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Scattered light noise in gravitational wave interferometric detectors: A statistical approach

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The sensitivity of interferometric detectors of gravitational waves such as LIGO or VIRGO could eventually be limited by the noise due to scattered light propagation and rescattering in the long vacuum pipes containing the laser beams. We propose a statistical method for evaluating scattered light noise and compare trapping systems, easy to implement in Monte Carlo simulation codes. [S0556-2821(97)07622-4]

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I. INTRODUCTION

Gravitational wave interferometric antennas such as the Laser Interferometric Gravitational Wave Observatory (LIGO), GEO or VIRGO [1–3] presently in construction, involve kilometer-long Fabry-Pérot optical cavities able to store high-power light beams in between mirrors (see Fig. 1). These devices are designed to be highly sensitive to very small changes in phase of the stored standing wave due to a change in the light distance between the mirrors caused by a passing gravitational wave. They are also highly sensitive to spurious mirror displacements, and to any interference effect caused by modulated stray light. Physical mirror displacements are minimized by a very efficient seismic isolation system. But the stray light issue remains, and can be summarized as follows. The optical cavities consist of high-quality mirrors with rms roughnesses of the order of 1 Å. These mirrors nevertheless scatter a finite amount (a few ppm) of the stored light power, then the resulting scattered light (SL) propagates in the steel pipe surrounding the optical system for maintaining an ultrahigh vacuum along the optical path, and may reach any other mirror, including the initial one, after various scattering interactions with the walls and the various objects fixed in the pipe. The vacuum pipe and all these objects are in a state of vibration sustained by the seismic activity and eventually acoustical coupling to the environment. A second scattering process on a mirror surface sends a finite amount of the incoming SL into the stored wave with which it interferes, thus transmitting its noisy phase modulation acquired from the vibrating walls (see Fig. 2). Being a second-order scattering process on weakly scattering optical elements, the resulting noise is very low; however, gravitational signals are also expected to be very small, and a careful evaluation is therefore necessary in order to check that the signal-to-noise ratio of the instrument remains essentially unchanged by that contribution to the overall noise. The initial ideas for this kind of evaluation have been formulated by Thorne [4,5]. It turns out that in the simplest tube configuration, even assuming a perfect stainless steel cylinder, the noise is too high compared to other fundamental sources of noise. Several systems of light traps generally called baffles have thus been devised in order to suppress, or sufficiently attenuate, the flux of SL reaching the mirrors.

The various modes of interaction of SL with the material inside the vacuum pipe will be called channels. With each channel can be associated its own noise, and a global evaluation of a given configuration requires first finding the significant channels, then computing and adding the noise of each.

The analysis of some very special channels, such as the direct diffraction or reflection coupling, can be carried out analytically using wave optics, as shown in a preceding paper [6]. But wave optics fails to allow a convenient way to treat complex channels involving multiple interactions. It is therefore appealing to use numerical codes propagating light rays or ''photons'' according to geometrical optics in the mechanical structure, and investigate the resulting SL flux on the mirrors. Several such codes have been developed by specialized companies for optical instruments design, and are commercially available. Our concern is not, however, the flux of SL by itself, but the resulting phase noise, which is entirely due to the wave nature of light, absent from the particle picture of the above-mentioned Monte Carlo type codes because this problem is specific to gravitational wave antennas.

We present here a statistical approach of SL propagation,

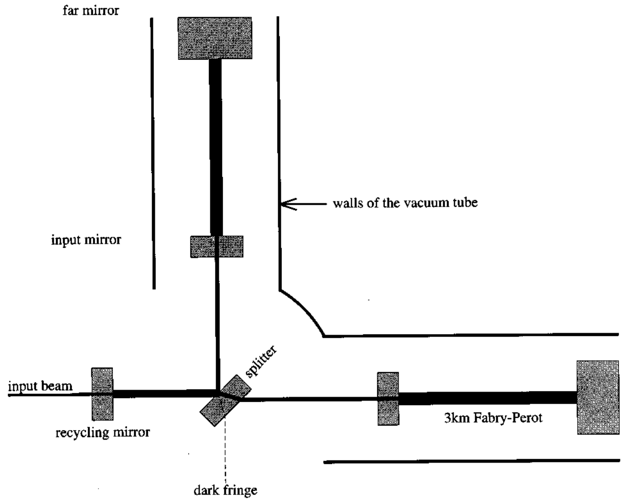

FIG. 1. General sketch of a gravitational wave interferometer.

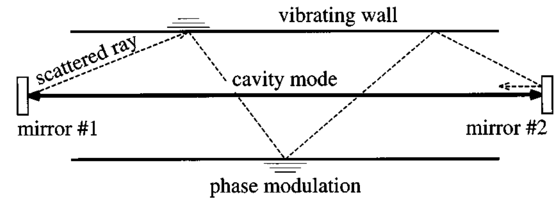

FIG. 2. Recombination of noisy stray light.

supplementing the information on trajectories obtained from a particle analysis by additional information regarding phase modulation coming from the wave picture. We represent SL by geometrical optics rays or ''photons,'' and we derive a method for extracting information on the noise from the statistics of photons received by the mirrors. We first give the theoretical basis of the method, then we show that the results obtained for the special backscattering channel are consistent with a wave optics approach, and with some results already obtained by Thorne [4]. In the last section, we report the results obtained concerning various options for a baffle system, using Monte Carlo codes.

II. BASIC THEORY OF INCOHERENT SCATTERED LIGHT

A. Emission of scattered photons

A rough mirror surface can be represented by the equation $z=f(\vec{x})$, where $\vec{x}$ represents the coordinates in the projection plane, z the height of the corresponding point of the surface, and $f(\vec{x})$ a two-dimensional stationary stochastic process having the following properties (angular brackets denoting the expectation value operator):

$$\langle f\rangle=0,$$

$$\langle f^2\rangle=\sigma^2,$$

$$\langle f(\vec{x})f(\vec{x}+\vec{y})\rangle=\sigma^2 C(|\vec{y}|).$$

The second equation determines the integrated scattering losses $P_{\mathrm{scatt}}/P_{\mathrm{in}}=\epsilon=16\pi^2\sigma^2/\lambda^2$ [6], provided that $\sigma\ll\lambda$; the third equation expresses the fact that the autocorrelation function is isotropic because there is no preferred direction on the mirror for the surface defects. The Fourier transform of the autocorrelation function $\tilde{C}(\vec{p})$ gives us the spectral density of the process f:

