Polarized $\Lambda_b \rightarrow X_c \tau \nu$ in the standard model and two-Higgs-doublet model

Chao-Shang Huang
Institute of Theoretical Physics, Academia Sinica, Beijing 100080, China
and International Centre for Theoretical Physics, P.O. Box 586, 341000 Trieste, Italy

Hua-Gang Yan
Institute of Theoretical Physics, Academia Sinica, Beijing 100080, China
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The inclusive rate and τ spectrum for a polarized Λ_b baryon to decay to charm hadronic final states and leptons $\tau\nu$ in the SM and a two-Higgs-doublet model are computed. The $O(\alpha_s)$ QCD corrections to the τ spectrum in the two-Higgs-doublet model are also given. [S0556-2821(97)06121-3]

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I. INTRODUCTION

The semileptonic decay $B \rightarrow X_c l \nu$ has been extensively studied in both the standard model (SM) and a two-Higgs-doublet model (THDM) [1–9]. Compared with the B decay, in addition to the spectrum of the lepton arising from the decay, the various spin correlation quantities are of interest for the decay of a polarized Λ_b. It is well known [10,11] that the heavy quarks produced in Z^0 decay are polarized and only charmed and flavored Λ baryons seem to offer a practical method to measure the polarization of the corresponding heavy quark. The polarization transferred from a heavy quark Q to the corresponding Λ_Q is 100% [12] in the limit $m_Q \rightarrow \infty$. Thus the angular distributions of charged leptons [13,14] from semileptonic decays of Λ_b and Λ_c can be used as spin analyzers for the decays of heavy quarks. The inclusive rate and l spectrum of polarized $\Lambda_b \rightarrow X_c l \nu (l = e, \mu)$ have been computed in the SM [3,15,16].

The $\Lambda_b \rightarrow X_c \tau \nu$ (and $B \rightarrow X_c \tau \nu$) decay is sensitive to new physics, in particular, models with charged Higgs bosons. Because the charged Higgs bosons contribute at the tree level, its contribution cannot be cancelled by other new particles in the models. Therefore, the calculations of charged Higgs boson-contributions with high accuracy will provide a strong bound on parameters of the models when experimental measurement of the decay is available.

In this paper we investigate the polarized inclusive decay $\Lambda_b \rightarrow X_c \tau \nu$ in both the SM and THDM. In the SM we extend the results of Manohar and Wise [3] to the case of a nonzero mass of the final state lepton (τ). We calculate the spin dependent form factors in the hadronic tensor to the $1/m_b^2$ order in heavy quark effective theory (HQET) which do not contribute to the decay rate when the mass of the final state lepton is neglected. In the THDM we compute the inclusive rate and τ spectrum for a polarized $\Lambda \rightarrow X_c \tau \nu$ included in the $\Lambda_{\rm QCD}^2/m_b^2$ nonperturbative corrections and the $O(\alpha_s)$ perturbative corrections to τ spectrum.

The paper is organized as follows. In Sec. II we calculate the τ spectrum from a polarized Λ_b decay to the order $1/m_b^2$ in the $1/m_b$ expansion in the SM. Section III is devoted to calculations in the THDM. The $O(\alpha_s)$ QCD corrections to the decay are included. In Sec. IV numerical results are given. Finally, a summary and discussion are presented in Sec. V.

II. τ SPECTRUM OF $\Lambda_b \rightarrow X_c \tau \nu$ IN THE SM

We consider the inclusive semileptonic decay $\Lambda_b \rightarrow X_c \tau \nu$ in the SM. At the tree level, for unpolarized leptons the partial decay width can be written as

$$d\Gamma = \frac{G_f^2 |V_{cb}|^2}{(2\pi)^5 E_H} L^{\mu\nu} W_{\mu\nu} \frac{d^3 p_\tau}{2E_\tau} \frac{d^3 p_\nu}{2E_\nu}, \tag{1}$$

where E_H is the energy of Λ_b, $L_{\mu\nu}$ is the leptonic tensor,

$$L^{\mu\nu} = 2[p_\tau^\mu p_\nu^\nu + p_\tau^\nu p_\nu^\mu - g^{\mu\nu} p_\tau . p_\nu + i\epsilon^{\mu\nu\alpha\beta}(p_\tau)_\alpha (p_\nu)_\beta], \tag{2}$$

and $W_{\mu\nu}$ is the hadronic tensor,

$$W_{\mu\nu} = (2\pi)^3 \sum_X \delta^4(p_{\Lambda_b} - q - p_X)\langle \Lambda_b(v,s)|J_\mu{}^\dagger|X\rangle \times \langle X|J_\nu|\Lambda_b(v,s)\rangle, \tag{3}$$

with $p_{\Lambda_b} = m_{\Lambda_b} v$, $q = p_\tau + p_\nu$, and s being the spin of Λ_b. $J_\mu = \overline{c}\gamma_\mu[(1-\gamma_5)/2]b$ in Eq. (3) is the hadronic current. The expansion of $W_{\mu\nu}$ in terms of Lorentz-invariant structure functions W_i is defined by

$$\begin{aligned} W_{\mu\nu} = & -g_{\mu\nu}W_1 + v_\mu v_\nu W_2 - i\epsilon_{\mu\nu\alpha\beta}v^\alpha q^\beta W_3 + q_\mu q_\nu W_4 \\ & + (q_\mu v_\nu + q_\nu v_\mu)W_5 + \{-q\cdot s[-g_{\mu\nu}W_1^s + v_\mu v_\nu W_2^s \\ & - i\epsilon_{\mu\nu\alpha\beta}v^\alpha q^\beta W_3^s + q_\mu q_\nu W_4^s + (q_\mu v_\nu + q_\nu v_\mu)W_5^s] \\ & + (s_\mu v_\nu + s_\nu v_\mu)W_6^s + (s_\mu q_\nu + s_\nu q_\mu)W_7^s \\ & + i\epsilon_{\mu\nu\alpha\beta}v^\alpha s^\beta W_8^s + i\epsilon_{\mu\nu\alpha\beta}q^\alpha s^\beta W_9^s\}. \end{aligned} \tag{4}$$

The structure functions W_i can be calculated in HQET [17] and results to the $1/m_b^2$ order are

$$W_1=\delta(z)\left(\frac{m_b}{2}+\frac{K_b m_b}{6}-\frac{q\cdot v}{2}\right)-\delta'(z)\left(\frac{2\,K_b m_b q^2}{3}+K_b m_b{}^2 q\cdot v-\frac{5\,K_b m_b q\cdot v^2}{3}\right)+\delta''(z)\times\left(\frac{-2\,K_b m_b{}^3(q^2-q\cdot v^2)}{3}+\frac{2\,K_b m_b{}^2 q\cdot v(q^2-q\cdot v^2)}{3}\right),$$

$$W_2=\delta(z)\left(m_b+\frac{5\,K_b m_b}{3}\right)-\frac{14\,\delta'(z)K_b m_b{}^2 q\cdot v}{3}+\delta''(z)\left(\frac{-4\,K_b m_b{}^3(q^2-q\cdot v^2)}{3}\right),$$

$$W_3=\frac{\delta(z)}{2}-\frac{5\,\delta'(z)K_b m_b q\cdot v}{3}+\delta''(z)\times\left(\frac{-2\,K_b m_b{}^2(q^2-q\cdot v^2)}{3}\right),$$

$$W_4=\frac{-4\,\delta'(z)K_b m_b}{3},$$

$$W_5=\frac{-\delta(z)}{2}-\delta'(z)\left(\frac{-4\,K_b m_b{}^2}{3}-\frac{5\,K_b m_b q\cdot v}{3}\right)+\delta''(z)\left(\frac{2\,K_b m_b{}^2(q^2-q\cdot v^2)}{3}\right),$$

