Further remarks on isospin breaking in charmless semileptonic B decays

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We consider the isospin-breaking corrections to charmless semileptonic decays of B mesons. Both the recently measured branching ratios of exclusive decays by the CLEO Collaboration and the end-point region of the inclusive lepton spectrum in form factor models can be affected by these corrections. Isospin corrections can affect the determination of $|V_{ub}|$ from exclusive semileptonic B decays at a level comparable to present statistical uncertainties. [S0556-2821(97)06821-5]

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The first measurements of the exclusive charmless semileptonic decays of B mesons have been reported recently by the CLEO Collaboration [1]. A comparison between CLEO's results for the branching ratios of $B^0\to\pi^- l^+\nu$ and $B^0\to\rho^- l^+\nu$ and the theoretical expressions for their decay rates allows a determination of the $|V_{ub}|$ entry of the Cabibbo-Kobayashi-Maskawa mixing matrix. Actually, the value $|V_{ub}|=(3.3\pm 0.2^{+0.3}_{-0.4}\pm 0.7)\times 10^{-3}$ has been estimated by combining the yields of five different channels of B decays measured by CLEO and using four different theoretical models to describe the form factors of these exclusive decays.

The small statistical uncertainty ($\approx 6\%$) quoted in CLEO's estimate of $|V_{ub}|$ is obtained by assuming the isospin symmetry relations

$$\Gamma(B^0\to\pi^- l^+\nu)=2\Gamma(B^+\to\pi^0 l^+\nu), \tag{1}$$

$$\Gamma(B^0\to\rho^- l^+\nu)=2\Gamma(B^+\to\rho^0 l^+\nu), \tag{2}$$

$$=2\Gamma(B^+\to\omega l^+\nu), \tag{3}$$

to combine the set of five channels in B^+ and B^0 decays into two independent measurements of $B(B^0\to\pi^- l^+\nu)$ and $B(B^0\to\rho^- l^+\nu)$.

There are two reasons to consider the effects of isospin symmetry breaking in Eqs. (1)–(3). First, as we have shown in a previous paper [2], the isospin breaking corrections due to ρ^0-ω mixing affect the relations (2) and (3) at the level of the statistical uncertainties reported for the $B^0\to\rho^- l^+\nu$ branching ratio [1]. Second, the precision for the branching ratios of exclusive charmless semileptonic B decays would certainly be improved in forthcoming measurements at projected B factories.

In order to further emphasize the importance of isospin breaking effects let us mention that the correction to the $K^+\to\pi^0 e^+\nu$ decay (K^+_{e3}) due to the π^0-η mixing affects its decay rate by around 3.4% [3]. The inclusion of this correction in semileptonic K^+ decays is important in order to achieve a determination of $|V_{us}|$ at the level of $\pm 1\%$ [3] by combining the semielectronic rates of K^+ and K^0_L in a consistent way. As it was discussed by Leutwyler and Roos [3], these isospin breaking corrections enter at first order in the charge of vector weak transitions without violating the Ademollo-Gatto theorem.

Although the charmless semileptonic rates of B mesons cannot be measured with a similar precision as K_{e3} decays, the several exclusive channels accessible to $B\to X_u l^+\nu$ decays ($X_u=\pi^-,\rho^-,\pi^0,\rho^0,\omega$) partially compensate the limited accuracy for the individual rates. In addition, the end-point region of the lepton spectrum in inclusive semileptonic B decays is another source for the extraction of $|V_{ub}|$ [4]. Those properties also suggest that is necessary to include isospin breaking corrections in semileptonic B decays.

In this Brief Report we first discuss the isospin-breaking corrections to the lepton spectrum in inclusive charmless semileptonic B decays as described by form factor models [5]. We also address some comments on the size expected for isospin-breaking effects in the recent measurements of exclusive charmless B decays as reported by CLEO [1].

