-flavor precession of neutrinos due to their transition magnetic moments [13,14] rather than neutrino oscillations.[1] We show here that our proposed mechanism can account for the observed large space velocities of pulsars provided that the

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[1]The resonantly enhanced spin-flavor precession of neutrinos in supernovae has been considered in the literature [14,15], but no implications for pulsar velocities were discussed there.

neutrino transition magnetic moment satisfies $\mu_\nu \gtrsim (10^{-15} - 10^{-14})\mu_B$, the neutrino mass is less than or about 16 keV and the supernova magnetic field $B \gtrsim 4\times 10^{15}$ G. Notice that the required magnetic field is slightly weaker than the one necessary for the KS mechanism [10]. Such large fields are presently considered possible in supernovae [16].

If the supernova magnetic field has a noticeable ''up-down'' asymmetry, neutrino spin or spin-flavor precession can affect differently neutrinos emitted in the upper and lower hemispheres. As was pointed out by Voloshin [7], this could also be the reason for the observed birth velocities of pulsars. In this paper we restrict ourselves to the case of symmetric magnetic fields; possible effects of neutrino conversions in asymmetric magnetic fields will be considered elsewhere [17].

II. NEUTRINO POTENTIALS IN POLARIZED MEDIA

In a medium containing a magnetic field the particles of matter have in general nonzero average spin. This spin polarization contributes to the neutrino potential energy in matter through the neutrino coupling with the axial-vector currents of the matter constituents [18–26]. For a very lucid and detailed discussion of matter polarization effects on neutrino propagation in a medium we refer the reader to [26]; here we summarize some results that will be relevant for our discussion and elaborate on some of them.

The contribution of the polarization of the medium to the potential energy of a test neutrino ν_j is of the order $\delta V_i(\nu_j) \sim (G_F/\sqrt{2}) g_A^i \langle \lambda_i \rangle_\parallel N_i$. Here G_F is the Fermi constant, $\langle \lambda_i \rangle_\parallel$, g_A^i, and N_i are the polarization along the test neutrino momentum, weak axial-vector coupling constant and number density of the particles of the ith type ($i = e,p,n$ or background neutrinos). Under supernova conditions the average spins of electrons, protons, and neutrons are rather small; this means that their polarizations are linear in the magnetic field strength B, i.e., $\delta V_i = c^i B_\parallel$, where $B_\parallel$ is the component of the magnetic field along the neutrino momentum. The electron spin polarization affects the potential of electron neutrinos and antineutrinos in matter through both charged-current and neutral-current interactions, whereas for ν_μ, ν_τ and their antineutrinos there are only neutral-current contributions c_Z^e. The polarizations of protons and neutrons contribute to the neutrino potentials in a medium only through the neutral-current interactions; therefore these contributions are the same for neutrinos of all flavors.

Thus, one arrives at the following expressions for the neutrino potentials in a magnetized medium:

$$V(\nu_e) = -V(\overline{\nu_e}) = \sqrt{2} G_F (N_e - N_n/2 + 2N_{\nu_e}) + (c_W^e + c_Z^e + c^p + c^n) B_\parallel, \quad (1)$$

$$V(\nu_{\mu,\tau}) = -V(\overline{\nu_{\mu,\tau}}) = \sqrt{2} G_F (-N_n/2 + N_{\nu_e}) + (c_Z^e + c^p + c^n) B_\parallel. \quad (2)$$

Here $N_e \equiv N_{e^-} - N_{e^+}$, etc., and we have taken into account that the number of muon and tauon neutrinos coincides with the number of their antineutrinos in supernovae. The electron neutrino number density is relatively small in the regions of interest to us in supernovae, and from now on we neglect it. Taking it into account would not change our estimates significantly.

The contributions of the electron spin polarization to the neutrino potentials $V(\nu_i)$ were calculated in [18–26]. The charged-current electron polarization contribution to $V(\nu_e)$, which we denoted by c_W^e, turns out to be twice as large as the neutral-current one, and has the opposite sign; as a result,

$$c^e = c_Z^e + c_W^e = c_Z^e - 2c_Z^e = -c_Z^e. \quad (3)$$

During the supernova explosion neutrinos and anitineutrinos of all flavors are thermally produced in the hot central part of the star. They are trapped in the dense core of the supernova and diffuse out on a time scale of ~ 10 s. When they reach the regions with density $\rho \sim 10^{11} - 10^{12}$ g/cm^3, they are no longer trapped and escape freely from the star. The surface at unit optical depth is called the neutrinosphere. It is located deeper inside the star for ν_μ, ν_τ and their antiparticles than for ν_e and $\overline{\nu_e}$ since the medium is more opaque for these latter particles, which have charged-current interactions as well as neutral-current ones. As a result, the nonelectron-type neutrinos are emitted at a higher temperature, about 6 MeV as against 3 MeV for ν_e and slightly higher for $\overline{\nu_e}$ (because the medium contains more neutrons than protons).