$$W_1^s=\frac{-\delta(z)(1+\epsilon_b)}{2}-\delta'(z)\frac{5\,K_b m_b(-q\cdot v)}{3}+\frac{2\,\delta''(z)K_b m_b{}^2(q^2-q\cdot v^2)}{3},$$

$$W_2^s=\frac{-4\,\delta'(z)K_b m_b{}^2}{3},$$

$$W_3^s=\frac{-2\,\delta'(z)K_b m_b}{3},$$

$$W_4^s=0,$$

$$W_5^s=\frac{2\,\delta'(z)K_b m_b}{3},$$

$$W_6^s=\delta(z)\left(\frac{-(1+\epsilon_b)m_b}{2}-\frac{5\,K_b m_b}{6}\right)-\delta'(z)\left(-\frac{5\,K_b m_b{}^2 q\cdot v}{3}\right)+\frac{2\,\delta''(z)K_b m_b{}^3(q^2-q\cdot v^2)}{3},$$

$$W_7^s=\frac{\delta(z)(1+\epsilon_b)}{2}-\delta'(z)\left(\frac{5\,K_b m_b{}^2}{3}-K_b m_b(m_b-q\cdot v)\right)-\frac{2\,\delta''(z)K_b m_b{}^2(q^2-q\cdot v^2)}{3},$$

$$W_8^s=\delta(z)\left(\frac{(1+\epsilon_b)m_b}{2}+\frac{K_b m_b}{6}\right)-\delta'(z)\left(\frac{5\,K_b m_b{}^2 q\cdot v}{3}\right)-\frac{2\,\delta''(z)K_b m_b{}^3(q^2-q\cdot v^2)}{3},$$

$$W_9^s=\frac{-\delta(z)(1+\epsilon_b)}{2}-\delta'(z)\times\left(\frac{-5\,K_b m_b{}^2}{3}+K_b m_b(m_b-q\cdot v)\right)+\frac{2\,\delta''(z)K_b m_b{}^2(q^2-q\cdot v^2)}{3}, \tag{5}$$

where

$$K_b=-\langle\Lambda_b(v,s)|\overline{b}_v\frac{(iD)^2}{2m_b^2}b_v|\Lambda_b(v,s)\rangle,$$

$$z=(m_b v-q)^2-m_c^2, \tag{6}$$

and ϵ_b is defined by [3]

$$\langle\Lambda_b(v,s)|\overline{b}\gamma^\lambda\gamma_5 b|\Lambda_b(v,s)\rangle=(1+\epsilon_b)\overline{u}(v,s)\gamma^\lambda\gamma_5 u(v,s). \tag{7}$$

K_b and ϵ_b are the only unknown two parameters at $O(m_b^{-2})$ for the Λ_b decay which parametrize the nonperturbative phenomena and are expected to be of order $(\Lambda_{\rm QCD}/m_b)^2$.We will discuss them in Sec. IV. Another parameter at $O(m_b^{-2})$,

$$G_b=Z_b\langle H_b(v,s)|\overline{b}_v\frac{gG_{\alpha\beta}\sigma^{\alpha\beta}}{4m_b^2}b_v|H_b(v,s)\rangle, \tag{8}$$

is equal to zero for $H_b=\Lambda_b$ due to the zero spin of the light degrees of freedom inside Λ_b. W_i ($i=1,2,3$) and W_i^s ($i=1,2,3,6,8,9$) have already been given by Manohar and Wise [3] and W_i ($i=4,5$) by Balk *et al.* [5]. We list them here only for completeness. From Eqs. (1), (2), (4), and (5) we get the differential decay rate

$$\frac{d\Gamma_W}{\Gamma_b dt dx dy d\cos\theta} = \hat{A}(x,t,y,\eta,\epsilon) + \hat{B}(x,t,y,\eta,\epsilon)\cos\theta, \tag{9}$$

here

$$\hat{A}(x,t,y,\eta,\epsilon) = [-12\,ty + 12\,xy - 12\,y\eta + K_b(-16\,t + 20\,xy + 16\,\eta)]\delta(z) - 4\,K_b(4\,t^2 - 4\,tx - 4\,ty - 5\,txy + 7\,x^2y - 5\,ty^2 + 7\,xy^2 + 4\,x\eta - 4\,y\eta - 5\,xy\eta - 5\,y^2\eta - 4\,\eta^2)\delta'(z) + 4\,K_b y(4\,t - x^2 - 2\,xy - y^2)(t - x + \eta)\delta''(z),$$

$$\hat{B}(x,t,y,\eta,\epsilon) = \frac{1}{\sqrt{x^2 - 4\eta}}\{[24\,t^2 - 24\,tx - 12\,txy + 12\,x^2y + 24\,x\eta - 12\,xy\eta - 24\,\eta^2 + K_b(-24\,tx + 20\,x^2y + 24\,x\eta - 32\,y\eta) + \epsilon_b(24\,t^2 - 24\,tx - 12\,txy + 12\,x^2y + 24\,x\eta - 12\,xy\eta - 24\,\eta^2)]\delta(z) - 4\,K_b(8\,t^2 + 6\,t^2x - 10\,tx^2 + 10\,t^2y - 18\,txy - 5\,tx^2y + 7\,x^3y - 5\,txy^2 + 7\,x^2y^2 + 10\,x^2\eta + 8\,ty\eta + 2\,xy\eta - 5\,x^2y\eta - 5\,xy^2\eta - 8\eta^2 - 6\,x\eta^2 - 2\,y\eta^2)\delta'(z) + 4\,K_b(4\,t - x^2 - 2\,xy - y^2)(t - x + \eta)(-2\,t + xy + 2\eta)\delta''(z)\}, \tag{10}$$

where

$$\Gamma_b = \frac{G_F^2|V_{cb}|^2 m_b^5}{192\pi^3}, \quad x = \frac{2E_\tau}{m_b}, \quad y = \frac{2E_\nu}{m_b}, \quad t = \frac{q^2}{m_b^2}, \quad \eta = \frac{m_\tau^2}{m_b^2}, \quad \epsilon = \frac{m_c}{m_b},$$

and θ is the angle between the τ direction and the Λ_b spin in the rest frame of the Λ_b. After integrating over t and y, one obtains the τ energy spectrum

$$\frac{d\Gamma_W}{\Gamma_b dx d\cos\theta} = A_W(x,\eta,\epsilon) + B_W(x,\eta,\epsilon)\cos\theta, \tag{11}$$

where $A_W(x,\eta,\epsilon)$ and $B_W(x,\eta,\epsilon)$ are given in the Appendix. The $\eta\to 0$ limits of A_W and B_W agree with the results of Manohar and Wise [3]. The total inclusive decay width of $\Lambda_b \to X_c \tau\nu$ can be obtained by integrating the spectrum formula over the range

$$2\sqrt{\eta} \leqslant x \leqslant 1 - \rho + \eta.$$

The result is not presented here because one can easily obtain it from Ref. [5] by taking $G_b = 0$. Perturbative $O(\alpha_s)$ QCD corrections to the double differential distribution of the τ energy and the invariant mass of the lepton system for $b\to c\tau\nu$ have been studied by Jeżabek and Motyka [18,19]. We will use Eq. (30) in Ref. [19] in our numerical analysis for the nonpolarized distribution of the τ energy.