$$\tilde{C}(\vec{p})=\int C(\vec{x})e^{i\vec{p}\cdot\vec{x}}d\vec{x}.$$

Owing to the obvious property $C(\vec{0})=1$, we have, consequently,

$$\frac{1}{4\pi^2}\int\tilde{C}(\vec{p})d\vec{p}=1$$

or, by identifying spatial frequencies with angular directions according to $\vec{p}\equiv(k\theta\cos\phi,k\theta\sin\phi,\ k\equiv2\pi/\lambda)$,

$$\frac{1}{\lambda^2}\int\tilde{C}(\theta,\phi)d\Omega=1.$$

The function

$$b(\theta,\phi)=\frac{1}{\lambda^2}\tilde{C}(\theta,\phi)$$

can therefore be interpreted as a relative angular density of scattering. Statistically, we shall consider $b(\theta,\phi)$ as the density of probability for the emission of a scattered photon. This function is equivalent to the bidirectional reflectance distribution function (BRDF) well known in photometric measurements: we have

$$\epsilon\times b=\cos(\theta)\times\mathrm{BRDF}.$$

In all processes we shall study, large angles will give negligible effects, so that the b function is practically proportional to the BRDF. In fact, for an isotropic two-dimensional process, the b function is independent of the azimuthal angle ϕ, so that

$$b(\theta,\phi)=\frac{1}{2\pi}b(\theta).$$

The function $b(\theta)$ is normalized according to

$$\int_0^{\pi/2}b(\theta)\sin\theta d\theta=1.$$

The BRDF (or b as well) has a shape depending on the correlation range of the dominant defects.

B. Interaction with the walls and phase modulation process

When a photon hits any part of the surrounding vacuum vessel, it can be absorbed or reemitted in any direction, due to scattering or reflection. For analyzing the phase modulation, we return to the wave picture, and interpret the photon as a plane wave of wavelength λ, propagating along the direction defined by the unit vector $\vec{w}$, and having the amplitude

$$A(x,y,z)\propto e^{ik(w_1x+w_2y+w_3z)},$$

where $k\equiv 2\pi/\lambda$. In the wave picture, the outgoing wave results from diffraction off the surface. This diffracted wave may be viewed as the sum of partial plane waves resulting from its Fourier transform, so that each partial plane wave may be interpreted as a possible new direction for the photon. In the particle picture, the probability of the photon taking this direction will be proportional to the observed scattering power angular distribution of the material. For this particular direction of unit vector $\vec{s}$, the corresponding partial wave will be represented by

$$B(x,y,z)\propto e^{ik(s_1x+s_2y+s_3z)}.$$

The phase of A on the scattering surface is given by $\phi_A=k\vec{w}\cdot\vec{r}_S$, where $\vec{r}_S$ is some parametrization of the surface. We have in the same way the phase of B as $\phi_B=k\vec{s}\cdot\vec{r}_S$, so that, on the surface where they coincide, the differential phase between the two waves is given by $\Delta\phi=k(\vec{s}-\vec{w})\cdot\vec{r}_S$.

Assume now the surface to be moving according to the displacement vector $\vec{X}(t)\equiv(\xi(t),\eta(t),\zeta(t))$; the preceding formula for the phase shift undergoes the following transformation:

$$x\to x+\xi(t),\quad y\to y+\eta(t),\quad z\to z+\zeta(t),$$

so that its equation becomes $\vec{r}_S(t)=\vec{r}_S+\vec{X}(t)$. This will induce in the $\Delta\phi$ a part depending only on time, or phase modulation, given by

$$\Delta\phi(t)=\frac{2\pi}{\lambda}(\vec{s}-\vec{w})\cdot\vec{X}(t).$$

This is the general formula, depending on the incidence direction, the polarization of the displacement, and the outgoing direction. The result is even simpler in two special cases.

Specular reflection. $\vec{n}$ being the normal to the surface, Snell's law reads $\vec{s}=\vec{w}-2(\vec{w}\cdot\vec{n})\vec{n}$, and consequently

$$\Delta\phi(t)=\frac{4\pi}{\lambda}\cos\theta X_n(t),$$

X_n being the projection of the displacement onto the normal, and θ the incidence angle of the photon.

Backscattering. With $\vec{s}=-\vec{w}$, we obtain

$$\Delta\phi(t)=\frac{4\pi}{\lambda}X_w(t),$$

where X_w is the projection of the displacement onto the incidence direction. In the case where nothing is known about the polarization of the displacement, which is likely to change with the location of the interacting zone on the wall, it is possible to assume a random direction uniformly distributed on the sphere, so that

$$\Delta\phi(t)=\frac{2\pi}{\sqrt{3}\lambda}\sqrt{(\vec{w}-\vec{s})^2}X(t),$$

where $X(t)$ is a global rms amplitude of displacement, averaged on all polarizations and all locations. The motion of the walls can be sustained by the seismic activity of the soil, or by atmospheric sound waves caused by natural events (storms, wind, rain, etc.) or human activity (agricultural engines, airplanes, etc.). For the high-frequency part of the acoustic spectrum, we expect a cutoff due to double insulation by the tunnel surrounding the tube then by the thick layer of thermal insulation material installed permanently for baking. Low-frequency sound waves could be less attenuated, and are probably present in the measurements carried out on site. In the foregoing applications we will use the measured spectral density, calling it ''seismic noise'' for the sake of brevity, knowing that it could include acoustic effects. We also neglect possible amplification of the motion by mechanical resonances, because the tube's mounting system avoids such resonances at low frequencies (a few tens of Hz); for higher frequencies, the seismic excitation is negligible.

C. Recombination process

After following various paths, some of the launched photons may reach a mirror where they are mainly reflected (and partially transmitted, in the case of the input mirror). They can, however, be marginally scattered in any direction, including a neighborhood of the optical axis of the cavity. When this happens, we say that the photon is recombined with the main beam. Let us show how phase modulation can be transferred from spurious light to stored light by this process.