As is well known [5], the inclusive lepton spectrum of B decays in form factor models can be seen as a sum over exclusive channels. The end-point region of the lepton spectrum in semileptonic B decays ($2.3\leqslant E_e\leqslant 2.6$ GeV, E_e is the lepton energy) is expected to be dominated by a few exclusive modes $[B\to(\pi+\rho+\omega)l^+\nu]$. Most of the commonly used form factor models [5] explicitly assume isospin symmetry in their calculations. Neglecting resonances heavier than the ρ and ω mesons, the lepton spectra in decays of neutral and charged B mesons are thus given by

$$\frac{d\Gamma^0(B^0)}{dE_e}=\sum_{X^-=\pi^-,\rho^-}\frac{d\Gamma^0(B^0\to X^- l^+\nu)}{dE_e}, \tag{4}$$

$$\frac{d\Gamma^0(B^+)}{dE_e}=\sum_{X^0=\pi^0,\rho^0,\omega}\frac{d\Gamma^0(B^+\to X^0 l^+\nu)}{dE_e}$$
$$=\frac{d\Gamma^0(B^0)}{dE_e}-\frac{d\Gamma^0(B^+\to\pi^0 l^+\nu)}{dE_e}, \tag{5}$$

where $d\Gamma^0(B\to X l\nu)/dE_e$ denotes the lepton spectrum of each channel in the limit of exact isospin symmetry.

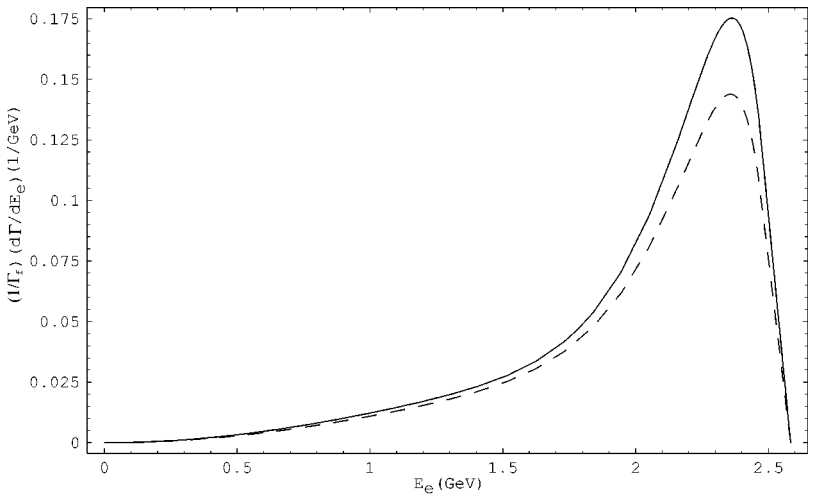

FIG. 1. Inclusive lepton spectrum in charmless semileptonic B^+ decays with (solid line) and without (dashed) isospin breaking corrections. These spectra are normalized to the tree level width Γ_f for $b\to u l^- \nu$.

After including isospin-breaking corrections due to π^0-η and ρ^0-ω mixing, the spectrum for the decay of B^+ mesons becomes [note that Eq. (4) remains unchanged]

$$\frac{d\Gamma(B^+)}{dE_e} = |1+\epsilon|^2 \frac{d\Gamma^0(B^+\to\pi^0 l^+\nu)}{dE_e} + |1+\epsilon'|^2 \frac{d\Gamma^0(B^+\to\rho^0 l^+\nu)}{dE_e} + |1+\epsilon''|^2 \frac{d\Gamma^0(B^+\to\omega l^+\nu)}{dE_e} \quad (6)$$

where ϵ (see Leutwyler and Roos in [3]) and ϵ', ϵ'' [2] are given by

$$\epsilon = \frac{3}{4}\frac{m_d - m_u}{m_s - \hat{m}} \approx 1.7\times 10^{-2}, \quad (7)$$

$$\epsilon' = \frac{m^2_{\rho\omega}}{m^2_\rho - m^2_\omega + i m_\omega \Gamma_\omega} \approx 0.160 + 0.051 i, \quad (8)$$

$$\epsilon'' = -\frac{m^2_{\rho\omega}}{m^2_\omega - m^2_\rho + i m_\rho \Gamma_\rho} \approx 0.006 - 0.031 i. \quad (9)$$

In the above expressions m_q $(q=u,d,s)$ denote current quark masses, $\hat{m}\equiv(m_u+m_d)/2$ and $m^2_{\rho\omega}=(-3.67\pm 0.30)\times 10^{-3}$ GeV^2 [6]. The values for the resonance parameters of ρ^0 and ω mesons are taken from [4]. As discussed in Ref. [2], the value of ϵ' is very sensitive to the specific value used for the mass of the ρ^0.