In the supernova environment in the vicinity of the neutrinosphere electrons are relativistic and degenerate. In this case [20,23,24],

$$c_Z^e \simeq \frac{e G_F}{2\sqrt{2}} \left(\frac{3 N_e}{\pi^4} \right)^{1/3}. \quad (4)$$

The effects of the polarization of protons have been calculated in [23] in the approximation of Dirac protons, i.e., treating protons essentially as positrons (though with a different mass). This is certainly not a valid approximation since the anomalous magnetic moment of the proton, which is neglected in the Dirac approximation, is even larger than its normal magnetic moment. Also, the strong-interaction renormalization of the proton axial-vector coupling constant was not taken into account. These shortcomings, however, can be readily removed. As we shall see, the nucleons are nonrelativistic and nondegenerate in the hot protoneutron star during the thermal neutrino emission stage. It is not difficult to calculate their polarizations directly using the well-known expressions for the Hamiltonian of a nonrelativistic fermion in a magnetic field and for the Boltzmann distribution function. This gives

$$c^p \simeq \frac{G_F}{\sqrt{2}} g_A \frac{\mu_p \mu_N}{T} N_p, \quad c^n \simeq -\frac{G_F}{\sqrt{2}} g_A \frac{\mu_n \mu_N}{T} N_n. \quad (5)$$

Here $\mu_p = 2.793$ and $\mu_n = -1.913$ are the proton and neutron magnetic moments in units of the nuclear magneton $\mu_N = e/2m_p = 3.152\times 10^{-18}$ MeV/G, and $g_A = 1.26$ is the axial-vector renormalization constant of the nucleon. The expressions in Eq. (5) coincide with the results obtained in the recent paper [26]. Notice that c^p, c^n, and c_Z^e all have the same sign. For nondegenerate particles, thermal fluctuations tend to destroy the polarization, hence c^p and c^n decrease with increasing temperature T. The polarization of the protons and neutrons in the medium can influence the oscillations between the active and sterile neutrinos as well as the neutrino spin and spin-flavor precession. However, these effects have not been taken into account in most of the previous analyses of neutrino conversions in supernovae (the only exception we are aware of is [26]). Probably, the reason for this was the general idea that heavy nucleons will be polarized to a lesser extent than the light electrons. We shall show now that this is not quite correct: even though the nucleon polarization contributions to neutrino potentials are typically smaller than that of the polarized electrons in the supernova environment, they are of the same order of magnitude. More importantly, as we shall see, for spin-flavor precession mediated by neutrino transition magnetic moments of Majorana type, the effects of polarized electrons nearly cancel out, and nucleon polarization constitutes the main magnetic field effect on the resonance conditions.

Let us consider now the degree of degeneracy of nucleons in the protoneutron star. The degeneracy parameter of nonrelativistic nucleons can be written as

$$\frac{|\kappa_i|}{T}=\ln\left[\frac{2}{N_i}\left(\frac{m_N T}{2\pi}\right)^{3/2}\right]\approx\ln\left[41.5Y_i^{-1}\left(\frac{10^{11}\ \text{g/cm}^3}{\rho}\right)\right.$$
$$\left.\times\left(\frac{T}{3\ \text{MeV}}\right)^{3/2}\right]. \tag{6}$$

Here m_N is the nucleon mass, N_i and κ_i $(i=p,n)$ are the nucleon number densities and chemical potentials. When $d_i \equiv \exp(\kappa_i/T)=\exp(-|\kappa_i|/T)\ll 1$ the nucleons are nondegenerate, i.e., form a classical gas, whereas in the opposite limit $d_i \gg 1$ the nucleons will be strongly degenerate. In the vicinity of the neutrinosphere ($\rho\sim 10^{11}-10^{12}$ g/cm^3, $T\sim 3-6$ MeV, $Y_e\sim 0.1-0.2$) we have $d_n\lesssim 0.08$, $d_p\lesssim 0.02$, i.e., the nucleons are strongly nondegenerate. Therefore the formulas of Eq. (5) are valid there. In the core of the star the densities are $\rho\gtrsim 10^{14}$ g/cm^3, and the temperatures are higher, too: $T\gtrsim 20$ MeV [27]. As a result, nucleons are weakly nondegenerate there with $d_i\sim 1$. This means that one can use Eq. (5) only for rough estimates of the nucleon polarization contribution to neutrino dispersion relations in the core of the supernova.

It is instructive to estimate the relative size of the nucleon and electron polarization contributions to the neutrino potentials. Let Y_i be the number of particles of the ith kind per baryon. Then $N_i=Y_i N$, where N is the total baryon number density. The electric neutrality of matter implies that $Y_p=Y_e$ (we neglect the nuclei since their fraction is very small in the region of interest to us). From Eqs. (4) and (5) we get

$$\frac{c^i}{c_Z^e}=g_A\mu_i\left(\frac{\pi}{6}\right)^{1/3}\left(\frac{Y_i}{Y_e}\right)^{1/3}\left[\frac{N_i}{2}\left(\frac{2\pi}{m_N T}\right)^{3/2}\right]^{2/3}$$
$$=g_A\mu_i\left(\frac{\pi}{6}\right)^{1/3}\left(\frac{Y_i}{Y_e}\right)^{1/3}d_i^{2/3}. \tag{7}$$

With increasing degeneracy, the relative contribution of the nucleon polarization increases. However, even in the nondegenerate case the nucleon polarization contributions may not be small provided the degeneracy parameter is not too small. To see this, let us rewrite c_p/c_e and c_n/c_e in the following form:

$$\frac{c^p}{c_Z^e}\approx 0.24\left(\frac{\rho}{10^{11}\ \text{g cm}^{-3}}Y_e\right)^{2/3}\left(\frac{3\ \text{MeV}}{T}\right), \tag{8}$$

$$\frac{c^n}{c_Z^e}\approx 0.16\left(\frac{\rho}{10^{11}\ \text{g cm}^{-3}}\right)^{2/3}\frac{Y_n}{Y_e^{1/3}}\left(\frac{3\ \text{MeV}}{T}\right). \tag{9}$$