III. τ SPECTRUM OF $\Lambda_b \to X_c \tau\nu$ IN THE THDM

We consider the THDM [20–22] in which up-type quarks get masses from Yukawa couplings to the one Higgs doublet H_2 (with the vacuum expectation value v_2) and down-type quarks and leptons get masses from Yukawa couplings to another Higgs doublet H_1 (with the vacuum expectation value v_1). Such a model occurs as a natural feature in supersymmetric theories. For the sake of simplicity we shall use the Feynman rules of the THDM in the minimal supersymmetric standard model (MSSM) [23]. In a THDM there are three diagrams contributing to the decay τ spectrum of $\Lambda_b \to X_c \tau\nu$ which correspond to W exchange, Goldstone boson exchange, and charged Higgs boson exchange, respectively, if one uses a nonphysical gauge. It is shown in Ref. [8] that in the Landau gauge the rate can be decomposed into the sum of two incoherent decays:

$$M = M_W + M_S \Rightarrow |M|^2 = |M_W|^2 + |M_S|^2,$$

where $M_S = M_G + M_H$. Here M_W, M_G, and M_H are the W-mediated, Goldstone-boson-mediated, and Higgs-boson-mediated decay amplitudes, respectively. This decomposition has an advantage to simplify calculations, in particular, the calculations of QCD corrections. We assume the Landau gauge hereafter. Then the new thing we need to do is to calculate M_S.

For the purpose of calculating M_S the hadronic current J_μ in Eq. (3) is replaced by

$$J_i = \overline{c}(a_i + b_i\gamma_5)b \quad (i = H,G), \tag{12}$$

with

$$a_H = m_b\tan\beta + m_c\cot\beta, \quad b_H = m_b\tan\beta - m_c\cot\beta,$$

$$a_G = -m_b + m_c, \quad b_G = -m_b - m_c.$$

Following the same steps as those in Sec. II, a straightforward calculation leads to

$$\frac{d\Gamma_H}{\Gamma_b dx d\cos\theta} = A_H(x,\eta,\epsilon,\xi,\tan\beta) + B_H(x,\eta,\epsilon,\xi,\tan\beta)\cos\theta, \tag{13}$$

$$\frac{d\Gamma_I}{\Gamma_b dx d\cos\theta} = A_I(x,\eta,\epsilon,\xi,\tan\beta) + B_I(x,\eta,\epsilon,\xi,\tan\beta)\cos\theta, \tag{14}$$

with $A_i(x,\eta,\epsilon,\xi,\tan\beta)$ and $B_i(x,\eta,\epsilon,\xi,\tan\beta)$ $(i=H,I)$ given in the Appendix. Here $d\Gamma_i/dx$ $(i=H,I)$ denotes the contributions to the τ spectrum from the Higgs-mediated amplitude and the interference term between Higgs-mediated and Goldstone boson-mediated amplitudes, respectively, and $\xi=m_H/m_b$.

For the spin-independent terms A_H and A_I, our results agree with those obtained by Grossman and Ligeti [7]. Note that $B_I=0$ at leading order of the $1/m_b$ expansion as $m_c\to 0$. This is due to the chiral difference of the vertices of W and Higgs bosons. As we can see, in the numerical analysis this makes B_I much smaller than B_W.

Combining Eqs. (11), (13), and (14), one obtains the τ spectrum of $\Lambda_b\to X_c\tau\nu$ in the THDM:

$$\frac{d\Gamma_{\rm THDM}}{\Gamma_b dx d\cos\theta}=(A_W+A_I+A_H)+(B_W+B_I+B_H)\cos\theta. \qquad (15)$$

We now come to the position to calculate $O(\alpha_s)$ QCD corrections. Making use of the results obtained by Czarnecki and Davidson [24], Grossman *et al.* got the $O(\alpha_s)$ corrections of the total width of $b\to c\tau\nu$ mediated by Higgs bosons [8]. To get the $O(\alpha_s)$ corrections of τ spectrum one can also use their results. We find that the relation between $d\Gamma^{H(I)}_{\alpha_s}/dxdt$ and $d\Gamma^{H(I)}_{\alpha_s}/dt$ is very simple, as expected. Here $d\Gamma^{H(I)}_{\alpha_s}/dxdt$ is independent of x because Goldstone and Higgs bosons are both scalar particles. Therefore, $d\Gamma^{H(I)}_{\alpha_s}/dxdt$ can be simply obtained by dividing $d\Gamma^{H(I)}_{\alpha_s}/dt$ by $x_{\rm max}-x_{\rm min}$, where $x_{\rm max}$ and $x_{\rm min}$ denote x's kinematical upper and lower limits, respectively. $d\Gamma^{H(I)}_{\alpha_s}/dt$ can be easily obtained from Eq. (8) of Ref. [24] by multiplying the lepton part. The results are

$$\frac{d\Gamma^H_{\alpha_s}}{dxdt}=\frac{\sqrt{2}m_b^2\eta(t-\eta)\tan^2\beta}{16\pi^2\xi^4p_3}\Gamma(c_H^1,c_H^2,c_H^3),$$

$$\frac{d\Gamma^I_{\alpha_s}}{dxdt}=\frac{\sqrt{2}m_b^2\eta(t-\eta)\tan\beta}{8\pi^2\xi^2tp_3}\Gamma(c_I^1,c_I^2,c_I^3), \qquad (16)$$

where

$$c_H^1=2\tan^2\beta+2\epsilon^2\cot^2\beta, \quad c_H^2=4\epsilon,$$

$$c_H^3=\tan^2\beta-\epsilon^2\cot^2\beta, \quad c_I^1=-2\tan\beta+2\epsilon^2\cot\beta,$$

$$c_I^2=2\epsilon(\tan\beta-\cot\beta), \quad c_I^3=-\tan\beta-\epsilon^2\cot\beta, \qquad (17)$$

and $\Gamma(c^1,c^2,c^3)$ is [24]

$$\Gamma(c^1,c^2,c^3)=\frac{\alpha_s}{6\pi^2}\frac{G_Fm_b^3|V_{cb}|^2}{\sqrt{2}}[c^1G_++c^2G_-+c^3G_0], \qquad (18)$$

with

$$G_+=p_0H+p_0p_3\left[\frac{9}{2}-2\ln\left(\frac{16p_3^4}{\epsilon^2t}\right)\right]+\frac{1}{4t}Y_p(2-t-4t^2+3t^3$$
$$-2\epsilon^2-2\epsilon^4+2\epsilon^6-4t\epsilon^2-5t\epsilon^4),$$

$$G_-=H+p_3\left[6-2\ln\left(\frac{16p_3^4}{\epsilon^2t}\right)\right]+\frac{1}{t}Y_p(1-t-2\epsilon^2+\epsilon^4$$
$$-3t\epsilon^2),$$

$$G_0=-6p_0p_3\ln\epsilon,$$

where

$$H=4p_0\left[\mathrm{Li}_2(p_+)-\mathrm{Li}_2(p_-)-2\mathrm{Li}_2\left(1-\frac{p_-}{p_+}\right)\right.$$
$$\left.+\frac{1}{2}Y_p\ln\left(\frac{16p_3^4t}{p_+^4}\right)-Y_w\ln\epsilon\right]+2Y_w(1-\epsilon^2)$$
$$+\frac{2}{t}p_3\ln\epsilon(1+t-\epsilon^2),$$

$$p_0\equiv\frac{1}{2}(1-t+\epsilon^2), \quad p_3\equiv\frac{1}{2}\sqrt{1+t^2+\epsilon^4-2(t+\epsilon^2+t\epsilon^2)},$$

$$p_\pm\equiv p_0\pm p_3, \quad Y_p\equiv\frac{1}{2}\ln\frac{p_+}{P_-},$$

$$W_0\equiv\frac{1}{2}(1+t-\epsilon^2), \quad W_\pm\equiv W_0\pm p_3,$$

$$Y_w\equiv\frac{1}{2}\ln\frac{W_+}{W_-}.$$

After integrating Eqs. (16) over t, we get τ spectrum numerically with only parameters $\tan\beta$ and ξ. As a check, we find that our numerical results of $\Gamma^H_{\alpha_s}$ and $\Gamma^I_{\alpha_s}$ agree with those obtained by Grossman *et al.* [8].

