Relativistic and binding energy corrections to heavy quark fragmentation functions

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We calculate the fragmentation function for a charm quark to decay inclusively into S-wave charmonium states, including relativistic and binding energy corrections in powers of the quark relative velocity v. We also use these fragmentation functions to estimate their contribution to the production rate of η_c and J/ψ in Z^0 decay. These corrections contribute about 38% to the integrated $c \to J/\psi + X$ fragmentation. For η_c, these corrections are found to be small. [S0556-2821(97)02421-1]

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INTRODUCTION

The fragmentation function of a c quark into η_c and J/ψ mesons has been calculated by Braaten *et al.* [1]. They assumed that the relative momentum of the quark and the antiquark is small compared to m_c and hence can be neglected. Since the estimated average value of v^2 is about 1/3 for charmonium, the relativistic corrections of order $(v^2)^n$ can be expected to be more important than perturbative corrections of order α_s^{2n}. In this Brief Report, we calculate the relativistic and binding energy corrections to the fragmentation functions of a charm quark to decay into η_c and J/ψ.

The lack of manifest gauge invariance in previous works on processes involving heavy quarks has earlier been addressed and resolved in Refs. [2–4], where this key property of a gauge theory has been systematically restored in the study of quarkonia decay. In this paper, we see that gauge invariance naturally leads to incorporation of next to leading order effects.

We calculate the fragmentation function $f(z,\mu)$ at the scale $\mu = 3m$. Switching off the relativistic and binding energy corrections, we reproduce the results obtained by Braaten *et al.* [1]. Altarelli-Parisi equations are then used to evolve these functions to the scale $\mu = M_z/2$ appropriate for Z^0 decay. The knowledge of the fragmentation functions is then used to find the relativistic and binding energy corrections to the branching ratios of Z^0 decay into η_c and J/ψ.

FRAGMENTATION FUNCTIONS

Our starting point is the definition of fragmentation function in terms of matrix elements of field operators at light cone separation [5]; i.e.,

$$\hat{f}(z) = \frac{z}{4} \sum_X \int \frac{d\lambda}{2\pi} e^{-i\lambda/z} \mathrm{Tr}[\not{n} \langle 0 | \psi(0) | PX \rangle \langle PX | \overline{\psi(\lambda n)} | 0 \rangle], \tag{1}$$

where z is the momentum fraction of the fragmenting quark carried by the quarkonium in the forward direction. n^μ is defined such that $n^2 = n^+ = 0$, and P^μ is the four-momentum of the quarkonium. We choose a frame in which its three-momentum is along the z direction and then define $P^\mu = p^\mu + \frac{1}{2} M^2 n^\mu$, M being the quarkonium mass and p^μ a null vector such that $p^- = 0$ and $p \cdot n = 1$. In lowest order of α_s, the inclusive sum over X is restricted to c and $\overline{c}$:

$$\hat{f}(z) = \frac{z^2}{8(2\pi)^3(1-z)} \int d^2 l_\perp \mathrm{Tr}[\not{n} \langle 0 | \psi(0) | Pl \rangle \times \langle Pl | \overline{\psi(0)} | 0 \rangle] \;, \tag{2}$$

where l is the four-momentum of the undetected outgoing quark and $l_\perp$ is the transverse component. The leading order contribution to the amplitude comes from Fig. 1 and is given by

$$\langle Pl | \overline{\psi(0)} | 0 \rangle = g^2 \int \frac{d^4k}{(2\pi)^4} \overline{u(l)} \gamma^\mu M(k) \gamma^\nu \times S_F(P+l)\, D_{\mu\nu}\left(k - \frac{P}{2} + l\right), \tag{3}$$

where the color indices have been suppressed. The gauge-invariant Bethe-Salpeter amplitude $M(k)$ is

$$M(k) = \int d^4x e^{ik\cdot x} \langle P | \psi(x/2) \exp\left(i \int_{-x/2}^{x/2} A(\xi) d\xi \right) \times \overline{\psi(-x/2)} | 0 \rangle. \tag{4}$$