Taking for an estimate $Y_e\approx 0.2$, $Y_n\approx 0.8$, $\rho\approx 10^{11}$ g/cm^3 and $T\approx 3$ MeV, we obtain $(c^p+c^n)/c^e\approx 0.3$, i.e., the nucleon polarization effect is about 30% of the electron polarization one. The relative contribution of neutrons and protons is $c^n/c^p\approx 0.68(Y_n/Y_e)$; since $Y_n\gg Y_e$ in the supernova, c^n dominates over c^p. Similar conclusions have been reached in [26].

III. RESONANT SPIN-FLAVOR PRECESSION IN MATTER AND MAGNETIC FIELDS

We shall summarize here the main features of the resonant spin-flavor precession (RSFP) of neutrinos in matter and a magnetic field [13,14] and compare them with the corresponding characteristics of the MSW effect [9]; for a more detailed discussion of the RSFP see [28].

Neutrinos with nonvanishing flavor–off-diagonal (transition) magnetic moments experience a simultaneous rotation of their spin and flavor in a transverse magnetic field (spin-flavor precession) [29]. In vacuum, such a precession is suppressed because of the kinetic energy difference of neutrinos of different flavors, $\Delta E_{\text{kin}}\simeq\Delta m^2/2E$ for relativistic neutrinos. At the same time, in matter this kinetic energy difference can be cancelled by the potential energy difference $V(\nu_i)-V(\overline{\nu_j})$, leading to a resonant enhancement of the spin-flavor precession [13,14]. The RSFP of neutrinos is similar to resonant neutrino oscillations (MSW effect [9]). For Dirac neutrinos, their transition magnetic moments cause transitions between left-handed neutrinos of a given flavor and right-handed (sterile) neutrinos of a different flavor. For Majorana neutrinos the spin-flavor precession due to their transition magnetic moments induces transitions between left-handed neutrinos of a given flavor and right-handed antineutrinos of a different flavor which are not sterile.

A. Resonance conditions

The resonance condition for a transition between left-handed neutrinos of the ith flavor and right-handed neutrinos or antineutrinos of the jth flavor ($i,j=e\mu$, τ or s, where s means the sterile neutrino) is

$$V(\nu_{iL})+\frac{m_{\nu_i}^2}{2E}=V(\overline{\nu_{jR}})+\frac{m_{\nu_j}^2}{2E}. \tag{10}$$

For antineutrinos of the ith type $V(\overline{\nu_{iR}})=-V(\nu_{iL})$. Mean potential energies of active neutrinos and antineutrinos including matter polarization effects are given in Eqs. (1) and (2); for sterile neutrinos $V(\nu_s)=0$.

The resonance conditions for various RSFP transitions can be written in the following generic form:

TABLE I. Summary of the parameters that enter into the resonance condition (11).

No.	Transition	$N_{\rm eff}$	$c_{\rm eff}$	Δm^2
1	$\nu_e \leftrightarrow \bar{\nu}_x$	$N_n - N_e$	$2(c^p + c^n)$	$m^2_{\nu_e} - m^2_{\nu_x}$
2	$\bar{\nu}_e \leftrightarrow \nu_x$	$N_n - N_e$	$2(c^p + c^n)$	$m^2_{\nu_x} - m^2_{\nu_e}$
3	$\nu_e \leftrightarrow \bar{\nu}_s$	$N_e - N_n/2$	$c^e_Z - c^p - c^n$	$m^2_{\nu_s} - m^2_{\nu_e}$
4	$\bar{\nu}_e \leftrightarrow \nu_s$	$N_e - N_n/2$	$c^e_Z - c^p - c^n$	$m^2_{\nu_e} - m^2_{\nu_s}$
5	$\nu_x \leftrightarrow \bar{\nu}_s$	$N_n/2$	$c^e_Z + c^p + c^n$	$m^2_{\nu_x} - m^2_{\nu_s}$
6	$\bar{\nu}_x \leftrightarrow \nu_s$	$N_n/2$	$c^e_Z + c^p + c^n$	$m^2_{\nu_s} - m^2_{\nu_x}$
7	$\nu_e \leftrightarrow \nu_x$	N_e	$2c^e_Z$	$m^2_{\nu_x} - m^2_{\nu_e}$
8	$\bar{\nu}_e \leftrightarrow \bar{\nu}_x$	N_e	$2c^e_Z$	$m^2_{\nu_e} - m^2_{\nu_x}$
9	$\nu_e \leftrightarrow \nu_s$	$N_e - N_n/2$	$c^e_Z - c^p - c^n$	$m^2_{\nu_s} - m^2_{\nu_e}$
10	$\bar{\nu}_e \leftrightarrow \bar{\nu}_s$	$N_e - N_n/2$	$c^e_Z - c^p - c^n$	$m^2_{\nu_e} - m^2_{\nu_s}$
11	$\nu_x \leftrightarrow \nu_s$	$N_n/2$	$c^e_Z + c^p + c^n$	$m^2_{\nu_x} - m^2_{\nu_s}$
12	$\bar{\nu}_x \leftrightarrow \bar{\nu}_s$	$N_n/2$	$c^e_Z + c^p + c^n$	$m^2_{\nu_s} - m^2_{\nu_x}$

$$\sqrt{2} G_F N_{\rm eff} - c_{\rm eff} B_\parallel = \frac{\Delta m^2}{2E}. \tag{11}$$