IV. NUMERICAL RESULTS

In order to do numerical calculations we need to discuss the values of the parameters K_b, ϵ_b, m_c, and m_b and the parameters in the THDM.

A. Constraints on the parameters of THDM

In Refs. [25,26], constraints on $\tan\beta$ from $K-\overline{K}$ and $B-\overline{B}$ mixing, $\Gamma(b\to s\gamma)$, $\Gamma(b\to c\tau\nu)$, and R_b have been given:

$$0.7<\tan\beta<0.6\frac{m_{H_\pm}}{1\ \mathrm{GeV}}.$$

Also the lower limit $m_{H_\pm}>200$ GeV has been given there. Taking the radiative correction and the $1/m_Q^2$ correction into account in B meson decay, Grossman *et al.* have an improved bound of R (defined by $R=\tan\beta/m_{H^\pm}$) which is

$$R<0.49\ \mathrm{GeV}^{-1}.$$

We will predict the τ spectrum and total width of Λ_b decay under these constraints.

B. About the parameters K_b, ϵ_b and m_b, m_c

K_b and ϵ_b characterize the $1/m_Q^2$ corrections to the decay distribution for $\Lambda_b \to X_c \tau \nu$ and are nonperturbative quantities independent of m_Q. Since quarks are not free physical particles, m_b and m_c cannot be determined directly by experiment. However, we can estimate them by the phenomenological analysis of the heavy hadron spectra to the order $1/m_Q$. From the effective Lagrangian in HQET, the mass of a heavy hadron can be written as [30–32]

$$m_{h_Q} = m_Q + \overline{\Lambda}(j_l^P, I, S) + \frac{a(j_l^P, I, S)}{m_Q} + \frac{b(j_l^P, I, S)}{m_Q} \langle \vec{S}_Q \cdot \vec{j}_l \rangle + \cdots, \tag{19}$$

where $\langle \vec{S}_Q \cdot \vec{j}_1 \rangle = \frac{1}{2}[J(J+1) - j_l(j_l+1) - \frac{3}{4}]$ with J and j_l being the spins of the hadron and the light degrees of freedom inside the hadron, respectively. The parameter $\overline{\Lambda}$ represents contributions coming from the effective Lagrangian in the $m_Q \to \infty$ limit, and a and b are, respectively, associated with the kinetic energy and the color magnetic energy of the heavy quark inside the hadron. In the present case, $a(0^+,0,0) = m_b^2 K_b$ and $b(0^+,0,0) = 0$. It is shown that $\epsilon_b \leq -2/3 K_b$ [29]. Furthermore, one can take $\epsilon_b = -2/3 K_b$ if one omits the contributions of terms arising from double insertions of the chromomagnetic operator [27–29]. Starting from Eq. (19), it is shown [32] that one can obtain $a(\frac{1}{2}^-, \frac{1}{2}, 0)$, m_c [and $\overline{\Lambda}(\frac{1}{2}^-, \frac{1}{2}, 0), b(\frac{1}{2}^-, \frac{1}{2}, 0)$] by using the observed masses of the doublets (B^*, B) and (D^*, D) if choosing m_b as input. Furthermore, $a(0^+,0,0)$, which is the parameter we need, is determined by [32,3]

$$a(0^+,0,0) = \frac{h m_b}{1-h}[(m_{\Lambda_c} - \overline{m}_D) - (m_{\Lambda_b} - \overline{m}_B)] + a\left(\frac{1^-}{2}, \frac{1}{2}, 0\right), \tag{20}$$

where

$$h = \frac{m_{B^*} - m_B}{m_{D^*} - m_D}, \tag{21}$$

and $\overline{m}_H = \frac{1}{4}(m_H + 3 m_{H^*})$, $H = B, D$. Therefore, if we choose $m_b = 5.1$ GeV, the other parameters will be[1]

$$m_c = h m_b = 1.65 \ \text{GeV},$$

$$K_b = \frac{0.142 m_c \ \text{GeV}}{m_b^2} \approx 0.009, \quad \epsilon_b \approx -0.006.$$

If we choose $m_b = 5.044$, which is a critical value based on Eq. (19) [32], other parameters will be

$$m_c = 1.63 \ \text{GeV}, \quad K_b \approx 0.006, \quad \epsilon_b \approx -0.004.$$

We will use these two sets of values and discriminate them by the first ($m_b = 5.044$ GeV) and second ($m_b = 5.1$ GeV) set, respectively, in the numerical computations.

Using the parameters given above, we obtain the total width in terms of tan β and m_H as follows:

$$\frac{\Gamma_W^1}{\Gamma_b} = C_W^1 + D_W^1 \alpha_s,$$

$$\frac{\Gamma_H^1 + \Gamma_I^1}{\Gamma_b} = C_H^1 + D_H^1 \alpha_s,$$

$$\frac{\Gamma_W^2}{\Gamma_b} = C_W^2 + D_W^2 \alpha_s,$$

$$\frac{\Gamma_H^2 + \Gamma_I^2}{\Gamma_b} = C_H^2 + D_H^2 \alpha_s, \tag{22}$$

where

$$C_W^1 = 0.109,$$

$$C_H^1 = -\frac{0.0141\,(0.253 + 1.16\tan^2\beta)}{\xi^2} + \frac{0.0141\,(0.025 + 0.112\tan^2\beta + 0.239\tan^4\beta)}{\xi^4},$$

$$D_W^1 = -0.0476,$$

$$D_H^1 = \frac{0.00804\,(-0.165 - 0.577\,\xi^2 - 0.374\tan^2\beta - \xi^2\tan^2\beta - 0.331\tan^4\beta)}{\xi^4},$$

$$C_W^2 = 0.112,$$

$$C_H^2 = -\frac{0.0139\,(0.262 + 1.19\tan^2\beta)}{\xi^2} + \frac{0.0139\,(0.0256 + 0.107\tan^2\beta + 0.245\tan^4\beta)}{\xi^4},$$

$$D_W^2 = -0.0493,$$

[1]The experimental data used in Ref. [32] have been improved since then. Our data are from Ref. [33].

$$D_H^2 = \frac{0.0082\,(-0.163 - 0.574\ ^2 - 0.37\tan^2\beta - \xi^2\tan^2\beta - 0.328\tan^4\beta)}{\xi^4}. \tag{23}$$

The superscript i ($i=1, 2$) in Eqs. (22) and (23) denotes that the ith set of values of K_b, ϵ_b, m_c, and m_b is used.

The explicit dependence of the total width $\Gamma_{\rm THDM} = \Gamma_W + \Gamma_H + \Gamma_I$ (normalized to the electron channel) on $\tan\beta$ and m_H is plotted in Fig. 1. It should be noted that the absolute value of $\Gamma_{\rm THDM}$ is very sensitive to m_b. Using the ratio between the width $\Gamma^\tau_{\rm THDM}$ and $\Gamma^e_{\rm THDM}$ separates the theoretical and experimental uncertainties and deletes the $|V_{cb}|^2 m_b^5$ factor which is not well known yet. From Fig. 1 the following remarks can be drawn: (1) The normalized total width is not sensitive to the values of the parameters in HQET. The value of the normalized width for the second set of parameters is 3% larger than that for the first set. (2) In the range of $\tan\beta$ which is interesting physically, say, $\tan\beta < 60$, the normalized width changes roughly 5–15% when $\tan\beta$ changes from 20 to 60 and $m_{H^\pm}$ is fixed. The normalized width changes the same order of magnitude for fixed $\tan\beta$ and changing $m_{H^\pm}$ from 200 to 400 GeV. From Eqs. (22) and (23) it follows that the α_s corrections decrease the total width by roughly 20% which is larger than that in the SM. Because, when $\tan\beta \gg 1$, C_H^i is proportional to $r^2(-1+0.2\,r^2)$ and D_H^i is proportional to $-r^2(1+0.3\,r^2)$, where $r = Rm_b$, one can obtain constraints on r from the measurement of the total width.