In order to illustrate these effects, in Fig. 1 we have plotted the lepton spectrum for decays of charged B mesons by using the model of Isgur-Scora-Grinstein-Wise, Ref. [5]. The dashed plot corresponds to the total spectrum in the limit of isospin symmetry, Eq. (5), and the corrected spectrum of Eq. (6) is represented by a solid line. As expected, the total spectrum corrected by isospin breaking is enhanced basically due to the second term in Eq. (6).

Now we focus on the exclusive B decays reported by CLEO [1]. Isospin-breaking corrections due to π^0-η and ρ^0-ω mixings affect only the decays of charged B mesons. After including these corrections, the relationships among the physical values of B decays are changed to[1] [2]

$$\Gamma(B^0\to\pi^- l^+\nu) = 2\Gamma(B^+\to\pi^0 l^+\nu)/|1+\epsilon|^2, \quad (10)$$

$$\Gamma(B^0\to\rho^- l^+\nu) = 2\Gamma(B^+\to\rho^0 l^+\nu)/|1+\epsilon'|^2, \quad (11)$$

$$= 2\Gamma(B^+\to\omega l^+\nu)/|1+\epsilon''|^2, \quad (12)$$

to be compared to Eqs. (1)–(3). According to Eqs. (7)–(9), the effects of isospin breaking on the $B^+\to\pi^0 l^+\nu$ and $\omega l^+\nu$ decay modes is negligible compared to form factors and present experimental uncertainties. The most important effect would be present in $B^+\to\rho^0 l^+\nu$.

In practice, it turns out to be impossible to compute the effects of Eqs. (10) and (11) in the extraction of $B(B^0\to\rho^- l^+\nu)$ and the value of $|V_{ub}|$ from the numbers quoted in Ref. [1]. Actually, the relative normalizations imposed by the isospin constraints, Eqs. (1)–(3) or (10)–(12), must be implemented in the simultaneous fit to the five semileptonic channels. In addition, the yields depend on the kinematical region included in the analysis because, for instance, the signal and the crossfeed among different modes smear outside the region considered in Ref. [1].

The CLEO Collaboration has redone the analysis of Ref. [1] by implementing Eqs. (11) and (12) [7]. Using the model of Ref. [8] as an example, CLEO's reanalysis found that $B(B^0\to\rho^- l^+\nu)$ shifts by -8.8% in the combined fit to the $\rho^-/\rho^0/\omega$ channels [7]. Therefore the value of $|V_{ub}|$ from vector modes shifts only by -4.4% from the value obtained

[1]Since vector mesons are resonances, a factorization approximation must be introduced to separate $B\to(\rho,\omega)l^+\nu$ from the subsequent decay processes $\rho\to\pi\pi$, $\omega\to 3\pi$. In fact, our results on isospin breaking can also be derived by using the mixed propagator matrix of the ρ^I-ω^I unstable system (I is for isospin eigenstates) to describe the full $B\to(2\pi,3\pi)l^+\nu$ decay processes.

when isospin symmetry is assumed [1]. If the $B \to \pi l \nu$ modes are also considered, the value of $|V_{ub}|$ changes by -3.3%.

In summary, even though isospin breaking corrections to the exclusive channels reported in Ref. [1] are smaller than present total uncertainties, they are important effects to be included in forthcoming improved measurements of inclusive and exclusive charmless semileptonic decays of B mesons. To conclude, let us mention that similar corrections due to ρ^0-ω mixing must be applied when combining different decay channels to quote an average for the flavor changing radiative decays $B \to \rho + \gamma$.

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