The resonance condition for neutrino oscillations in matter (MSW effect) has almost the same form, the difference being that the right-hand side (RHS) of Eq. (11) is multiplied by the cosine of the double vacuum mixing angle, $\cos 2\theta_0$. The effective parameters of the resonance condition depend on the nature of the transition in question. In Table I we summarize the parameters that enter into the resonance condition (11) for the neutrino conversions of interest to us. Here ν_s is a sterile neutrino which is assumed to be left-handed ($\bar{\nu}_s$ is right-handed), $\nu_x = \nu_\mu$ or ν_τ, and for the sake of comparison we have also included the parameters for the neutrino conversions due to the MSW effect (lines 7–12). For $c_{\rm eff} B_\parallel < \sqrt{2} G_F N_{\rm eff}$, the neutrino transitions listed in Table I can only be resonantly enhanced if the corresponding Δm^2 and $N_{\rm eff}$ are of the same sign. For given signs of $N_{\rm eff}$, only 6 of the 12 transitions can be resonant, depending on the signs of the respective Δm^2. We shall comment on the $c_{\rm eff} B_\parallel > \sqrt{2} G_F N_{\rm eff}$ case later on. It is interesting to notice that the parameters of the resonance conditions for the RSFP transitions involving sterile neutrinos or antineutrinos and those for the corresponding MSW transitions are the same (lines 3–6 and 9–12, respectively). The reason for this is that $V(\nu_s) = 0 = V(\bar{\nu}_s)$.

Several remarks are in order. The authors of [22] pointed out that the electron polarization contribution cancels out in the resonance condition for the RSFP transitions $\nu_e \leftrightarrow \bar{\nu}_x$ and $\bar{\nu}_e \leftrightarrow \nu_x$. They therefore concluded that magnetic fields have no effect on these resonance conditions. We would like to emphasize that the cancellation noticed in [22] is not exact; it holds only up to the electroweak radiative corrections. In particular, this cancellation is a consequence of Eq. (3) which is based on the tree-level relation between the masses of the $W^\pm$ and Z^0 gauge bosons, $M_W^2 = M_Z^2 \cos^2 \theta_W$. This relation is known to receive radiative corrections of the order of 0.5% (mainly due to the heavy top quark). There are other electroweak corrections that would also lead to an incomplete cancellation, e.g., from the anomalous magnetic moment of the electron. They are typically of the same order of magnitude, $\lesssim 0.5\%$. However, for supernovae more important contributions come from the polarization of nucleons in the magnetized medium. As was demonstrated in Sec. II, they are quite sizable and can be comparable with c^e.

In [12] it has been claimed that the potential of electron neutrinos $V(\nu_e)$ does not receive any matter-polarization contributions. The authors, following [23], claimed that the electron and proton polarization effects cancel each other in $V(\nu_e)$ and therefore concluded that the $\nu_e \leftrightarrow \nu_s$ and $\bar{\nu}_e \leftrightarrow \bar{\nu}_s$ oscillations in the supernova are not affected by the magnetic field and so cannot be the cause of the pulsar birth velocities. The cancellation was the result of the assumption that protons are strongly degenerate in the protoneutron star; as we have shown in Sec. II, this assumption is incorrect. Moreover, even for degenerate protons the cancellation takes place only if they are treated as Dirac particles, i.e., when the strong-interaction renormalization of the proton's magnetic moment and axial-vector coupling constant are neglected. In addition, the effects of neutron polarization, which can be comparable with those of polarized electrons, were not considered in [12]. It should be noted, however, that the above shortcomings have no effect on the $\nu_x \leftrightarrow \nu_s$ transitions which were the main topic of [12].

In [25,22] it was claimed that in strong enough magnetic fields the term $c^e B_\parallel$ can overcome the $\sqrt{2} G_F N_e$ term in the resonance condition of the $\nu_e \leftrightarrow \nu_x$ and $\bar{\nu}_e \leftrightarrow \bar{\nu}_x$ oscillations, leading to the possibility of new resonances. However, it has been demonstrated in [26] that this is incorrect: the electron polarization contribution can never exceed the electron density contribution, essentially because the mean polarization of electrons cannot exceed unity. The expressions for the polarization of matter constituents (4) and (5) that are linear in the magnetic field are valid only when the induced polarizations of the particles in matter are small. Nevertheless, as was stressed in [26], for the $\nu_e \leftrightarrow \nu_s$ and $\bar{\nu}_e \leftrightarrow \bar{\nu}_s$ oscillations the $c_{\rm eff} B_\parallel$ term can indeed overcome the $\sqrt{2} G_F N_{\rm eff}$ term in the resonance condition, since the effective number density $N_{\rm eff} = N_e - N_n/2$ can become small provided that there is a compensation between the electron and neutron contributions. In this case new resonances are indeed possible, and the resonant channel will depend on whether the neutrinos are emitted along the magnetic field or in the opposite direction. Obviously, the same argument applies for the RSFP transitions involving sterile neutrinos or antineutrinos (lines 3–6 of Table I). We would like to point out here that in the case of the RSFP transitions the same is also true even for the $\nu_e \leftrightarrow \bar{\nu}_x$ and $\bar{\nu}_e \leftrightarrow \nu_x$ oscillations (lines 1 and 2 of Table I) that involve only active neutrinos; the matter polarization term can overcome the $\sqrt{2} G_F N_{\rm eff}$ term provided that the effective density $N_{\rm eff} = N_n - N_e$ becomes small because of the compensation of the electron and neutron number densities. Thus, in the case of the RSFP of neutrinos, the matter polarization term in the resonance condition can, in principle, exceed the effective matter density term leading to the possibility of new resonances *for all types of neutrino transitions*.