Consider again our rough mirror (as in Sec. II A) and a photon impinging on it along a direction corresponding to the spatial frequency $\vec{p}$. The plane wave associated with it is

$$A(x,y,z)=ae^{i\vec{p}\cdot\vec{x}}.$$

Its coupling with the main beam is given by the projection of the scattered wave onto the main transverse electromagnetic (TEM_{00}) wave of amplitude $\phi_0(\vec{x})$ [6]:

$$\gamma=\int ae^{i\vec{p}\vec{x}}2kf(\vec{x})\phi_0(\vec{x})d\vec{x}$$

so that

$$\langle\gamma\gamma^*\rangle=\epsilon aa^*\int C(\vec{x}-\vec{x}')e^{i\vec{p}\cdot(\vec{x}-\vec{x}')}\phi_0(\vec{x})\phi_0^*(\vec{x}')d\vec{x}d\vec{x}'.$$

We have as well

$$\langle\gamma\gamma^*\rangle=\frac{\epsilon aa^*}{4\pi^2}\int\tilde{C}(\vec{q})|\tilde{\phi}_0(\vec{p}-\vec{q})|^2d\vec{q}.$$

In fact, the angular distribution of the main beam is so sharply peaked (e.g., about 20 μrad for the Virgo cavities) that we replace it by a Dirac distribution in the preceding integral, so that finally

$$\langle\gamma\gamma^*\rangle=\epsilon aa^*\frac{\lambda^2}{2\pi}b(\theta).$$

The elementary phase change in the main wave due to interference with the recombined photon is given by [6]

$$\Delta\phi(t)=\mathrm{Im}[\gamma(t)]/\sqrt{P},$$

where P is the power stored in the main beam, and

$$\gamma(t)=\sqrt{\langle\gamma\gamma^*\rangle}e^{i[\phi_0+\Delta\phi(t)]},$$

where ϕ_0 is a static phase depending on the mean optical path, and $\Delta\phi(t)$ the sum of all phase modulations undergone at every encounter with the walls. Referring to the spectral density of the process $\sin[\Delta\phi(t)]$ by $n(f)$ we have

$$\Delta\phi(f)=\sqrt{\langle\gamma\gamma^*\rangle}n(f)/\sqrt{P}.$$

For low frequencies (up to a kHz) where the seismic noise is significant, a gravitational strain $h(t)$ causes a phase change of $\Delta\phi(t)=4\pi Lh(t)/\lambda$, L being the length of one arm. Above 1 kHz, the relation no longer holds, but the seismic noise is negligible. Thus, in order to be compared with the sensitivity of the interferometer, the spectral density of phase can be converted in spectral density of gravitational signal by $h(f)=(\lambda/4\pi L)\Phi(f)$, so that we get

$$h(f)=\frac{\lambda}{4\pi L}\sqrt{\frac{\epsilon aa^*\lambda^2}{2\pi P}b(\theta)n(f)},$$

n_s being the number of scattered photons sent per unit of time, and E the energy of each, we have obviously $n_s=\epsilon P/E$. On the other hand, R_m being the mirror's radius, we can normalize the amplitude a by $aa^*=E/\pi R_m^2$ so that

$$h(f)=\frac{\lambda^2\epsilon}{2^{2.5}\pi^2LR_m}\sqrt{\frac{b(\theta)}{n_s}}n(f).$$

We now consider a flux of n photons per unit time arriving at our mirror, each having a different history resulting in a random phase ϕ_0. We have to compute the rms value of the sum, assuming the random variable ϕ_0 uniformly distributed. This leads to the summation formula

$$h_{\mathrm{tot}}(f)=\frac{\lambda^2\epsilon}{2^{2.5}\pi^2LR_m}\sqrt{\frac{1}{n_s}\sum_{j=1}^{n}b(\theta_j)n_j(f)^2},\qquad(1)$$

where θ_j is the arrival angle of the jth photon, and $n_j(f)$ its phase noise.

D. Reflection noise in an empty tube

As a first example of application of the preceding summation formula, a very crude order of magnitude may be given in the case of a perfect straight tube. The calculation is made easy by assuming a centered beam, so that an axial symmetry may be imposed, by assuming a constant reflection coefficient of the steel, which is not completely unrealistic, because it will turn out that only grazing rays contribute to the noise in pratice, and at grazing incidence the steel's reflection coefficient is near 1. We see from formula 1 that we need to compute

$$\rho=\frac{1}{n_s}\sum_{j=1}^{n}b(\theta_j)n_j(f)^2.$$

Instead of summing over the individual photons, we may sum over classes of emission angles, because photons having the same emission angle have the same trajectory up to an azimuthal rotation. The modulation depends on the vibration mode of the tube: it is clear that in general, it will depend on the azimuthal angle. In the absence of information about the vibration mode (which probably varies along the several km of tube and with time), we take a global rms value $X(f)$ for the displacement spectral density, so that the modulation factor appears as azimuthally independent (though the physical motion is not). In the emission angular region around θ, the number of emitted photons (per unit time) is $dn=n_sb(\theta)\theta d\theta$. Calling L the length between opposite mirrors of a cavity, and R_t the radius of the tube, the number of reflections in the tube is given by $N(\theta)=\theta/\theta_{\mathrm{min}}$ where $\theta_{\mathrm{min}}=2R_t/L$. The transmission coefficient for that elementary channel is $T(\theta)=R^{N(\theta)}=\exp[\ln(R)\theta/\theta_{\mathrm{min}}]$, The modulation for 1 reflection is $n(\theta,f)=4\pi X(f)\theta/\lambda\sqrt{6}$ and adding incoherently the contributions of the $N(\theta)$ reflections yields $N(\theta)\times n^2(\theta,f)$ for the total modulation spectral density, so that Eq. (1) is equivalent to

$$\rho=\int_{\theta_{\mathrm{min}}}\theta d\theta b(\theta)T(\theta)gN(\theta)n^2(\theta)b(\theta),$$

where g is the probability that at the end of the tube the photon reaches the mirror. With our pointlike source, given an emission angle, the arrival point on the opposite end is strictly determined. But we consider the fact that real trajectories are not strictly radial, that the emitter mirror has a finite extent, and that, due to a small amount of scattering, the reemission angle may differ slightly from specular at each encounter with the wall, so that it is more realistic to consider the arrival point as randomly distributed on the end plane and we take $g=R_a^2/R_t^2$. For the BRDF, we consider a conservative model of the form

$$b(\theta)=\frac{\kappa}{\theta^2}\quad\text{for}\quad\theta>\theta_{\mathrm{min}},\qquad(2)$$

which is supported by direct measurements of various supermirrors, at least in the region $\theta>10^{-2}$. The constant κ is chosen such that

$$\int_{\theta_{\mathrm{min}}}^{\pi/2}b(\theta)\sin\theta d\theta=1.$$

The minimum angle θ_{min} corresponds to the radius of a mirror seen from the other one (about 55 μrad), so that $\kappa\sim0.1$. Other models can hold for very small angles, but result in less scattered light reaching the walls. We get

$$\rho=\frac{R_a^2}{R_t^2}\kappa^2\left(\frac{4\pi X(f)}{\lambda\sqrt{6}}\right)\frac{R}{\ln(1/R)}.$$