The light-cone gluon propagator is

$$D_{\mu\nu}(q) = \left(-g_{\mu\nu} + \frac{q_\mu n_\nu + q_\nu n_\mu}{q \cdot n} \right) \frac{1}{q^2} \,. \tag{5}$$

Since the relative velocity of heavy quarks is much less than the scale set by their mass, we can expand $D_{\mu\nu}(k - P/2 + l)$ in powers of k, the relative momentum,

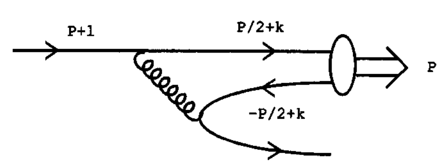

FIG. 1. Feynman diagram contributing to the fragmentation of a c quark into charmonium.

$$\langle Pl|\overline{\psi(0)}|0\rangle = \overline{u(l)}\,\gamma^\mu M(0)\,\gamma^\nu S_F(P+l)D_{\mu\nu}(P/2-l) + \overline{u(l)}\,\gamma^\mu M^\alpha(0)\,\gamma^\nu S_F(P+l)D_{\mu\nu,\alpha}(P/2-l) + \overline{u(l)}\,\gamma^\mu M^{\alpha\beta}(0)\,\gamma^\nu S_F(P+l) \times D_{\mu\nu,\alpha\beta}(P/2-l), \tag{6}$$

where

$$M(0)\equiv\langle P|\psi\overline{\psi}|0\rangle,$$
$$M^\alpha(0)\equiv\langle P|\psi i\overleftrightarrow{D}^\alpha\overline{\psi}|0\rangle,$$
$$M^{\alpha\beta}(0)\equiv\frac{1}{2!}\langle P|\psi i\overleftrightarrow{D}^\alpha i\overleftrightarrow{D}^\beta\overline{\psi}|0\rangle, \tag{7}$$

with

$$\overleftrightarrow{D}^\alpha=\frac{1}{2}(\vec{\partial}^\alpha-\overleftarrow{\partial}^\alpha)-igA^\alpha \tag{8}$$

and

$$D_{\mu\nu,\alpha}\equiv\partial_\alpha D_{\mu\nu},$$
$$D_{\mu\nu,\alpha\beta}\equiv\partial_\alpha\partial_\beta D_{\mu\nu}. \tag{9}$$

Evaluation of the hadronic matrix elements for the decay of mesons has been detailed in Refs. [3,4,6]. Their Hermitian conjugation and trivial algebraic manipulation [6] yields the matrix elements considered in this paper. For η_c,

$$\langle P|\psi\overline{\psi}|0\rangle = \frac{M^{1/2}}{2}\gamma_5\left(1+\frac{\nabla^2}{M^2}\right)\phi(0)\left(1+\frac{\not P}{M}\right) - \frac{M^{1/2}}{2}\gamma_5\frac{\nabla^2\phi(0)}{M^2}\left(1-\frac{\not P}{M}\right),$$

$$\langle P|\psi\; i\overleftrightarrow{D}^\alpha\overline{\psi}|0\rangle = \frac{i}{3}M^{1/2}\frac{\nabla^2\phi(0)}{M^2}\gamma_5\sigma^{\alpha\beta}P_\beta,$$

$$\langle P|\psi\; i\overleftrightarrow{D}^\alpha i\overleftrightarrow{D}^\beta\overline{\psi}|0\rangle = \frac{1}{6}M^{5/2}\frac{\nabla^2\phi(0)}{M^2}\gamma_5\left(g^{\alpha\beta}-\frac{P^\alpha P^\beta}{M^2}\right)\times\left(1+\frac{\not P}{M}\right), \tag{10}$$

where $\phi(0)$ is the quarkonium wave function at the origin. Using these values of the matrix elements, we compute the fragmentation function, including the binding energy correction coming from $m=M/2+\epsilon_B/2$:

$$\hat{f}_{c\to\eta_c}(z,3m)=\frac{64\alpha_s^2(3m)}{81\pi}\frac{|R(0)|^2}{M^3}[f_0(z)+\eta_B f_B(z)+\eta_W f_W(z)] = \hat{f}_0(z,3m)+\eta_B\hat{f}_B(z,3m)+\eta_W\hat{f}_W(z,3m) \tag{11}$$

where

$$f_0(z)=\frac{z\;(1-z)^2(48+8z^2-8z^3+3z^4)}{(2-z)^6},$$

$$f_B(z)=\frac{4\,z^2\;(1-z)^2(-48+48z-40z^2+12z^3-5z^4)}{(2-z)^8},$$

$$f_W(z)=\frac{8\,z\;(1-z)^2(96+144z-528z^2+296z^3-102z^4+43z^5-9z^6)}{3\,(2-z)^8},$$

$$\hat{f}_i(z,3m)=\frac{64\alpha_s^2(3m)}{81\pi}\frac{|R(0)|^2}{M^3}f_i(z)\qquad(i=0,B,W), \tag{12}$$

and

$$\eta_B=\frac{\epsilon_B}{M},\qquad \eta_W=\frac{\nabla^2R(0)}{M^2R(0)}\quad, \tag{13}$$

where $R(0)$ is the radial wave function, related to $\phi(0)$ as $\phi(0)=R(0)/4\pi$. Setting $\eta_B=\eta_W=0$ gives the result obtained by Braaten *et al.* [1]. We now use Altarelli-Parisi evolution equations to evolve the fragmentation function evaluated at the scale $\mu=3m$ to $\mu=M_z/2$. Taking into consideration only the contribution of charm quark and antiquark, the total decay rate for inclusive η_c production at leading order in α_s through Z^0 decay is given by [1]

$$\Gamma(Z^0\to\eta_c+X)=2\Gamma(Z^0\to c\,\overline{c})\int_0^1 dz\,\hat{f}_{c\to\eta_c}(z,3m), \tag{14}$$

where we have used the fact that at leading order in α_s, the fragmentation probability $\int_0^1 dz\hat{f}_{c\to\eta_c}(z,\mu)$ does not evolve with scale μ. It is straightforward to obtain the fragmentation probability by integrating Eq. (11) over z :

$$\int_0^1 dz\ \hat{f}_{c\to\eta_c}(z,3m)=\frac{64\alpha_s^2}{27\pi}\frac{|R(0)|^2}{M^3}(F_0+\eta_B F_B+\eta_W F_W), \tag{15}$$

where

$$F_0=\frac{773}{30}-37\ln 2,$$

$$F_B=-\frac{5639}{105}+\frac{232}{3}\ln 2,$$

$$F_W=-\frac{100304}{315}+\frac{4136}{9}\ln 2. \tag{16}$$

In an identical fashion, one can repeat the above calculation for the fragmentation of a c quark to the 1^{--} state. The corresponding matrix elements can be derived, as before, from the ones evaluated for the relevant decay process in Ref. [4]:

$$\langle P,\epsilon|\psi\overline{\psi|}0\rangle=\frac{1}{2}M^{1/2}\left(1+\frac{\nabla^2}{M^2}\right)\phi\not{\epsilon}^*\left(1+\frac{\not{P}}{M}\right)$$
$$-\frac{1}{6}M^{1/2}\frac{\nabla^2\phi}{M^2}\not{\epsilon}^*\left(1-\frac{\not{P}}{M}\right),$$

$$\langle P,\epsilon|\psi\ i\overleftrightarrow{D}^{\alpha}\overline{\psi|}0\rangle=\frac{1}{3}M^{3/2}\frac{\nabla^2\phi}{M^2}\epsilon_\beta^*\left[g^{\alpha\beta}+\frac{i}{M}\epsilon^{\mu\nu\alpha\beta}P_\nu\gamma_\mu\gamma_5\right],$$

$$\langle P,\epsilon|\psi\ i\overleftrightarrow{D}^{\alpha}i\overleftrightarrow{D}^{\beta}\overline{\psi|}0\rangle=\frac{1}{6}M^{5/2}\frac{\nabla^2\phi}{M^2}\left(g^{\alpha\beta}-\frac{P^\alpha P^\beta}{M^2}\right)\times\not{\epsilon}^*\left(1+\frac{\not{P}}{M}\right). \tag{17}$$