B. Adiabaticity parameters and transition probabilities

The probability of an RSFP-induced neutrino transition depends on the degree of its adiabaticity and is generally large when the adiabaticity parameter γ is large enough:

$$\gamma=4\,\frac{(\mu_\nu B_{\perp r})^2}{\sqrt{2}G_F N_{\rm eff}}\,L_{\rho r}\approx 0.81 Y_{\rm eff}^{-1}\left(\frac{10^{11}\ {\rm g/cm^3}}{\rho}\right)\left[\frac{\mu_\nu}{10^{-13}\mu_B}\,\frac{B_{\perp r}}{3\times 10^{14}\ {\rm G}}\right]^2\left(\frac{L_{\rho r}}{10\ \ {\rm km}}\right)>1. \tag{12}$$

Here μ_ν is the neutrino transition magnetic moment, $B_{\perp r}$ is the strength of the transverse component of the magnetic field at the resonance, $Y_{\rm eff}$ is defined through $N_{\rm eff}=Y_{\rm eff}N$, and $L_\rho \equiv |(1/\rho)(d\rho/dr)|^{-1}$ is the characteristic length over which the matter density varies significantly in the supernova, $L_{\rho r}$ being its value at the resonance.

The spin-flavor precession is typically strongly suppressed far below and far above the resonance point; in this case the transition probability is to a very good accuracy approximated by

$$P_{tr}\approx(1-P'),\quad P'\equiv\exp\left(-\frac{\pi}{2}\gamma\right). \tag{13}$$

The probability of neutrino transitions due to the MSW effect is given by the same expression with the adiabaticity parameter γ replaced by $\gamma_{\rm MSW}=(\sin^2 2\theta_0/\cos 2\theta_0)\times(\Delta m^2/2E)L_{\rho r}$.[2] For adiabatic transitions ($\gamma\gg 1$) the transition probability is close to one. In the vicinity of the neutrinosphere, $L_{\rho r}\sim 10$ km; therefore the RSFP transitions will be adiabatic for

$$B_{\perp r}\gtrsim 3\times 10^{14}(10^{-13}\mu_B/\mu_\nu)\ \ {\rm G}. \tag{14}$$

This constraint can in principle be somewhat relaxed since $Y_{\rm eff}$ is typically <1 and can also be $\ll 1$ in some cases; however, we will need the RSFP adiabaticity condition to be satisfied with some margin and so shall continue to use Eq. (14). For the MSW transitions to be adiabatic near the neutrinosphere the vacuum mixing angle should satisfy $\theta_0 \gtrsim 10^{-4}$.

IV. KICK MOMENTA OF PULSARS

In [8] the following two conditions for generating the pulsar kick velocity through resonant neutrino oscillations were formulated: (1) The neutrino conversion takes place between the neutrinospheres of two different neutrino species; (2) the resonance coordinate depends on the angle between the directions of the magnetic field and the neutrino momentum.

These conditions apply to the RSFP-induced neutrino conversions as well. Consider, e.g., the $\nu_\tau \leftrightarrow \bar{\nu}_e$ conversions above the ν_τ sphere but below the $\bar{\nu}_e$ sphere. A ν_τ propagates freely until it gets transformed into $\bar{\nu}_e$ through the RSFP conversion. The resulting $\bar{\nu}_e$ cannot escape easily and gets trapped since the resonance point is below its neutrinosphere. At the same time, a $\bar{\nu}_e$ which initially was trapped and diffused out slowly will be converted into ν_τ when it reaches the resonance surface. The resulting ν_τ escapes freely since the resonant conversion occurred above its neutrinosphere. Thus, the resonance surface becomes the new "neutrinosphere" for the ν_τ's. It is, however, not a sphere. The matter polarization in the supernova magnetic field leads to the resonance taking place at different distances from the core of the star for neutrinos emitted parallel and antiparallel to the magnetic field; as a result, the resonance surface has different temperatures in these two directions leading to an asymmetry of the momenta of the emitted neutrinos.