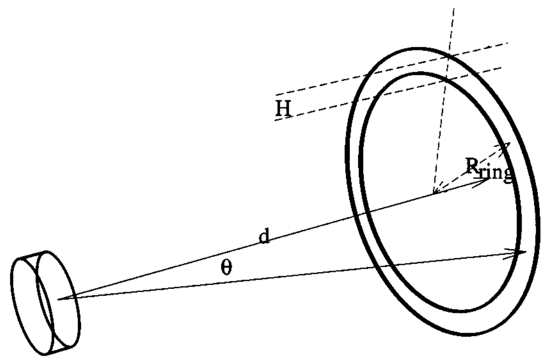

FIG. 3. Scattering ring.

Substituting this result in the square root appearing in Eq. (1), and taking into account the four mirrors defining the interferometer cavities, we obtain

$$h(f)=\frac{\epsilon\kappa}{\pi\sqrt{3}}\frac{\lambda X(f)}{L\ R_t}\sqrt{\frac{R}{\ln(1/R)}},$$

with $R=0.99$, we have $h(10\ \mathrm{Hz})=1.3\times10^{-23}\ \mathrm{Hz}^{-1/2}$. This is two orders of magnitude lower than the presently (thermal noise limited) foreseen sensitivity for VIRGO ($\sim10^{-21}\ \mathrm{Hz}^{-1/2}$) at 10 Hz. A reasonable estimation of the future improvements in the thermal noise level by using, for instance, sapphire instead of silica for the mirror substrates is at least 1 order of magnitude. We see from the preceding result that the safety margin even in this oversimplified case would be too narrow. Moreover because the actual tube is not a perfect cylinder, and has not only shape imperfections, but also ports and bellows, able to directly reflect SL to the emitter, it will likely be noisier than this rough calculation. Thus it is clearly necessary some system of baffles be installed.

III. BACKSCATTERING

Another special channel is the direct backscattering, where light is scattered off a mirror as before, reaches a rough surface, and is scattered back to the mirror where a third scattering recombines it with the main beam. This channel is especially interesting because, firstly, formulas for computation of backscattering noise have been given by Thorne [5] and it is possible to check on agreement with these, and secondly, some very simple cases of backscattering can be solved both by the photonic approach summarized by Eq. (1) and by the coherence function technique outlined in [6], and it is essential to test (at least in this special case) the consistency of the two methods.

A. Statistical approach and consistency with the Thorne backscattering formula

Consider a scattering object (target, for brevity) at a distance d from a mirror. Assuming n_s scattered photons sent per unit time leads to the number of photons received within the small solid angle $d\Omega$ by the target:

$$dn_1=n_s\frac{b(\theta)}{2\pi}d\Omega,$$

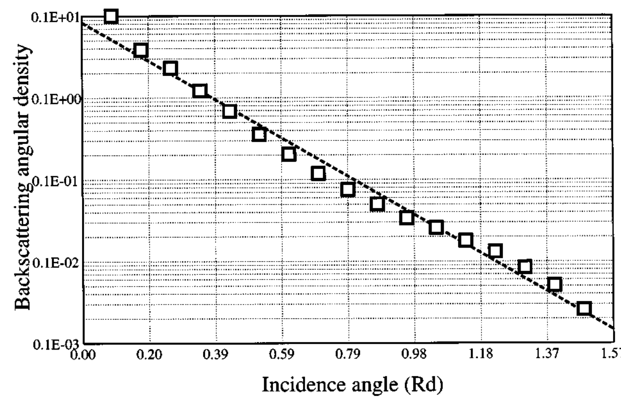

FIG. 4. BRDF of stainless steel for backscattering. Dashed line: exponential fit.

R_m being the mirror's radius, the solid angle corresponding to the mirror disk seen from the current point of the scatterer is $\pi R_m^2/d^2$, so that $dP_{\mathrm{scatt}}/d\Omega$ being the value of the BRDF of the scatterer in the direction of the mirror, we have the number of backscattered photons returning to the emitter:

$$dn_2=n_s\frac{b(\theta)}{2\pi}d\Omega\frac{dP_{\mathrm{scatt}}}{d\Omega}\frac{\pi R_m^2}{d^2}.$$

Now we can use Eq. (1), which gives:

$$dh^2(f)=\frac{\lambda^4\epsilon^2}{64\pi^4L^2d^2}b(\theta)^2d\Omega\frac{dP_{\mathrm{scatt}}}{d\Omega}n^2(f).$$

If we express the BRDF of the mirror as

$$\frac{dP_{\mathrm{mirr}}}{d\Omega}=\epsilon\frac{b(\theta)}{2\pi}$$

we obtain

$$dh^2(f)=\frac{\lambda^4}{16\pi^2L^2d^2}\left(\frac{dP_{\mathrm{mirr}}}{d\Omega}\right)^2\frac{dP_{\mathrm{scatt}}}{d\Omega}n^2(f)d\Omega,\qquad(3)$$

which is the Thorne-Flanagan formula 19 in [5].