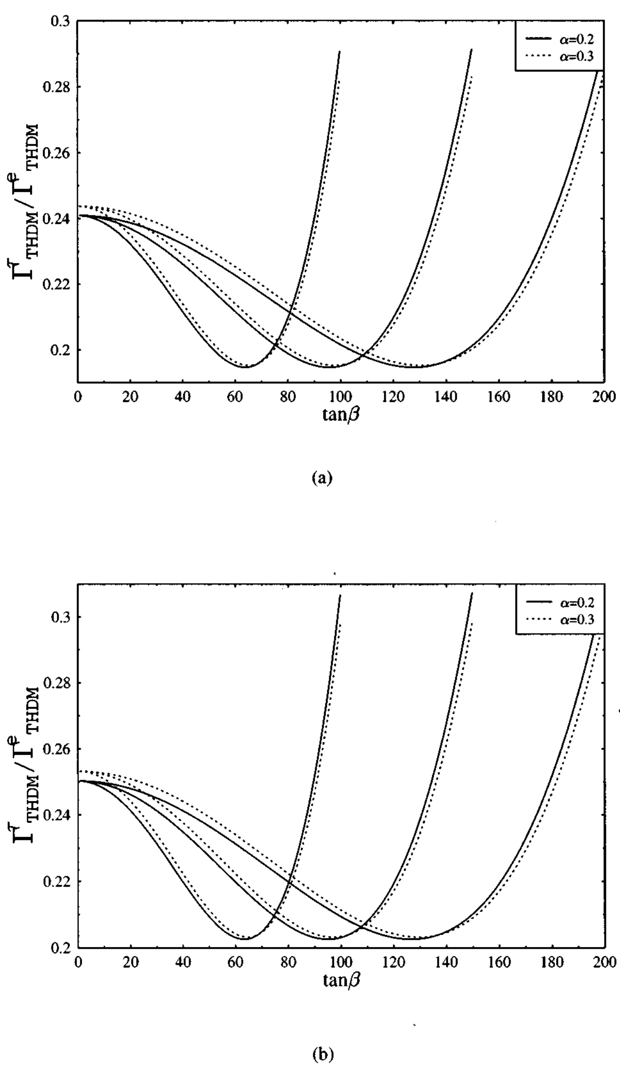

FIG. 1. Total width (normalized to the electron channel) in terms of $\tan\beta$ and m_H, using the (a) first ($m_b = 5.044$ Gev) and (b) second ($m_b = 5.1$ GeV) set of values. The curves terminating at $\tan\beta = 100$, 150, and 200 correspond to $m_H = 200$, 300, and 400 GeV, respectively.

The τ spectrum for some typical values of $\tan\beta$ and m_H is calculated and the result is plotted in Figs. 2–5. The predictions in the SM are also plotted in the figures. Here the α_s corrections for the spin-dependent term are not considered. We can see from Figs. 4 and 5 that the spin-dependent spectrum is quite different for $r \leqslant 1$ and $r \geqslant 2$ (note that we have the constraints $r < 0.49$ GeV^{-1}, $m_b \approx 2.5$, and $m_{H^\pm} \geqslant 200$ GeV from experiments, as mentioned before). The reason is as follows. We know from the Appendix that $B_H = (-Jr^2\eta/8\,\xi^2)B_W \approx -r^4\eta B_W/4$. That is, it depends on r^4. For $r \leqslant 1$, $B_H \ll B_W$. As pointed out in Sec. III, B_I is negligibly small compared with B_W due to the chiral difference of b-quark couplings to W and $H_\pm$ which is deduced from model II of the THDM. Therefore, the spectrum is almost the same as that in the SM, as can be seen from Figs. 4 and 5. For $r \geqslant 2$, B_H is as the same order of magnitude as B_W so that B_H and B_W tend to cancel each other, which makes the spin-dependent distribution of the τ energy very small and a little dependent on the τ energy, as shown in Figs. 4 and 5. Thus one can say that if the τ spectrum is somewhat more isotro-

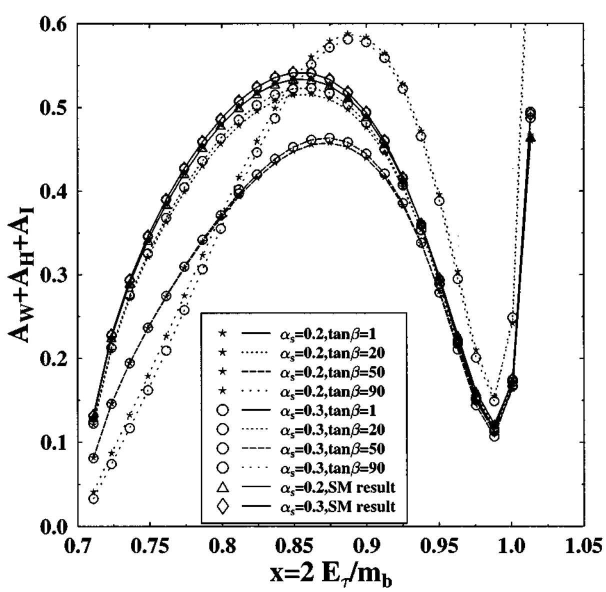

FIG. 2. τ spectrum for different α_s and $\tan\beta$, $m_H = 200$ GeV. The first set of parameter values ($m_b = 5.044$ GeV) is used.

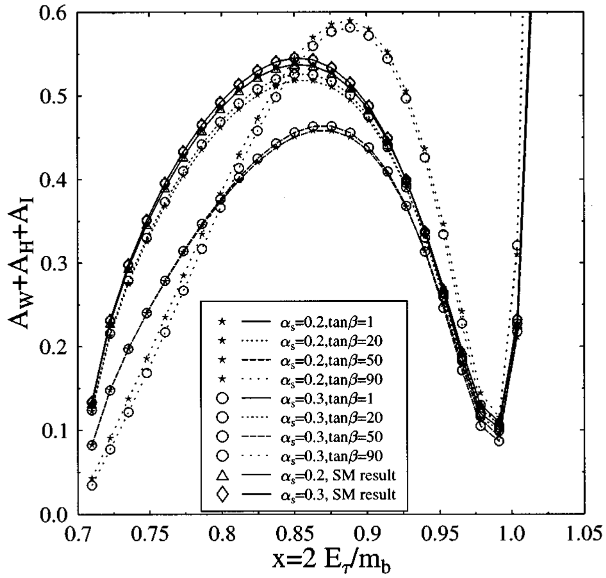

FIG. 3. τ spectrum for different α_s and tan β, m_H=200 GeV. The second set of parameter values (m_b=5.1 GeV) is used.