Let us estimate the magnitudes of Δm^2 that are necessary for various neutrino conversions to occur in the regions of interest to us. Since the neutrino mean energy is $\langle E\rangle \approx 3.15T$, from Eq. (11) one finds

$$\Delta m^2\approx 1.4\times 10^5 Y_{\rm eff}\left(\frac{\rho}{10^{11}\ \ {\rm g/cm^3}}\right)\left(\frac{T}{3\ \ {\rm MeV}}\right). \tag{15}$$

This depends on $Y_{\rm eff}$ and therefore on the neutrino transition in question. In the vicinity of the neutrinosphere the electron fraction Y_e is typically of the order of 0.1–0.2 at the time when neutrinos are copiously produced in the supernova (a few seconds after the core bounce). It decreases towards smaller densities and reaches the value of about 0.46. At the same time, in the dense core of the supernova there is still a significant amount of trapped ν_e's which hinder the neutronization process. Therefore Y_e can be close to 0.4 in the supernova's core (see [30] for a more detailed discussion).

For our estimates we will assume a hierarchical pattern of neutrino masses. For MSW transitions between active neutrinos or antineutrinos (lines 7 and 8 of Table I) the required heavier neutrino mass is in general in the range $m_2 \sim 100-800$ eV. For the RSFP transitions between active neutrinos and antineutrinos one would need $m_2 \sim 300-1500$ eV, whereas for transitions between muon or tauon neutrinos or antineutrinos and sterile neutrino states (lines 5, 6, 11, and 12 of Table I) m_2 should be in the range $200 \lesssim m_2 \lesssim 1.6\times 10^4$ eV. In all these cases neutrinos do not satisfy the cosmological bounds on the mass of stable neutrinos and would have to decay sufficiently fast. However, for the RSFP and MSW transitions between electron neutrinos or antineutrinos and sterile neutrino states (lines 3, 4, 9, and 10 of Table I) m_2 can be considerably smaller since $Y_{\rm eff}$ can be very small in this case. Indeed, the parameter Y_e passes through the value 1/3 somewhere between the supernova core and the neutrinosphere, i.e., $Y_{\rm eff}$ passes through zero. This means that the required value of $\Delta m^2 \approx m_2^2$ can be very small, too: $0 \leqslant m_2 \lesssim 10$ keV. For example, m_2 can well be in the ranges of a few eV or few tens of eV, which are both cosmologically safe and interesting (in particular, the allowed ranges of neutrino mass and transition magnetic moment are consistent with the predictions of the decaying neutrino theory of the ionization of the interstellar medium [31]). The mass m_2 can also be in the range $10^{-3}-10^{-4}$ eV which of interest for the solar neutrino problem; moreover, in this case the transition can be resonant even for massless

[2]We note in passing that the MSW adiabaticity condition was formulated incorrectly in [8,12]: The oscillation length at resonance must be compared with the resonance width $\Delta r = 2\tan 2\theta_0 L_{\rho r}$ and not with $L_{\rho r}$ itself.

neutrinos. For $m_2 \lesssim 4$ keV transitions including both electron neutrinos and antineutrinos can be resonantly enhanced;[3] this would lead to an additional factor of 2 increase of the pulsar velocities for a given value of the supernova magnetic field.

We will now derive the generalized expression for the neutrino momentum asymmetry which will be valid for the RSFP as well as for neutrino oscillation transitions. The relative recoil momentum of a pulsar can be estimated as [8]

$$\frac{\Delta k}{k} \propto \frac{T^4(r_0-\delta)-T^4(r_0+\delta)}{T^4(r_0)}, \tag{16}$$

where r_0 is the position of the resonance in the absence of the magnetic field and $\pm\delta$ is the shift of the resonance coordinate for the neutrinos emitted parallel and antiparallel to the magnetic field. From the generic resonance condition (11) one can estimate the value of δ as

$$\delta \simeq \frac{c_{\rm eff} B}{\sqrt{2} G_F (dN_{\rm eff}/dr)}, \tag{17}$$

where B is the magnetic field strength. For the neutrino momentum asymmetry we get

$$\frac{\Delta k}{k} \approx \frac{1}{6} \times 2 \times 4 \times R \, \frac{1}{T} \frac{dT}{dr} \, \delta \approx \frac{4}{3} R \left(\frac{c_{\rm eff} B}{\sqrt{2} G_F} \right) \frac{1}{T} \frac{dT}{dN_{\rm eff}}. \tag{18}$$

Here the factor 1/6 takes into acount that only one neutrino or antineutrino species out of 6 acquires a momentum asymmetry, and R is a geometrical factor to be discussed below. The desirable value of $\Delta k/k$ is about 10^{-2}; this can be achieved for a temperature asymmetry of the order of 10^{-2} (we are assuming here the neutrino transition probability $P_{\rm tr} \approx 1$).

For resonant neutrino oscillations, the geometrical factor R in Eq. (18) takes into account the fact that for the neutrinos emitted in directions orthogonal to the supernova magnetic field the resonance condition is not affected by the field. Therefore such neutrinos do not contribute to the kick velocity of the pulsar. In order to find this velocity one has to calculate the net momentum of neutrinos emitted in the direction of the magnetic field. KS estimated the resulting geometrical factor as 1/2; however, their result was criticized by Qian [10] who showed that in fact $R \approx 1/6$.