It is now easy to compute, for instance, the backscattering noise due to a flat ring of weakly scattering material (e.g., glass) normal with respect to the optical axis and at a distance d; see Fig. 3 for notation. H being the ring's width and R_{ring} its mean radius, the solid angle seen from the mirror is $\Delta\Omega=2\pi R_{\mathrm{ring}}H/d^2$, so that using Eq. (1) we obtain

$$h(f)=\frac{\epsilon\lambda^2}{2^{2.5}\pi^2LR_m}\sqrt{\rho}n(f),$$

where

$$\rho=\frac{p(\theta)}{2\pi}\frac{2\pi R_{\mathrm{ring}}H}{d^2}\frac{dP_{\mathrm{scatt}}}{d\Omega}\frac{\pi R_m^2}{d^2}b(\theta)$$

so that

$$h(f)=\frac{\epsilon\lambda^2}{2^{2.5}\pi^{1.5}Ld^2}b(\theta)\sqrt{R_{\text{ring}}H}\sqrt{\frac{dP_{\text{scatt}}}{d\Omega}}n(f)$$

or

$$h(f)=\frac{1}{\sqrt{8\pi}}\frac{\lambda^2\sqrt{R_{\text{ring}}H}}{Ld^2}\frac{dP_{\text{mirr}}}{d\Omega}\sqrt{\frac{dP_{\text{scatt}}}{d\Omega}}n(f). \tag{4}$$

B. Consistency with the coherence function approach

The same problem can be analyzed within the wave optics framework. Roughness of the scattering surface will be represented by the stochastic process $f(\vec{y})$, $\vec{y}$ being the coordinates in the plane $z=d$. The self-coupling of the mirror due to the scatterer is expressed [6] by

$$\gamma=\int d\vec{y}\,2kf(\vec{y})\Psi_1(\vec{y})\Psi_2(\vec{y}),$$

where γ expresses the contribution of the channel to the main beam amplitude, Ψ_1 and Ψ_2 are two independent realizations of a wave scattered by the rough mirror and diffracted at a distance d. We have

$$\langle\gamma\gamma^*\rangle=\int d\vec{y}d\vec{y}'\epsilon' C(\vec{y}-\vec{y}')\mathbf{C}(d;\vec{y},\vec{y}')^2,$$

where $C(\vec{x})$ refers to the autocorrelation function of the process $f(\vec{x})$. ϵ' represents the integrated scattering rate of the scattering surface of the ring. **C** refers to the coherence function of the scattering process off the emitter mirror defined by [6]

$$\mathbf{C}(d;\vec{y},\vec{y}')=\frac{\epsilon b(\theta)}{2\pi d^2}\exp\left(-\frac{(\vec{y}-\vec{y}')^2}{2d^2\theta_g^2}\right)\exp\left(i\frac{k}{2d}(y^2-y'^2)\right)$$

in which θ_g is the Gaussian divergence of the primary beam, and θ the mean angle defined by the optical axis and the neighborhood of $(\vec{y},\vec{y}')$. Here we take $\theta=R_{\text{ring}}/d$ as a constant for the ring (assumed the ring thickness H is small with respect to R_{ring}). We can replace C by its Fourier transform, closely related to the mirror's BRDF:

$$\langle\gamma\gamma^*\rangle=\frac{1}{4\pi^2}\int d\vec{p}\;d\vec{y}\;d\vec{y}'\,\epsilon'\tilde{C}(\vec{p})\left(\frac{\epsilon b(\theta)}{2\pi}\right)^2\exp\left(-\frac{(\vec{y}-\vec{y}')^2}{d^2\theta_g^2}\right)\exp\left(i\frac{k(y^2-y'^2)}{d}\right)e^{-i\vec{p}\cdot(\vec{y}-\vec{y}')}.$$

We have the relation $\tilde{C}(\vec{p})=\lambda^2 dP_{\text{scatt}}/d\Omega$ and we can use again

$$\frac{\epsilon b(\theta)}{2\pi}\equiv\frac{dP_{\text{mirr}}}{d\Omega}.$$

In polar coordinates $[\vec{p}=(-k\alpha\cos\beta,-k\alpha\sin\beta)]$, and with the change of coordinates $\vec{u}\equiv(\vec{y}-\vec{y}')/2,\vec{v}\equiv(\vec{y}+\vec{y}')/2$ we get $(\vec{w}\equiv\vec{p}/k)$

$$\langle\gamma\gamma^*\rangle=\frac{4}{\lambda^2d^4}\int\alpha d\alpha d\beta\frac{dP_{\text{scatt}}}{d\Omega}(\alpha,\beta)\frac{dP_{\text{mirr}}}{d\Omega}(\theta)^2\int d\vec{u}d\vec{v}\exp[-2ik\vec{u}\cdot(\vec{w}-2\vec{v}/d)]\exp\left(-\frac{4u^2}{d^2\theta_g^2}\right).$$

After performing the various integrations, we find that the angles (α,β) correspond to backscattering $(\alpha=2\theta)$ and

$$\langle\gamma\gamma^*\rangle=\frac{2\pi\lambda^2R_{\text{ring}}H}{d^4}\left(\frac{dP_{\text{mirr}}}{d\Omega}\right)^2\frac{dP_{\text{scatt}}}{d\Omega}.$$

The equivalent gravitational density of noise is now:

$$h(f)=\frac{\lambda}{4\pi L}\langle\gamma\gamma^*\rangle^{1/2}n(f).$$

Substituting the value found for $\langle\gamma\gamma^*\rangle$ leads to Eq. (4).

Having verified the compatibility of the proposed photon method with already existing ones when it is possible, we use it now to obtain noise evaluations in a few elementary channels where an analytical approximate calculation can be carried out.

C. Analytical results on backscattering off the baffles set

The main component of the vacuum system is the pair of 3-km-long tubes containing the Fabry-Pérot cavities. In these cavities, which can be seen as gravito-optical tranducers, several kW light power are stored. The walls of these tubes will be made of stainless steel, and direct photometric measurements of steel samples have been carried out. A first result is that the backscattering angular density, i.e., the BRDF taken at angle $-2\times\theta_{\text{incidence}}$, is a decreasing function of the incidence angle denoted by $B(\theta)$, as can be seen in Fig. 4, and an exponential fit of the form

$$B(\theta)=0.83\times\exp(-5.5\theta) \tag{5}$$

is the most conservative model, probably overestimating the backscattering at grazing incidence. The actual vacuum tube of an antenna of the VIRGO (LIGO) type, is obviously not a