With these values of matrix elements, we obtain

$$\hat{f}_{c\to J/\psi}(z)=\frac{64}{27\pi}\alpha_s(3m_c)^2\frac{|R(0)|^2}{M^3}[f_0(z)+\eta_B f_B(z)+\eta_W f_W(z)]$$
$$=\hat{f}_0(z,3m)+\eta_B\hat{f}_B(z,3m)+\eta_W\hat{f}_W(z,3m), \tag{18}$$

where

$$f_0(z)=\frac{z(1-z)^2(16-32z+72z^2-32z^3+5z^4)}{(2-z)^6},$$

$$f_B(z)=-\frac{4z^2(1-z)^2(48-144z+152z^2-28z^3+13z^4-2z^5)}{3(z-2)^8},$$

$$f_W(z)=\frac{8z(1-z)^2(288-912z+2176z^2-2280z^3+1438z^4-471z^5+58z^6)}{9(2-z)^8}, \tag{19}$$

and all the other symbols have the same meaning as before with only the difference that M now stands for the mass of J/ψ. Using the Altarelli-Parisi equation, we have evolved the fragmentation function at the scale $\mu=3m$ to the scale $\mu=M_z/2$ (see Fig. 2). As before, the evaluation of the decay rate of Z^0 to J/ψ would require the total fragmentation probability, which can be obtained by integrating Eq. (18) over z:

$$\int_0^1 dz\ \hat{f}_{c\to J/\psi}(z,3m)=\frac{64\alpha_s^2}{27\pi}\frac{|R(0)|^2}{M^3}(F_0+\eta_B F_B+\eta_W F_W), \tag{20}$$

where

$$F_0=\frac{1189}{30}-57\ln 2,$$

$$F_B=\frac{2327}{35}-96\ln 2,$$

$$F_W=\frac{54308}{63}-\frac{3728}{3}\ln 2. \tag{21}$$

In the present treatment, the parameters η_B and η_W are independent of each other. Note that the condition $\eta_B=2\eta_W$ generally imposed is equivalent to $(1/M)\nabla^2\phi(0)=(1/2)\epsilon_B\phi(0)$ which is the Schrödinger equation for quark relative motion in a potential vanishing at zero relative sepa-

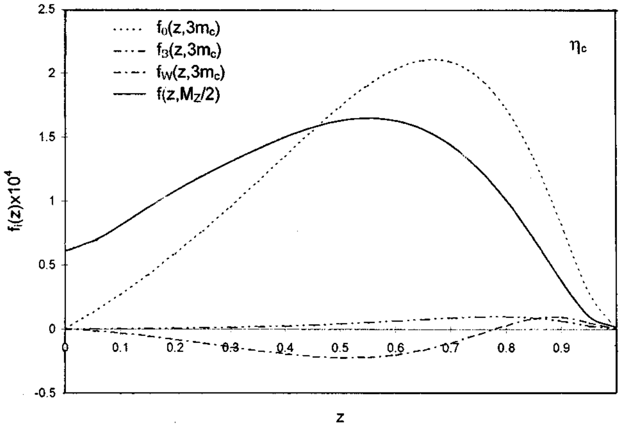

FIG. 2. The functions f_0, f_B, and f_W at $Q^2=(3m)^2$ for $c\to\eta_c$. The solid line shows the complete fragmentation function f at $Q^2=(M_Z/2)^2$.