For spin-flavor precession the situation is somewhat different. The magnetic field plays a dual role in this process: first, its component $B_\perp$ transverse to the neutrino momentum mixes the left-handed and right-handed neutrino states and causes the spin-flavor precession itself; second, the longitudinal component $B_\parallel$ affects the resonance condition, as discussed in Sec. III A. Both roles are important for the purposes of our discussion. In fact, only the neutrinos emitted in directions different from that of the magnetic field or orthogonal to it can contribute to $\Delta k/k$. For neutrinos emitted exactly along the magnetic field, the RSFP does not occur; for those emitted in the orthogonal plane the field has no effect on the resonance condition and therefore does not lead to any momentum asymmetry. For this reason, in the case of the RSFP, the geometrical factor in Eq. (18) can be written as $R = R_1 R_2$, where $R_1 \approx 1/6$ as for neutrino oscillations, while the factor R_2 takes into account the reduction of the RSFP transition probability $P_{\rm tr}$ for neutrinos emitted close to the magnetic field directions. Basically, this reduction excludes the zenith angles close to 0° and 180° in the angular integration over the neutrino momenta. The numerical value of R_2 depends on the extent of the excluded region, which in turn depends on the magnitude of the adiabaticity parameter γ. If the adiabaticity condition (12) is satisfied with a large margin (i.e., $\gamma \gg 1$), even a strong reduction of $B_\perp = B \sin\theta$ because of the zenith angle θ being close to 0° or 180° will not suppress the RSFP transition probability significantly. In this case $R_2 \approx 1$ and $R \approx R_1 \approx 1/6$. In our estimates we will assume that this is the case.

An important parameter that enters into the neutrino momentum asymmetry (18) is the derivative $dT/dN_{\rm eff} = (dN_{\rm eff}/dT)^{-1}$. The effective density $N_{\rm eff}$ is in general a linear combination of N_e and N_n, depending on the type of the neutrino conversion. The derivative dN_e/dT was estimated by KS as $(\partial N_e/\partial T)_{\kappa_e}$ using the relativistic Fermi distribution function for the electrons. This gave

$$dN_e/dT \approx \frac{2}{3} (3\pi^2 N_e)^{1/3} T. \tag{19}$$

However, this approach was criticized by Qian [10] who pointed out that the chemical potential of electrons cannot be considered as temperature independent in the supernova. He suggested using instead the results of numerical simulations of matter density and temperature profiles, which typically give $N \propto T^3$ [32]. We adopt this approach here.

Using $N_i = Y_i N$ $(i = e, n)$ it is easy to show that

$$\frac{dN_i}{dT} \approx \frac{N_i}{T} \left(3 + \frac{d \ln Y_i}{d \ln T} \right). \tag{20}$$

The electron fraction Y_e decreases (and therefore Y_n increases) with increasing r below the ν_e sphere. For this reason for electrons the expression in the parentheses in Eq. (20) is in fact larger than 3 whereas for neutrons it is slightly smaller than 3. Estimates of the logarithmic derivatives using the Y_e profile from [33] give

$$\frac{dN_e}{dT} \approx 4 \frac{N_e}{T}, \quad \frac{dN_n}{dT} \approx 2.8 \, \frac{N_n}{T} \tag{21}$$

in the neutrinospheric region. Notice that numerically dN_e/dT in Eq. (21) is about an order of magnitude larger than the corresponding KS value (19) [10].

It is instructive to estimate the relative sizes of dN_n/dT and dN_e/dT in the supernova environment:

$$\frac{dN_n/dT}{dN_e/dT} \approx 0.7 (Y_n/Y_e). \tag{22}$$

[3] This includes the case $\Delta m^2 = 0$ which corresponds to the resonance transition due to the ordinary (flavor-diagonal) magnetic moments of neutrinos [7,13]).

Thus, dN_n/dT is typically a factor of 3 to 6 larger than dN_e/dT. Notice that the dN_e/dT contribution to $dN_{\rm eff}/dT$ has the opposite sign compared to the dN_n/dT one (lines 1–4, 9, and 10 of Table I) and so will tend to increase the kick, especially for transitions of electron neutrinos and antineutrinos into sterile states.

We shall first estimate the asymmetry $\Delta k/k$ for the RSFP-induced transitions between active neutrinos $\nu_e \leftrightarrow \overline{\nu_x}$ and $\overline{\nu_e} \leftrightarrow \nu_x$ due to the Majorana neutrino transition magnetic moments (lines 1 and 2 of Table I):

$$\frac{\Delta k}{k} \approx \frac{2}{9}\left[\frac{2(c^p+c^n)B}{\sqrt{2}G_F}\right]\frac{1}{T}\frac{dT}{d(N_n-N_e)}$$
$$\approx 1.2\times 10^{-4}\left(\frac{B}{3\times 10^{14}\ \mathrm{G}}\right)\frac{3\ \mathrm{MeV}}{T}. \qquad (23)$$

Let us compare Eq. (23) with the corresponding expression for the case of the $\nu_e \leftrightarrow \nu_x$ oscillations derived in [8]. We first notice that the numerical coefficient in Eq. (10) of [8] was overestimated (and so the requisite magnetic field underestimated) by about a factor of 40, where a factor ~ 3 comes from the geometrical factor R and a factor ~ 13 from dN_e/dT [10]. Apart from the different numerical coefficient, their expression for $\Delta k/k$ falls with increasing temperature as T^{-2} and not as T^{-1}. Comparing Eq. (23) with the corrected Eq. (10) of [8] we find that in order to obtain the same effect on the pulsar velocities one would need about a factor of 2 stronger magnetic field in the case of the RSFP of active neutrinos than in the case of the resonant oscillation of active neutrinos. For example, in order to produce $\Delta k/k \simeq 1\%$ a magnetic field $B \gtrsim 2.5\times 10^{16}$ G is necessary. This field is of the same order of magnitude as that needed to explain the pulsar birth velocities by asymmetric neutrino production [6]. By contrast, in our case, neutrinos carrying an asymmetric momentum will experience very few interactions with matter, and so the asymmetry is unlikely to be suppressed by such interactions.