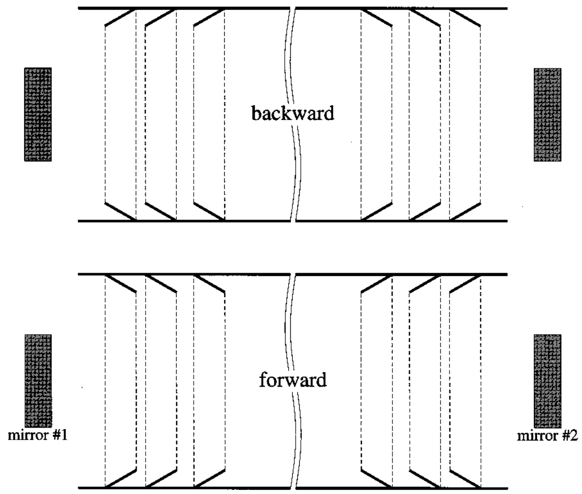

FIG. 5. Possible orientations of the baffles system.

perfect circular cylinder, but can be expected to present a number of special locations such as bellows, junctions of pumping systems, able to behave as reflectors with respect to the scattered light. We have seen above, and it will be confirmed below (Sec. IV) that even a perfect circular cylinder would generate too high a level of noise, with a realistic scattering model of stainless steel. It is therefore necessary to hide completely the tube from a direct line of sight to any mirror. This is done by a series of baffles distributed along the tube and covering the entire solid angle. Because more than one light beam may be installed in the 1.2-m-diameter tube, the height of the baffles must remain of the order of 10 cm in order to preserve a free aperture of 1 m, and consequently about $N=100$ baffles are needed to protect the mirrors from the tube. We call first series the $N/2$ elements located within the first half of the tube (the origin being taken at the corner mirror), and second series the $N/2$ elements located within the second half of the tube.

1. Noise due to backscattering off baffles

The simplest shape for a baffle is a truncated cone having its apex directed either towards the farthest mirror (forward orientation), or towards the nearest one (backward orientation) (see Fig. 5). The material out of which it could be made is either a dark glass having a polished surface, a null transmission factor, and a reflection factor analogous to regular glass, or the same stainless steel as the tube itself, which would give a number of benefits in the manufacturing. The global direct backscattering of the baffle system can be easily computed analytically using Eq. (1). For the noise corresponding to 1 mirror we have:

$$h(f)=\frac{\lambda^2\epsilon}{2^{5/2}\pi^2 L R_m}\sqrt{\rho}, \tag{6}$$

where

$$d\rho=\frac{b(\theta)}{2\pi}d\Omega\,\frac{1}{P_{\rm inc}}\frac{dP_s}{d\Omega}\,\frac{\pi R_m^2}{d^2}b(\theta)n^2(f).$$

d is the distance of the surface element seen under the angle θ, and $dP_s/d\Omega$ the backscattering BRDF of the baffle material. The mirror's BRDF will be assumed of the form 2, and $\theta=R_t/d$. This mirror will be assumed centered on the tube axis. We shall consider that $dP_s/P_{\rm inc}d\Omega$ takes a constant value, B, because the variations of incidence angle from the first baffle to the last one are small, and the backscattering angle does not depend on the orientation of the baffle (forward or backward) We have thus

$$\rho=\frac{\pi R_m^2\kappa^2}{R_t^2}n^2(f)B\ln(\theta_1/\theta_0),$$

where $\theta_0\equiv R_t/z_0$, and $\theta_1\equiv R_t/L$. The modulation here comes from only one interaction with the wall, and assuming a ground motion small compared to the wavelength, we have

$$n(f)=\frac{1}{\sqrt{2}}\frac{1}{\sqrt{3}}\frac{4\pi X(f)}{\lambda},$$

where the modulation expressed by the last factor has been multiplied by $1/\sqrt{3}$ for averaging the direction of the motion of the walls, and by $1/\sqrt{2}$ for averaging the cosine term coming from the static phase offset. Now, considering the effect of 4 mirrors, we get

$$h(f)=\sqrt{\frac{1}{3\pi}}\epsilon\kappa\frac{\lambda X(f)}{LR_t}\sqrt{B\ \ln(\theta_0/\theta_1)},$$

with the following reference parameters (assuming stainless steel baffles):

λ	wavelength	1.06×10^{-6} m
$\epsilon\kappa$	scattering	10^{-6}
R_t	tube radius	0.6 m
L	arm length	3×10^3 m
B	BRDF (60 deg)	3×10^{-3}
$X(f)$	seis. noise	10^{-8} mHz$^{-1/2}\times(10\ \text{Hz}/f)^2$

we find

$$h(10\ \text{Hz})\simeq3\times10^{-25}\ \text{Hz}^{-1/2}.$$

In the case of glass baffles, the constant B is very low (comparable to a mirror's) and the corresponding noise level is negligible.

2. Noise due to backscattering off bare ends of the tube

Even with any baffling system, there remains a short length of bare tube able to backscatter. In the VIRGO design, due to the valve installation scheme, there is a 6-m-long tube section between the input cavity mirror and the first valve, which we consider as bare in our calculation, as well as in the foregoing numerical estimations, despite the fact that special light traps will probably be installed there in order to hide the valve. The corresponding noise can be computed in a similar manner as above, using Eq. 6, with now

$$d\rho = \frac{b(\theta)}{2\pi}\sin(\theta)d\theta d\phi B(\theta)\frac{\pi R_m^2}{d^2}b(\theta)n^2(f)$$

by using Eqs. (2), (5), and $1/d = \tan(\theta)/R_t$, the preceding equation becomes

$$\rho = \frac{\pi R_m^2}{R_t^2}\,\kappa^2 B_0 n^2(f)\int_{\theta_0}^{\theta_1}\frac{\sin^3\theta}{\theta^4\cos^2\theta}\exp(5.5\theta)d\theta,$$

with $B_0 \sim 1.47\times 10^{-4}$ sr^{-1}. Assuming the first baffle located 6m from the closer end of the tube, the tube's end being at 1 m from the mirror, the angles are $\theta_0 \simeq 0.54$ rad and $\theta_1 \simeq 0.1$ rad, the numerical value of the integral is 9.84, so that

$$\rho = 4.6\times 10^{-3}\frac{R_m^2\kappa^2 B_0 n^2(f)}{R_t^2}$$

and finally, using the same parameters as in the preceding calculation,

$$h(10\ \mathrm{Hz}) \simeq 9\times 10^{-26}\ \mathrm{Hz}^{-1/2}.$$

Let us emphasize that these results are obtained assuming a single interaction with the steel. It is thus a partial evaluation of the global noise. The complete calculation should involve multi-interaction channels, and will be done in the next section, we keep these results here because they can give comparison points for controlling the purely numerical calculation.