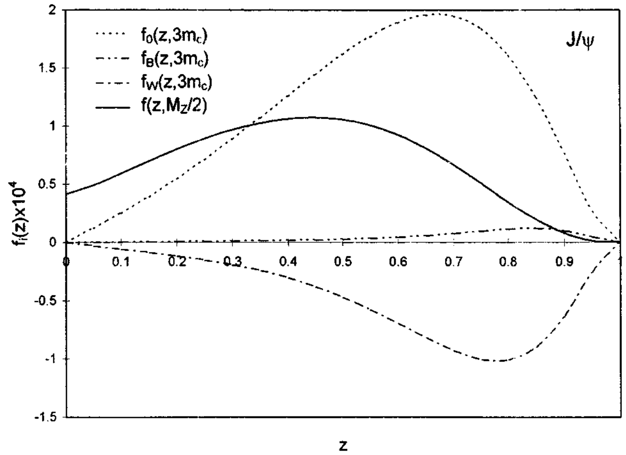

FIG. 3. Same as Fig. 2 for $c\to J/\psi$.

ration. There is no principle in our treatment which constrains η_B and η_W to bear a fixed relation with each other. Hence both are regarded as adjustable parameters [4].

In order to produce a numerical estimation for corrections to fragmentation functions, the values used for the various parameters are $\alpha_s=0.19$, $m=1.43$ GeV, $|R(0)_{\eta_c}|^2=0.936$ GeV^3, $|R(0)_{J/\psi}|^2=0.978$ GeV^3, $\nabla^2R/R=-0.7$ GeV^2, $M_Z=91$ GeV, $M_{\eta_c}=2.98$ GeV, $M_{J/\psi}=3.097$ GeV. The choice of the parameters α_s, $|R(0)|^2$, and ∇^2R/R is discussed in Ref. [3]. The fragmentation functions have been depicted in Figs. 2 and 3.

One of the first applications of the fragmentation ideas was to charmonia production at the CERN e^+e^- collider LEP. After calculating the lowest order fragmentation functions, Braaten, Cheung, and Yuan [1] calculated the branching fraction $B(Z^0\to\eta_c(J/\psi))=\Gamma(Z^0\to\eta_c(J/\psi)+X)/\Gamma(Z^0\to c\bar{c})$. We generalize the above results to incorporate the binding energy and relativistic corrections. We get, for η_c,

$$B(Z^0\to\eta_c)=[\Gamma_0/\Gamma(Z^0\to c\bar{c})][1-0.84\eta_B+0.95\eta_W], \tag{22}$$

where Γ_0 is the color-singlet decay rate in the absence of relativistic and binding energy corrections. For the values of parameters chosen above, the branching ratio without and with the corrections is 2.32×10^{-4} and 2.22×10^{-4}, respectively, a negligible difference. For J/ψ,

$$B(Z^0\to J/\psi)=[\Gamma_0/\Gamma(Z^0\to c\bar{c})][1-0.45\eta_B+5.5\eta_W]\ . \tag{23}$$

The binding and relativistic corrections modify the color-singlet branching ratio of Braaten *et al.* from 2.22×10^{-4} to 1.36×10^{-4}, an effect of around 38%, which is not surprising because v^2/c^2 is expected to be around 1/3 for charmonium. Note that similar correction for η_c is small. There, though the contributions arising from various individual terms are of the order 1/3 as in the case of J/ψ but, owing to different matrix elements, they happen to cancel each other out to give a negligible effect.

In conclusion, we incorporated relativistic and binding energy corrections of $O(v^2)$ to the fragmentation functions for charm quark splitting into η_c and J/ψ, and showed how these corrections can be expressed in terms of various bound state matrix elements of gauge-invariant quark and gluon operators. In the absence of the said corrections, these results reduce to the leading order result of Braaten *et al.* [1], as expected. We then used the modified fragmentation functions to estimate the contribution of relativistic and binding energy corrections to the corresponding branching ratios in $Z^0\to\psi c\bar{c}$ decays.

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