Next, we consider the RSFP-induced transitions between active and sterile neutrino states due to the Dirac neutrino transition magnetic moments (lines 3–6 of Table I). For the transitions $\nu_x \leftrightarrow \overline{\nu_s}$ and $\overline{\nu_x} \leftrightarrow \nu_s$ we obtain

$$\frac{\Delta k}{k} \approx (2/9)\{[c_Z^e+(c^p+c^n)]B/\sqrt{2}G_F\}(1/T)[dT/d(N_n/2)] \approx \left[3.7\times 10^{-4}\ \frac{Y_e^{1/3}}{Y_n}\left(\frac{10^{11}\ \mathrm{g/cm^3}}{\rho}\right)^{2/3} + 7.6\times 10^{-5}\left(\frac{3\ \mathrm{MeV}}{T}\right)\right]$$
$$\times\left(\frac{B}{3\times 10^{14}\ \mathrm{G}}\right). \qquad (24)$$

From Eq. (24) it follows that in order to get $\Delta k/k \simeq 1\%$ one would need $B \gtrsim 9\times 10^{15}$ G. This field is of the same order of magnitude as the one that is needed in the case of the neutrino flavor oscillations. Moreover, for a hierarchical neutrino mass pattern $m_{\nu_s} \gg m_{\nu_\mu}, m_{\nu_\tau}$ the transitions between both $\overline{\nu_\mu}$ and $\overline{\nu_\tau}$ and sterile neutrino states can be resonant and contribute to $\Delta k/k$.[4] In this case a factor of two weaker field would be able to produce the desired kick.

For the transitions $\nu_e \leftrightarrow \overline{\nu_s}$ and $\overline{\nu_e} \leftrightarrow \nu_s$ we obtain

$$\frac{\Delta k}{k} \approx (2/9)\{[c_Z^e-(c^p+c^n)]B/\sqrt{2}G_F\}(1/T)[dT/d(N_n/2-N_e)] \approx \left[3.7\times 10^{-4}\ \frac{Y_e^{1/3}}{Y_n}\left(\frac{10^{11}\ \mathrm{g/cm^3}}{\rho}\right)^{2/3}\right.$$
$$\left. -7.6\times 10^{-5}\left(\frac{3\ \mathrm{MeV}}{T}\right)\right]\frac{Y_n}{Y_n-2.86Y_e}\left(\frac{B}{3\times 10^{14}\ \mathrm{G}}\right). \qquad (25)$$

For these transitions in order to get $\Delta k/k \simeq 1\%$ one would typically need $B \gtrsim 4\times 10^{15}$ G. This field is about a factor of 2 weaker than the one that is needed in the case of the KS mechanism. We would like to emphasize, however, that our consideration is rather simplified and can only yield order-of-magnitude estimate of the requisite magnetic field strengths.

It should be noticed that the results in Eqs. (24) and (25) apply to the case of resonant oscillations between active and sterile neutrinos as well. This case was studied in [12]; however the result obtained in that paper differs from ours. The reason for this is that the authors of [12] erroneously considered neutrons as strongly degenerate in the hot protoneutron star. This resulted in a suppressed value of $\Delta k/k$, and in order to save the situation, they had to assume that the resonant oscillations take place deep in the core of the supernova. However, as follows from our considerations, the nondegeneracy of neutrons increases $\Delta k/k$ so that there is no need to assume that the resonance takes place in the supernova's core. Moreover, the asymmetry decreases with the resonance density.

The lower bounds on the supernova magnetic fields B were derived here assuming that the RSFP transitions are adiabatic. The adiabaticity condition puts another lower bound on B, Eq. (14). The bounds obtained in this section are more restrictive provided the neutrino transition magnetic moments satisfy $\mu_\nu \gtrsim 10^{-14}\mu_B$ ($10^{-15}\mu_B$) for Dirac (Majorana) neutrinos.

[4]This has been pointed out for the case of oscillations into sterile neutrinos in [12].

V. CONCLUSION

We have studied the effects of the spin polarization of matter in a supernova magnetic field on the resonance conditions for spin-flavor precession of Dirac and Majorana neutrinos in supernovae. The magnetic field distorts the resonance surface resulting in an asymmetric neutrino emission, which can explain the observed space velocities of pulsars and their possible correlation with the pulsar magnetic fields. Our estimates for the case of spin and spin-flavor precession of Dirac neutrinos also apply to oscillations into sterile neutrinos and correct the results of [12] where the effect was underestimated. In the case of resonant spin-flavor precession into sterile neutrino states due to Dirac transition magnetic moments of neutrinos, the requisite supernova magnetic field strengths are $B \gtrsim 4\times10^{15}$ G. This is about a factor of 2 smaller than the field necessary in the case of neutrino flavor oscillations. Such fields are considered possible in supernovae [16]. For resonant spin-flavor precession between active neutrinos and antineutrinos due to Majorana transition magnetic moments of neutrinos, magnetic field strengths $B \gtrsim 2\times10^{16}$ G would be needed.

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