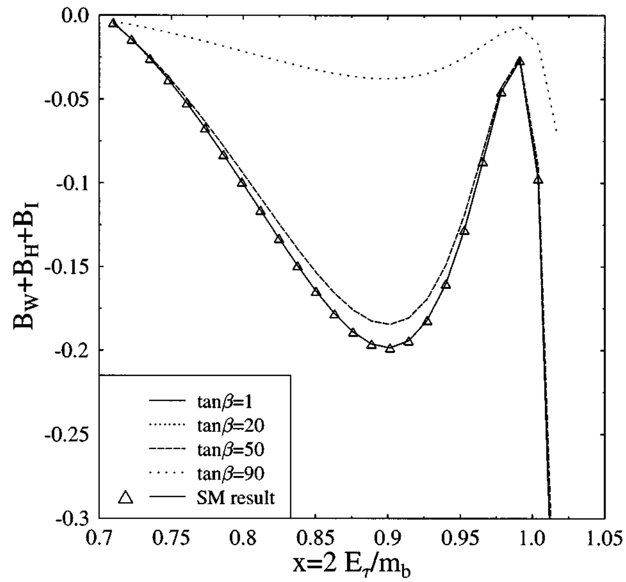

FIG. 5. τ spectrum for different tan β, m_H=200 GeV. The second set of parameter values (m_b=5.1 GeV) is used. $O(\alpha_s)$ corrections are not considered here.

pic than what the SM predicts, the THDM with large tanβ ($>$80) and $m_H \geqslant$200 GeV is preferred in describing its nature. For the nonpolarized term, as can be seen from Figs. 4 and 5, and the spectrum is very similar to that of B decay since the difference between Λ_b decay and B decay comes from the $1/m_b^2$ corrections.

V. SUMMARY

In summary, we have calculated the rate and τ spectrum of the inclusive semileptonic decay for a polarized $\Lambda_b \to X_c \tau\nu$ to $1/m_b^2$ order in the $1/m_b$ expansion in the SM.

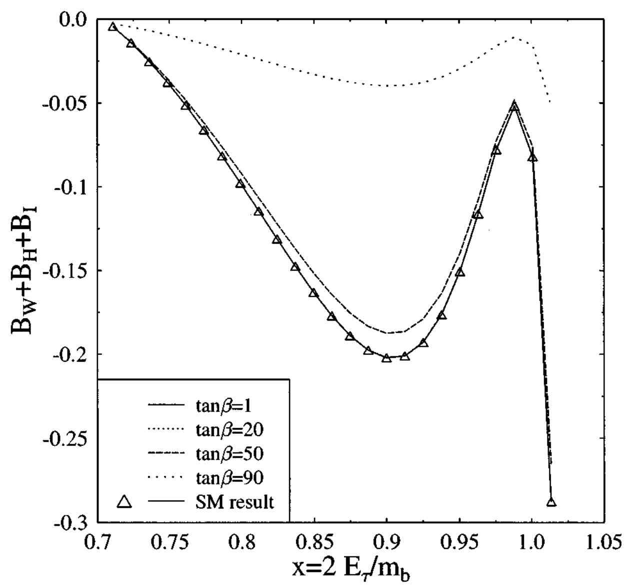

FIG. 4. τ spectrum for different tan β, m_H=200 GeV. The first set of parameter values (m_b=5.044 GeV) is used. $O(\alpha_s)$ corrections are not considered here.

The α_s corrections are included in the numerical computations for the spin-independent terms of the τ spectrum. Our results show that the spin-dependent τ spectrum is significant enough to be seen.

We have also calculated the same quantities in a THDM. For the spin-independent terms of the τ spectrum arising from the Higgs-mediated amplitude and the interference term, we have calculated the $O(\alpha_s)$ QCD corrections to the double differential distribution. Together with the α_s corrections in the SM given in Ref. [18], we obtained all the α_s corrections to the nonpolarized double differential distribution (and so the total width) in the THDM. The numerical results show that the branching ratio of $\Lambda_b \to X_c \tau\nu$ in the THDM is of approximately 25% of that in the electron channel and the spin-dependent τ spectrum can be used to estimate the size of tan β and $m_{H^\pm}$. The spectrum depends dominantly on R if tan $\beta \gg 1$ so that from the measurement of the angular distribution of a polarized $\Lambda_b \to X_c \tau\nu$ in B factories within the coming years one can obtain constraints on R.

It is obvious that substituting the u-quark mass m_u=0 for m_c one immediately obtains the decay rate and τ spectrum for a polarized $\Lambda_b \to X_u \tau\nu$. And with minor changes one can extend the results in the paper to the inclusive semileptonic decay of a polarized $\Lambda_c \to X_{s,d} \tau\nu$. It is interesting to calculate the α_s corrections to the spin-dependent term of the τ spectrum.

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APPENDIX: EXPRESSIONS OF A_i AND B_i $(i=W,H,I)$

$$A_W=\frac{\sqrt{x^2-4\,\eta}}{(1-x+\eta)^3}(-1+\epsilon^2+x-\eta)^2(3\,x+3\,\epsilon^2x-5\,x^2-\epsilon^2x^2+2\,x^3-4\,\eta-8\,\epsilon^2\eta+10\,x\,\eta+3\,\epsilon^2x\,\eta-5\,x^2\eta-4\,\eta^2+3\,x\,\eta^2)$$
$$+2\,K_b\frac{\sqrt{x^2-4\,\eta}}{3(1-x+\eta)^5}(-5\,x^2-15\,\epsilon^4x^2+20\,\epsilon^6x^2+25\,x^3+21\,\epsilon^4x^3-10\,\epsilon^6x^3-50\,x^4-6\,\epsilon^4x^4+2\,\epsilon^6x^4+50\,x^5-25\,x^6$$
$$+5\,x^7+14\,\eta+6\,\epsilon^4\eta-20\,\epsilon^6\eta-70\,x\,\eta+78\,\epsilon^4x\,\eta-80\,\epsilon^6x\,\eta+115\,x^2\eta-147\,\epsilon^4x^2\eta+44\,\epsilon^6x^2\eta-40\,x^3\eta+60\,\epsilon^4x^3\eta$$
$$-10\,\epsilon^6x^3\eta-80\,x^4\eta-6\,\epsilon^4x^4\eta+86\,x^5\eta-25\,x^6\eta+70\,\eta^2-126\,\epsilon^4\eta^2+152\,\epsilon^6\eta^2-280\,x\,\eta^2+300\,\epsilon^4x\,\eta^2-80\,\epsilon^6x\,\eta^2$$
$$+370\,x^2\eta^2-147\,\epsilon^4x^2\eta^2+20\,\epsilon^6x^2\eta^2-130\,x^3\eta^2+21\,\epsilon^4x^3\eta^2-80\,x^4\eta^2+50\,x^5\eta^2+140\,\eta^3-126\,\epsilon^4\eta^3-20\,\epsilon^6\eta^3$$
$$-420\,x\,\eta^3+78\,\epsilon^4x\,\eta^3+370\,x^2\eta^3-15\,\epsilon^4x^2\eta^3-40\,x^3\eta^3-50\,x^4\eta^3+140\,\eta^4+6\,\epsilon^4\eta^4-280\,x\,\eta^4+115\,x^2\eta^4+25\,x^3\eta^4$$
$$+70\,\eta^5-70\,x\,\eta^5-5\,x^2\eta^5+14\,\eta^6), \tag{A1}$$