IV. MONTE CARLO CODES AND NUMERICAL RESULTS

A. Principles of Monte Carlo codes

We briefly describe here the main procedures of the Monte Carlo codes developed for evaluating the scattered light noise in more general configurations involving various materials.

1. Generation of photons

Photons are randomly generated at a point of one mirror, which is a two-dimensional random variable obeying a Gaussian probability density identical to the normalized intensity distribution in the optical beam stored inside the cavity, i.e.,

$$\frac{dp_{\mathrm{mirr}}}{dS} = \frac{2}{\pi w_0^2}\exp(-2r^2/w_0^2).$$

The direction of emission is defined by the spherical angles (θ,ϕ). The angle ϕ is a random variable having a uniform density in the interval $[0,2\pi]$, and the angle θ is a random variable having a probability density proportional to the mirror's BRDF.

2. Interaction with a wall

When a photon hits a wall, it may be absorbed, reflected, or scattered. The distinction between reflection and scattering is not physically relevant for incoherent photons, but it can help to get efficient codes in the case of weakly scattering materials for which scattering is treated as an exception. For an incoming photon, knowing its incidence angle with respect to the normal, a random choice is made according to the statistical weights of the different options. The weights are fixed by empirico-theoretical laws of materials. For instance, for a black absorbing glass, it is reasonable to neglect scattering, owing to the very small roughness of the surface, and the reflection-transmission laws are given by well-known formulas for dielectrics (averaged over the two polarizations). Transmission in this case is equivalent to absorption. For stainless steel, the models depend strongly on the exact kind of steel and its processing. For samples of the VIRGO-type steel, absorption has been measured for several incidence angles not too close from grazing. For grazing incidence, the absorption rate can be thought to be zero, by analogy to the reflection coefficient of a conducting wall. The essential parameter is absorption at normal incidence, which depends in particular on the material bake processing. Measurements show that this absorption can vary between 40% (factory clean steel) and 60% (oxidized after baking in air). In the simulations, we vary this parameter to get a realistic confidence interval. The reflection coefficient was also measured and found to be almost constant even for large incidence angles, and near unity for grazing incidence. The total integrated scattering rate is then defined as $1-$absorption$-$reflection. A particular model, corresponding to 60% absorption at normal incidence is presented in Fig. 6. During the Monte Carlo run, if the absorption case is selected, the photon disappears and another is launched.

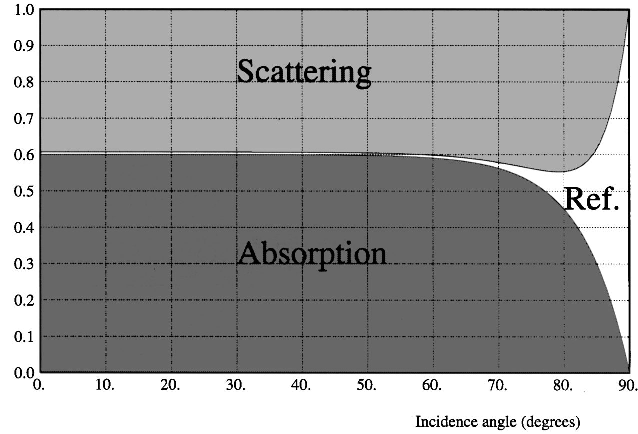

FIG. 6. Probabilities of reflection, absorption and scattering vs incidence angle for a stainless steel sample.

An alternative method is to leave the photon alive, to update the product of the survival probabilities after each encounter with material, and finally to weight its contribution by the last value of that product.

In the case of reflection , the new direction is computed according to the law of reflection. If the scattering case is selected, the new direction is a two-dimensional random variable (θ,ϕ), representing the angular deviation with respect to the specular reflection direction. ϕ is assumed uniformly distributed over the admissible part of $[0,2\pi]$, whereas θ is statistically distributed according to the BRDF of measured samples of stainless steel: an example is given in Fig. 7.

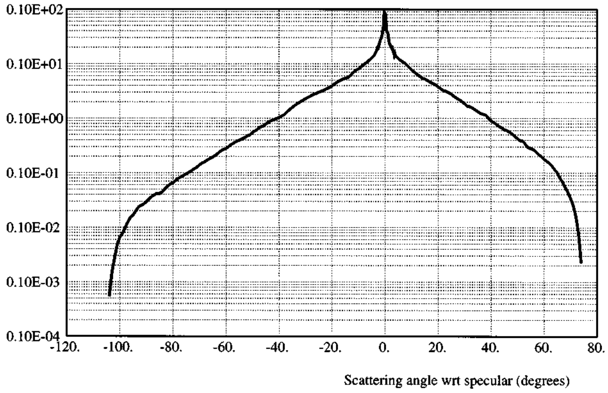

FIG. 7. Angular distribution of light scattered off a stainless steel sample for 15 degrees incidence angle.

3. Updating the modulation

At each interaction with a material object linked to the vacuum pipe, the global spectral density of modulation is incremented (starting from zero) by the quantity $||\vec{s}_{\mathrm{out}}-\vec{s}_{\mathrm{in}}||^2$, where $\vec{s}_{\mathrm{in}}$ and $\vec{s}_{\mathrm{out}}$ are unit vectors corresponding to the incidence direction and the outgoing direction obtained after reflection or scattering, respectively. At any time of a photon's life, its current modulation status is thus defined.

4. Interaction with a mirror

When a photon hits a far mirror (reflectivity near unity), it is only reflected, because we can neglect absorption or scattering (too low probabilities). But the impact point, the arrival angle and the current modulation status are recorded for further statistics.

5. Summing the noise

At the end of the run, we get the list of all photon impacts on the mirrors, and for each, the arrival angle and the modulation status. It is easy to compute the global noise spectral density by using the summation formula (1).

B. Numerical evaluation of scattered light noise generated by a complex vacuum system

1. Monte Carlo results

We give in Table I the significant results obtained after several runs of a code based on the principles described above. The two series of results for the forward orientation of baffles were obtained by two codes developed separately and independently by two different teams for the sake of reliability of the results. The aim was to aid in deciding among various options for the baffle system design, concerning shape, and material. These results represent the contribution of incoherent processes, and are partial results. To get a global estimation of noise, extra channels must be added, as will be discussed below. Let us recall that the VIRGO cavities are 3 km long, and surrounded by a 1.2-m-diameter stainless steel tube, with the inner diameter of the 80 baffles being 1 m.