$$B_W=\frac{(x^2-4\,\eta)}{(1-x+\eta)^3}(-1+\epsilon^2+x-\eta)^2(1-\epsilon^2-3\,x-\epsilon^2x+2\,x^2+4\,\eta+3\,\epsilon^2\eta-5\,x\,\eta+3\,\eta^2)+\epsilon_b\frac{(x^2-4\,\eta)}{(1-x+\eta)^3}(-1+\epsilon^2+x$$
$$-\eta)^2(1-\epsilon^2-3\,x-\epsilon^2x+2\,x^2+4\,\eta+3\,\epsilon^2\eta-5\,x\,\eta+3\,\eta^2)+2\,K_b\frac{(-x^2+4\,\eta)}{3\,(1-x+\eta)^5}(5\,x-15\,\epsilon^4x+10\,\epsilon^6x-25\,x^2+21\,\epsilon^4x^2$$
$$+4\,\epsilon^6x^2+50\,x^3-6\,\epsilon^4x^3-2\,\epsilon^6x^3-50\,x^4+25\,x^5-5\,x^6+36\,\epsilon^4\eta-36\,\epsilon^6\eta+25\,x\,\eta-51\,\epsilon^4x\,\eta-22\,\epsilon^6x\,\eta-100\,x^2\eta$$
$$+10\,\epsilon^6x^2\eta+150\,x^3\eta+6\,\epsilon^4x^3\eta-100\,x^4\eta+25\,x^5\eta+60\,\epsilon^6\eta^2+50\,x\,\eta^2+51\,\epsilon^4x\,\eta^2-20\,\epsilon^6x\,\eta^2-150\,x^2\eta^2-21\,\epsilon^4x^2\eta^2$$
$$+150\,x^3\eta^2-50\,x^4\eta^2-36\,\epsilon^4\eta^3+50\,x\,\eta^3+15\,\epsilon^4x\,\eta^3-100\,x^2\eta^3+50\,x^3\eta^3+25\,x\,\eta^4-25\,x^2\eta^4+5\,x\,\eta^5), \tag{A2}$$

$$A_H=\frac{\eta\,\tan^2\beta\sqrt{x^2-4\,\eta}}{8\,\xi^4(1-x+\eta)^3}(1-\epsilon^2-x+\eta)^2[L(6\,\epsilon x-6\,\epsilon x^2-12\,\epsilon\eta+18\,\epsilon x\,\eta-12\,\epsilon\eta^2)+F(3\,x+3\,\epsilon^2x-5\,x^2-\epsilon^2x^2+2\,x^3-4\,\eta$$
$$-8\,\epsilon^2\eta+10\,x\,\eta+3\,\epsilon^2x\,\eta-5\,x^2\eta-4\,\eta^2+3\,x\,\eta^2)]+K_b\,\eta\frac{\tan^2\beta\sqrt{x^2-4\,\eta}}{12\,\xi^4(1-x+\eta)^5}[L(-18\,\epsilon x+36\,\epsilon^3x-18\,\epsilon^5x+54\,\epsilon x^2$$
$$-120\,\epsilon^3x^2+66\,\epsilon^5x^2-36\,\epsilon x^3+132\,\epsilon^3x^3-60\,\epsilon^5x^3-36\,\epsilon x^4-48\,\epsilon^3x^4+12\,\epsilon^5x^4+54\,\epsilon x^5-18\,\epsilon x^6+72\,\epsilon\eta-96\,\epsilon^3\eta$$
$$+24\,\epsilon^5\eta-342\,\epsilon x\,\eta+480\,\epsilon^3x\,\eta-210\,\epsilon^5x\,\eta+504\,\epsilon x^2\eta-720\,\epsilon^3x^2\eta+264\,\epsilon^5x^2\eta-180\,\epsilon x^3\eta+360\,\epsilon^3x^3\eta-60\,\epsilon^5x^3\eta$$
$$-144\,\epsilon x^4\eta-24\,\epsilon^3x^4\eta+90\,\epsilon x^5\eta+288\,\epsilon\eta^2-384\,\epsilon^3\eta^2+192\,\epsilon^5\eta^2-900\,\epsilon x\,\eta^2 1080\,\epsilon^3x\,\eta^2-390\,\epsilon^5x\,\eta^2+756\,\epsilon x^2\eta^2$$
$$-792\,\epsilon^3x^2\eta^2+102\,\epsilon^5x^2\eta^2+36\,\epsilon x^3\eta^2+84\,\epsilon^3x^3\eta^2-180\,\epsilon x^4\eta^2+432\,\epsilon\eta^3-480\,\epsilon^3\eta^3+168\,\epsilon^5\eta^3-828\,\epsilon x\,\eta^3$$
$$+672\,\epsilon^3x\,\eta^3-54\,\epsilon^5x\,\eta^3+216\,\epsilon x^2\eta^3-96\,\epsilon^3x^2\eta^3+180\,\epsilon x^3\eta^3+288\,\epsilon\eta^4-192\,\epsilon^3\eta^4-234\,\epsilon x\,\eta^4+36\,\epsilon^3x\,\eta^4-90\,\epsilon x^2\eta^4$$
$$+72\,\epsilon\eta^5+18\,\epsilon x\,\eta^5)+F(-5\,x^2-15\,\epsilon^4x^2+20\,\epsilon^6x^2+25\,x^3+21\,\epsilon^4x^3-10\,\epsilon^6x^3-50\,x^4-6\,\epsilon^4x^4$$
$$+2\,\epsilon^6x^4+50\,x^5-25\,x^6+5\,x^7+14\,\eta+6\,\epsilon^4\eta-20\,\epsilon^6\eta-70\,x\,\eta+78\,\epsilon^4x\,\eta-80\,\epsilon^6x\,\eta+115\,x^2\eta-147\,\epsilon^4x^2\eta+44\,\epsilon^6x^2\eta$$
$$-40\,x^3\eta+60\,\epsilon^4x^3\eta-10\,\epsilon^6x^3\eta-80\,x^4\eta-6\,\epsilon^4x^4\eta+86\,x^5\eta-25\,x^6\eta+70\,\eta^2-126\,\epsilon^4\eta^2+152\,\epsilon^6\eta^2-280\,x\,\eta^2$$
$$+300\,\epsilon^4x\,\eta^2-80\,\epsilon^6x\,\eta^2+370\,x^2\eta^2-147\,\epsilon^4x^2\eta^2+20\,\epsilon^6x^2\eta^2-130\,x^3\eta^2+21\,\epsilon^4x^3\eta^2-80\,x^4\eta^2+50\,x^5\eta^2+140\,\eta^3$$
$$-126\,\epsilon^4\eta^3-20\,\epsilon^6\eta^3-420\,x\,\eta^3+78\,\epsilon^4x\,\eta^3+370\,x^2\eta^3-15\,\epsilon^4x^2\eta^3-40\,x^3\eta^3-50\,x^4\eta^3+140\,\eta^4+6\,\epsilon^4\eta^4-280\,x\,\eta^4$$
$$+115\,x^2\eta^4+25\,x^3\eta^4+70\,\eta^5-70\,x\,\eta^5-5\,x^2\eta^5+14\,\eta^6)], \tag{A3}$$

$$B_H=-\frac{Jr^2\eta}{8\,\xi^2}B_W, \tag{A4}$$

$$A_I = -\frac{3\,\eta\sqrt{x^2-4\,\eta}(-1+\epsilon^2+x-\eta)^2}{\xi^2(1-x+\eta)^2}(2\,\epsilon^2+2\,\tan^2\beta-\epsilon^2 x-2\,x\tan^2\beta+2\,\tan^2\beta\,\eta)-K_b\frac{\eta\sqrt{x^2-4\,\eta}}{\xi^2(1-x+\eta)^4}[6\,\epsilon^2-12\,\epsilon^4+6\,\epsilon^6$$
$$-24\,\epsilon^2 x+24\,\epsilon^4 x+36\,\epsilon^2 x^2-14\,\epsilon^4 x^2-24\,\epsilon^2 x^3+6\,\epsilon^2 x^4+2\,\epsilon^4 x^4+24\,\epsilon^2\eta-4\,\epsilon^4\eta-36\,\epsilon^6\eta-72\,\epsilon^2 x\eta+24\,\epsilon^6 x\eta$$
$$+72\,\epsilon^2 x^2\eta+8\,\epsilon^4 x^2\eta-6\,\epsilon^6 x^2\eta-24\,\epsilon^2 x^3\eta-8\,\epsilon^4 x^3\eta+36\,\epsilon^2\eta^2-4\,\epsilon^4\eta^2+6\,\epsilon^6\eta^2-72\,\epsilon^2 x\eta^2+8\,\epsilon^4 x\eta^2+36\,\epsilon^2 x^2\eta^2$$
$$+6\,\epsilon^4 x^2\eta^2+24\,\epsilon^2\eta^3-12\,\epsilon^4\eta^3-24\,\epsilon^2 x\eta^3+6\,\epsilon^2\eta^4+\tan^2\beta(-6+12\,\epsilon^2-6\,\epsilon^4+24\,x-48\,\epsilon^2 x+24\,\epsilon^4 x-36\,x^2+72\,\epsilon^2 x^2$$
$$-26\,\epsilon^4 x^2+24\,x^3-48\,\epsilon^2 x^3+8\,\epsilon^4 x^3-6\,x^4+12\,\epsilon^2 x^4-30\,\eta+48\,\epsilon^2\eta-34\,\epsilon^4\eta+96\,x\eta-144\,\epsilon^2 x\eta+64\,\epsilon^4 x\eta-108\,x^2\eta$$
$$+144\,\epsilon^2 x^2\eta-26\,\epsilon^4 x^2\eta+48\,x^3\eta-48\,\epsilon^2 x^3\eta-6\,x^4\eta-60\,\eta^2+72\,\epsilon^2\eta^2-34\,\epsilon^4\eta^2+144\,x\eta^2-144\,\epsilon^2 x\eta^2+24\,\epsilon^4 x\eta^2$$
$$-108\,x^2\eta^2+72\,\epsilon^2 x^2\eta^2+24\,x^3\eta^2-60\,\eta^3+48\,\epsilon^2\eta^3-6\,\epsilon^4\eta^3+96\,x\eta^3-48\,\epsilon^2 x\eta^3-36\,x^2\eta^3-30\,\eta^4+12\,\epsilon^2\eta^4$$
$$+24\,x\eta^4-6\,\eta^5)], \tag{A5}$$

$$B_I=\frac{3\,\eta\epsilon^2(-1+\epsilon^2+x-\eta)^2(x^2-4\,\eta)}{\xi^2(1-x+\eta)^2}+\epsilon_b\frac{3\,\eta\epsilon^2(-1+\epsilon^2+x-\eta)^2(x^2-4\,\eta)}{\xi^2(1-x+\eta)^2}-K_b\frac{\eta(1-\epsilon^2-x+\eta)}{\xi^2(1-x+\eta)^4}[-22\,\epsilon^2 x+20\,\epsilon^4 x$$
$$+2\,\epsilon^6 x+62\,\epsilon^2 x^2-30\,\epsilon^4 x^2-8\,\epsilon^6 x^2-52\,\epsilon^2 x^3+4\,\epsilon^4 x^3+4\,\epsilon^2 x^4+6\,\epsilon^4 x^4+10\,\epsilon^2 x^5-2\,\epsilon^2 x^6+92\,\epsilon^2\eta-104\,\epsilon^4\eta+12\,\epsilon^6\eta$$
$$-360\,\epsilon^2 x\eta+236\,\epsilon^4 x\eta+20\,\epsilon^6 x\eta+448\,\epsilon^2 x^2\eta-126\,\epsilon^4 x^2\eta+8\,\epsilon^6 x^2\eta-176\,\epsilon^2 x^3\eta-4\,\epsilon^4 x^3\eta-2\,\epsilon^6 x^3\eta-12\,\epsilon^2 x^4\eta$$
$$+2\,\epsilon^4 x^4\eta+8\,\epsilon^2 x^5\eta+272\,\epsilon^2\eta^2-152\,\epsilon^4\eta^2-72\,\epsilon^6\eta^2-660\,\epsilon^2 x\eta^2+156\,\epsilon^4 x\eta^2+18\,\epsilon^6 x\eta^2+420\,\epsilon^2 x^2\eta^2+6\,\epsilon^4 x^2\eta^2$$
$$-4\,\epsilon^6 x^2\eta^2-20\,\epsilon^2 x^3\eta^2-8\,\epsilon^4 x^3\eta^2-12\,\epsilon^2 x^4\eta^2+264\,\epsilon^2\eta^3-56\,\epsilon^4\eta^3+12\,\epsilon^6\eta^3-328\,\epsilon^2 x\eta^3+4\,\epsilon^4 x\eta^3+32\,\epsilon^2 x^2\eta^3$$
$$+6\,\epsilon^4 x^2\eta^3+8\,\epsilon^2 x^3\eta^3+80\,\epsilon^2\eta^4-8\,\epsilon^4\eta^4-6\,\epsilon^2 x\eta^4-2\,\epsilon^2 x^2\eta^4-4\,\epsilon^2\eta^5+\tan^2\beta(2\,x-4\,\epsilon^2 x+2\,\epsilon^4 x-6\,x^2+10\,\epsilon^2 x^2$$
$$-4\,\epsilon^4 x^2+4\,x^3-6\,\epsilon^2 x^3+2\,\epsilon^4 x^3+4\,x^4-2\,\epsilon^2 x^4-6\,x^5+2\,\epsilon^2 x^5+2\,x^6-16\,\eta+16\,\epsilon^2\eta+74\,x\eta-60\,\epsilon^2 x\eta+2\,\epsilon^4 x\eta$$
$$-120\,x^2\eta+66\,\epsilon^2 x^2\eta+76\,x^3\eta-16\,\epsilon^2 x^3\eta-2\,\epsilon^4 x^3\eta-8\,x^4\eta-6\,\epsilon^2 x^4\eta-6\,x^5\eta-64\,\eta^2+48\,\epsilon^2\eta^2+212\,x\eta^2$$
$$-108\,\epsilon^2 x\eta^2-2\,\epsilon^4 x\eta^2-228\,x^2\eta^2+54\,\epsilon^2 x^2\eta^2+4\,\epsilon^4 x^2\eta^2+76\,x^3\eta^2+6\,\epsilon^2 x^3\eta^2+4\,x^4\eta^2-96\,\eta^3+48\,\epsilon^2\eta^3+212\,x\eta^3$$
$$-52\,\epsilon^2 x\eta^3-2\,\epsilon^4 x\eta^3-120\,x^2\eta^3-2\,\epsilon^2 x^2\eta^3+4\,x^3\eta^3-64\,\eta^4+16\,\epsilon^2\eta^4+74\,x\eta^4-6\,x^2\eta^4-16\,\eta^5+2\,x\eta^5)], \tag{A6}$$

where

$$F=2\,\tan^2\beta+2\epsilon^2\cot^2\beta,\quad J=2\,\tan^2\beta-2\epsilon^2\cot^2\beta,\quad L=4\epsilon.$$

When $\tan\beta \gg 1$, Eqs. (3), (4), and (5) reduce to

$$A_H=\frac{r^4\eta}{4}A_W+O(r^4\tan\beta^{-2}), \tag{A7}$$

$$B_H=-\frac{r^4\eta}{4}B_W+O(r^4\tan\,\beta^{-4}), \tag{A8}$$

$$A_I=-\frac{6\,\eta r^2\sqrt{x^2-4\,\eta}}{(1-x+\eta)}(1-x+\eta-\epsilon^2)^2+O(r^2\tan\beta^{-2}). \tag{A9}$$

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