2. Remarks on the consistency and the dispersion of the results

We may compare the preceding results with those obtained by analytical calculations in the preceding sections. For instance, for the bare tube, the agreement is about 30%, rather good owing to the oversimplification done in the analytical calculation.

Successive runs of the Monte Carlo codes give a standard deviation of about 12% of the output value, which allows for the maximum observed departure of less than 20% of team 1 with respect to team 2. Owing to our accuracy on photometric parameters, our simplification of the state of the different surfaces inside the vacuum pipe and of the geometry, a better accuracy is not needed.

TABLE I. Some results obtained by Monte Carlo simulation of scattered light propagation and recombination in the Fabry-Pérot cavities under various configurations of baffles. Results are expressed in $\mathrm{Hz}^{-1/2}$ at 10 Hz, and are supposed to scale as $1/f^2$. Monte Carlo results of teams 1 and 2 have standard deviations of about 12%.

Configuration	Analytic calculation	Results by team 1	Results by team 2
Ideal empty tube	1.3×10^{-23}	10^{-23}	
Backscattering baffles (30 deg)	3×10^{-25}		
Tube+glass baffles (backward)		1.1×10^{-25}	
Steel baffles (backward)			
40% abs, 30 deg aper.		1.1×10^{-24}	
50% abs, 30 deg aper.		7.6×10^{-25}	
60% abs, 30 deg aper.		4.8×10^{-25}	
Steel baffles (forward)			
40% abs, 40 deg aper.			1.5×10^{-24}
50% abs, 40 deg aper.			1.4×10^{-24}
60% abs, 40 deg aper.			6.8×10^{-25}
40% abs, 30 deg aper.		7.8×10^{-25}	8.9×10^{-25}
50% abs, 30 deg aper.		5.2×10^{-25}	6.3×10^{-25}
60% abs, 30 deg aper.		3.5×10^{-25}	2.9×10^{-25}

TABLE II. Some results of wave optics calculations, expressed in $\mathrm{Hz}^{-1/2}$ at 10 Hz, and supposed to scale as $1/f^2$. The SQL scales as $1/f$.

Channel	Noise level
Standard quantum Limit (100 kg):	1.54×10^{-23}
Diffraction by edges	
(1) Centered baffles	4.1×10^{-25}
(2) 20 cm offset	3.9×10^{-25}
Reflection by edges	
(1) Tilted baffle (1 cm),100 μm edge (glass)	1.4×10^{-25}
(2) Tilted baffle (1 cm),100 μm edge (steel)	7.1×10^{-25}
(3) Serrated edge (steel)	0

3. Global noise evaluation

In order to estimate the global noise due to scattered light, we must add to the preceding estimations, evaluations of specific extra channels related to coherent effects, by nature excluded from the statistical method discussed above. For instance, diffraction noise caused by interaction of the scattered light with baffle edges has been evaluated in [6] in the case of a nearly centered baffle (with respect to the optical axis). The case of a large offset can be treated similarly, and must be considered, being the actual situation when more than one interferometer are installed in the vacuum pipe. Even if the initial operation will involve only one optical beam, the baffle design must allow addition of extra beams in the future. Reflection from baffle edges was also analyzed in the same work, but it can be shown that serrations reduce it to essentially zero. We present in Table II the main numerical results. It is also worth recalling the value of the standard quantum limit (SQL):

$$h_{\mathrm{SQL}}(f)=\frac{1}{2\pi fL}\sqrt{\frac{8\hbar}{m}}$$

obtained by assuming an optimal light power flux, which minimizes at frequency f the root sum square of shot noise (which is decreasing with power), and of radiation pressure noise (which is increasing with power). This represents the ultimate noise level, below which it will be difficult to go without use of squeezed light. Reaching this level requires having reduced the thermal noise by two orders of magnitude, i.e., having enhanced the mechanical Q factor of the suspended mirror by four orders of magnitude. For the time being, we are unable to suggest how to reach these figures, but we must take these possibilities into account and design our baffling system with the idea that the sensitivity of the antenna could eventually reach $10^{-23}\ \mathrm{Hz}^{-1/2}$ at 10 Hz. In this case, in order to maintain the ability of operating the antenna at this level, the SL noise should remain 1 order of magnitude below the sensitivity limit, and the requirement principle is therefore that the overall SL noise must be less than $10^{-24}\ \mathrm{Hz}^{-1/2}$ at 10 Hz. Note that the diffraction noise decreases as the offset from center grows, reaches a minimum, and then increases again when going close to the edge: the two values quoted in Table II are on the two sides of the U-shaped curve. We may compare two solutions, assuming the central value of 50% for the absorption by steel at normal incidence.

(1) Glass baffles with smooth edges.

In a configuration involving 80 glass baffles located along the tube at $z_k=(1+\tau)^k\times z_0$, the resulting noise is essentially the noise of diffraction, reflection off the edges (serrations are hardly conceivable on a glass edge), and backscattering from the bare ends of the tube:

$$h(10\ \ \mathrm{Hz})=4.3\times10^{-25}.$$

Let us note that the noise contribution due to the bare ends could be still reduced by a foreseen specific baffle series protecting the valves.

(2) Stainless steel baffles with serrated edges.

In the same configuration of baffles out of a stainless steel identical to the tube's, with 30 degrees aperture angle, we have the contribution of the diffraction, the backscattering by the tube and by the baffles (the reflection noise is assumed zero due to serrations):

$$h(10\ \ \mathrm{Hz})=6.5\times10^{-25}$$

provided that the serrations do not produce small facets facing the mirror. More precisely, any facet should have a characteristic dimension less than 20 μm to keep the reflection noise less than $h(10\ \ \mathrm{Hz})=10^{-26}$.

V. CONCLUSION

The presented statistical method allows computation of the scattered light noise in a Fabry-Pérot cavity surrounded by a vibrating vacuum system. This is of great importance to the design of gravitational wave interferometric antennas, in which seismic isolation is essential. This method is based on ray optics, but was found consistent with a wave optics method developed earlier, in the regime where they overlap. The two methods are complementary for evaluating a general configuration. The method discussed is especially useful when coupled with a Monte Carlo numerical code for evaluating a realistic vacuum system including light traps made of various materials. Some numerical results obtained during a study of the VIRGO configuration have been presented.

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